HOLOGRAM-LIKE TRANSMISSION OF PICTURES

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ABSTRACT

A new digital representation of pictures is proposed. The main feature of this representation is: Given a string of data Z representing a picture with full resolution, various substrings of Z represent the same picture with appropriately lower resolution. This is analogous to a well-known property of holograms. The paper presents the principle of the new representation (based on a particular picture traversal algorithm), analyzes its overhead, and provides examples of picture reconstruction. An application of the hologram-like representation to the transmission of pictures with progressive resolution is indicated.

RÉSUMÉ

Dans cet article on propose une nouvelle représentation digitale des images. Cette représentation est le mieux caracterisée par la propriété suivante: Soit Z une séquence des données qui représente une image avec résolution maximale; différentes sous-séquences de Z représentent alors la même image avec des résolutions réduites. Ceci est analogique à une propriété bien connue des hologrammes. L'article présente l'idée principale de la nouvelle représentation (fondée sur un algorithme particulier de balayage des images), discute sa redondance et fournit des exemples de reconstruction des images. L'application de cette méthode à la transmission des images est indiquée.

KEYWORDS: progressive transmission, sampling and reconstruction.

1. INTRODUCTION

Numerous methods for the transmission of pictures have been studied in the past. In the simplest case, a picture is represented as an array of samples. Its transmission and display proceed along rows or columns, referred to as scan lines. If t_c is the time necessary to display the whole picture, in $1/k \cdot T_c$ time only one kth of the picture will be displayed. This part will be presented at full resolution. In some applications, for example slow transmission of visual information (videotex), or browsing through a database of stored images [2], it may be desirable for the resolution of a picture, rather then its visible area, to increase while the transmission proceeds. To this end, methods of progressive transmission of pictures were developed. They use quad trees [8] or binary trees [3] as underlying data structures. Consequently, various initial substrings of the string of data representing the entire picture can be used for its reconstruction, with the resolution proportional to the length of the substring. However, non-initial substrings are meaningless.

This paper describes a new representation of pictures suitable for their transmission with progressive resolution. The main feature of this representation is: Given a string of data Z representing a picture with full resolution, various (not necessarily initial) substrings of Z represent the same picture with an appropriately lower resolution. This is analogous to the well-known feature of holograms: Any portion of a hologram represents the same picture as the whole hologram, but with a lower resolution. The new representation is based on a traversal algorithm (i.e. the selection and ordering of sampling points) with the following key property: Given a string of sampling points, $P = \langle p_0, p_1, p_2, \ldots \rangle$, various substrings of P consist of points uniformly distributed in the sampling region Q. Hence the longer a substring of sampled values is, the better the resolution of the reconstruction of the original picture can be achieved. An earlier version of this traversal algorithm was described in [5, 6].

This paper is organized as follows. In Section 2 the *traversal algorithm* is formally defined, and its essential properties are stated. A suitable *sampling technique*, and the corresponding *reconstruction method* are described in Section 3. Section 4 provides an *analysis of the overhead* of the hologram-like representation.

2. PICTURE TRAVERSAL ALGORITHM

Intuitively, the traversal algorithm is based on two observations (Fig. 1):

- A translation of a set of sampling points uniformly distributed in a region of a plane is a set of uniformly distributed points;
- The union of appropriately translated sets of uniformly distributed points is also a set of uniformly distributed points, with a reduced distance between the adjacent points.

The string (sequence) of sampling points is defined recursively, by translating and concatenating previous strings. Consequently, various substrings consist of points uniformly distributed in the plane.

A formalization follows.

Let $P = \langle p_0, p_1, p_2, \ldots \rangle$ be a string of points in a plane. By $P(n,h)$ we denote the following substring of P:

$$P(n,h) = \langle p_{h \cdot 4^n}, p_{h \cdot 4^n + 1}, \ldots, p_{(h+1) \cdot 4^n - 1} \rangle$$

Furthermore, by $S(\langle p_0, p_1, \ldots, p_m \rangle, \vec{c})$ we denote translation of the substring $\langle p_0, p_1, \ldots, p_m \rangle$ by the vector $\vec{c}$:

$$S(\langle p_0, \ldots, p_m \rangle, \vec{c}) = \langle p_0 + \vec{c}, \ldots, p_m + \vec{c} \rangle$$

We will represent the translation vector $\vec{c}$ as $c_x \vec{1}_x + c_y \vec{1}_y$, where $\vec{1}_x$ and $\vec{1}_y$ are the unit vectors in the directions of axes x and y, respectively.

