A COMPUTER AIDED SYSTEM FOR CAR BODY DESIGN

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ABSTRACT

In order to manufacture the tools for car bodies production, one requires information which can be either replicas of a hand made model and drawings or a list of numbers.

The numerical process can be aimed at translating, as faithfully as possible, a refined version of a hand made "clay model", produced by stylists and plasterers. Another solution is to rely on the styling or drawing offices to provide data describing completely with figures the shape of the body, including inner panels.

Such a system, now widely used in the French car industry, is described.

1. Introduction

For a long time, the shape of the parts manufactured in mechanical industries have been defined with segments of lines and circles.

Many systems have been devised in order to express these forms with data for controlling NC machine tools, and we may consider that this problem is solved.

Before the advent of NC, the surfaces which could not be described with the aforesaid curves had to be copied either with lathes, milling machines or grinders. Hence, it was compulsory to provide those very accurate machines with templates or masters; the camshafts grinders are a good example of such a technique.

But the manufacturing of a master is generally difficult, expensive and time consuming. Consequently, as soon as NC appeared it has been endeavoured to use it in order to solve that problem. Very likely, the first application ever, circa 1942, was for manufacturing prototype camoids. At that time, it was expected that such a solution would, at the same time improve accuracy and cut lagtime.

2. Example

2.1. Problem

One must first observe that the non-analytic surfaces that industry must produce belong to one of the three different types of species:

1) parts playing a basic role in technical phenomena; their shapes have been arrived at by successive approximations. Many examples can be given: propellers, boat hulls, aircraft wings, turbine foils, engine manifolds, etc.

2) parts which only have to fulfill esthetic requirements. The "skin" of the car bodies is one of the most obvious cases, together with pieces of furniture, household implements, glassware, etc.

3) Parts which are not seen, hence have no aesthetical function, the only requirement being of not colliding with other parts,

either moving or steady, or of being somewhat easy to produce, by stamping, forging, casting or machining. Inner panels of a car body belong to that category.

2.1.1. Technical surfaces.

In the first case it is required to translate the shape of an existing part with a very good accuracy. The Coons' method [4,5] is very well adapted to solving this problem, though it has some limitations and may need complementing by the operator's decision.

2.1.2. Aesthetic surfaces.

Directly conceiving the shape of an aesthetic object must evidently be carried on the interactive mode. The operator, stylist or designer must be provided with adequate means ie hardware and software.

Hardware means are widely varied: CRT's give a fast answer, but their dimensions, up to now, are limited to a few decimeters; their screens are not plane and this does not help aesthetic judgement; hence, their use is restricted to the first sketching period, or for the study of tools related with an already defined shape.

The car industry (Fig. 1) used drawing machines 7.5 x 1.8 meters, and larger ones are found in shipyards. Their tracing speed ranges from 300 to 500 mm/s (60 to 100 ft/min) when they are motor driven. Those which are equipped with a magnetic control are much faster, and electrostatic systems give a nearly instant answer.

One should keep in mind that in the actual conception period, a good part of time is devoted to reflection, to choosing or typing data, to comparing and considering results, and a medium speed, say 300 mm/s seems quite sufficient. On the contrary, speed is directly related with the output or efficiency when it comes to tracing.

But it is totally impossible for a stylist to express a final opinion on a three-dimensional shape when it is only defined as a group of projections in a drawing, or even by perspective views. This is why we believe that a styling or drawing office must have the possibility of using a milling machine, fast enough to manufacture a 3D object in a short time. In fact, it can be considered a "3D tracing machine", working soft material such as styrofoam, plaster or resin. Those which are used at Peugeot and Renault (fig. 2) have a capacity of 2.2 x 1.7 x 1.2 meters and their feed is 150 mm/s (30 ft/min).

About the software which is to be used for direct conception, it is imperative that it can be used by operators having a mathematical experience equivalent to that of a designer ie a good knowledge of geometry and very little analysis.

The system known by the name of Unisurf, which is used by Renault and Peugeot [1,2] or some other similar solutions seem to be well adapted to that sort of problems of generating free-form curves and surfaces.

2.1.3. General purpose surfaces.

For the last type of surfaces, which only have to contain a certain number of points, the Coons method can be used but it is not, in its principle, purely algorithmic; besides, it is unable to deal with some special cases eg degenerate patches.

The Riesenfeld's method [6,7] and that which is briefly summarized in Appendix I seem able to give an acceptable answer to that problem, at the cost of an increase of the computer workload.

2.2. Solution.

To give a correct image of a possible use of CAD/CAM, the best is to try and sketch briefly the solution which is employed in a significant part of the French car industry for concieving a car body with the assistance of a computer; the succession of elementary operations is rather different from the traditional one.

The basic principle is to give, as early as possible, a numerical definition of the shape of the various parts, instead of using drawings, templates or 3D models. This does not imply that drawings have become obsolete and useless; in fact their significance is only indicative and not definitive. Obviously, a drawing is the best means for explaining which is the shape of a part, but numbers are the final definition.

2.2.1. Styling period.

In the first stage of the styling work, a few mock-ups are prepared for the management to choose which one will be developed. They are small scale (1/5 or 1/10) simplified versions of a future car. The one which has been selected is then amplified as a full scale so called "clay model", then a prototype till the final choice is made.

Some stylists insist on making them by hand but it is quite possible (fig. 3) to have them milled in a block of styrofoam, polyurethane or clay (fig. 4) though milling of clay, which is a sticky material, is far from easy.

From that moment on, basic information will be carried via numbers.

It is even possible to mill large blocks of foam, bolt them on a central cube and obtain directy a full scale mock-up (fig. 5).

2.2.2. Clay model.

When a small scale mock-up has been selected, the next step is to build a 1/1 clay model.

As a preliminary, a drawing is made on a NC drawing machine, either starting from the numerical definition of the mock-up or from dimensions picked-up on a hand made one.

That drawing can disclose some defect that could not be detected from the mock-up, due mainly to its small scale or to some weakness inevitable with hand made objects.

When the drawing is completed, it only defines the "skin" of the car. The clay model is then divided into a few plaster blocks - hood, top, grill, fenders, doors, trunk - each of which is milled on the "3D drawing machine" ; then they are scraped and assembled on a central cube, the model is lacquered, equipped with panes, windshield, chrome trimmings, headlamps, wheels, until it looks exactly like a real car (fig. 6). Time and again, it is scrutinized, small corrections are made. Each alteration is defined with numbers and when, at last, the model is accepted, the shape of the "skin" is entirely expressed with figures. It is complete, undisputable and it is the basis for the total definition of the body, including inner panels, upholstery, and location of the mechanical parts: engine, transmission, axles, steering mechanisms, etc.

Some stylists still prefer to have the clay model made and corrected by hand. The obvious disadvantage is, first, that it is by no means symmetrical, which is a cause for discussion, and, second that it leaves a lot of work to do before actual drawing begins: measure coordinates of many points with a styling bridge (fig. 7), define character lines and correct the small defects which are inevitable with hand made models. It is evident that the continuity of a line traced with a spline or a french curve can not be, geometrically speaking, as continuous as a mathematical curve.

2.2.3. Final drawing.

Whichever be the method for producing the "clay model", the final drawing is strictly obtained via computer. Every descriptive operation - rotation, scaling, intersection, etc. - is performed by the computer. As soon as a part is properly defined, the relevant data are stored in the memory; this ensures that the shapes so defined are strictly in accordance with that of the clay which has been agreed upon.

