

GLOBAL JOURNAL

OF SCIENCE FRONTIER RESEARCH: F

Mathematics and Decision Science

Higher Genus Solution

Systems of Differential Equations

Highlights

Exploring Torus Black-Holes

Creation of a Homogeneous 5D Universe

Discovering Thoughts, Inventing Future

VOLUME 24 ISSUE 2 VERSION 1.0



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: F
MATHEMATICS & DECISION SCIENCES



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: F
MATHEMATICS & DECISION SCIENCES

VOLUME 24 ISSUE 2 (VER. 1.0)

OPEN ASSOCIATION OF RESEARCH SOCIETY

© Global Journal of Science
Frontier Research. 2024.

All rights reserved.

This is a special issue published in version 1.0
of "Global Journal of Science Frontier
Research." By Global Journals Inc.

All articles are open access articles distributed
under "Global Journal of Science Frontier
Research"

Reading License, which permits restricted use.
Entire contents are copyright by of "Global
Journal of Science Frontier Research" unless
otherwise noted on specific articles.

No part of this publication may be reproduced
or transmitted in any form or by any means,
electronic or mechanical, including
photocopy, recording, or any information
storage and retrieval system, without written
permission.

The opinions and statements made in this
book are those of the authors concerned.
Ultraculture has not verified and neither
confirms nor denies any of the foregoing and
no warranty or fitness is implied.

Engage with the contents herein at your own
risk.

The use of this journal, and the terms and
conditions for our providing information, is
governed by our Disclaimer, Terms and
Conditions and Privacy Policy given on our
website [http://globaljournals.us/terms-and-condition/
menu-id-1463/](http://globaljournals.us/terms-and-condition/menu-id-1463/)

By referring / using / reading / any type of
association / referencing this journal, this
signifies and you acknowledge that you have
read them and that you accept and will be
bound by the terms thereof.

All information, journals, this journal,
activities undertaken, materials, services and
our website, terms and conditions, privacy
policy, and this journal is subject to change
anytime without any prior notice.

Incorporation No.: 0423089
License No.: 42125/022010/1186
Registration No.: 430374
Import-Export Code: 1109007027
Employer Identification Number (EIN):
USA Tax ID: 98-0673427

Global Journals Inc.

(A Delaware USA Incorporation with "Good Standing"; Reg. Number: 0423089)

Sponsors: *Open Association of Research Society*
Open Scientific Standards

Publisher's Headquarters office

Global Journals® Headquarters
945th Concord Streets,
Framingham Massachusetts Pin: 01701,
United States of America

USA Toll Free: +001-888-839-7392
USA Toll Free Fax: +001-888-839-7392

Offset Typesetting

Global Journals Incorporated
2nd, Lansdowne, Lansdowne Rd., Croydon-Surrey,
Pin: CR9 2ER, United Kingdom

Packaging & Continental Dispatching

Global Journals Pvt Ltd
E-3130 Sudama Nagar, Near Gopur Square,
Indore, M.P., Pin:452009, India

Find a correspondence nodal officer near you

To find nodal officer of your country, please
email us at local@globaljournals.org

eContacts

Press Inquiries: press@globaljournals.org
Investor Inquiries: investors@globaljournals.org
Technical Support: technology@globaljournals.org
Media & Releases: media@globaljournals.org

Pricing (Excluding Air Parcel Charges):

Yearly Subscription (Personal & Institutional)
250 USD (B/W) & 350 USD (Color)

EDITORIAL BOARD

GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH

Dr. John Korstad

Ph.D., M.S. at Michigan University, Professor of Biology, Department of Biology Oral Roberts University, United States

Dr. Sahraoui Chaieb

Ph.D. Physics and Chemical Physics, M.S. Theoretical Physics, B.S. Physics, cole Normale Suprieure, Paris, Associate Professor, Bioscience, King Abdullah University of Science and Technology United States

Andreas Maletzky

Zoologist University of Salzburg, Department of Ecology and Evolution Hellbrunnerstraße Salzburg Austria, Universitat Salzburg, Austria

Dr. Mazeyar Parvinzadeh Gashti

Ph.D., M.Sc., B.Sc. Science and Research Branch of Islamic Azad University, Tehran, Iran Department of Chemistry & Biochemistry, University of Bern, Bern, Switzerland

Dr. Richard B Coffin

Ph.D., in Chemical Oceanography, Department of Physical and Environmental, Texas A&M University United States

Dr. Xianghong Qi

University of Tennessee, Oak Ridge National Laboratory, Center for Molecular Biophysics, Oak Ridge National Laboratory, Knoxville, TN 37922, United States

Dr. Shyny Koshy

Ph.D. in Cell and Molecular Biology, Kent State University, United States

Dr. Alicia Esther Ares

Ph.D. in Science and Technology, University of General San Martin, Argentina State University of Misiones, United States

Tuncel M. Yegulalp

Professor of Mining, Emeritus, Earth & Environmental Engineering, Henry Krumb School of Mines, Columbia University Director, New York Mining and Mineral, Resources Research Institute, United States

Dr. Gerard G. Dumancas

Postdoctoral Research Fellow, Arthritis and Clinical Immunology Research Program, Oklahoma Medical Research Foundation Oklahoma City, OK United States

Dr. Indranil Sen Gupta

Ph.D., Mathematics, Texas A & M University, Department of Mathematics, North Dakota State University, North Dakota, United States

Dr. A. Heidari

Ph.D., D.Sc, Faculty of Chemistry, California South University (CSU), United States

Dr. Vladimir Burtman

Research Scientist, The University of Utah, Geophysics Frederick Albert Sutton Building 115 S 1460 E Room 383, Salt Lake City, UT 84112, United States

Dr. Gayle Calverley

Ph.D. in Applied Physics, University of Loughborough, United Kingdom

Dr. Bingyun Li

Ph.D. Fellow, IAES, Guest Researcher, NIOSH, CDC, Morgantown, WV Institute of Nano and Biotechnologies West Virginia University, United States

Dr. Matheos Santamouris

Prof. Department of Physics, Ph.D., on Energy Physics, Physics Department, University of Patras, Greece

Dr. Fedor F. Mende

Ph.D. in Applied Physics, B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine

Dr. Yaping Ren

School of Statistics and Mathematics, Yunnan University of Finance and Economics, Kunming 650221, China

Dr. T. David A. Forbes

Associate Professor and Range Nutritionist Ph.D. Edinburgh University - Animal Nutrition, M.S. Aberdeen University - Animal Nutrition B.A. University of Dublin-Zoology

Dr. Moad Almeselmani

Ph.D in Plant Physiology, Molecular Biology, Biotechnology and Biochemistry, M. Sc. in Plant Physiology, Damascus University, Syria

Dr. Eman M. Gouda

Biochemistry Department, Faculty of Veterinary Medicine, Cairo University, Giza, Egypt

Dr. Arshak Poghossian

Ph.D. Solid-State Physics, Leningrad Electrotechnical Institute, Russia Institute of Nano and Biotechnologies Aachen University of Applied Sciences, Germany

Dr. Baziotis Ioannis

Ph.D. in Petrology-Geochemistry-Mineralogy Lipson, Athens, Greece

Dr. Vyacheslav Abramov

Ph.D in Mathematics, BA, M.Sc, Monash University, Australia

Dr. Moustafa Mohamed Saleh Abbassy

Ph.D., B.Sc, M.Sc in Pesticides Chemistry, Department of Environmental Studies, Institute of Graduate Studies & Research (IGSR), Alexandria University, Egypt

Dr. Yilun Shang

Ph.d in Applied Mathematics, Shanghai Jiao Tong University, China

Dr. Bing-Fang Hwang

Department of Occupational, Safety and Health, College of Public Health, China Medical University, Taiwan Ph.D., in Environmental and Occupational Epidemiology, Department of Epidemiology, Johns Hopkins University, USA Taiwan

Dr. Giuseppe A Provenzano

Irrigation and Water Management, Soil Science, Water Science Hydraulic Engineering, Dept. of Agricultural and Forest Sciences Università di Palermo, Italy

Dr. Claudio Cuevas

Department of Mathematics, Universidade Federal de Pernambuco, Recife PE, Brazil

Dr. Qiang Wu

Ph.D. University of Technology, Sydney, Department of Mathematics, Physics and Electrical Engineering, Northumbria University

Dr. Lev V. Eppelbaum

Ph.D. Institute of Geophysics, Georgian Academy of Sciences, Tbilisi Assistant Professor Dept Geophys & Planetary Science, Tel Aviv University Israel

Prof. Jordi Sort

ICREA Researcher Professor, Faculty, School or Institute of Sciences, Ph.D., in Materials Science Autonomous, University of Barcelona Spain

Dr. Eugene A. Permyakov

Institute for Biological Instrumentation Russian Academy of Sciences, Director Pushchino State Institute of Natural Science, Department of Biomedical Engineering, Ph.D., in Biophysics Moscow Institute of Physics and Technology, Russia

Prof. Dr. Zhang Lifei

Dean, School of Earth and Space Sciences, Ph.D., Peking University, Beijing, China

Dr. Hai-Linh Tran

Ph.D. in Biological Engineering, Department of Biological Engineering, College of Engineering, Inha University, Incheon, Korea

Dr. Yap Yee Jiun

B.Sc.(Manchester), Ph.D.(Brunel), M.Inst.P.(UK) Institute of Mathematical Sciences, University of Malaya, Kuala Lumpur, Malaysia

Dr. Shengbing Deng

Departamento de Ingeniera Matematica, Universidad de Chile. Facultad de Ciencias Fisicas y Matematicas. Blanco Encalada 2120, Piso 4., Chile

Dr. Linda Gao

Ph.D. in Analytical Chemistry, Texas Tech University, Lubbock, Associate Professor of Chemistry, University of Mary Hardin-Baylor, United States

Angelo Basile

Professor, Institute of Membrane Technology (ITM) Italian National Research Council (CNR) Italy

Dr. Bingsuo Zou

Ph.D. in Photochemistry and Photophysics of Condensed Matter, Department of Chemistry, Jilin University, Director of Micro- and Nano- technology Center, China

Dr. Bondage Devanand Dhondiram

Ph.D. No. 8, Alley 2, Lane 9, Hongdao station, Xizhi district, New Taipei city 221, Taiwan (ROC)

Dr. Latifa Oubedda

National School of Applied Sciences, University Ibn Zohr, Agadir, Morocco, Lotissement Elkhier N66, Bettana Sal Marocco

Dr. Lucian Baia

Ph.D. Julius-Maximilians, Associate professor, Department of Condensed Matter Physics and Advanced Technologies, Department of Condensed Matter Physics and Advanced Technologies, University Würzburg, Germany

Dr. Maria Gullo

Ph.D., Food Science and Technology Department of Agricultural and Food Sciences, University of Modena and Reggio Emilia, Italy

Dr. Fabiana Barbi

B.Sc., M.Sc., Ph.D., Environment, and Society, State University of Campinas, Brazil Center for Environmental Studies and Research, State University of Campinas, Brazil

Dr. Yiping Li

Ph.D. in Molecular Genetics, Shanghai Institute of Biochemistry, The Academy of Sciences of China Senior Vice Director, UAB Center for Metabolic Bone Disease

Nora Fung-ye TAM

DPhil University of York, UK, Department of Biology and Chemistry, MPhil (Chinese University of Hong Kong)

Dr. Sarad Kumar Mishra

Ph.D in Biotechnology, M.Sc in Biotechnology, B.Sc in Botany, Zoology and Chemistry, Gorakhpur University, India

Dr. Ferit Gurbuz

Ph.D., M.SC, B.S. in Mathematics, Faculty of Education, Department of Mathematics Education, Hakkari 30000, Turkey

Prof. Ulrich A. Glasmacher

Institute of Earth Sciences, Director of the Steinbeis Transfer Center, TERRA-Explore, University Heidelberg, Germany