Definition 1. Let $T>0$ denote the edge size of the square sampling region $Q(T)$ (its vertices are: $(0,0), (0,T), (T,T)$, and $(T,0)$). The string of sampling points is then defined as follows:

$$P(0,0) = \langle p_0 \rangle = \langle T,T \rangle$$
$$P(n,1) = S(P(n,0),\ -T \cdot 2^{-n-1} \vec{1}_x)$$
$$P(n,2) = S(P(n,0),\ -T \cdot 2^{-n-1} \vec{1}_y)$$
$$P(n,3) = S(P(n,0),\ -T \cdot 2^{-n-1} (\vec{1}_x + \vec{1}_y))$$
$$P(n+1,0) = P(n,0) \circ P(n,1) \circ P(n,2) \circ P(n,3)$$

where $\circ$ denotes concatenation of strings, and $n = 0,1,2,\ldots$

Theorem 1. Let the binary word $r_{2n+1} r_{2n} \ldots r_1 r_0$ represent index k of a sample point $p_k = (x_k, y_k)$ in the pure binary number system:

$$k = \sum_{i=0}^{2n+1} r_i 2^i$$

The coordinates of point p_k are then defined as:

$$x_k = T\left(1 - \sum_{i=0}^{n} r_{2i}\, 2^{-(i+1)}\right)$$

$$y_k = T\left(1 - \sum_{i=0}^{n} r_{2i+1}\, 2^{-(i+1)}\right)$$

See [5] for the proof.

Theorem 1 provides an explicit (non-recurrent) relationship between the index of a sampling point and its coordinates. This relation can also be used as a definition of sequence P [6]. It lacks, however, the intuitive flavor of definition 1.

The central property of the string of sampling points P is given by Theorem 2. It refers to definition 2 [8], formalizing the notion of $m \times m$ points uniformly distributed in the square $Q(T)$.

Definition 2. Given a sampling region $Q(T)$, the sampling lattice of $m \times m$ elements, with the origin in point (c_x, c_y), is the set:

$$M_{m \times m}(c_x, c_y) =$$

$$\{ (c_x + (i-1)\frac{T}{m},\ c_y + (j-1)\frac{T}{m}) : i,j = 1,2,\ldots,m \}$$

Theorem 2. Let $\hat{P}(n,h)$ denote the (unordered) set of elements of the substring $P(n,h)$:

$$\hat{P}(n,h) = \{ p_{h \cdot 4^n}, p_{h \cdot 4^n + 1}, \ldots, p_{(h+1) \cdot 4^n - 1} \}$$

For any $n, h = 0,1,2,\ldots$, the set $\hat{P}(n,h)$ is a sampling lattice in the region $Q(T)$:

$$(\forall n, h = 0,1,2,\ldots)$$

$$(\exists c_x, c_y \in [0, \frac{T}{2^n})\ )\ \ \hat{P}(n,h) = M_{2^n \times 2^n}(c_x, c_y)$$

See [5] for the proof.

3. SAMPLING AND RECONSTRUCTION

The sampling method used for the hologram-like transmission of pictures should make it possible to reconstruct a picture f from any number of uniformly distributed samples, with the resolution proportional to the number of samples considered.

The simplest sampling method — Shannon's sampling — does not meet this requirement. Since the number of samples which will actually be used to reconstruct picture f is not known when sampling, it is not possible to adequately filter out high frequency components of f. Consequently, a reconstruction of f from a small number of samples may be totally misleading, due to aliasing [7].