If the performing of the manufacturing process - stamping, assembly, lacquering, etc. - requires a slight alteration, the storing of the new data is immediate and the recorded information is permanently kept up to date, which is practically impossible when models and drawings are made by hand.

Numerical definition brings another major advantage: on the one hand, it is extremely easy to define the grids which are necessary to perform finite elements calculations. On the other, it is possible to simulate distortion or buckling due to stress or shocks, or to compute the free vibration frequency for such or such panel, thus helping to reduce the noise level long before any practical experience can be carried on actual parts.

2.2.4. Master.

The numbers which define the shape of a part are used for manufacturing the related tooling. If the master model has not completely become useless, the only reason is that the tool shop is still partly equipped with some copy-milling machine which, unfortuately, are not completely written-off. The parts of the master which are still required are NC milled.

2.2.5. Tooling engineering.

In many respects, CAD plays a major part in tooling engineering: tool design begins with the definition of the different steps of the process. Some work has been carried in view of simulating the behavior of a metal sheet during the stroke of the punch; some results have already been obtained but one must admit that the specialist's experience is still widely used in this respect.

To design a stamping tool, one needs first tip the part in order to facilitate its stamping and then define the surface, nearly developable, on which the blank will be held. Both tasks, evidently, are performed with the help of the computer. For the first one, time can be reduced by 90%. Of course, it is easy to modify the shape of the tool in order to compensate for the springback of the sheet, provided someone is able to predict it.

Regarding the jigs, NC is used to produce the small pads on which stamped parts are clamped for assembling, welding, etc. Previously, their fitting was a difficult, tedious and lengthy task. Now, these pads are put in place, dowelled and screwed as rough from milling.

The rest of the drawing is obtained from a program which defines automatically the walls, ribbing, ejectors, clamps, etc., the standard form of which is stored in the computer memory.

2.2.6. Pattern.

The shape of the tool being totally defined, it is easy to obtain an accurate pattern, including compensation for shrinking and warping.

The allowance for cutting being totally defined, this reduces at the same time the amount of energy required for melting steel or iron, and for removing then the excess of metal on NC milling machines. The far from negligible benefit is two fold, as milling time is reduced too.

2.2.7. Methods.

The active part of a stamping tool being an ensemble of patches, it is the task of Methods to choose which trajectory the cutting tool will follow for milling.

It is admitted that to make the best use of the capacities of the computer, the tool must run along isoparametric lines and not along plane sections. The orientation of the normal, which is needed for tool radius compensation, is real-time computed; consequently, the cutter radius needs not being known beforehand; in the meantime, the same data are used for roughing and finishing cuts, and even for a part as well as for its counterpart, eg a punch and a die.

Owing to the cutting conditions of the tool and to the stiffness of the machine, one has to decide which of the two families of isoparametric lines the cutter must follow: it is better to choose those which are about level rather than steep.

The methods man must define the order in which the patches which constitute the active surface of the tool must be scanned, and the number of roughing and finishing cuts which are required.

Any error in this field can have very harmful consequences. In order to detect them, the trajectory of the tool is simulated on a CRT (fig. 9). Hard copies can be obtained, and one is included in the written information sent to the tool shop.

2.2.8. NC tool milling

When tools were copy-milled, the cuts were generally widely spaced and, due to the limited accuracy of the process, a large amount of metal was left to be removed by hand, with pneumatic chisels, portable grinders, files and-stone. This task was performed by very highly skilled operators. This was a difficult and painstaking work; the man was guided by the bottom of the cuts, then on the reflection of a linear lamp on the surface of the tool. To obtain the ultimate finish, the operator relied on the sensitivity of the palm of his hand to detect the smallest defect on the highly polished surface.

For the hollow part, it was adjusted on the convex one by a process called "die spotting".

On the contrary, NC milled parts have so good an accuracy that hand work is limited to a simple polishing operation, the crests of the cuts being limited to .02 millimeter (fig. 10).

2.2.9. Inspection.

The inspection department is in charge of:

1) checking the shape of each tool before it is delivered to the production shop.

2) checking parts going out of the production line, or when they are delivered by the subcontractors.

The tools are inspected whilst they are still on the table of the NC milling machine. The cutting tool is replaced with an electronic sensing device, and the co-ordinates of a certain number of predetermined points are measured. It is a very fast work, and the obvious advantage of that practice is that there is no time wasted in locating correctly the part on a measuring machine, or back on the milling machine in case a retouch is needed, which is in fact seldom required, due to the accuracy of the milling operation.

For the inspection of stamped parts, templates have been replaced by 3D frames bearing surfaces milled 2 millimeters apart from the correct shape, the conformity of which is controlled by a wire gauge inserted between the part and the frame (fig. 11). The process is much more effective than the use of thin templates.

3. Conclusion.

In the sequence of operations ranging from the production of styling small scale mock-ups till the assembly of finished parts, CAD and CAM play their part from end to end.

The system, in the French car industry, has been operative for eight years and, since 1977, it is considered irreplaceable; it is even now used for defining some mechanical parts such as manifolds, crankshafts, con'rods, cylinder heads, levers, steering knuckles, etc.

It is now granted that, at the end of a learning period which does not last more than a few months, one can learn how to use CAD.

CAD/CAM bring undisputable benefits in lagtime, and cost which are sometimes above 50%. Moreover, accuracy is significantly improved.

Besides, some difficult and exhausting tasks have disappeared or become much easier to perform, such as the tracing of very large drawings or the die- spotting of stamping tools.

To state it briefly, objectivity has replaced subjectivity. Neither technician nor scientist would doubt that it is a sure and desirable progress.

4. Bibliography.

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APPENDIX I
SEMI-ALGORITHMIC SYSTEM
FOR DEFINING FREE-FORM CURVES AND SURFACES

1. INTRODUCTION.

The system described hereafter is aimed at: obtaining an algorithm defining piecewise curves and surfaces made of segments or patches connecting points previously located in space. Theoretically, these segments or patches blend up to any order.

Expressing the solution in such a way that an operator can modify it at his will till he obtains a convenient shape (slope, curvature, twist, etc.) in case it is not correctly represented by the pre-defined points.

In the present work, it has been made use of some basic properties described in item [2] of the bibliography attached to the principal text. In the appendix, a number between character ⸨ ⸩ indicates the specific paragraph of [2] to which it refers.

2. DESCRIPTION.

2.1. Curves

2.1.1 Tangential blending

Let P be a collection of points (Fig. 1) that a continuous piecewise curve must contain.

These points P_{i-2}, ..., P_{i+2} define vectors such that:

$$\left.\begin{aligned} \bar{a}_{i-1} &= \bar{P}_{i-1} - \bar{P}_{i-2} \\ &\cdots\cdots\cdot \\ \bar{a}_{i+2} &= \bar{P}_{i+2} - \bar{P}_{i+1} \end{aligned}\right\} \qquad (1)$$

Each point P is related with a parameter "u", the value of which is:

$$u_k = \frac{\sum_{j=i-1}^{k} |\bar{a}_j|}{\sum_{j=i-2}^{i+2} |\bar{a}_j|} \qquad (2)$$

Consequently:

$$u_{i-2} = 0 \quad \text{and} \quad u_{i+2} = 1 \tag{3}$$

A curve containing points $P_{i-2}, \ldots, P_{i+2}$ is expressed by:

$$\overline{P}(u) = \sum_{k=0}^{4} \overline{b}_k \cdot u^k \qquad u \in [0, 1] \tag{4}$$

and the five "b_k";s are the roots of a set of five linear equations.