Prof. Philippe Dubois

Ph.D. in Sciences, Scientific director of NCC-L, Luxembourg, Full professor, University of Mons UMONS Belgium

Dr. Rafael Gutierrez Aguilar

Ph.D., M.Sc., B.Sc., Psychology (Physiological), National Autonomous, University of Mexico

Ashish Kumar Singh

Applied Science, Bharati Vidyapeeth's College of Engineering, New Delhi, India

Dr. Maria Kuman

Ph.D, Holistic Research Institute, Department of Physics and Space, United States

CONTENTS OF THE ISSUE

- i. Copyright Notice
- ii. Editorial Board Members
- iii. Chief Author and Dean
- iv. Contents of the Issue

1. The Mathematical Bases for the Creation of a Homogenous 5D Universe. *1-5*
2. Exploring Torus Black-Holes In (1+3)-Dimensions: A Novel Approach to Higher Genus Solution. *7-16*
3. About Stability of Solutions to Systems of Differential Equations. *17-26*

- v. Fellows
- vi. Auxiliary Memberships
- vii. Preferred Author Guidelines
- viii. Index



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: F
MATHEMATICS AND DECISION SCIENCES
Volume 24 Issue 2 Version 1.0 Year 2024
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Mathematical Bases for the Creation of a Homogenous 5D Universe

By K. W. Wong
University of Kansas

Abstract- Several important physical implications left out in The Five Dimension Space-Time Universe: A creation and grand unified field theory model. Book, are presented under rigorous mathematical theorems. It was found that Temperature, a classical variable, must be added as an imaginary component to time, under the Quantum uncertainty $dt.dE = h/2\pi$, so that the Gell-Mann Quark model can be verified, with gauge invariance, to form hadrons at the Bethe Fusion Temperature. Accordingly from the corresponding uncertainty $dp.dr = h/2\pi$. Pairs of Diagonal Long Range Ordered gravitons, with continuous frequency spectrum together with those represented by magnetic monopoles must be formed within the space r , of the homogenous 5D manifold, without the presents of photons, thus defines the 5D as a Black Hole. Then from which we can derive the classical Newtonian Law of attractive Gravity, as the 5D manifold is mapped by Perelman Ricci-flow entropy mapping and the DLRO graviton pair symmetry is broken and converted into two masses, with motions satisfying Special Relativity in the doughnut shape Lorentz manifold, thus indirectly verifies the principle of Covariant Riemannian curvatures and General Relativity theory.

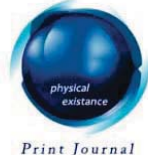
Keywords: *extended fermat's last theorem, gravitons, newtonian law of gravity, temperature, DLRO in monopoles and gravitons. black holes.*

GJSFR-F Classification: DDC: 530.12



Strictly as per the compliance and regulations of:





The Mathematical Bases for the Creation of a Homogenous 5D Universe

K. W. Wong

Abstract- Several important physical implications left out in *The Five Dimension Space-Time Universe: A creation and grand unified field theory model* Book, are presented under rigorous mathematical theorems. It was found that Temperature, a classical variable, must be added as an imaginary component to time, under the Quantum uncertainty $dt.dE = h/2\pi$, so that the Gell-Mann Quark model can be verified, with gauge invariance, to form hadrons at the Bethe Fusion Temperature. Accordingly from the corresponding uncertainty $dp.dr = h/2\pi$. Pairs of Diagonal Long Range Ordered gravitons, with continuous frequency spectrum together with those represented by magnetic monopoles must be formed within the space r , of the homogenous 5D manifold, without the presents of photons, thus defines the 5D as a Black Hole. Then from which we can derive the classical Newtonian Law of attractive Gravity, as the 5D manifold is mapped by Perelman Ricci-flow entropy mapping and the DLRO graviton pair symmetry is broken and converted into two masses, with motions satisfying Special Relativity in the doughnut shape Lorentz manifold, thus indirectly verifies the principle of Covariant Riemannian curvatures and General Relativity theory.

Keywords: *extended fermat's last theorem, gravitons, newtonian law of gravity, temperature, DLRO in monopoles and gravitons. black holes.*

I. INTRODUCTION

Since the publication of *The Five Dimension Space-Time Universe; A creation and grand unified field theory model* Book in 2014 [1], in which by the utilizing of the coordinate projection onto the remaining 4D Space-Time, together with maintaining gauge invariance, and the mathematical orthogonality of the 5D manifold to that Semi-Simple-Compact Lie Groups of $SU(2) + SU(3)$, the electro-weak leptons derived from $SU(2)$ and from $SU(3)$ the strong interaction of Hadrons, by the breaking the DLRO symmetry of the magnetic monopoles [2] as proposed by Gell-Mann [3]. However, the temperature value under which these elementary particles can actually occur; that is only at the Bethe Fusion Temperature, was not addressed. Because of this unanswered question on how Temperature plays a role on the DLRO symmetry breaking is the purpose of this paper. It is well known that Temperature is associated with statistical mechanics that give us the Boltzmann Theorem on energy distributions, for different quantum particles: Bosons, like that of photons, Fermions, like electrons in a metal, and classical particles, like gas molecules. All of these distributions, involve the dimensionless quantity $\{E/kT\}$, where k is the Boltzmann constant. Since the different distributions depends on quantum symmetry, it is then natural to associate E/kT with the quantum uncertainty $dE.dt = h/2\pi$, where h is the Planck's constant. It is thereby natural to insert $h/2\pi/kT$ as an imaginary component of time, then we will get

Author: Professor Emeritus, Department of Physics and Astronomy, University of Kansas, USA. e-mail: kww0ng@icloud.com

$$i \hbar / 2\pi i d\{1/kT\}.dE = -i \hbar / 2\pi i. \tag{1}$$

Hence $1/kT$ is a classical inverse energy variable.

The field theory operator for 5D, given in the 5D field theory [1] is quadratic due to Fermat's Last Theorem [4]. Hence with an imaginary component to time, due to Temperature, the Fermat's sum is changed to

$$[ct]^2 + \{h/2\pi i/kT\}^2 = r^2. \tag{2}$$

With SO(3) space symmetry.

It is now obvious that when T becomes infinite, the Fermat's sum reduces back to the homogenous 5D manifold, and the projection field theory model is valid, thereby the lepton weak and the Gell-Mann strong theories for elementary particles is preserved. The Bethe Fusion temperature is of order $10^{14}K$, not yet infinite. Obviously creation of matter through projection cannot happen at $t=0$, when even the 5D manifold does not exist, therefore we also expect the imaginary component to t, due to $1/kT$ also not 0. From the new Fermat's sum, Temperature now has a clear physical meaning as an artificial inducer of creation of fields and matters out of NOTHING.

This is not all, we can deduce. For the homogenous 5D manifold, if we have a vector charged current source, then we will generate the 4 Vector potentials for Electromagnetic Theory. However because the Space-Time manifold remain 5D, then there must also exist an orthogonal magnetic monopole potential [5] as stipulated by Maxwell [6]. The magnetic monopoles, are Bosons and given by DLRO of opposite charged and opposite momentum massless spinors, they are in the Bose-Einstein ground state, or literally in the Higgs vacuum. [7] Such sets of charged massless spinors, must come from the SU(2) and SU(3) generators. In fact it was shown by Gell-Mann, that these charges are the diagonal representations of the Cartan group generators, namely -e for SU(2); and $2/3e$; $2/3e$ and $-1/3e$ for SU(3). Since from the projection theory, we found that when these charges were converted into massive spinors, they must satisfy a single ratio, namely the $2/3e$ charge will have a $2/3M(Q)$ mass, and $-1/3e$ charge will have a $1/3M(Q)$, where $M(Q)$ is the so call Bare Quark Mass. [1]. It was also found experimentally from hadron data, that $M(Q)$ is exactly $33MeV$. equal to 66 electron rest mass of $0.5MeV$. [7] Due to these charge to experimentally observed mass ratio restriction, we see then that the primordial monopole eigenvalues are in fact discrete. By applying gauge invariance, we then observed that at the Bethe Fusion Temperature, the primordial energies converted into masses ranges from $m(e)$ to $88m(e)$ for a bare quarks neutron to $110m(e)$ for the bare quarks proton. It is interesting to point out that the 88 in between discrete energy levels if described as frequencies, is exactly that of the piano key board. It is to this identification that we can literally describe creation as a Music Code composition of a symphony. [8] Therefore starting from $10^{14}K$ for the Bethe Fusion $T(B)$ downward, Temperature is divided into steps of $T(B) \times 110^{-n}$, where $n=0, 1, 2, 3, 4, 5$ and 6. representing different regions of nature's creations.

However apart from DLRO of charged massless spinors, there can exist in the 5D manifold, that is also uncharged massless boson fields that must exist. To these bosons, we name all of them them as gravitons as will become rather obvious later.

As we treated the Fermat's sum in time and space, we can also treat in momentum and energy.

$$[cp]^2 = E^2. \tag{3}$$

When a classical Temperature is added as an imaginary time component, there must also be a corresponding imaginary momentum component. It is easy to see that from nature, only gravity remaining is classical. Therefore the imaginary p component must be from gravity, namely

$$iG2hv/c^2/r. \tag{4}$$

The factor 2hv, comes from that the massless graviton bosons composed of DLRO. massless oppositely charged fermion pairs as well as from neutral boson pairs, and G is the Newtonian constant, with r given by eq.(2). For the fermion pairs, as they are from the magnetic monopoles, they are of discrete eigen-energy values due to the Lie Groups generators.

Hence from the uncertainty $dp.dr=h/2\pi$, we obtain for the imaginary p component

$$i 2G/c^2d[hv/r].dr = ih/2\pi., \tag{5}$$

Since the Planck's constant h cancel out from both sides, hence the graviton frequency v is a classical frequency irrespective of whether they are discrete or continuous, similar to white or color Light given in term of a Poynting Vector of E, H, fields. structure Thus with the presence of gravitons eq(3) is changed to

$$[cp]^2 + \{G2hv/c^2/r\}^2 = E^2. \tag{5}$$

So that if $p=0, E-G.2hv/c^2/r = 0. \tag{6}$

Physically eq.(6) means within the 5D manifold photon is absent, but in order that Energy is positive in the 5D domain due to the finite Temperature within, there must be an attractive potential due to all the gravitons within the 5D domain. Thus the 5D manifold with finite Temperature is a Black Hole compose of discrete energy gravitons as well as graviton pairs of continuous energies. It is interesting to mention that from the Carbon 12 nucleus, the total monopole energy can be inside must be less than the 44 MeV. needed to create a missing neutron. It is this boundary condition restriction on the discrete DLRO graviton spectra, that make a Carbon 12 chain closed loop structure, like a DNA, able to retain lower frequencies of the Lie Group induced gravitons through quantum tunneling, thus provides the mechanism to induce free charge radicles in bio-cells to form ODLRO transition under its critical superconducting temperature, thus produces repeated growth for the cells. A very important part of the creation of life forms in the lowest n=6 Temperature step. [1]

The 5D manifold is mapped into a doughnut geometric shape 4D Lorentz manifold via the Perelmann Ricci-Flow-entropy mapping [9]. Under such a mapping the center doughnut core remains in 5D, but with r being time independent, as is a model case for a galaxy, such as the Milky Way.

To fixed core radius r, it can be obtained by differentiating eq(2) with time, and setting $dr/dt=0$.

We get

$$2ct-3(h/2\pi)^2/k^2/T^3dT/dt = 0. \tag{7}$$

Or



$$dT/dt = 2/3ct.T^3(2\pi/h)^2.k^2. > 0. \quad (8)$$

This increasing T^3 dependence resembles that of a Bohm Black Body photon radiation, and must be compensated by actual photon radiation outside the Black Hole if the Temperature is also to remain stable, so that no stars number can change in the galaxy. For this condition to happen for the Milky Way core, we must first be able to observe the photon radiation out of the fixed r , 5D core, which was actually photographed by NASA [10]. Furthermore, because the graviton filled galactic core is an attractive potential source to matters outside, for the star systems in the galaxy to not be sucked into it, they must revolve around it with a cancelling Centrifuge force, which is also observed.