Let us define the area sample z_k in point $p_k = (x_k, y_k)$ as the total amount of light which falls

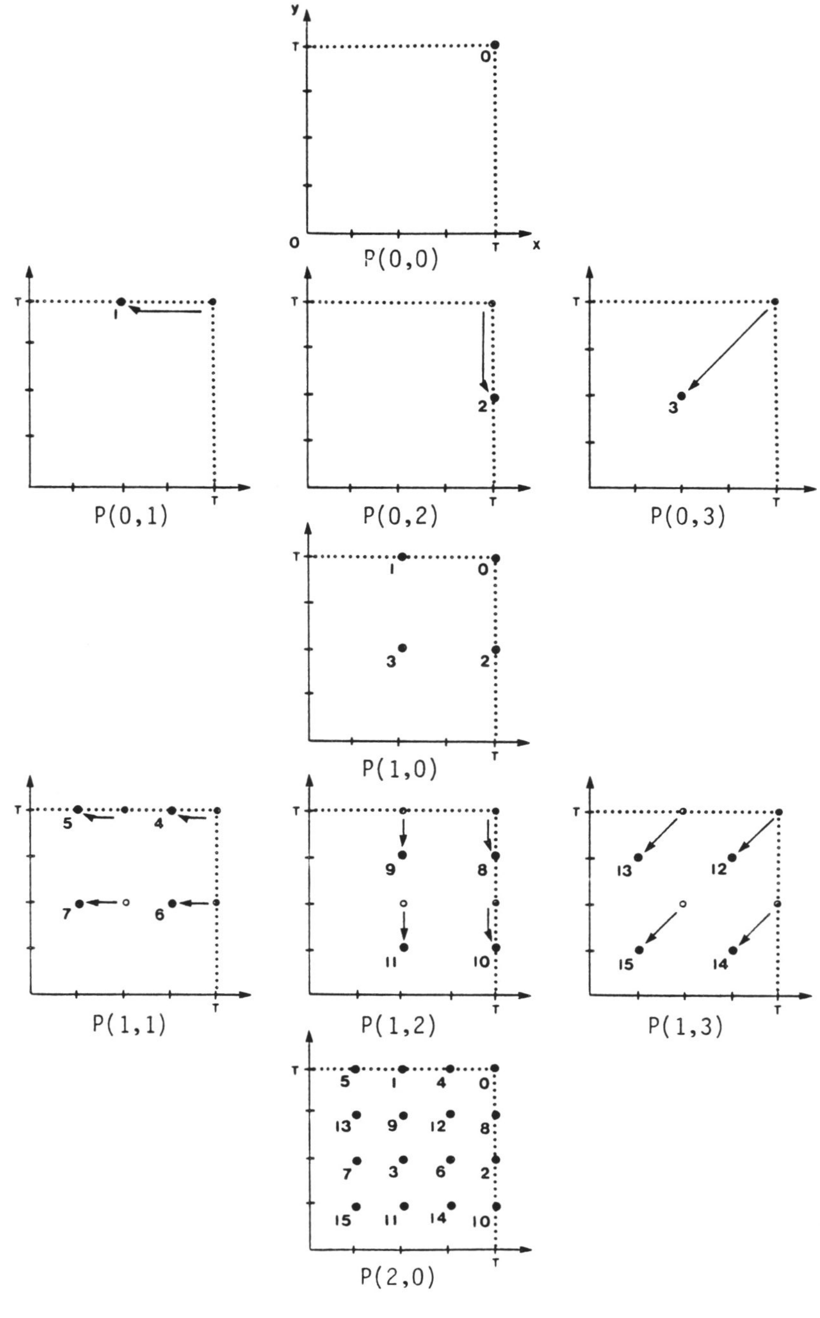

Fig. 1. Definition of the sequence of sampling points *P*.