From (4) we deduce:

$$\frac{d\overline{P}}{du} = \sum_{k=1}^{4} \overline{b}_k \cdot k \cdot u^{k-1} \tag{5}$$

Let us now consider a segment of this curve limited by points P_i and P_{i+1}; its points Q are:

$$\overline{Q}(v) = \sum_{k=0}^{4} \overline{b}_k \cdot [u_i + (u_{i+1} - u_i)v]^k \qquad v \in [0,1] \tag{6}$$

From (4), (5) and (6), we deduce:

$$\overline{\frac{dQ}{dv}} = (u_{i+1} - ui) \cdot \sum_{k=1}^{4} \overline{b}_k \cdot k \cdot [u_i + (u_{i+1} - u_i)v]^{k-1}$$

and, in particular:

$$\overline{Q}(0) = \overline{P}_i = \overline{P}(u_i)$$

$$\overline{\frac{dQ}{dv}}(0) = (u_{i+1} - u_i) \cdot \sum_{k=1}^{4} \overline{b}_k \cdot k \cdot u_i^{k-1} = (u_{i+1} - u_i) \cdot \overline{\frac{dP}{du}}(u_i) \tag{7}$$

Considering, in the same manner, a segment of the same curve, limited by P_{i-1} and P_i, the points of which are R(t), one finds:

$$\overline{R}(1) = \overline{P}(u_i) = \overline{P}_i$$

and

$$\overline{\frac{dR}{dt}} = (u_i - u_{i-1}) \cdot \overline{\frac{dP}{du}}(u_i) \tag{8}$$

Hence, the vectors $\frac{dQ}{dv}$ and $\frac{dR}{dt}$ are collinear, and

$$\overline{\frac{dR}{dt}}(1) = \overline{\frac{dQ}{dv}}(0) \cdot \frac{u_i - u_{i-1}}{u_{i+1} - u_i} \tag{9}$$

From points P_{i-1}, ..., P_{i+3}, we define a curve a segment of which, limited by P_i and P_{i+1}, is the locus of points S(w); that segment does not coincide with the locus of Q(v), but it does not differ much from it.

The point S(1) coincides with P_{i+1}, and its first derivative is obtained by applying a formula similar to (8).

So, between P_i and P_{i+1}, we shall define a segment, locus of T(z), such that:

$$\left.\begin{aligned} \overline{T}(0) &= \overline{P}_i = \overline{Q}(0) \\ \overline{T}(1) &= \overline{P}_{i+1} = \overline{S}(1) \\ \overline{\frac{dT}{dz}}(0) &= \overline{\frac{dQ}{dv}}(0) \\ \overline{\frac{dT}{dz}}(1) &= \overline{\frac{dS}{dw}}(1) \end{aligned}\right\} \tag{10}$$

The tangents in T(0) and T(1) are respectively collinear with those of segments defined in the same manner between P_{i-1} and P_i as well as P_{i+1} and P_{i+2}.

As a special case, the segments comprised between P_0 and P_2 are parts of the curve containing P_0 .... P_4. In the same manner, between P_{n-2} and P_n, the segments are part of the curve running through P_{n-4} .... P_n.

If P_0 and P_n coincide, the curve is a loop and the conditions in P_0 are deduced from that of the curve running through P_{n-2} ... P_2.

2.1.2 Osculatory blending

From (6) we deduce:

$$\overline{\frac{d^2Q}{dv^2}} = (u_{i+1} - u_i)^2 \cdot \sum_{k=2}^{4} \overline{b}_k \cdot k(k-1) \left[u_i + (u_{i+1} - u_i)v\right]^{k-2} \tag{11}$$

and

$$\overline{\frac{d^2Q}{dv^2}}(0) = (u_{i+1} - u_i)^2 \cdot \sum_{k=2}^{4} b_k \cdot k(k-1) \cdot u_i^{k-2} \qquad (12)$$

and, in the same manner

$$\frac{d^2R}{dt^2}(1) = (u_i - u_{i-1})^2 \cdot \sum_{k=2}^{4} \overline{b}_k \cdot k(k-1) \cdot u_i^{k-2} \qquad (13)$$

In the point Q(0), the vectors $\frac{dQ}{dv}(0)$ and $\frac{d^2Q}{dv^2}$ define the center of curvature, in P_i, of the curve running from P_{i-2} to P_{i+2}. The same applies to point R(1).

A segment $\overset{\frown}{P_iP_{i+1}}$, defined by:

$$\left.\begin{matrix} \overline{Q}(0) & \overline{T}(1) \\ \overline{\frac{dQ}{dv}}(0) & \overline{\frac{dT}{dz}}(1) \\ \overline{\frac{d^2Q}{dv^2}}(0) & \overline{\frac{d^2T}{dz^2}}(1) \end{matrix}\right\} \qquad (14)$$

is of the fifth order and osculates the adjacent segments defined in the same manner.

One should observe that the curve $\overset{\frown}{P_{i-2}P_{i+2}}$, which is of the fourth order, yields the end conditions for segments $\overset{\frown}{P_{i-1}P_i}$ and $\overset{\frown}{P_iP_{i+1}}$ which are either of third or of fifth order, according to the level of continuity sought with the adjacent segments.

Let us not forget, too, that tangency does not require that the derivative be identical since collinearity is sufficient.

Same applies to osculation. Identity of first and second derivatives is sufficient, but not compulsory (2.§ 2.4.1.1. and § 2.4.1.2.).

2.1.3 Higher order blending

From (6), we can deduce the values of $\frac{d^3Q}{dv^3}(0)$ and $\frac{d^3R}{dt^3}(1)$, which would define segments such as $\overset{\frown}{P_iP_{i+1}}$ having a continuity of the fourth order. In such a case, it would probably be better to define first a curve of order six running through points $P_{i-3}, \ldots. P_{i+3}$.

The maximal order of continuity is only limited by the capacity of the computer.

2.1.4 Definition by vectors

To alter the shape of a curve, it is useful to make use of the characteristic polygon (2.§2.2.1.1.2).

For a curve of the third order, we have (Fig. 2)

$$\left.\begin{aligned} \overline{a}_1 &= \overline{Q(0)M} = \frac{1}{3} \cdot \overline{\frac{dQ}{dv}}(0) \\ \overline{a}_3 &= \overline{NQ(1)} = \frac{1}{3} \cdot \overline{\frac{dQ}{dv}}(1) \end{aligned}\right\} \quad (15)$$

and, in case it is of the fifth order (Fig. 3):

$$\left.\begin{aligned} \overline{a}_1 &= \overline{Q(0)M} = \frac{1}{5} \cdot \overline{\frac{dQ}{dv}}(0) \\ \overline{a}_2 &= \overline{MN} = \overline{a}_1 + \frac{1}{20} \cdot \overline{\frac{d^2Q}{dv^2}}(0) \\ \overline{a}_4 &= \overline{RS} = \overline{a}_5 - \frac{1}{20} \cdot \overline{\frac{d^2Q}{dv^2}}(1) \\ \overline{a}_5 &= \overline{SQ(1)} = \frac{1}{5} \cdot \overline{\frac{dQ}{dv}}(1) \end{aligned}\right\} \quad (16)$$

2.1.5 Arbitrary modification

A curve segment being previously defined, the operator can alter its shape without changing the end conditions.