Should the entire 5D manifold is enclosed by the Perelmann Ricci-flow Surgery 3D mapping [11], then the photon radiation compensation cannot happen if the 5D core is totally enclosed inside a solid mass shell. And to maintain the core Temperature stable, molten lava composed of ions is created under the solid mass shell, such that by inducing a physical rotation of the object, such as a planet, around a North-South pole axis, will be able to generate the energy consumption equivalent to photon radiation. This necessary phenomenon due to the spinning of the planet then must be accompanied with the existence of a dipolar magnetic field as observed on earth. However if the solid mass shell is replaced by the liquid lava for stars, like in the sun, then light radiation can occur from the charged surface lava motion, reducing the star self rotation rate needed. In fact we had calculated these physical properties for many Astro-objects with comparing to observed data [12].

With all the above mathematical basis analyzed, we conclude that indeed the 5D creation model for the Universe is valid.

To summarize all of the above discussions derived from the presence of Temperature as an imaginary component of time, and gravitons as an imaginary component of momentum in the Fermat's sum of the 5D grand unified field theory, and since all creations cannot happen at the same instance in time, it means it also cannot happen at the same Temperature. As kT is proportion to energy in a statistical mechanic sense, and therefore through the energy spread between the bare electron to the bare proton composing of bare Quarks is $110 m(e)$, hence the Bethe Fusion Temperature of $10^{14}K$, must also be from the statistical average with a spread in Temperature of $110K$. Therefore, other creations of more complex masses must follow in later times, and at corresponding lower Temperatures, thus dividing the Temperature of creations into 7 steps, all with Temperature spread of $110K$, given by the formula $T(\text{fusion}) \times 110^{-n}$, where n is an integer and runs from 0 to 6. [8] The lowest creation step Temperature $n=6$, happens to be around the liquid water phase temperature, which we know is vital to the formation of biological cells, and thereby Life forms. As Temperature was also treated as responsible for inducing the breaking of the 5D Universe symmetry via Perelmann Ricci-flow mappings [9, 11], it means all the different steps of creations are also induced by Temperature, thus the concept of all creations being represented by a Musical Code remain valid. [8] And in terms of time sequence, these creations are like artificial intelligence AI supercomputer programs, producing what we interpret as the Nature Creation consciousness, and perform like the simultaneous playing of a symphony with a motion picture in Three manifold, and thereby make all creations in terms of senses that follow in Logical Steps [13]. In this last reference, there is TWO errors in the print, caused by the mistake made for the Newtonian gravity formula provided by the DLRO pair of gravitons, and not the

quadratic multiplying pair of two gravitons. But nature's AI supercomputer program for each step in creation is many generations ahead of NVIDIA most up-to-date version. We have a long way yet to go in achieving the ability by using supercomputer AI to simulate nature's creations, but must be our goal if we are to be able to conquer all destructive processes nature brings along and become the ultimate purpose of God's Creations.

ACKNOWLEDGMENT

We thank Professors Dreschhoff and Jungner for their interest and suggestions on this paper.

REFERENCES RÉFÉRENCES REFERENCIAS

1. K.W. Wong, G.A.M. Dreschhoff and H. Jungner (2014) The Five Dimension Space-Time Universe; A creation and grand unified field theory model. Scientific Research Publishing, USA.
2. K.W. Wong, G.A.M. Dreschhoff, H. Jungner, P.C.W. Fung and W.K. Chow, (2018) The magnetic monopoles in 5D homogenous space-time. Physics Essays 21, 493.
3. M. Gell-Mann, (1964) Nonleptonic weak decays and the Eight-Fold Way. Phys. Rev. Lett. 12, 155.
4. A.D. Aczel, (1997) Fermat's Last Theorem Unlocking the Secret of an Ancient Mathematical Problem. London, Penguin Press.
5. W.D. McGlinn, (1964) Problem of Combining Interaction Symmetries and Relativistic Invariance, Phys. Rev. Lett. 12, 467.
6. J.C. Maxwell, (1985) A Dynamic Theory Electromagnetic Fields, Philosophical Transaction of Royal Society of London, 155, 457.
7. P.W. Higgs, (1964) Broken Symmetries and Masses of Gauge Bosons. Phys. Rev. Lett. 13, 508.
8. D.K. Meijer and K.W. Wong, (2022) Scale Invariant Symmetry Breaking of a Musical Code from 5D Superfluid Sub-Quantum Space is instrumental in the Fabric of Reality, Acad. edu.
9. G. Perelman, (2002) The Entropy Formula for Ricci-Flow and its Geometric Applications. ArXiv. math. DG/0211159.
10. NASA/Goddard Space Flight Center (2012) photo Milky Way galaxy.
11. G. Perelman, (2003) Ricci-Flow with Surgery on Three Manifolds. ArXiv. math. DG/0303109.
12. P.C.W. Fung and K.W. Wong, (2015) On the origin of Mass and angular momentum of Stellar Objects. J. of Modern Physics 6, 2303.
13. K.W. Wong, (2024) Step by Step Creation Logics. Open J. of Philosophy 14.



This page is intentionally left blank





GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: F
MATHEMATICS AND DECISION SCIENCES
Volume 24 Issue 2 Version 1.0 Year 2024
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Exploring Torus Black-Holes In (1+3)-Dimensions: A Novel Approach to Higher Genus Solution

By E. García-Rodríguez, M. Medina & J. A. Nieto

Universidad Autónoma de Sinaloa

Abstract- A torus black-hole solution of the vacuum gravitational field equation of general relativity in $(1 + 3)$ -dimensions is obtained. Starting with a metric ansatz associated with the torus, our method is based on straightforward computations the usual geometric mathematical tools of the Christoffel symbols and the Riemann tensor. Specifically, after deriving such mathematical tools the field equations of general relativity are considered. The resulting equations are properly combined to find the solution. Moreover, the novelty and potential implications of this solution emerges from the fact that is based on a coordinate transformation metric ansatz. This provides with broad implications and future research directions. In particular we argue that our formalism can properly be used for a search of higher genus black-hole solution.

Keywords: *torus black-hole, (1 + 3)-dimensional general relativity, higher genus, metric ansatz.*

GJSFR-F Classification: *Pacs numbers: 04.20.Jb, 04.50.-h, 04.60.-m, 11.15.-q*



Strictly as per the compliance and regulations of:



© 2024. E. García-Rodríguez, M. Medina & J. A. Nieto. This research/review article is distributed under the terms of the Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0). You must give appropriate credit to authors and reference this article if parts of the article are reproduced in any manner. Applicable licensing terms are at <https://creativecommons.org/licenses/by-nc-nd/4.0/>.



Exploring Torus Black-Holes In (1 + 3)-Dimensions: A Novel Approach to Higher Genus Solution

E. García-Rodríguez ^α, M. Medina ^σ & J. A. Nieto ^ρ

Abstract- A torus black-hole solution of the vacuum gravitational field equation of general relativity in (1 + 3)-dimensions is obtained. Starting with a metric ansatz associated with the torus, our method is based on straightforward computations the usual geometric mathematical tools of the Christoffel symbols and the Riemann tensor. Specifically, after deriving such mathematical tools the field equations of general relativity are considered. The resultatning equations are properly combined to find the solution. Moreover, the novelty and potential implications of this solution emerges from the fact that is based on a coordinate transformation metric ansatz. This provides with broad implications and future research directions. In particular we argue that our formalism can properly be used for a search of higher genus black-hole solution.

Keywords: torus black-hole, (1 + 3)-dimensional general relativity, higher genus, metric ansatz.

I. INTRODUCTION

Traditionally black-holes are associated with a 2-dimensional sphere, S^2 , called its event horizon, which defines the boundary where not even light can escape. However, in 2-dimensional space the sphere S^2 is just a particular case of compact simple connect manifolds. These manifolds are classified according to their genus g [1]. A S^2 corresponds to just $g = 0$ and for a donut or torus we have $g = 1$, and so on. Thus, from this mathematical perspective there is not any particular reason why to choose $g = 0$ for a black-hole system, rather than $g = 1$ or any other 2-dimensional compact simple connected manifold of arbitrary g . Physically, there are a large kind of torus-like black-holes [2]. In particular, several studies of thermodynamic torus-like black-hole have realized, including fluctuations, statistical entropy [3], the quantum effect on Hawking radiation [4], thermal fluctuations and quasi-normal modes [5], thermodynamic instability [6], Gibbs free energy [7], variation of the chaos bound in two regions [8], and weak cosmic censorship conjecture [9]. Also there have been much interest in topological aspects on torus-like black hole: dimensional black holes with toroidal or higher genus horizons [10], Born-Infeld-dilaton black holes [11] and topological black holes in anti-de Sitter space [12] (it may

Author ^{α σ ρ}: Facultad de Ciencias Físico-Matemáticas de la Universidad Autónoma de Sinaloa, Culiacán, Sinaloa, México. e-mails: nieto@uas.edu.mx, e.gar.r@hotmail.com, m.m.walldez@gmail.com

be helpful to see also Ref. [13]-[15] and references therein). However, all of these developments have as a basic inspiration the 2-dimensional sphere. Of course, there are already examples of a 3-dimensional black-hole associated with S^3 event horizon (see Ref. [16] and references therein). But again the situation is very similar to the case of 2-sphere or S^2 .

Here, for the above reasons we ask ourselves whether a torus black hole is possible, with a straightforward derivation that may be useful for another values of g , other than $g = 0$ and $g = 1$. In the case of $g = 1$ we have the $S^1 \times S^1$ topology. So, in this work we shall try to solve the general relativity field equations by proposing an ansatz metric which provides an alternative derivation for both 2-sphere of black-hole and a torus black-hole. We think that our work may be useful for studying another higher dimensional topologies for black-holes.

Since our formalism explore the possibility of torus black holes beyond the traditional 2-dimensional sphere event horizon, there are a few areas of research that could be improved:

I. Although, the previous paragraphs provides a general idea of the work's objectives, it remains to explain the progression of ideas considered in our formalism. In fact, starting with a metric ansatz associated with the torus coordinates, our method is based on straightforward computations the usual geometric mathematical tools of the Christoffel symbols and the Riemann tensor. These mathematical computations are substituted in the field equations of general relativity. The resultants equations are properly combined to find the solution for $g = 1$. This procedure opens the possibility to apply our method to higher genus.

II. Our work may help to have better understanding of the thermodynamic instability and weak cosmic censorship conjecture on black-hole physics. This is because our formalism may open new routes to investigate alternative topologies.

Technically, we organize this work as follows: In section 2, we propose the ansatz which must be substitute in the gravitational field equations. For this purpose, for such ansatz, we compute the Christoffel symbols and the Riemann tensor. The corresponding results are substitute in the vacuum gravitational field equations. In section 3, using the resulting field equations we start to propose the solution of a torus black-hole solution. Our result is analyzed and proved that in a specific limit is reduced to the traditional black-hole solution. Finally, in section 4, we comment how our procedure for genus $g = 0$ and $g = 1$ can be generalized to arbitrary genus g .