in the rectangle (0, 0), $(x_k, 0)$, (x_k, y_k), $(0, y_k)$:

$$z_k = \int_0^{x_k} \int_0^{y_k} f(x,y)\,dx\,dy \qquad (*)$$

A reconstruction of f can be performed, given area samples corresponding to any set of sampling points $\hat{P}(n,h)$. Following Theorem 1, the set $\hat{P}(n,h)$ forms the sampling matrix $M_{m\times m}(c_x, c_y)$ for some m, c_x, c_y. Consequently, each sampling point p_k can be expressed as

$$(c_x + (i-1)\frac{T}{m},\ c_y + (j-1)\frac{T}{m})$$

where $i, j \in 1,2,...,m$. We will use i and j as indices, and write $p_{i,j}$ instead of p_k. Similarly, if $p_{i,j} = p_k$, we will write $z_{i,j}$ instead of z_k. By referring to the definition of the area sample, we then obtain:

$$\frac{m^2}{T^2} \int_{x_i}^{x_i+\frac{T}{m}} \int_{y_i}^{y_i+\frac{T}{m}} f(x,y)\,dx\,dy =$$

$$\frac{m^2}{T^2}(z_{i+1,j+1} - z_{i+1,j} - z_{i,j+1} + z_{i,j})$$

where $i, j = 1,2,...,m-1$ (an extension to $i=0$ or $j=0$ is straightforward). The above equation is the basis of picture reconstruction. The left side represents the average value of function f (average gray level) in the square $Q_{i,j}$ with vertices:

$$(x_i, y_j),\ (x_{i+\frac{T}{m}}, y_j),\ (x_{i+\frac{T}{m}}, y_{j+\frac{T}{m}}),\ (x_i, y_{j+\frac{T}{m}})$$

This value (known as standard sample [7]) can be directly used as an approximation of f in $Q_{i,j}$. Due to averaging, the reconstruction of f based on standard samples will be automatically antialiased.

Examples of the reconstruction of pictures from the hologram-like representations are shown in Fig. 2 and Fig. 3. These figures were obtained on a laser printer, with an 8×8 dither matrix [1] used to simulate 64 gray levels.

4. ANALYSIS OF OVERHEAD

Suppose that the original picture is sampled using a lattice of $2^n \times 2^n$ points. Furthermore, suppose that the gray level function f takes values from the set $0,1,...,2^d-1$ (after quantization). From the formula (*), it follows that the area sample $z_{i,j}$ in point $p_{i,j}$ can take any value from 0 to $i \cdot j \cdot (2^d - 1)$. The number of bits necessary to represent this sample is equal to:

$$\lceil \log_2(i \cdot j \cdot (2^d - 1)) \rceil \approx d + \lceil \log_2(i) + \log_2(j) \rceil$$

where $\lceil x \rceil$ denotes the ceiling function. Consequently, the total length of the hologram-like representation of f (in bits) is equal to:

$$N_1 = 4^n d + \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} \lceil \log_2(i) + \log_2(j) \rceil$$

Since the number of bits required to send 4^n d-bit samples is equal to $4^n \cdot d$, the overhead related to the hologram-like representation can be expressed as:

$$O_1 = \frac{N_1 - 4^n \cdot d}{4^n \cdot d}$$

Value N_1 is calculated under the assumption that representations of area samples $z_{i,j}$ may have variable lengths. Decoding of the picture can actually be simplified if all samples are represented by words of equal length: $d + 2n$. The length of the hologram-like representation is then equal to $N_2 = 4^n \cdot (d + 2n)$. The corresponding overhead is equal to:

$$O_2 = \frac{N_2 - 4^n \cdot d}{4^n \cdot d} = 2\frac{n}{d}$$

The overheads O_1 and O_2 calculated for various values of n and d are shown in Fig. 4. The overheads are approximately proportional to the logarithm of the number of samples n, and inversely proportional to the number of bits per standard sample d. The variable-length representation of samples does not significantly reduce the overhead.

5. CONCLUDING REMARKS

A new method for the transmission of pictures has been proposed. The main property of this method is: given a data string Z representing a picture f with full resolution, various substrings of Z represent f with a resolution proportional to the length of the substring. Analysis of the overhead of the method is given. Examples of the reconstruction of pictures from the hologram-like representation are presented.

The method is applicable, for example, to browsing through a set of pictures sent round robin over a communications channel (in a videotex system). Pictures can be quickly reconstructed from a small number of samples, allowing for previewing before the full resolution reconstruction of the selected picture proceeds.