If he wants to keep the slope, he only has to change the magnitude of vectors a_1 or a_n. To keep the curvature, after moving point M to M' (Fig. 4), one needs only locate N' on a line parallel to a_1, which is obtained as shown in fig. 4 (2 §2.2.1.2.1.3).

2.2 Surfaces

By similarity with curves, one must first determine, in each given point, the value that could be assigned to the parameters.

2.2.1 Computation of parameters

One surface being expressed by:

$$\bar{P}(u, v) = \sum_{i=0}^{m} \sum_{i=0}^{n} \bar{b}_{ij} \cdot u^i v^j \qquad (17)$$

let P be a collection of points through which we want to run a piecewise surface. Those points are the apices of a network

Among them let us select twenty five points such as:

$$\begin{matrix} P_{i-2,\ j-2} & \cdots\cdots\cdots & P_{i+2,\ j-2} \\ \vdots & & \vdots \\ P_{i-2,\ j+2} & \cdots\cdots\cdots & P_{i+2,\ j+2} \end{matrix} \qquad (18)$$

Vectors a's and α's are defined by

$$\left.\begin{aligned} \bar{a}_{k,\ell} &= \bar{P}_{k,\ell} - \bar{P}_{k-1,\ell} \\ \bar{\alpha}_{k,\ell} &= \bar{P}_{k,\ell} - \bar{P}_{k,\ell-1} \end{aligned}\right) \qquad (19)$$

with:

$$k \in [i-2,\ i+2],\ \ell \in [j-2,\ j+2]$$

The values of "u" are next computed by

$$u_{k,j} = \frac{\sum_{m=i-1}^{k} \left|\bar{a}_{m,j}\right|}{\sum_{m=i-1}^{i+2} \left|\bar{a}_{m,j}\right|} \quad \text{and} \quad \frac{\sum_{n=j-1}^{\ell} \left|\bar{\alpha}_{i,n}\right|}{\sum_{n=j-1}^{j+2} \left|\bar{\alpha}_{i,n}\right|} = v_{i,\ell} \qquad (20)$$

2.2.2 Blending conditions

2.2.2.1 Average parameter method

The simplest method, provided the shape of the surface is not too bumpy and when the given points are properly scattered, is to assign the same value to the u_{ij}'s having the same subscript "i". Same applies to the $v_{i,j}$'s having the same subscript "j". Consequently:

$$u_{i_0,j} = \frac{\sum_{k=i-2}^{i+2} u_{k,j}}{5} \quad \text{and} \quad v_{i,j_0} = \frac{\sum_{k=j-2}^{j+2} v_{i,k}}{5} \qquad (21)$$

and the twenty five coefficients $b_{i,j}$'s of (17) are the roots of a system of linear equations.

2.2.2.1.1 Tangential blending

The sub-patch having points $P_{i,j}$, $P_{i+1,j}$, $P_{i,j+1}$, $P_{i+1,j+1}$ for apices is defined by (Fig. 5):

$$\bar{M}(w,z) = \sum_{k=i-2}^{i+2} \sum_{\ell=j-2}^{j+2} \bar{b}_{k\ell} \cdot [u_{ij} + (u_{i,j+1} - u_{ij})w]^k \cdot [v_{ij} + (v_{i,j+1} - v_{ij})z]^\ell \qquad (22)$$

from what we deduce:

$$\overline{\frac{\delta M}{\delta w}} = \sum_{k=1-2}^{i+2} \sum_{\ell=j-2}^{j+2} \bar{b}_{k\ell} k(u_{i+1} - u_i) [u_i + (u_{i+1} - u_i)w]^{k-1} \cdot [v_j + (v_{j+1} - v_j)z]^\ell$$

$$\overline{\frac{\delta M}{\delta z}} = \sum_{k=i-2}^{i+2} \sum_{\ell=j-2}^{j+2} \bar{b}_{k\ell} \cdot \ell\, (v_{j+1} - v_j)[u_i + (u_{i+1} - u_i)w]^k [v_j + (v_{j+1} - v_j)z]^{\ell-1}$$

$$\overline{\frac{\delta^2 M}{\delta w \cdot \delta z}} = \sum_{k=i-2}^{i+2} \sum_{\ell=i-2}^{j+2} \bar{b}_{k\ell} \cdot k\ell \cdot (u_{i+1} - u_i) \cdot (v_{j+1} - v_j) \cdot [u_i + (u_{i+1} - u_i)w]^{k-1} \cdot [v_j + (v_{j+1} - v_j)z]^{\ell-1} \qquad (23)$$

The end conditions at M(0, 0), deduced from (17) and (23), are:

$$\bar{M}\,(0,0) = \bar{P}_{ij}$$

$$\overline{\frac{\delta M}{dw}}\,(0,0) = (u_{i+1} - u_i) \cdot \overline{\frac{\delta P}{\delta u}}\,(u_i,\ v_j)$$

$$\overline{\frac{\delta M}{\delta z}}\,(0,0) = (v_{j+1} - v_j)\, \overline{\frac{\delta P}{\delta v}}\,(u_i,\ v_j) \qquad (24)$$

$$\overline{\frac{\delta^2 M}{\delta w \times \delta z}}\,(0,0) = (u_{i+1} - u_i) \cdot (v_{j+1} - v_j)\, \overline{\frac{d^2 P}{\delta u \cdot \delta v}}\,(u_i,\ v_j)$$

In the same manner, for sub-patches having for apices the points:

$$\begin{matrix} P_{i-1,j}, & P_{ij}, & P_{i-1,j+1} & \text{and } P_{i,j+1} \\ P_{i-1,j-1}, & P_{i,j-1}, & P_{i-1,j} & \text{and } P_{i,j} \\ P_{i,j-1}, & P_{i+1,j-1}, & P_{i,j} & \text{and } P_{i+1,j} \end{matrix} \quad (25)$$

and the points of which are N, Q and R, one finds in the same manner:

$$\left.\begin{aligned} \overline{N}(1,0) &= \overline{P}(u_i, v_j) \\ \overline{\frac{\delta N}{\delta w}}(1,0) &= (u_i - u_{i-1}) \cdot \overline{\frac{\delta P}{\delta u}}(u_i, v_j) \\ \overline{\frac{\delta N}{\delta t}}(1,0) &= (v_{j+1} - v_j) \cdot \overline{\frac{\delta P}{\delta t}}(u_i, v_j) \\ \overline{\frac{\delta^2 N}{\delta w \cdot \delta t}} &= (u_i - u_{i-1}) \cdot (v_{j+1} - v_j) \overline{\frac{\delta^2 P}{\delta u \cdot \delta v}}(u_i, v_j) \end{aligned}\right\} \quad (26)$$

and similar expressions would yield the values related with points Q(1,1) and R(0,1).

It is evident that the values related with M(0,0), N(1,0), Q(1,1) and R(0,1) express the same geometric conditions such as slope, curvature and twist since the four sub-patches are parts of the same patch having $P_{i-2,j-2}, \dots P_{i+2,j+2}$ for apices.