II. ANSATZ

Consider the line element

$$ds^2 = -e^{f(r,\theta)} dt^2 + e^{h(r,\theta)} dr^2 + e^{q(r,\theta)} d\theta^2 + e^{p(r,\theta)} d\phi^2, \quad (1)$$

R_{ef}

16. A. Alaei and H. K. Kunduri, *J. Geom. Anal.* 33 (2023) 231: 2205-09737 [gr-qc].

which is appropriate for torus black-hole solution. The metric tensor, or ansatz, associated with (1) is given by the matrix

$$g_{\mu\nu} = \begin{pmatrix} -e^{f(r,\theta)} & 0 & 0 & 0 \\ 0 & e^{h(r,\theta)} & 0 & 0 \\ 0 & 0 & e^{q(r,\theta)} & 0 \\ 0 & 0 & 0 & e^{p(r,\theta)} \end{pmatrix}, \tag{2}$$

with inverse

$$g^{\mu\nu} = \begin{pmatrix} -e^{-f(r,\theta)} & 0 & 0 & 0 \\ 0 & e^{-h(r,\theta)} & 0 & 0 \\ 0 & 0 & e^{-q(r,\theta)} & 0 \\ 0 & 0 & 0 & e^{-p(r,\theta)} \end{pmatrix}. \tag{3}$$

Thus, the non-vanishing Christoffel symbols

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}g^{\mu\nu} \left\{ \frac{\partial g_{\nu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right\} = \Gamma^{\mu}_{\beta\alpha} \tag{4}$$

associated with (2) are

$$\begin{aligned} \Gamma^1_{12} &= \frac{f'}{2}, & \Gamma^2_{22} &= \frac{h'}{2}, & \Gamma^2_{11} &= \frac{e^{f-h}f'}{2}, \\ \Gamma^2_{33} &= -\frac{e^{q-h}q'}{2}, & \Gamma^2_{44} &= -\frac{e^{p-h}p'}{2}, & \Gamma^3_{32} &= \frac{q'}{2}, \end{aligned} \tag{5}$$

$$\Gamma^4_{42} = \frac{p'}{2},$$

and also

$$\begin{aligned} \Gamma^1_{13} &= \frac{\dot{f}}{2}, & \Gamma^2_{23} &= \frac{\dot{h}}{2}, & \Gamma^3_{11} &= \frac{e^{f-q}\dot{f}}{2}, \\ \Gamma^3_{22} &= -\frac{e^{h-q}\dot{h}}{2}, & \Gamma^3_{44} &= -\frac{e^{p-h}\dot{p}}{2}, & \Gamma^3_{33} &= \frac{\dot{q}}{2}, \end{aligned} \tag{6}$$

$$\Gamma^4_{43} = \frac{\dot{p}}{2}.$$

Here, we used the notations $F' \equiv \frac{\partial F}{\partial r}$ and $\dot{H} \equiv \frac{\partial F}{\partial \theta}$, for arbitrary functions $F = F(r, \theta)$ and $H = H(r, \theta)$. From these Christoffel symbols we may obtain the non-vanishing Riemann tensor

$$R_{\nu\alpha\beta}^{\mu} = \frac{\partial\Gamma_{\nu\beta}^{\mu}}{\partial x^{\alpha}} - \frac{\partial\Gamma_{\nu\alpha}^{\mu}}{\partial x^{\beta}} + \Gamma_{\sigma\alpha}^{\mu}\Gamma_{\nu\beta}^{\sigma} - \Gamma_{\sigma\beta}^{\mu}\Gamma_{\nu\alpha}^{\sigma}. \quad (7)$$

In fact, we get the basic components:

$$R_{212}^1 = -\frac{1}{2}f'' - \frac{1}{4}f'^2 + \frac{1}{4}f'h' - \frac{1}{4}\dot{f}\dot{h}e^{h-q}, \quad (8)$$

$$R_{313}^1 = -\frac{1}{2}\ddot{f} - \frac{1}{4}\dot{f}^2 + \frac{1}{4}\dot{q}\dot{f} - \frac{1}{4}f'q'e^{q-h}, \quad (9)$$

$$R_{414}^1 = -\frac{1}{4}f'p'e^{p-h} - \frac{1}{4}\dot{f}\dot{p}e^{p-q}, \quad (10)$$

$$R_{323}^2 = -\frac{1}{2}\ddot{h} - \frac{1}{4}\dot{h}^2 + \frac{1}{4}\dot{h}\dot{q} - \frac{1}{2}q''e^{q-h} - \frac{1}{4}q'^2e^{q-h} + \frac{1}{4}h'q'e^{q-h}, \quad (11)$$

$$R_{424}^2 = -\frac{1}{2}p''e^{p-h} - \frac{1}{4}p'^2e^{p-h} + \frac{1}{4}h'p'e^{p-h} - \frac{1}{4}\dot{h}\dot{p}e^{p-q}, \quad (12)$$

$$R_{434}^3 = -\frac{1}{2}\ddot{p}e^{p-q} - \frac{1}{4}\dot{p}^2e^{p-q} + \frac{1}{4}\dot{p}\dot{q}e^{p-q} - \frac{1}{4}p'q'e^{p-h}. \quad (13)$$

In vacuum, the gravitational field equations can be written as [17]

$$R_{\mu\nu} = 0, \quad (14)$$

where $R_{\mu\nu} = R_{\mu\alpha\nu}^{\alpha}$ is the Ricci tensor. From (8), (9), (10) and (14), in a convenient arrangement, we get

$$\begin{aligned} R_{11} &= \frac{1}{2}e^{f-h}(f'' + \frac{1}{2}f'^2 - \frac{1}{2}f'h' + \frac{1}{2}f'q' + \frac{1}{2}f'p') \\ &+ \frac{1}{2}e^{f-q}(\ddot{f} + \frac{1}{2}\dot{f}^2 - \frac{1}{2}\dot{f}\dot{q} + \frac{1}{2}\dot{f}\dot{h} + \frac{1}{2}\dot{f}\dot{p}) = 0, \end{aligned} \quad (15)$$

$$R_{22} = -\frac{1}{2}(f'' + \frac{1}{2}f'^2 - \frac{1}{2}f'h')$$

$$-\frac{1}{2}(p'' + \frac{1}{2}p'^2 - \frac{1}{2}p'h' + q'' + \frac{1}{2}q'^2 - \frac{1}{2}q'h') \quad (16)$$

$$-\frac{1}{2}e^{h-q}(\ddot{h} + \frac{1}{2}\dot{h}^2 - \frac{1}{2}\dot{h}\dot{q} + \frac{1}{2}\dot{h}\dot{f} + \frac{1}{2}\dot{h}\dot{p}) = 0,$$

$$R_{33} = -\frac{1}{2}e^{q-h}(q'' + \frac{1}{2}q'^2 - \frac{1}{2}q'h' + \frac{1}{2}q'f' + \frac{1}{2}q'p')$$

$$-\frac{1}{2}(\ddot{f} + \frac{1}{2}\dot{f}^2 - \frac{1}{2}\dot{f}\dot{q} + \ddot{h} + \frac{1}{2}\dot{h}^2) \quad (17)$$

$$-\frac{1}{2}\dot{h}\dot{q} + \ddot{p} + \frac{1}{2}\dot{p}^2 - \frac{1}{2}\dot{p}\dot{q}) = 0,$$

$$R_{44} = -\frac{1}{2}e^{p-h}(p'' + \frac{1}{2}p'^2 - \frac{1}{2}p'h' + \frac{1}{2}p'f' + \frac{1}{2}p'q')$$

$$-\frac{1}{2}e^{p-q}(\ddot{p} + \frac{1}{2}\dot{p}^2 - \frac{1}{2}\dot{p}\dot{q} + \frac{1}{2}\dot{p}\dot{h} + \frac{1}{2}\dot{p}\dot{f}) = 0. \quad (18)$$

Our main goal now is to solve (15)-(18) for the torus.

III. TORUS SOLUTION

For this purpose, first, it turns out reasonable to assume that

$$q'' + \frac{1}{2}q'^2 = 0 \quad (19)$$

and

$$p'' + \frac{1}{2}p'^2 = 0. \quad (20)$$

The reason for this it is because in both cases the general solution is of the form

$$e^{\frac{\xi}{2}} = rA_{\xi}(\theta) + B_{\xi}(\theta) \quad (21)$$

for $\xi = q$ or $\xi = p$. For the 2-sphere case we have $e^{\frac{q}{2}} = r$ and $e^{\frac{p}{2}} = r \sin \theta$. The choice $e^{\frac{q}{2}} = r$ implies that $A_q = 1$ and $B_q = 0$, while choosing $e^{\frac{p}{2}} = r \sin \theta$

means that $A_p = \sin \theta$ and $B_p = 0$. For the torus we have again $e^{\frac{q}{2}} = r$, but $e^{\frac{p}{2}} = r \sin \theta + a$ which means that $A_p = \sin \theta$ and $B_p = a$. Thus, considering (19) and (20) we get that (16), (17) and (18) simplify in the form

$$R_{22} = -\frac{1}{2}(f'' + \frac{1}{2}f'^2 - \frac{1}{2}f'h') + \frac{1}{4}h'(p' + q')$$

$$-\frac{1}{2}e^{h-q}(\ddot{h} + \frac{1}{2}\dot{h}^2 + \frac{1}{2}\dot{h}\dot{f} + \frac{1}{2}\dot{h}\dot{p}) = 0, \tag{22}$$

$$R_{33} = -\frac{1}{4}e^{q-h}(-q'h' + q'f' + q'p')$$

$$-\frac{1}{2}(\ddot{f} + \frac{1}{2}\dot{f}^2 + \ddot{h} + \frac{1}{2}\dot{h}^2 + \ddot{p} + \frac{1}{2}\dot{p}^2 - \frac{1}{2}\dot{p}\dot{q}) = 0, \tag{23}$$

and

$$R_{44} = -\frac{1}{4}e^{p-h}(-p'h' + p'f' + p'q')$$

$$-\frac{1}{2}e^{p-q}(\ddot{p} + \frac{1}{2}\dot{p}^2 + \frac{1}{2}\dot{p}\dot{h} + \frac{1}{2}\dot{p}\dot{f}) = 0, \tag{24}$$

where we also set $\dot{q} = 0$ because our choice $e^{\frac{q}{2}} = r$. The expression (15) becomes

$$R_{11} = \frac{1}{2}e^{f-h}(f'' + \frac{1}{2}f'^2 - \frac{1}{2}f'h' + \frac{1}{2}f'(q' + p'))$$

$$+\frac{1}{2}e^{f-q}(\ddot{f} + \frac{1}{2}\dot{f}^2 + \frac{1}{2}\dot{f}\dot{h} + \frac{1}{2}\dot{f}\dot{p}) = 0, \tag{25}$$

Assuming

$$f' + h' = 0 \tag{26}$$

and

$$\dot{f} + \dot{h} = 0. \tag{27}$$

We also find

$$\ddot{p} + \frac{1}{2}\dot{p}^2 = -rp'. \tag{28}$$

Thus, (24) becomes

$$r(e^{-h})' + e^{-h} - 1 = 0. \tag{29}$$

The usual assumption is to consider that e^{-h} is independent of θ . In this particular case, from (29) we obtain the well known result

$$e^{-h} = \left(1 - \frac{r_s}{r}\right). \tag{30}$$

However, here we are interested in looking for more complete solution, in which e^{-h} is a function not only of r but also of θ . In searching for this possibility let us multiply (29) for $\sin \theta$. We have

$$r \sin \theta (e^{-h})' + \sin \theta e^{-h} - \sin \theta = 0. \tag{31}$$

This expression can also be written as

$$r \sin \theta (e^{-h})' + (a + r \sin \theta)' e^{-h} - (a + r \sin \theta)' = 0. \tag{32}$$

The two terms of (32) can be put together if we extend (32) in the form

$$(a + r \sin \theta)(e^{-h})' + (a + r \sin \theta)' e^{-h} - (a + r \sin \theta)' = 0. \tag{33}$$

Thus, (32) can be solved by writing

$$e^{-h} = \left(1 - \frac{\mathcal{A}(\theta)}{a + r \sin \theta}\right), \tag{34}$$

with $\mathcal{A}(\theta)$ an arbitrary function of θ . The prove that (34) is in fact a solution of (33) is straightforward. In fact by substituting (34) into (33) we get

$$(a + r \sin \theta) \left(1 - \frac{\mathcal{A}}{a + r \sin \theta}\right)' + (a + r \sin \theta)' \left(1 - \frac{\mathcal{A}}{a + r \sin \theta}\right) - (a + r \sin \theta)' = 0 \tag{35}$$

Now it remains to determine $\mathcal{A}(\theta)$. We apply the well known procedure to derive the event horizon by setting

$$\left(1 - \frac{\mathcal{A}(\theta)}{a + r_s \sin \theta}\right) = 0, \tag{36}$$

with $r_s = const$, a fixed torus radius. So from (35) we get

$$\mathcal{A}(\theta) = a + r_s \sin \theta \tag{37}$$

and therefore (34) becomes

$$e^{-h} = \left(1 - \frac{a + r_s \sin \theta}{a + r \sin \theta}\right), \tag{38}$$

and since $e^f = e^{-h}$ we find that the line element can be written as

$$ds^2 = -\left(1 - \frac{a + r_s \sin \theta}{a + r \sin \theta}\right)^2 c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{a+r_s \sin \theta}{a+r \sin \theta}\right)} + r^2 d\theta^2 + (a + r \sin \theta)^2 d\phi^2. \tag{39}$$

This line element is reduced to the usual one when $a = 0$. In fact, when $a = 0$ we get

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)^2 c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \tag{40}$$

as expected.