ACKNOWLEDGMENT

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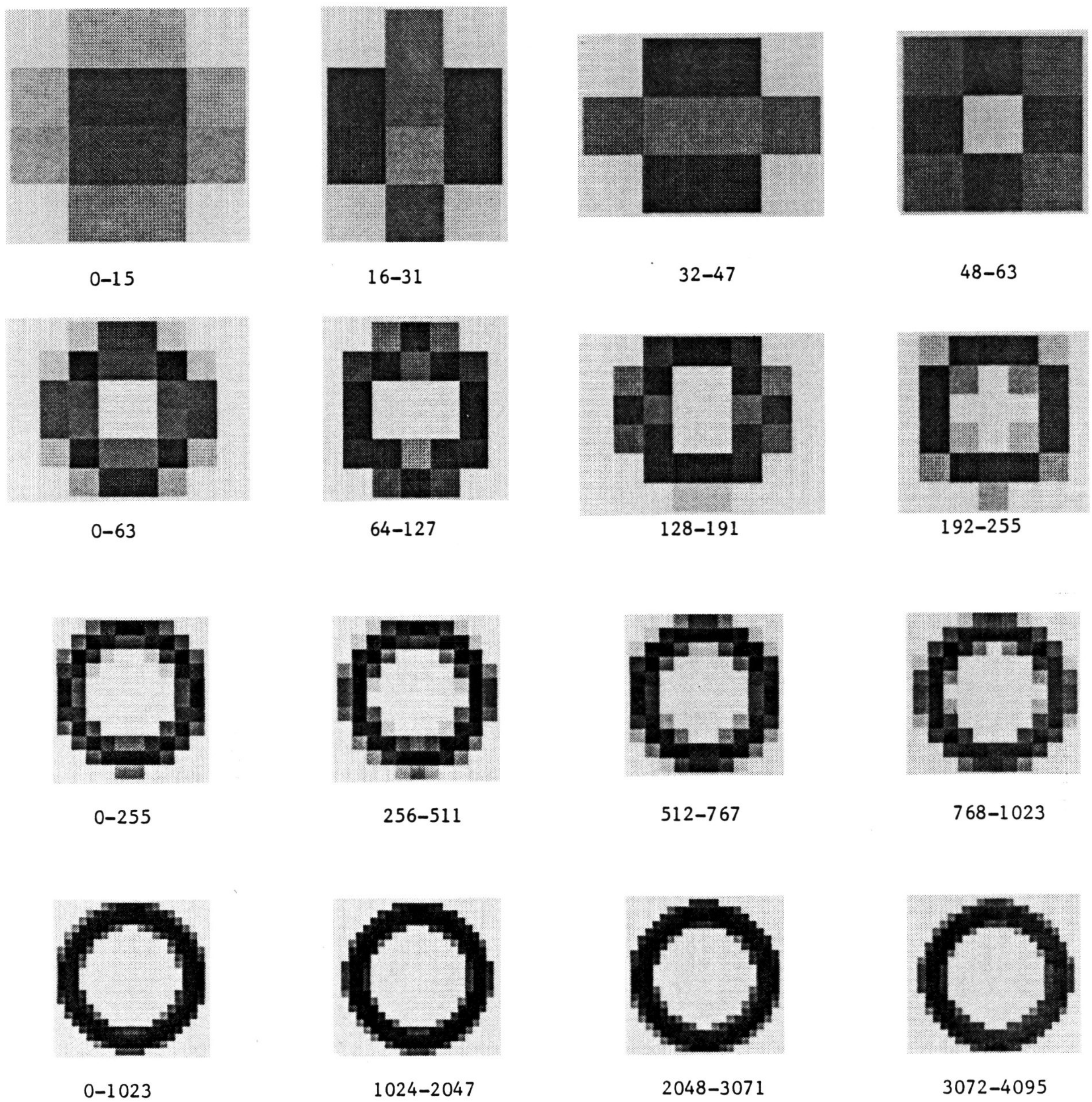