A bicubic patch having $P_{i,j}$, $P_{i+1,j}$, $P_{i,j+1}$ and $P_{i+1,j+1}$ for apices is defined by end conditions such as those expressed in (24), obtained from patches having respectively for apices:

$$\begin{matrix} P_{i-2,j-2}, & P_{i+2,j-2}, & P_{i-2,j+2}, & P_{i+2,j+2} \\ P_{i-1,j+2}, & P_{i+3,j-2}, & P_{i-1,j+2}, & P_{i+3,j+2} \\ P_{i-2,j-1}, & P_{i+2,j-1}, & P_{i-2,j+3}, & P_{i+2,j+3} \\ P_{i-1,j-1}, & P_{i+3,j-1}, & P_{i-1,j+3}, & P_{i+3,j+3} \end{matrix}$$

Consequently, the sub-patch having $P_{i,j}$, $P_{i+1,j}$, $P_{i,j+1}$, $P_{i+1,j+1}$ for apices blends up to the second order with the eight patches having either one or two apices in common with it.

2.2.2.1.2 Osculatory blending

A bi-quartic patch being defined as in (2.2.1.), the conditions in the middle point $P_{i,j}$ are:

$$
\left.
\begin{array}{c}
\overline{P}(i,j) \\[2ex]
\overline{\frac{\delta P}{\delta u}}(i,j), \quad \overline{\frac{\delta P}{\delta v}}(i,j) \\[2ex]
\overline{\frac{\delta^2 P}{\delta u^2}}(i,j), \quad \overline{\frac{\delta^2 P}{\delta u \cdot \delta v}}(i,j), \quad \overline{\frac{\delta^2 P}{\delta v^2}}(i,j) \\[2ex]
\overline{\frac{\delta^3 P}{\delta u^2 \delta v}}(i,j), \quad \overline{\frac{\delta^3 P}{\delta u \delta v^2}}(i,j) \\[2ex]
\overline{\frac{\delta^4 P}{\delta u^2 \cdot \delta v^2}}(i,j)
\end{array}
\right\} \qquad (27)
$$

Consequently the thirty-six coefficients are available in order to define a bi-quintic sub-patch which osculates the adjacent ones.

2.2.2.1.3 Higher order blending

To obtain a higher order blending, one defines first a patch of degree (5 × 5), the apexes of which have subscripts i-3, i+3, j-3, j+3. From them are deduced sixteen conditions ranging from $P_{i,j}$ up to $\delta^6 P/\delta^3 u \cdot \delta^3 v$. Each bi-quintic sub-patch is then defined by sixty-four coefficients.

2.2.2.1.4 Definition by vectors

It is useful to have the characteristic network of a surface, especially in case one needs to alter it.

From the end conditions, the vector coefficients of the complete expression of the surface are deduced. The apices of the network are then obtained with the help of a well known matrix (2.§2.2.2.2.3).

In the special case of bi-cubic patches, there is an immediate solution: P(0,0), A, C and D being the apices (Fig. 6) of the mesh related to P(0,0),

$$
\left. \overline{PA} = \frac{1}{3} \cdot \overline{\frac{\delta P}{\delta u}}(0,0) \right\} \qquad (28)
$$

$$\overline{PB} = \frac{1}{3} \cdot \overline{\frac{\delta P}{\delta v}} (0,0)$$

and, PABD being a parallelogram,

$$\overline{DC} = \frac{1}{9} \cdot \overline{\frac{\delta^2 P}{\delta u \cdot \delta v}}$$

2.2.2.1.5 Arbitrary modification

Let be a collection of bicubic patches, blending together, the apices of which are points P_{ij}.

It is known ⟬ 2§ 2.4.2.3. ⟭ that in an apex belonging to four patches (Fig. 7), the limits of the four meshes of the characteristic networks are the generatrices of a hyperbolic paraboloid, and that those meeting in $P_{i,j}$, i.e. BF and DH, divide the others by a ratio

$$\frac{u_i - u_{i-1}}{u_{i+1} - u_i} \text{ and } \frac{v_j - v_{j-1}}{v_{j+1} - v_j} \qquad (29)$$

Hence, it is possible to displace points such as P, A, ····, H, provided condition (29) is complied with.

2.2.2.2 Order raising method

If the shape of the surface and the distribution of the predetermined points is such that averaging the parameters is not acceptable, the previous solution is not valid.

It is known that ⟬ 2 §2.4.1.2.1 ⟭ if two bicubic patches are tangent along a line $\widehat{P(0)P(1)}$ (Fig. 8), the derivatives $\frac{\delta M}{\delta v}(u,1)$ and $\frac{\delta N}{\delta w}(u,0)$ can generally be collinear only if at P(0) and P(1) the derivatives are collinear and if, at the same time:

$$\frac{\left|\overline{\frac{\delta M}{dv}}(0,1)\right|}{\left|\overline{\frac{\delta N}{\delta w}}(0,0)\right|} = \frac{\left|\overline{\frac{\delta M}{\delta v}}(1,1)\right|}{\left|\overline{\frac{\delta N}{\delta w}}(1,0)\right|}$$

2.2.2.2.1 Definition of auxiliary directrices

The basic problem is (Fig. 9) to obtain that along the line $\widehat{P_{ij}P_{i+1,j}}$, the patches be tangent. Let them be A and B, their respective points being M(u,v) and N(u,w); in each apex, the values of P, $\delta P/\delta u$, $\delta P/\delta v$ and $\delta^2 P/\delta u \cdot \delta v$ are defined as in 2.2.2.1.1.

In the general case,

$$\frac{u_{i,j+1} - u_{i,j}}{u_{i,j} - u_{i,j+1}} \quad \frac{u_{i+1,j+1} - u_{i+1,j}}{u_{i+1,j} - u_{i+1,j+1}} \tag{30}$$

hence, the collinearity of $\frac{\delta M}{\delta v}$ (u,1) and $\frac{\delta N}{\delta w}$ (u,0) is not automatically granted, except in points $P_{i,j}$ and $P_{i+1,j}$ where u = 0 or 1.

Patch A, locus of M, being bi-cubic, the characteristic polygon of $\overset{\frown}{P_{i,j}P_{i+1,j}}$, which coincides with $\overset{\frown}{M(0,1)M(1,1)}$ and $\overset{\frown}{N(0,0)N(1,0)}$ and that of the first auxiliary directrix $\overset{\frown}{S(0)S(1)}$, (Fig. 10) have three vectors.

For simplicity, let be:

$$\frac{v_{i,j+1} - v_{i,j}}{v_{i,j} - v_{i,j-1}} = \lambda_0 \text{ and } \frac{v_{i+1,j+1} - v_{i+1,j}}{v_{i+1,j} - v_{i+1,j-1}} = \lambda_1 \tag{31}$$

In patch B, locus of N, let us define the first auxiliary directrix, locus of T, by:

$$\overline{T}(u) = \overline{N}(u,0) + \frac{1}{3}\,\overline{\frac{\delta M}{\delta v}}(u,1)\left[\lambda_0 + (\lambda_1 - \lambda_0)\cdot(-2u^3 + 3u^2)\right] \tag{32}$$

The locus of T is obviously of degree six with respect to u, and its characteristic polygon contains six vectors. In order to define the network of the modified patch, we must describe the other directrices, principal or auxiliary, with six-sided polygons [2.§ 2.3.1.3] though they remain actually of degree three.