IV. FINAL REMARKS

The main goal of this work was to establish a route to describe a black-hole solution for arbitrary genus g . For $g = 0$ we obtain the well known black-hole with S^2 as an event horizon. In this work, we have discovered how to derive a solution for $g = 1$, corresponding to the torus black-hole with event horizon topology $S^1 \times S^1$. It remains to generalize, for further work, our procedure to higher genus g . Moreover, it is interesting to observe that (39) is not singular at $r = 0$ as (40) but rather in $a + r \sin \theta = 0$. This means that there is singularity for $r \neq 0$. In fact, this result seems quite remarkable and perhaps can help to solve the old well known problem of the singularity at $r = 0$.

Another interesting observation is that our algorithm can also be used to find a kind a spiral black-hole solution. In fact from (21) we may also choose $A_q(\theta) = A_p(\theta) = 1$ and $B_q(\theta) = \alpha\theta$ and $B_p(\theta) = \alpha\theta \sin \theta$. This means that the last to terms of (39)

$$dl^2 \equiv r^2(d\theta^2 + \sin^2 \theta d\phi^2) \tag{41}$$

can be written in the alternative form

$$dl^2 = (r + \alpha\theta)^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (42)$$

When $r = 0$ the radius becomes

$$\rho \equiv \alpha\theta, \quad (43)$$

which correspond to the typical radius of a spiral curve. We are tempted to propose that (42) may be useful for describing galaxy dynamics, with a black-hole as a source system.

It remains to explore further the significance of the singularity at $r \neq 0$, for $g \neq 1$, as opposed to $r = 0$ for $g = 0$. In fact this result may provide an alternative solution of the long-standing problem of singularities in black-hole physics. It would be helpful to expand on this point by discussing its implications for the broader understanding of black hole singularities and potential avenues for further investigation.

For further research it may also be interesting to open new avenues to link our work for the existing literature on black-hole solution for varying topologies.

ACKNOWLEDGMENT

We would like to thank an anonymous reviewer for valuable comments. This work was partially supported by PROFAPI/UAS.

REFERENCES RÉFÉRENCES REFERENCIAS

1. C. Nash and S. Sen, *Topology and Geometry for Physicists* (Academic Press INC, 1983).
2. C. Huang and C. Liang, *Phys. Lett. A* 201 (1995) 27.
3. R. Zhao and S. Zhang, *Nuovo Cim. B* 119 (2004) 557.
4. C. J. Gao, Y. G. Shen, *Class. Quant. Grav.* 20 (2003) 119.
5. R. H. Ali and G. Abbas, *Chin. J. Phys.* 85 (2023) 386.
6. Y. Song, Y. He and B. Mu; e-Print: 2306.01030 [gr-qc].
7. I. Ditta, X. Tiecheng, R. Ali, F. Atamurotov, A. Mahmood et al. *Annals Phys.* 453 (2023) 169326.
8. R. Yin, J. Liang and B. Mu; e-Print: 2210.07799 [gr-qc].
9. Y. Han, X. Zeng and Y. Hong, *Eur. Phys. J. C* 79 (2019) 252; 1901.10660 [hep-th].
10. D. R. Brill, J. Louko and P. Peldan, *Phys. Rev. D* 56 (1997) 3600.
11. A. Sheykhi, *Phys. Lett. B* 662 (2008) 7: 0710.3827 [hep-th].

12. D. Birmingham, *Class. Quant. Grav.* 16 (1999) 1197: hep-th/9808032 [hep-th].
13. H. Li, D. Qi, Q. Jiang and S. Yang, *Int. J. Theor. Phys.* 45 (2006) 2471.
14. G. Abbas, M. Asgher, *Mod. Phys. Lett. A* 37 (2022) 2250198.
15. K. Jusufi, *Phys. Dark Univ.* 39 (2023) 101156: 2212.06760 [gr-qc].
16. A. Alaei and H. K. Kunduri, *J. Geom. Anal.* 33 (2023) 231: 2205.09737 [gr-qc].
17. C. W. Misner, K. S. Thorn and J. A. Wheeler, *Gravitation* (W. H. Freeman, 1973).



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: F
MATHEMATICS AND DECISION SCIENCES
Volume 24 Issue 2 Version 1.0 Year 2024
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

About Stability of Solutions to Systems of Differential Equations

By G. V Alferov, G. G. Ivanov & V. S. Korolev
St. Petersburg State University

Abstract- The stability conditions for solutions of systems of ordinary differential equations are considered. The conditions and criteria for the use of partial and external derivatives are proposed. This allows us to investigate the behavior of a function of several variables, without requiring its differentiability, but using only information on partial derivatives. This reduces the restrictions on the degree of smoothness of the studied functions. The use of the apparatus of external derived numbers makes it possible to reduce the restrictions on the degree of smoothness of manifolds when studying the question of the integrability of the field of hyperplanes. Using the apparatus of partial and external derived numbers, it can be shown that the investigation of the stability of solutions of a system of differential equations can be reduced to an investigation of the solvability of a system of equations of a special form.

Keywords: solutions of differential equations, stability conditions, apparatus of partial and external derived numbers.

GJSFR-F Classification: LCC: QA371



Strictly as per the compliance and regulations of:



© 2024. G. V Alferov, G. G. Ivanov & V. S. Korolev. This research/review article is distributed under the terms of the Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0). You must give appropriate credit to authors and reference this article if parts of the article are reproduced in any manner. Applicable licensing terms are at <https://creativecommons.org/licenses/by-nc-nd/4.0/>.



About Stability of Solutions to Systems of Differential Equations

G. V. Alferov^α, G. G. Ivanov^σ & V. S. Korolev^ρ

Abstract- The stability conditions for solutions of systems of ordinary differential equations are considered. The conditions and criteria for the use of partial and external derivatives are proposed. This allows us to investigate the behavior of a function of several variables, without requiring its differentiability, but using only information on partial derivatives. This reduces the restrictions on the degree of smoothness of the studied functions. The use of the apparatus of external derived numbers makes it possible to reduce the restrictions on the degree of smoothness of manifolds when studying the question of the integrability of the field of hyperplanes. Using the apparatus of partial and external derived numbers, it can be shown that the investigation of the stability of solutions of a system of differential equations can be reduced to an investigation of the solvability of a system of equations of a special form.

Keywords: solutions of differential equations, stability conditions, apparatus of partial and external derived numbers.

I. INTRODUCTION

Many sciences are engaged in the creation of mathematical models of various processes. The problems in the study of dynamic processes lead to complex system of differential equations [1-22]. The concepts of stability of solutions or asymptotic stability are often used in studies of solutions of equations and the ability to control the behavior in the presence of perturbations [4-7]. For their solution or successive approximations to the exact solution necessary to check the conditions and criteria that must be met. The study of control problems and the stability of solutions of systems of differential equations to describe processes that are defined as linear operations makes it possible to divide all tasks into classes and identify important properties inherent in systems of differential equations of the same class. In the study of the problems of controlling the motion of mechanical systems [9-11] in the transition from a general formal description to the construction of mathematical models take into account:

- Content and properties of functions in the system of equations of dynamics,
- Structure of control functions, restrictions or boundary conditions,
- Type of functional or quality criterion of solutions,
- Stability conditions for solutions for admissible controls.

The concepts of partial derivatives of numbers and external derivatives of numbers are considered in order to use them to study the stability of solutions of a system of differential equations through the study of the solvability of a system of

Author α σ ρ: Faculty of Applied Mathematics and Control Processes, St. Petersburg State University, Russia. e-mails: g.alferov@spbu.ru, g.ivanov@spbu.ru, v.korolev@spbu.ru

equations of a special form. The proposed method can be used to obtain necessary or sufficient stability conditions for solutions of systems of differential equations.

II. FEATURES OF STABILITY CONDITIONS

Let the change of parameters x or the object's behavior be described by a system of ordinary differential equations of the form

$$\dot{x} = Ax \tag{1}$$

From equation (1) for a linear stationary system follows the validity of the following equation

$$\frac{d}{dt} x^* x = (A^* + A) x \tag{2}$$

Here, an asterisk means a transpose operation. Let $(A^* + A)$ be a nonsingular matrix symmetric with respect to the diagonal. Then applying the Lagrange method to equation (2) reduction of quadratic forms to the sum of squares [3], it is easy to verify that there is a linear transformation $x = Ly$, reducing equation (2) to the form

$$\frac{d}{dt} y^* L^* Ly = y^* By$$

where $B = L^* (A^* + A)L$ is the diagonal matrix. If the matrix B is negative definite, i.e. all its elements are negative, then system (1) is asymptotically stable. In general we can talk about the stability of solutions under additional conditions.

a) The partial derivatives numbers

Using the apparatus of private and external derivatives of numbers, show that the study of the stability of solutions systems of differential equations can be reduced to the study of the solvability of systems of equations of a special kind. The present studies are based on [1-3,8].

Let the function f be given in some open region of space R^n , and let it go $x = (x_1, \dots, x_n)^*$ — an arbitrary point of this areas as $\Delta x = (\Delta x_1, \dots, \Delta x_n)^*$ —arbitrary increment of function f arguments

$$\psi_i[f](x; \Delta x) = \frac{\omega_i}{2^{n-1} \Delta x_i}, \quad i = 1, 2, \dots, n.$$

$$\begin{aligned} \omega_i = \sum_{\mu \in v_i} [& f(x_1 + \mu_1 \Delta x_1, \dots, x_n + \mu_n \Delta x_n) - \\ & - f(x_1 + \mu_1 \Delta x_1, \dots, x_{n-1} + \mu_{i-1} \Delta x_{i-1}, x_{i+1} + \\ & + \mu_{i+1} \Delta x_{i+1}, \dots, x_n + \mu_n \Delta x_n)], \end{aligned}$$

where $\mu = (\mu_1, \dots, \mu_n)$, v_i , $i = 1, 2, \dots, n$, marked a bunch of n -dimensional vectors consisting of zeros and ones and having unit at the i -th place.