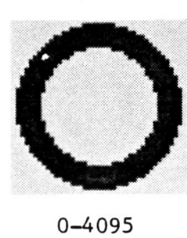
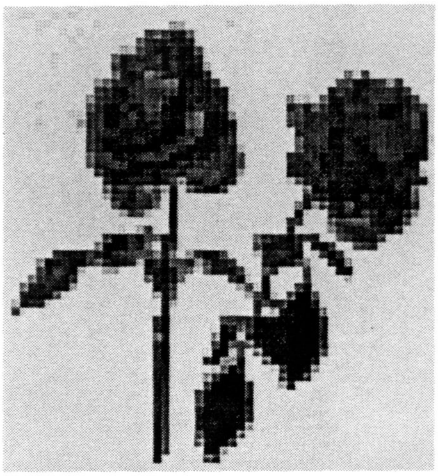
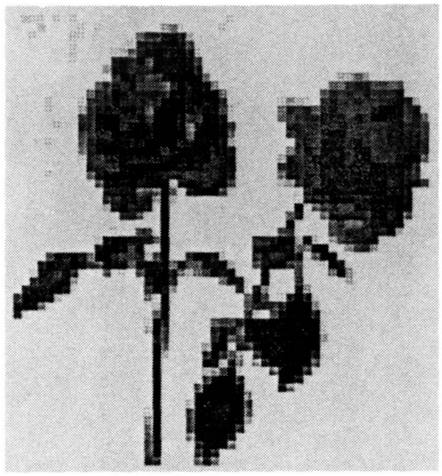


Fig. 2. Example of the hologram-like transmission of a picture.

Numbers indicate indices of area samples z_k used for reconstruction.



0–4095



4096–8191

8192–12287

12288–16383

0–16383

Fig. 3. Example of the hologram-like transmission of a picture.

Numbers indicate indices of area samples z_k used for reconstruction.

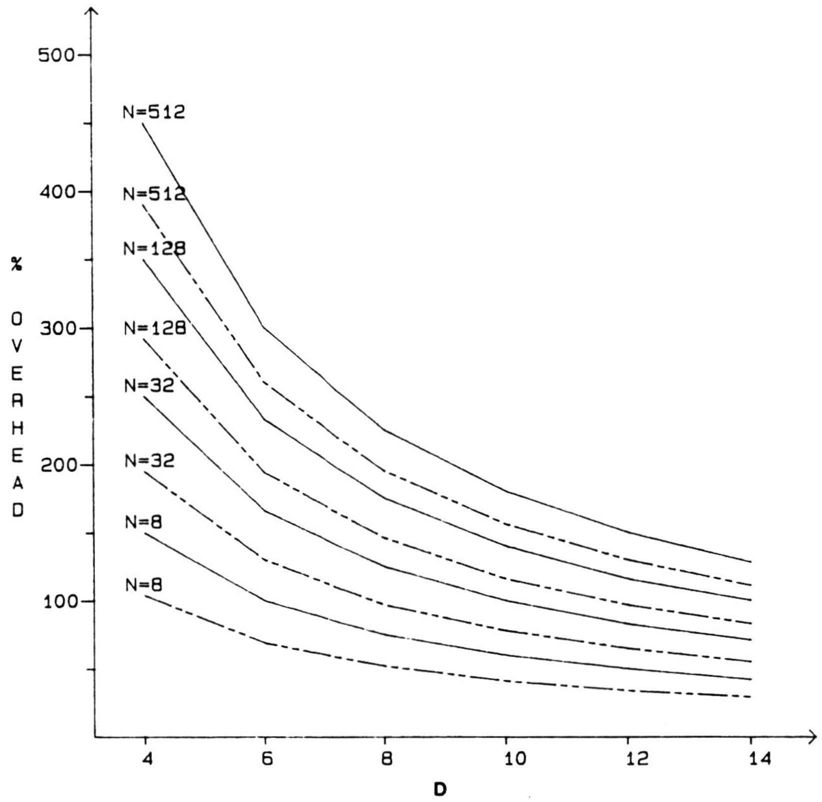


Fig. 4. Overhead of the hologram-like representation of pictures. N indicates the total number of area samples taken (4^N). D is the number of bits per area sample (pixel). Dashed lines correspond to the variable-length representation of samples (overhead O_1). Continuous lines correspond to the fixed-length representation (overhead O_2).

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