In expression (32), the quantity between square brackets varies from λ_0 to λ_1 when u varies from 0 to 1. Besides, its first derivative is nil when u = 0 or 1.

Let us observe that patch B is now of degree six relative to u, and three relative to v.

Let us call B' the patch after it has been modified, and N'(u,w) its points.

It is known that

$$\overline{\frac{\delta N'}{\delta w}}(u,0) = 3[\overline{T}(u) - \overline{N}'(u,0)] \tag{33}$$

but:

$$\overline{N}'(u,0) = \overline{N}(u,0) \tag{34}$$

consequently

$$\overline{\frac{\delta N'}{\delta w}}\ (0,0) = 3\ [T(0) - N(0,0)] = \overline{\frac{\delta M}{\delta v}}\ (0,i) \cdot \lambda_0 = \overline{\frac{\delta N}{\delta w}}\ (0,0) \tag{35}$$

consequently, in the same manner:

$$\overline{\frac{\delta N}{\delta w}}\ (u,0) = \overline{\frac{\delta M}{\delta v}}\ (u,1)[\lambda_0 + (\lambda_1 - \lambda_0) \cdot (-2u^3 + 3u^2)] \tag{36}$$

In the same manner, taking the derivative of (36) with respect to u:

$$\left.\begin{aligned} \overline{\frac{\delta^2 N'}{\delta u \cdot \delta w}}\ (u,0) &= \overline{\frac{\delta^2 M}{\delta u \cdot \delta v}}\ (u,1)[\lambda_0 + (\lambda_1 - \lambda_0) \cdot (-2u^3 + 3u^2)] + \\ &+ \overline{\frac{\delta M}{\delta v}}\ (u,1) \cdot \frac{d[\lambda_0 + (\lambda_1 - \lambda_0)(-2u^3 + 3u^2)]}{du} \end{aligned}\right\} \tag{37}$$

but

$$\frac{d(-2u^3 + 3u^2)}{du} = 6u \cdot (1-u) \tag{38}$$

and this quantity is nil when u = 0 or 1, consequently:

$$\left.\begin{aligned} \overline{\frac{\delta^2 N'}{\delta u \cdot \delta w}}\ (0,0) &= \lambda_0 \cdot \overline{\frac{\delta^2 M}{\delta u \cdot \delta v}}\ (0,1) = \overline{\frac{\delta^2 N}{\delta u \cdot \delta w}}\ (0,0) \\ \overline{\frac{\delta^2 N'}{\delta u \cdot \delta w}}\ (1,0) &= \lambda_1 \cdot \overline{\frac{\delta^2 M}{\delta u \cdot \delta v}}\ (1,1) = \overline{\frac{\delta^2 N}{\delta u \cdot \delta w}}\ (1,0) \end{aligned}\right\} \tag{39}$$

So, the conditions existing in N'(0,0) and N'(1,0) are identical to those which would exist if patch B had not been modified.

2.2.2.2 General Blending

When a bi-cubic patch has been blended with an adjacent bi-cubic path, its auxiliary directrix locus of T (Fig. 10) becomes a curve of degree six, and the patch has become a (6 × 3) patch.

So, we must ensure that, from patch to patch, the degree is not increased up to an unmanageable value.

Let us consider (Fig. 11) the characteristic network of a bi-cubic patch; the polygons of limits AB, BD, DC and CA have three sides, and so have those of the auxilary directrices EF and GH.

After blending with the patch which is adjacent along AB, the auxiliary directrix EF has a degree six.

In the same manner after blending with the patch which is adjacent along CD, the characteristic polygons of EF and GH have six sides.

Raising artificially the degree of AB and CD up the sixth order, the characteristic network (Fig. 12) becomes of degree 6 × 3.

After blending with the patches which are adjacent along AC and BD, the characteristic polygons of the auxiliary directrices IJ and HL become too of degree six; the network of the patch is then of degree 6 × 6, but the patches which blend along it limits remain of degree 3 × 3.

Consquently the patches are alternately of degree 3 × 3 and 6 × 6, and the alternations described in 2.2.2.1.5 are still possible.

3. CONCLUSION

The process which has been described is not intended to deal with the problems already solved with systems such as Coons', Riesenfeld's or Unisurf, to name only a few.

Its field is restricted to surfaces which must contain predetermined points, but play no part whatsoever in aesthetics.

The process having yielded a solution, it is easy, for the operator, to improve the result, either changing the location of some points or altering the successive derivatives, provided some simple rules are complied with.