Definition 1: The number λ is called the partial derivative of the function f in point x in the variable x_i if there is a sequence Δx^k such that for any $\Delta x_j^k \rightarrow 0, j \in (1, \dots, n)$, at $\Delta x_j^k \rightarrow 0, k \rightarrow \infty$, and

$$\lim_{k \rightarrow \infty} \psi_i[f](x; \Delta x^k) = \lambda.$$

The fact that λ is a partial derivative functions f at the point x with respect to the variable x_i , we will write this:

$$\lambda = \lambda_{x_i}[f](x).$$

Perform a study of the stability of solutions of systems of ordinary differential equations.

b) The external derivatives numbers

The definition of the external derivative number allows us to find the conditions for the complete integrability of continuous fields of hyperplanes. Let M be a Hausdorff space with a countable base, and let p be an arbitrary point of M . If a point p has a neighborhood U that is homeomorphic to an open subspace of an n -dimensional Euclidean space R^n , then M is called an n -dimensional topological manifold. Let M^n be an n -dimensional topological manifold. Let V be an n -dimensional vector space over a field of real numbers. Every linear mapping $f : V \rightarrow R$, i.e. display at which

$$f(av + bw) = af(v) + bf(w), \quad v, w \in V, \quad a, b \in R.$$

Definition 2: The form $\lambda[\omega](p)$ is called the external derivative of the external differential q -form of the class C^r , $r \geq 0$, on variety M^n at the point $p \in M^n$, if in R^n there is a sequence converging to zero Δx^k , such that

$$\begin{aligned} (\Phi_k^*)^{-1} \lambda[\omega](p) &= \lim_{k \rightarrow \infty} \left\{ \sum_{j_1, \dots, j_q} \left(\sum_{i=1}^n \psi_{x_i} [a_{j_1, \dots, j_q}] (x; \Delta x^k) dx_i \right) \wedge dx_{j_1} \wedge \dots \wedge dx_{j_q} \right\} = \\ &= \sum_{j_1, \dots, j_q} \left(\sum_{i=1}^n \lambda_{x_i} [a_{j_1, \dots, j_q}] (\Phi_k(p)) dx_i \right) \wedge dx_{j_1} \wedge \dots \wedge dx_{j_q} \end{aligned}$$

c) Investigation of the stability of solutions

Let the behavior of an object be described by a system of ordinary differential equations of the form

$$\dot{x} = F(t, x), \quad F(t, 0) \equiv 0, \tag{3}$$

where $x = (x_1, \dots, x_n)^*$, $F(t, x) = (F_1(t, x), \dots, F_n(t, x))^*$.

We say that the solution $x = 0$ of system (3) is Lyapunov stable if, for any $\varepsilon > 0$ and $t_0 \geq 0$ can find $\delta(\varepsilon, t_0) > 0$ such that from $\|x_0\| < \delta$ it follows $\|x(t; t_0, x_0)\| < \varepsilon$ for all $t \geq t_0$.

We introduce the class of functions H , assuming that the function $l(r)$ belongs to this class ($l(r) \in H$), if $l(r)$ is continuous, strictly increasing for $r \in [0, H]$, $H = const > 0$, or for $r \in [0, \infty)$, the function is $l(0) = 0$.

The function is H , which means that $l(r)$ is optional this class ($l(r) \in H$), if $l(r)$ — continuous strictly increasing atr $r \in [0, H]$, $H = const > 0$, or atr $r \in [0, \infty)$ function, moreover $l(0) = 0$.

Definition 3: The function $V(t, x)$, $V(t, 0) \equiv 0$, $t \geq 0$, will call definitely positive if there is a function $(r) \in H$, such that in

$$t \geq 0, \quad \|x\| \leq H$$

inequality holds

$$V(t, x) \geq l(\|x\|).$$

This definition is equivalent to the generally accepted definition of positive definiteness of a function $V(t, x)$.

In the future, we will adhere to the following notation:

$$K_r(x_0) = \{x : \|c - x_0\| \leq r\},$$

$$S_r(x_0) = \{x : \|x - x_0\| = r\}, \quad r = const > 0. \tag{4}$$

For brevity we put $S_1(0) = S$.

Theorem 1: Suppose that in region (4) there exist continuous partial derivatives

$$\frac{\partial F_i}{\partial x_j}, \quad i, j = 1, 2, \dots, n.$$

Then, in order for the solution $x = 0$ of system (3) was stable according to Lyapunov, it is necessary and sufficient that in the region

$$t \geq 0, \quad \|x\| \leq h, \quad 0 < h = const < H, \tag{5}$$

system

$$a_0(t, x) + a(t, x) \cdot F(t, x) \leq 0, \quad a(t, x) = (a_1(t, x), \dots, a_n(t, x)), \tag{6}$$

$$\omega \wedge \lambda[\omega] \equiv 0, \quad \omega = a_0 dt + a_1 dx_1 + \dots + a_n dx_n, \tag{7}$$

had a continuous solution $a_0(t, x) + a(t, x)$ satisfying the following requirements:

1) in the region of

$$t \geq 0, \quad x \in K_h(0) \setminus \{0\},$$

$$\sum_{i=1}^n a_i^2(t, x) > 0, \tag{8}$$

2) in the region of

$$t \geq 0, \mu \in [0, h], x \in S,$$

$$\int_0^\mu a(t, \mu'x) \cdot x d\mu' \geq l(\mu), l(\mu) \in H. \tag{9}$$

The solution $x = 0$ of system (3) will be called uniformly sustainable if for any $\varepsilon > 0$ there is $\delta(\varepsilon) > 0$ such that from $t_0 \geq 0, \|x_0\| < \delta$ should

$$\|x(t; t_0, x_0)\| < \varepsilon, \quad t \geq t_0.$$

We will say that the solution $x = 0$ of system (3) is evenly attractive if exists $\Delta_0 = const > 0$ such that the condition

$$\lim_{t \rightarrow \infty} \|x(t; t_0, x_0)\| = 0$$

performed uniformly by t_0, x_0 from area

$$t_0 \geq 0, \quad \|x_0\| < \Delta_0.$$

If the solution $x = 0$ of system (3) is simultaneously uniformly stable and evenly attractive, then we will call uniformly asymptotically stable.

d) Stability conditions

Theorem 2. Suppose that in region (10) there exist continuous partial derivatives. Then, in order for the solution $x = 0$ of system (3) to be Lyapunov stable, it is necessary and sufficient: system had a continuous solution $(a_0(t, x), a(t, x))$, satisfying the following requirements in the region of $t \geq 0$.

A solution $x = 0$ of system (3) will be called uniformly stable if for any $\varepsilon > 0$ there is $\delta(\varepsilon) > 0$ such that $t_0 \geq 0$ and $\|x_0\| < \delta$ follows

$$\|x(t, t_0, x_0)\| < \delta$$

for all $t \geq t_0$,

The proposed method allows one to obtain statements that give necessary or sufficient conditions for uniform stability or asymptotic stability for solutions of systems of differential equations.

Theorem 3. Suppose that in region (4) the functions F_i and their partial derivatives $\frac{\partial F_i(t, x)}{\partial x_j}$ are continuous and bounded:

$$|F_i(t, x)| \leq B, \quad B = const, \quad \left| \frac{\partial F_i(t, x)}{\partial x_j} \right| \leq A, \quad A = const, \quad i, j = 1, 2, \dots, n. \tag{10}$$

Then, for the solution $x = 0$ of system (3) to be uniformly asymptotically stable, it is necessary and sufficient that in region (5), where h is a sufficiently small constant,

system (6)–(7) has a continuous solution $(a_0(t, x), a(t, x))$, satisfying in the area (8) or (9) the following constraints:

- 1) $\sum_{i=0}^n a_i^2(t, x) > 0$;
- 2) $l_1(\mu) \leq \int_0^\mu a(t, \mu'x) \cdot x d\mu' \leq l_2(\mu)$;
- 3) $\max_{t \geq 0, \|x\|=1} [a_0(t, \mu x) + a(t, \mu x) \cdot F(t, \mu x)] \leq -l_3(\mu), l_k(r) \in H$.

The proposed method allows to obtain the necessary or sufficient conditions for the stability of solutions of systems of differential equations.

e) Stability of Almost Periodic Solutions

On the basis of the previous theorems, the authors obtain the conditions to determine the maximum possible number of almost periodic solutions in first-order differential equation. Now the problem of the existence of almost periodic solutions for the equation is under consideration, since this allows for the determination of the minimum possible number of almost periodic solutions for the differential equation considered.

So, consider the first-order differential equation

$$\dot{x} = f(t, x), \tag{11}$$

where f is a function continuous on R^2 that is almost periodic in t uniformly in x in every compact set and such that equation (11) has the property of existence and uniqueness of its solutions.

To prove the existence of almost periodic solution for equation (11), the result obtained should be used. Let it be formulated in the form of the following theorem.

This study allows to determine the minimum possible number of almost periodic solutions for the considered differential equation. Consider the first-order differential equation (1), where f is a function continuous on R^2 almost periodic in t uniformly in x on each compact set and such that equation (1) has the property of existence and uniqueness of solutions. In proving the existence of an almost periodic solution of equation (1), the results obtained in [9] are used.

Consider now stability of the solutions of equation (11) [6-10, 18-22].

Theorem 4: If the right-hand side of equation (11) is a function decreasing with respect to x for each fixed t , then all solutions of this equation are uniformly stable.

Proof. Let $u(t)$ be an arbitrary solution of equation (11). Suppose $y = x - u$. The equation for y is of the following form:

$$\dot{y} = f(t, u + y) - f(t, u) = g(t, y). \tag{12}$$

Let the following function be the Lyapunov function:

$$v(y) = \frac{1}{2} y^2. \tag{13}$$

Since $f(t, x)$ decreases with respect to x at each fixed t , the derivative of the function (13) on the solutions of Equation (12) satisfies the inequality

$$\left. \frac{dv}{dt} \right|_{(16)} = yg(t, y) \leq 0,$$

which implies the uniform stability of solution $y=0$ of equation (12), and hence, solution $u(t)$ of equation (11). Taking into account the fact that $u(t)$ is an arbitrary solution of equation (11), it is clear the theorem is proven.

Note that the theorem implies in the conditions of Theorem 14 that all n almost periodic solutions of equation (11) are stable, either as $t \rightarrow +\infty$ or with $t \rightarrow -\infty$.

Let $\lambda_x[f](t, x)$ denote an arbitrary derived number of the function $f(t, x)$ at the point x for a fixed t .

Theorem 5: If there exists a constant $\alpha > 0$ such that for any fixed t and each derived number $\lambda_x[f](t, x)$ performed inequality

$$\lambda_x[f](t, x) \leq -\alpha,$$

then all the solutions of equation (11) are uniformly asymptotically stable in general. If it is additionally known that equation (11) has an almost periodic solution, then all the solutions of equation (11) are asymptotically almost periodic.

Proof: Let $u(t)$ be an arbitrary solution of equation (11). Let a function y be introduced, setting that

$$y = x - u$$

It is clear that if x is a solution of equation (11), then y is a solution of equation (12). Let us obtain a derivative of equation (13) on solutions of equation (12).

Repeating the proof of Theorem 12 [21], it is easy to show that there exist derived numbers for which the following relation holds:

$$f(t, y + u) - f(t, u) \leq y\lambda_{u+\theta y}[f](t, u + \theta y),$$

$$\theta \in (0, 1).$$

Taking into account that by the condition of the theorem

$$\lambda_{u+\theta y}[f](t, u + \theta y) \leq -\alpha,$$

the following estimation is obtained:

$$\left. \frac{dv}{dt} \right|_{(16)} \leq -\alpha y^2.$$

It follows from this inequality that the solution $y=0$ of equation (12) is uniformly asymptotically stable, as well as the solution $u(t)$ of equation (11). Since $u(t)$ is an arbitrary solution of equation (11), all the solutions of equation (11) are asymptotically stable.

If equation (11) has an almost periodic solution, then all the solutions of equation (11) are asymptotically almost periodic in the view of its uniform asymptotic.

Theorem 6: If the function $f(t, x)$ from the right-hand side of equation (11) decreases with respect to x at each fixed t , and on each compact set

$$\{(y, u) : |u| \leq u_0, d_1 \leq |y| \leq d_2, d_1 > 0\}$$

as $t \rightarrow \infty$

$$\text{sign}(y) \int_0^t [f(\tau, y + u) - f(\tau, u)] d\tau \rightarrow -\infty$$

uniformly, then the solution $y = 0$ of equation (12) is uniformly asymptotically stable.