Pierre BEZIER
03.02.1982

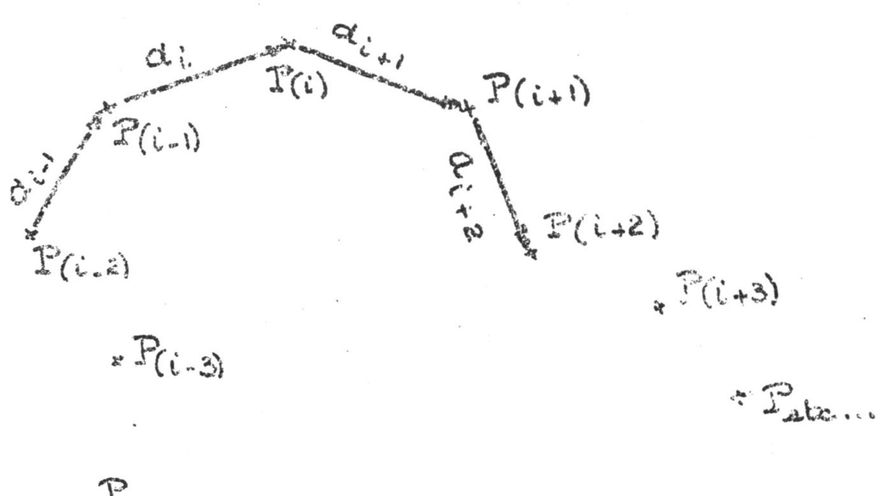


Fig. 1

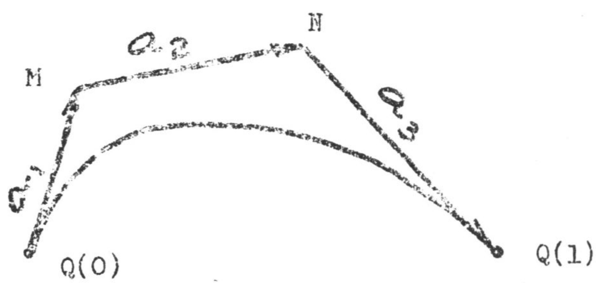


Fig. 2

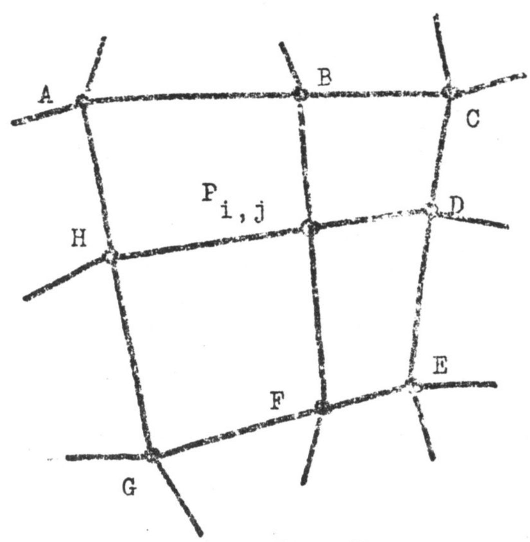
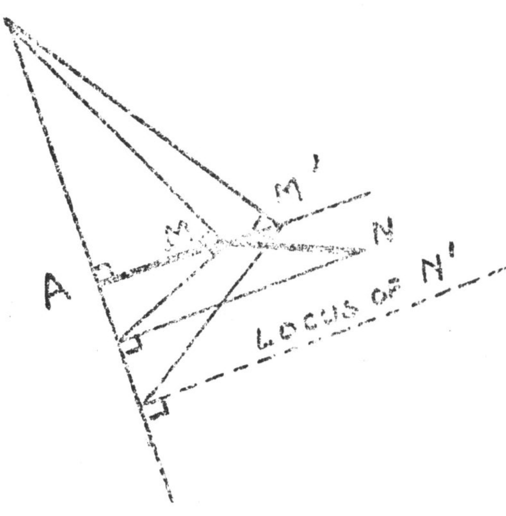


Fig. 7



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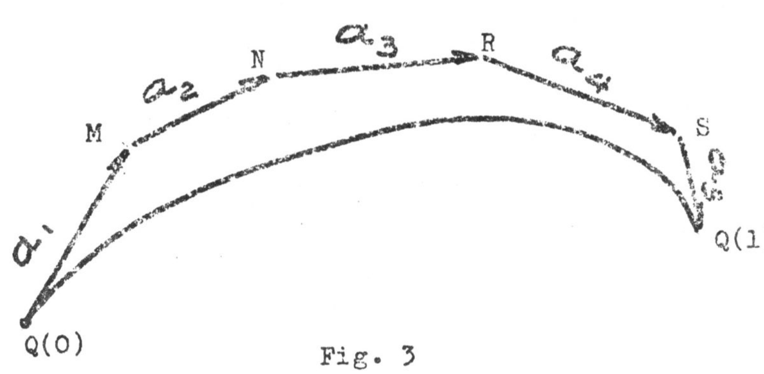

Fig. 3

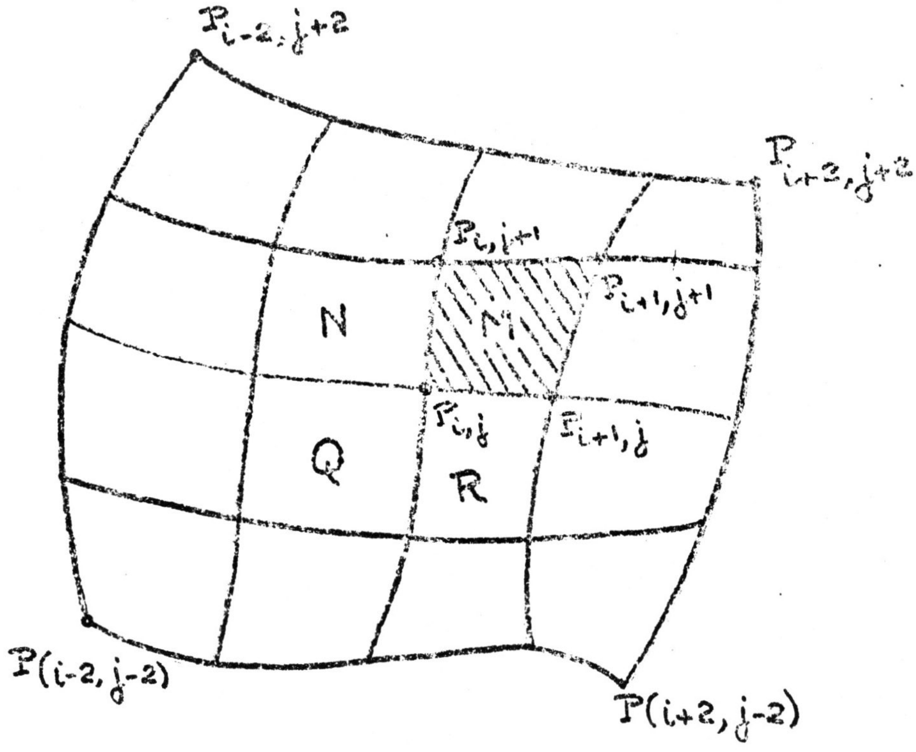

Fig. 5

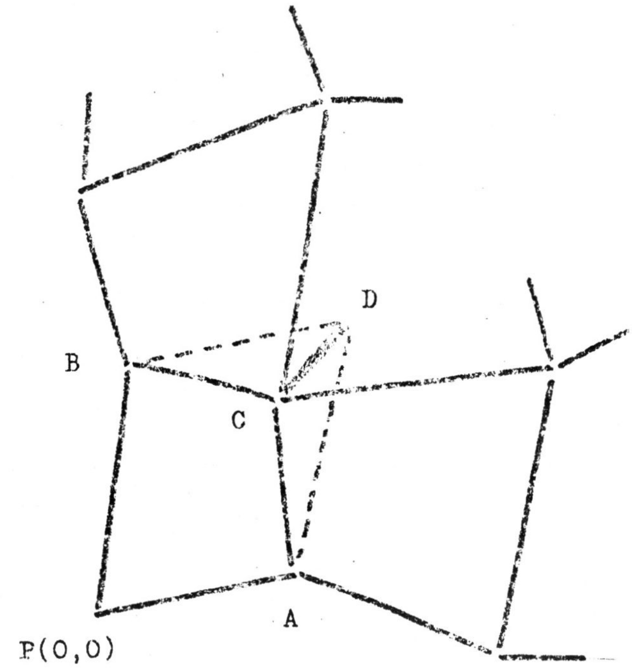

Fig. 6

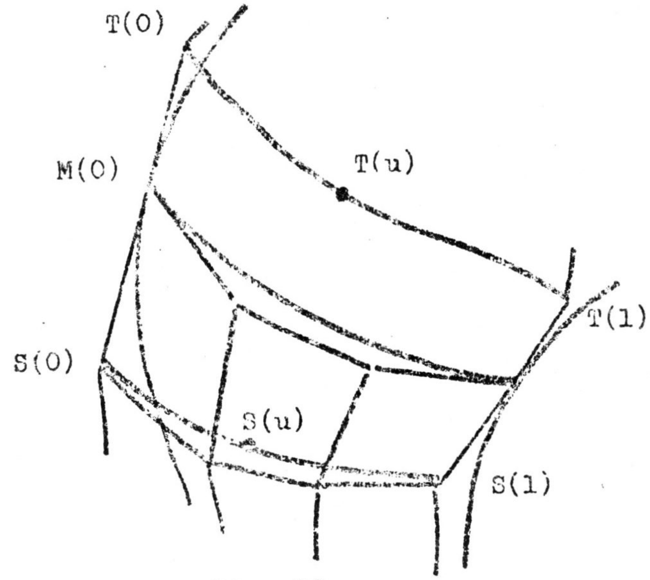

Fig. 10

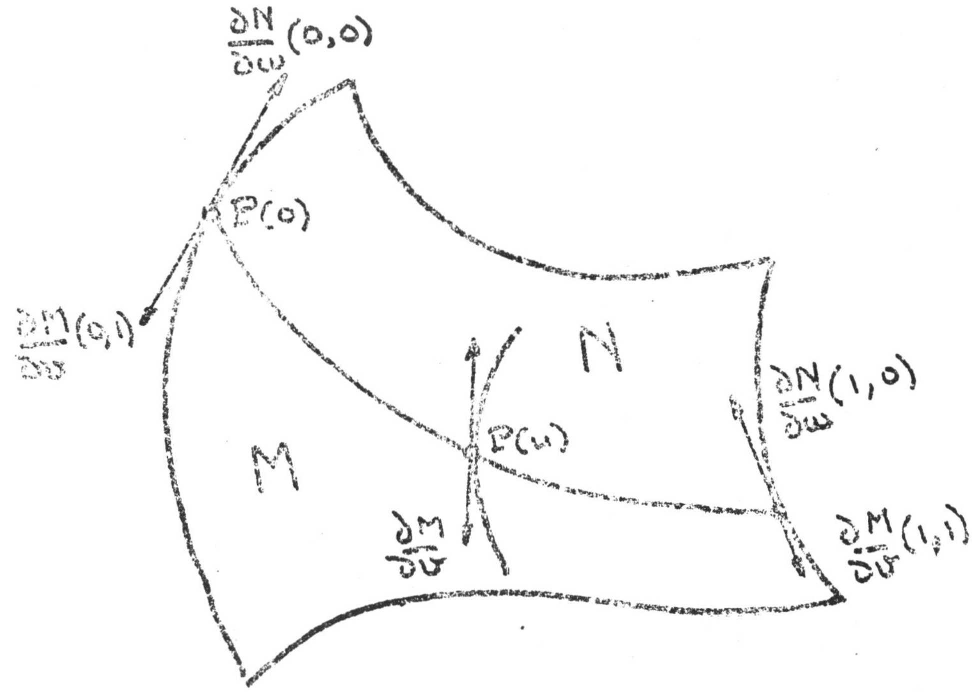
∂N/∂w (0,0)
P(0)
∂M/∂w (0,1)
N
∂N/∂w (1,0)
P(u)
M
∂M/∂w
∂M/∂w (1,1)

Fig. 8

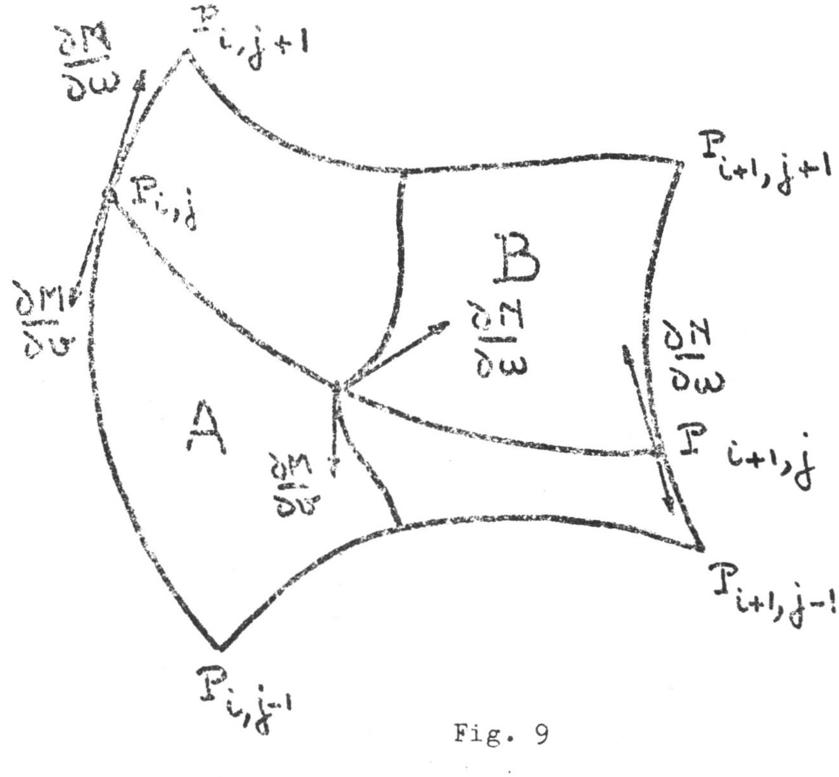
∂M/∂w
P i,j+1
P i+1,j+1
P i,j
B
∂M/∂w
∂N/∂w
∂N/∂w
A
P i+1,j
∂M/∂w
P i+1,j-1
P i,j-1

Fig. 9

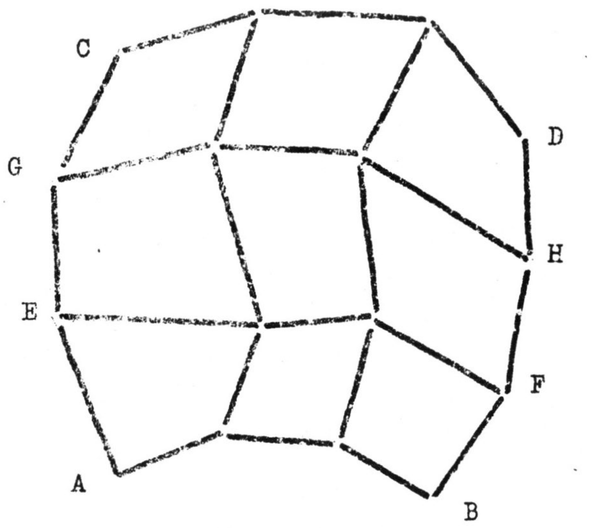

Fig. 11

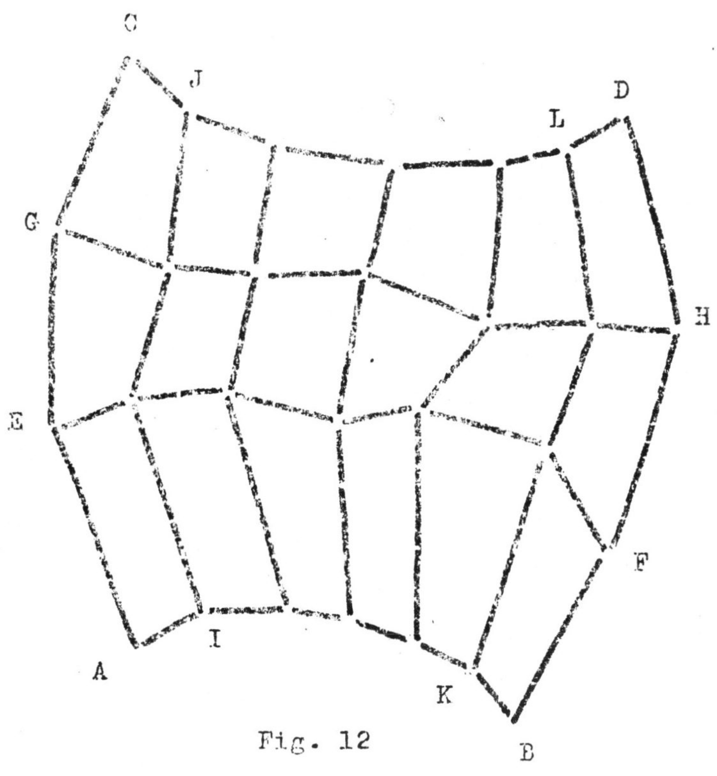

Fig. 12