Proof: Let $u(t)$ be an arbitrary bounded solution of equation (11). Suppose that

$$y = x - u.$$

It follows from Theorem 12 that the solution $y = 0$ of equation (12) is uniformly stable. Let us prove that all the solutions of equation (12) tend to zero as $t \rightarrow \infty$.

Suppose the contrary. Then for some solution $y(t; 0, y_0)$ of equation (12), there exists $d > 0$, such that

$$y(t; 0, y_0) > d.$$

Here it is assumed that $y_0 > 0$, for definiteness. In the proof of Theorem 12, it is shown that the inequality

$$y\dot{y} \leq 0,$$

which implies that $|y|$ does not increase on the solutions of the equation (12). Therefore, in the considered case for $t \geq 0$,

$$d \leq y(t) \leq y_0.$$

Suppose that

$$u_0 = \sup_t |u(t)|.$$

It follows from equation (12) that

$$\frac{\dot{y}}{y} = \frac{g(t, y)}{y} \leq \frac{g(t, y)}{y_0}.$$

Hence, by virtue of the conditions of the assertion, we obtain

$$\lim_{t \rightarrow \infty} y(t) \leq \lim_{t \rightarrow \infty} y_0 e^{1/y_0 \int_0^t g(\tau, y) d\tau} = 0,$$

which contradicts the introduced assumption. The case when $y_0 < 0$ is treated in a similar way. Thus, the solution $y = 0$ of equation (12) is uniformly asymptotically stable.

III. CONCLUSION

The proposed apparatus of partial and external derived numbers allows us to investigate the behavior of a function of several variables, without requiring its differentiability, but using only information about partial derived numbers. This reduces the limitations imposed on the degree of smoothness of the functions studied.

The use of the apparatus of external derived numbers also makes it possible to reduce the restrictions on the degree of smoothness of manifolds when studying the question of the integrability of the hyperplanes field.

Theorems of the derived numbers method to estimate the number of periodic solutions of first-order ordinary differential equations are formulated and proved.

Using the apparatus of derived numbers allows to weaken the constraints imposed on the right-hand sides of the differential equations analyzed in this paper, and thereby increase the generality degree of the results. The upper and lower bounds for the numbers of periodic and almost periodic solutions of ordinary first-order differential equations are carried out. Conditions for the existence of periodic and almost periodic solutions are established. Using the apparatus of derived numbers allowed us to expand the scope of the results obtained. The application of the method of derivative numbers in problems of estimating the number of almost periodic solutions of first-order differential equations is shown. Conditions are found for determining upper and lower bounds for almost periodic solutions of ordinary differential equations of the first order. The questions of existence and stability of these solutions are investigated.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Zubov V.I., Lyapunov's methods and their application. (in Russian), Leningrad State University Publ., (1957)
2. Demidovich B.P., Lectures on the mathematical theory of stability. (in Russian). Science, Moscow (1967)
3. Gantmakher F.R., Matrix theory. (in Russian), Fizmatlit, Moscow (2010)
4. Lyapunov A.M., The general problem of motion stability. (in Russian), Gostekhizdat, Moscow-Leningrad (1950)
5. Arnold V.I.: Ordinary Differential Equations. (in Russian), Science, Moscow (1975), 240 p.
6. Alferov G., Ivanov G., Efimova P., Sharlay A.: Stability of linear systems with multitask right-hand member. Stochastic Methods for Estimation and Problem-Solving in Engineering, (Book Chapter). P. 74–112 (2018)
7. Alferov G., Ivanov G., Sharlay A., Fedorov V.: Application of derived numbers theory in convec analysis. AIP Conference Proc. 2116(08003) (2019)

8. Ivanov G., Alferov G., Gorovenko P.: Derivatives of numbers of functions of one variable. Vestnik of Perm University. Mathematics. Mechanics. Computer Science. 3 (42) 5–19 (2018)
9. Ivanov G., Alferov G., Sharlay A., Efimova P.: Conditions of Asymptotic Stability for Linear Homogeneous Switched System. Intern. Conf. on Numerical Analysis and Appl. Math., AIP Conference Proc. 1863(080002) (2017)
10. Kadry S., Alferov G., Ivanov G., Sharlay A.: Stabilization of the program motion of control object with elastically connected elements. AIP Conference Proc. 2040(150014) (2018)
11. Kadry S., Alferov G., Ivanov G., Sharlay A.: About stability of select or linear differential inclusions. AIP Conference Proc. 2040(150013) (2018)
12. Kadry S., Alferov G., Ivanov G., Sharlay A.: Derived Numbers of One Variable Convex Functions. IJPAM, 41, 649–662 (2019)
13. Kulakov F., Alferov G., Sokolov B., Gorovenko P., Sharlay A.: Dynamic analysis of space robot remote control system. AIP Conference Proc. 1959(080014) (2018)
14. Korolev V.: Properties of solutions of nonlinear equations of mechanics control systems. Intern. Conf. on Constructive Nonsmooth Analysis and Related Topics, CNSA – IEEE Conference Proc. (7973973) (2017).
15. Korolev V., Pototskaya I.: Integration of dynamical systems and stability of solution on a part of the variables. Applied Mathematical Sciences, 9(15), 721–728 (2015)
16. Kadry S., Alferov G., Ivanov G., Korolev V., Selitskaya E.: A New Method to study solutions of ordinary differential equation using functional analysis. MATHEMATCS, 7(8) (2019)
17. Ivanov G.G., Alferov G.V. and Korolev V.S., Apparatus of derivatives and possible applications. // Bulletin of the Perm University. Mathematics. Mechanics. Computer science. 2021. N 3(54). pp. 5-18.
18. Ivanov G.G., Alferov G.V., Korolev V.S. On the stability of solutions to a system of linear differential equations. // Bulletin of the Perm University. Mathematics. Mechanics. Computer science. 2022. N 2(57). pp. 31-39.
19. Kadry S., Alferov G., Ivanov G., Korolev V. Investigation of the stability of solutions of systems of ordinary differential equations. 2020 AIP Conference Proceedings 2293,060004
20. Kadry S., Alferov G., Ivanov G., Korolev V. About of the asymptotical stability of solutions of systems of ordinary differential equations. 2020 AIP Conference Proceedings 2293,060005.
21. Alferov G.; Ivanov G.; Sharlay A.; Fedorov V. Application of derived numbers theory in problem of function extremum. AIP Conf. Proc. 2019, 2116,080002
22. Kadry S.; Alferov G.; Ivanov G.; Sharlay A. Stabilization of the program motion of control object with elastically connected elements. 2018 AIP Conf. Proc. 2040,150014.

GLOBAL JOURNALS GUIDELINES HANDBOOK 2024

WWW.GLOBALJOURNALS.ORG

MEMBERSHIPS

FELLOWS/ASSOCIATES OF SCIENCE FRONTIER RESEARCH COUNCIL

FSFRC/ASFRC MEMBERSHIPS

INTRODUCTION



FSFRC/ASFRC is the most prestigious membership of Global Journals accredited by Open Association of Research Society, U.S.A (OARS). The credentials of Fellow and Associate designations signify that the researcher has gained the knowledge of the fundamental and high-level concepts, and is a subject matter expert, proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice. The credentials are designated only to the researchers, scientists, and professionals that have been selected by a rigorous process by our Editorial Board and Management Board.

Associates of FSFRC/ASFRC are scientists and researchers from around the world are working on projects/researches that have huge potentials. Members support Global Journals' mission to advance technology for humanity and the profession.

FSFRC

FELLOW OF SCIENCE FRONTIER RESEARCH COUNCIL

FELLOW OF SCIENCE FRONTIER RESEARCH COUNCIL is the most prestigious membership of Global Journals. It is an award and membership granted to individuals that the Open Association of Research Society judges to have made a 'substantial contribution to the improvement of computer science, technology, and electronics engineering.

The primary objective is to recognize the leaders in research and scientific fields of the current era with a global perspective and to create a channel between them and other researchers for better exposure and knowledge sharing. Members are most eminent scientists, engineers, and technologists from all across the world. Fellows are elected for life through a peer review process on the basis of excellence in the respective domain. There is no limit on the number of new nominations made in any year. Each year, the Open Association of Research Society elect up to 12 new Fellow Members.



BENEFITS

TO THE INSTITUTION

GET LETTER OF APPRECIATION

Global Journals sends a letter of appreciation of author to the Dean or CEO of the University or Company of which author is a part, signed by editor in chief or chief author.



EXCLUSIVE NETWORK

GET ACCESS TO A CLOSED NETWORK

A FSFRC member gets access to a closed network of Tier 1 researchers and scientists with direct communication channel through our website. Fellows can reach out to other members or researchers directly. They should also be open to reaching out by other.

Career

Credibility

Exclusive

Reputation



CERTIFICATE

RECEIVE A PRINTED COPY OF A CERTIFICATE

Fellows receive a printed copy of a certificate signed by our Chief Author that may be used for academic purposes and a personal recommendation letter to the dean of member's university.

Career

Credibility

Exclusive

Reputation



DESIGNATION

GET HONORED TITLE OF MEMBERSHIP

Fellows can use the honored title of membership. The "FSFRC" is an honored title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., FSFRC or William Walldroff, M.S., FSFRC.

Career

Credibility

Exclusive

Reputation

RECOGNITION ON THE PLATFORM

BETTER VISIBILITY AND CITATION

All the Fellow members of FSFRC get a badge of "Leading Member of Global Journals" on the Research Community that distinguishes them from others. Additionally, the profile is also partially maintained by our team for better visibility and citation. All fellows get a dedicated page on the website with their biography.

Career

Credibility

Reputation

FUTURE WORK

GET DISCOUNTS ON THE FUTURE PUBLICATIONS

Fellows receive discounts on future publications with Global Journals up to 60%. Through our recommendation programs, members also receive discounts on publications made with OARS affiliated organizations.

Career

Financial



GJ INTERNAL ACCOUNT

UNLIMITED FORWARD OF EMAILS

Fellows get secure and fast GJ work emails with unlimited forward of emails that they may use them as their primary email. For example, john [AT] globaljournals [DOT] org.

Career

Credibility

Reputation



PREMIUM TOOLS

ACCESS TO ALL THE PREMIUM TOOLS

To take future researches to the zenith, fellows and associates receive access to all the premium tools that Global Journals have to offer along with the partnership with some of the best marketing leading tools out there.

Financial

CONFERENCES & EVENTS

ORGANIZE SEMINAR/CONFERENCE

Fellows are authorized to organize symposium/seminar/conference on behalf of Global Journal Incorporation (USA). They can also participate in the same organized by another institution as representative of Global Journal. In both the cases, it is mandatory for him to discuss with us and obtain our consent. Additionally, they get free research conferences (and others) alerts.

Career

Credibility

Financial

EARLY INVITATIONS

EARLY INVITATIONS TO ALL THE SYMPOSIUMS, SEMINARS, CONFERENCES

All fellows receive the early invitations to all the symposiums, seminars, conferences and webinars hosted by Global Journals in their subject.

Exclusive



PUBLISHING ARTICLES & BOOKS

EARN 60% OF SALES PROCEEDS

Fellows can publish articles (limited) without any fees. Also, they can earn up to 60% of sales proceeds from the sale of reference/review books/literature/publishing of research paper. The FSFRC member can decide its price and we can help in making the right decision.

Exclusive

Financial

REVIEWERS

GET A REMUNERATION OF 15% OF AUTHOR FEES

Fellow members are eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get a remuneration of 15% of author fees, taken from the author of a respective paper.

Financial

ACCESS TO EDITORIAL BOARD

BECOME A MEMBER OF THE EDITORIAL BOARD

Fellows may join as a member of the Editorial Board of Global Journals Incorporation (USA) after successful completion of three years as Fellow and as Peer Reviewer. Additionally, Fellows get a chance to nominate other members for Editorial Board.

Career

Credibility

Exclusive

Reputation

AND MUCH MORE

GET ACCESS TO SCIENTIFIC MUSEUMS AND OBSERVATORIES ACROSS THE GLOBE

All members get access to 5 selected scientific museums and observatories across the globe. All researches published with Global Journals will be kept under deep archival facilities across regions for future protections and disaster recovery. They get 10 GB free secure cloud access for storing research files.

ASSOCIATE OF SCIENCE FRONTIER RESEARCH COUNCIL

ASSOCIATE OF SCIENCE FRONTIER RESEARCH COUNCIL is the membership of Global Journals awarded to individuals that the Open Association of Research Society judges to have made a 'substantial contribution to the improvement of computer science, technology, and electronics engineering.

The primary objective is to recognize the leaders in research and scientific fields of the current era with a global perspective and to create a channel between them and other researchers for better exposure and knowledge sharing. Members are most eminent scientists, engineers, and technologists from all across the world. Associate membership can later be promoted to Fellow Membership. Associates are elected for life through a peer review process on the basis of excellence in the respective domain. There is no limit on the number of new nominations made in any year. Each year, the Open Association of Research Society elect up to 12 new Associate Members.



BENEFITS

TO THE INSTITUTION

GET LETTER OF APPRECIATION

Global Journals sends a letter of appreciation of author to the Dean or CEO of the University or Company of which author is a part, signed by editor in chief or chief author.



EXCLUSIVE NETWORK

GET ACCESS TO A CLOSED NETWORK

A ASFRC member gets access to a closed network of Tier 1 researchers and scientists with direct communication channel through our website. Associates can reach out to other members or researchers directly. They should also be open to reaching out by other.

Career

Credibility

Exclusive

Reputation



CERTIFICATE

RECEIVE A PRINTED COPY OF A CERTIFICATE

Associates receive a printed copy of a certificate signed by our Chief Author that may be used for academic purposes and a personal recommendation letter to the dean of member's university.

Career

Credibility

Exclusive

Reputation



DESIGNATION

GET HONORED TITLE OF MEMBERSHIP

Associates can use the honored title of membership. The "ASFRC" is an honored title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., ASFRC or William Walldroff, M.S., ASFRC.

Career

Credibility

Exclusive

Reputation

RECOGNITION ON THE PLATFORM

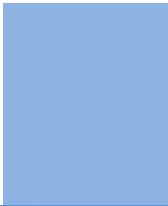
BETTER VISIBILITY AND CITATION

All the Associate members of ASFRC get a badge of "Leading Member of Global Journals" on the Research Community that distinguishes them from others. Additionally, the profile is also partially maintained by our team for better visibility and citation. All associates get a dedicated page on the website with their biography.

Career

Credibility

Reputation

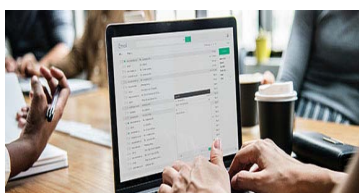


FUTURE WORK

GET DISCOUNTS ON THE FUTURE PUBLICATIONS

Associates receive discounts on the future publications with Global Journals up to 60%. Through our recommendation programs, members also receive discounts on publications made with OARS affiliated organizations.

Career Financial



GJ INTERNAL ACCOUNT

UNLIMITED FORWARD OF EMAILS

Associates get secure and fast GJ work emails with unlimited forward of emails that they may use them as their primary email. For example, john [AT] globaljournals [DOT] org.

Career Credibility Reputation



PREMIUM TOOLS

ACCESS TO ALL THE PREMIUM TOOLS

To take future researches to the zenith, fellows receive access to almost all the premium tools that Global Journals have to offer along with the partnership with some of the best marketing leading tools out there.

Financial

CONFERENCES & EVENTS

ORGANIZE SEMINAR/CONFERENCE

Associates are authorized to organize symposium/seminar/conference on behalf of Global Journal Incorporation (USA). They can also participate in the same organized by another institution as representative of Global Journal. In both the cases, it is mandatory for him to discuss with us and obtain our consent. Additionally, they get free research conferences (and others) alerts.

Career Credibility Financial

EARLY INVITATIONS

EARLY INVITATIONS TO ALL THE SYMPOSIUMS, SEMINARS, CONFERENCES

All associates receive the early invitations to all the symposiums, seminars, conferences and webinars hosted by Global Journals in their subject.

Exclusive





PUBLISHING ARTICLES & BOOKS

EARN 30-40% OF SALES PROCEEDS

Associates can publish articles (limited) without any fees. Also, they can earn up to 30-40% of sales proceeds from the sale of reference/review books/literature/publishing of research paper.

Exclusive

Financial

REVIEWERS

GET A REMUNERATION OF 15% OF AUTHOR FEES

Associate members are eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get a remuneration of 15% of author fees, taken from the author of a respective paper.

Financial

AND MUCH MORE

GET ACCESS TO SCIENTIFIC MUSEUMS AND OBSERVATORIES ACROSS THE GLOBE

All members get access to 2 selected scientific museums and observatories across the globe. All researches published with Global Journals will be kept under deep archival facilities across regions for future protections and disaster recovery. They get 5 GB free secure cloud access for storing research files.



ASSOCIATE	FELLOW	RESEARCH GROUP	BASIC
<p>\$4800 lifetime designation</p> <hr/> <p>Certificate, LoR and Momento 2 discounted publishing/year Gradation of Research 10 research contacts/day 1 GB Cloud Storage GJ Community Access</p>	<p>\$6800 lifetime designation</p> <hr/> <p>Certificate, LoR and Momento Unlimited discounted publishing/year Gradation of Research Unlimited research contacts/day 5 GB Cloud Storage Online Presense Assistance GJ Community Access</p>	<p>\$12500.00 organizational</p> <hr/> <p>Certificates, LoRs and Momentos Unlimited free publishing/year Gradation of Research Unlimited research contacts/day Unlimited Cloud Storage Online Presense Assistance GJ Community Access</p>	<p>APC per article</p> <hr/> <p>GJ Community Access</p>



PREFERRED AUTHOR GUIDELINES

We accept the manuscript submissions in any standard (generic) format.

We typeset manuscripts using advanced typesetting tools like Adobe In Design, CorelDraw, TeXnicCenter, and TeXStudio. We usually recommend authors submit their research using any standard format they are comfortable with, and let Global Journals do the rest.

Alternatively, you can download our basic template from <https://globaljournals.org/Template.zip>

Authors should submit their complete paper/article, including text illustrations, graphics, conclusions, artwork, and tables. Authors who are not able to submit manuscript using the form above can email the manuscript department at submit@globaljournals.org or get in touch with chiefeditor@globaljournals.org if they wish to send the abstract before submission.

BEFORE AND DURING SUBMISSION

Authors must ensure the information provided during the submission of a paper is authentic. Please go through the following checklist before submitting:

1. Authors must go through the complete author guideline and understand and *agree to Global Journals' ethics and code of conduct*, along with author responsibilities.
2. Authors must accept the privacy policy, terms, and conditions of Global Journals.
3. Ensure corresponding author's email address and postal address are accurate and reachable.
4. Manuscript to be submitted must include keywords, an abstract, a paper title, co-author(s) names and details (email address, name, phone number, and institution), figures and illustrations in vector format including appropriate captions, tables, including titles and footnotes, a conclusion, results, acknowledgments and references.
5. Authors should submit paper in a ZIP archive if any supplementary files are required along with the paper.
6. Proper permissions must be acquired for the use of any copyrighted material.
7. Manuscript submitted *must not have been submitted or published elsewhere* and all authors must be aware of the submission.

Declaration of Conflicts of Interest

It is required for authors to declare all financial, institutional, and personal relationships with other individuals and organizations that could influence (bias) their research.

POLICY ON PLAGIARISM

Plagiarism is not acceptable in Global Journals submissions at all.

Plagiarized content will not be considered for publication. We reserve the right to inform authors' institutions about plagiarism detected either before or after publication. If plagiarism is identified, we will follow COPE guidelines:

Authors are solely responsible for all the plagiarism that is found. The author must not fabricate, falsify or plagiarize existing research data. The following, if copied, will be considered plagiarism:

- Words (language)
- Ideas
- Findings
- Writings
- Diagrams
- Graphs
- Illustrations
- Lectures



- Printed material
- Graphic representations
- Computer programs
- Electronic material
- Any other original work

AUTHORSHIP POLICIES

Global Journals follows the definition of authorship set up by the Open Association of Research Society, USA. According to its guidelines, authorship criteria must be based on:

1. Substantial contributions to the conception and acquisition of data, analysis, and interpretation of findings.
2. Drafting the paper and revising it critically regarding important academic content.
3. Final approval of the version of the paper to be published.

Changes in Authorship

The corresponding author should mention the name and complete details of all co-authors during submission and in manuscript. We support addition, rearrangement, manipulation, and deletions in authors list till the early view publication of the journal. We expect that corresponding author will notify all co-authors of submission. We follow COPE guidelines for changes in authorship.

Copyright

During submission of the manuscript, the author is confirming an exclusive license agreement with Global Journals which gives Global Journals the authority to reproduce, reuse, and republish authors' research. We also believe in flexible copyright terms where copyright may remain with authors/employers/institutions as well. Contact your editor after acceptance to choose your copyright policy. You may follow this form for copyright transfers.

Appealing Decisions

Unless specified in the notification, the Editorial Board's decision on publication of the paper is final and cannot be appealed before making the major change in the manuscript.

Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

Declaration of funding sources

Global Journals is in partnership with various universities, laboratories, and other institutions worldwide in the research domain. Authors are requested to disclose their source of funding during every stage of their research, such as making analysis, performing laboratory operations, computing data, and using institutional resources, from writing an article to its submission. This will also help authors to get reimbursements by requesting an open access publication letter from Global Journals and submitting to the respective funding source.

PREPARING YOUR MANUSCRIPT

Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.



Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

- a) A title which should be relevant to the theme of the paper.
- b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
- c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
- d) An introduction, giving fundamental background objectives.
- e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
- f) Results which should be presented concisely by well-designed tables and figures.
- g) Suitable statistical data should also be given.
- h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

- i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
- j) There should be brief acknowledgments.
- k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.



FORMAT STRUCTURE

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.



Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

PREPARATION OF ELETRONIC FIGURES FOR PUBLICATION

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

TIPS FOR WRITING A GOOD QUALITY SCIENCE FRONTIER RESEARCH PAPER

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.



6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.



20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium through which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

INFORMAL GUIDELINES OF RESEARCH PAPER WRITING

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.



Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
- Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.



The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.



Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."



Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

THE ADMINISTRATION RULES

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.



CRITERION FOR GRADING A RESEARCH PAPER (COMPILATION)
BY GLOBAL JOURNALS

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	C-D	E-F
<i>Abstract</i>	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
<i>Introduction</i>	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
<i>Methods and Procedures</i>	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
<i>Result</i>	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
<i>Discussion</i>	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
<i>References</i>	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring



INDEX

A

Ansatz · 7, 8, 9
Arbitrary · 7, 8, 9, 13, 14, 18, 19, 22, 23, 24, 25
Assertion · 25
Asterisk · 18
Asymptotic · 17, 21, 25
Avenues · 15

C

Consumption · 4

E

Emerges · 7

H

Hadrons · 1

I

Instability · 7, 8

L

Lagrange · 18

P

Perturbations · 17
Primordial · 2

T

Tunneling · 3



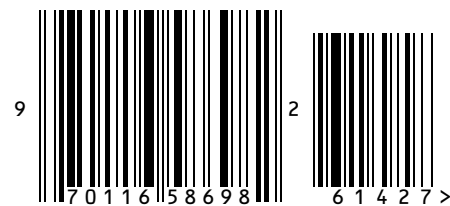
save our planet



Global Journal of Science Frontier Research

Visit us on the Web at www.GlobalJournals.org | www.JournalofScience.org
or email us at helpdesk@globaljournals.org

ISSN 9755896



© Global Journals