Supplementary Document on Revectorization-Based Shadow Mapping

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In this supplementary document we provide additional information about the visibility functions of both SMSR and RSMSS techniques. This document is not self-contained and is to be understood as an appendix of the paper.

1 Additional Definitions

For convenience, let us rewrite $v(x,y)$ as a function of three parameters $v(d_c, d_{on}, p)$: the compressed discontinuity d_c, the oriented normalized discontinuity d_{on}, and the relative coordinates of the screen-space pixel in the projected light space p. For the parameters d_c and p, the description given in the Section 3.1 of the paper is sufficient for this supplementary document. However, we need to include more details about d_{on}.

The parameter d_{on} stores the oriented and normalized discontinuity for vertical and horizontal axes of the 2-D discontinuity space. Let us extend the definition given in the paper and denote d_{on} as a four-channel vector which stores the 2-D relative position of the fragment in the edge discontinuity in the first two components $(d_{on})_r, (d_{on})_g$ and the type of the edge discontinuity for both horizontal and vertical axes in the last two components $(d_{on})_b, (d_{on})_a$.

The relative position of a fragment in an edge discontinuity lies in the unit interval $(d_{on})_{rg} \in [0,1]$, where 0 belongs to the edge discontinuity beginning and 1 belongs to the edge discontinuity end. $(d_{on})_{rg}$ is computed by using the Equation 1 of the paper. In fact, we compute the oriented normalized discontinuity only for the opposite axis of the dominant discontinuity. In this sense, we store the result of the Equation 1 for $(d_{on})_r$ if the opposite axis is the horizontal axis, and for $(d_{on})_g$ otherwise.

An edge discontinuity can be classified into three different types: positive-negative, dual positive and dual negative. This classification is based on the signed distances α_1 and α_2 between the fragment and the ends of the edge. As mentioned in the paper, α is positive to the discontinuity end, and negative to the discontinuity beginning. On the basis of this assumption, an edge discontinuity is classified as positive-negative if we can measure positive and negative distances for every fragment inside the edge discontinuity. The dual positive edge discontinuity does not have a beginning (i.e., α_1 and α_2 are positive). Conversely, the dual negative edge discontinuity does not have end (i.e., α_1 and α_2 are negative). Each one of these types of edge discontinuity must be handled separately in the visibility function. We use -1, 0 and 1 to label the dual negative, positive-negative and dual positive edge discontinuities, respectively. These values are stored in $(d_{on})_b$ for the horizontal axis and $(d_{on})_a$ for the vertical axis of the edge discontinuity.

To compute d_{on}, each edge discontinuity must be traversed in the opposite axis of the dominant discontinuity axis. For most of the fragments, the dominant discontinuity is simply d_c. However, fragments typically located at the corner of the edge discontinuity have a discontinuity in both axes. In this case, the dominant direction is computed considering the discontinuity of the closest shadow map samples in the light space. We compute the discontinuities for the neighbours in the opposite directions of the current discontinuity and assume their discontinuities as dominant if $(d_c)_{rg} \neq 0$. As we will see later, our visibility functions handle scenarios with one or two dominant directions.

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2 SMSR Visibility Function

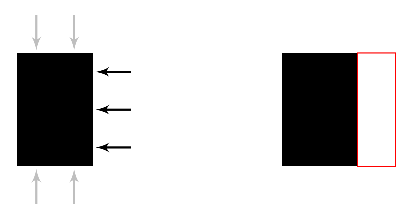
In this document, we show how SMSR works in 12 different scenarios. For the cases not shown here (e.g., $d_c = 0$), we assume $v_{\text{SMSR}}(d_c, d_{on}, p) = 1$ by default. In this subsection, we present and discuss each one of the 12 cases, including:

- An illustration of the scenario, with the shadows (black rectangles), the discontinuity directions (arrows) and the revectorization effect (red shape);
- A description of the case, including relevant information about the handling of edge discontinuities;
- A formalization of how v_{SMSR} solves aliasing. The terminology $v_{\text{SMSR}}(d_c, d_{on}, p)_C$ is merely used to identify the visibility function for a specific case numbered by the index C.

It is worthy to mention that each one of the following definitions of the visibility function can be efficiently implemented in a GLSL shader by the use of *step* and *mix* functions.

Case 1: $(d_{on})_b = -1$ or $(d_{on})_a = -1$

Illustration:



Description:

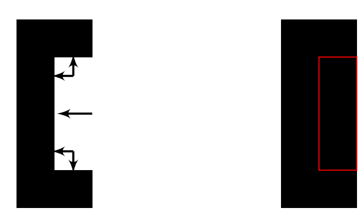
A dual negative edge discontinuity does not end in a shadowed fragment. In this case, the edge discontinuity does not consist of a jagged shadow edge. Therefore, it is not revectorized.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_1 = 1$$

Case 2: $(d_{on})_b = 1$ or $(d_{on})_a = 1$

Illustration:



Description:

A dual positive edge discontinuity consists of a shadow edge in which there are shadowed fragments on both ends of the edge discontinuity. To close the shadow boundary, we revectorize all the edge.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_2 = 0$$

Case 3: $(d_c)_r = 0.75$ or $(d_c)_g = 0.75$

Illustration:

Description:

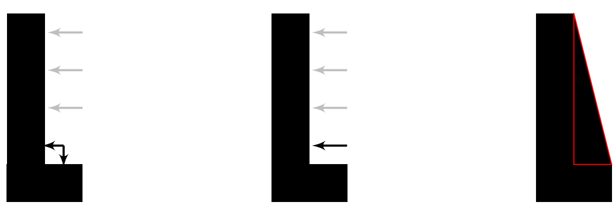
We can use d_c to directly estimate v_{SMSR}. In this case, if there is a discontinuity to the left and right or top and bottom directions, we close the shadow edge, similarly as done in **Case 2**.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_3 = 0$$

Case 4: Dominant $(d_c)_r > 0$ and $(d_c)_g > 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$

Illustration:



Description:

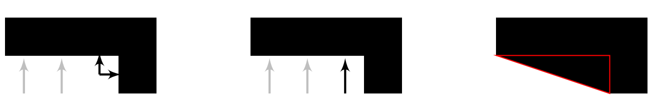
Let us assume the case where the shadow map sample is located at the corner of the jagged shadow edge and the dominant discontinuity axis of the edge discontinuity is the horizontal axis. In this case, we set $(d_c)_g = 0$ and determine the visibility function.

Visibility Function: Set $(d_c)_g = 0$, then evaluate v_{SMSR} as follows:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_4 = \begin{cases} v_{\text{SMSR}}(d_c, d_{on}, p)_{11} & \text{if } (d_c)_r = 0.5, \\ v_{\text{SMSR}}(d_c, d_{on}, p)_{12} & \text{otherwise.} \end{cases}$$

Case 5: $(d_c)_r > 0$ and dominant $(d_c)_g > 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$

Illustration:



Description:

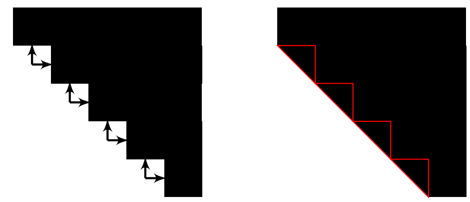
Let us assume the case where the shadow map sample is located at the corner of the jagged shadow edge and the dominant discontinuity axis of the edge discontinuity is the vertical axis. In this case, we set $(d_c)_r = 0$ and determine the visibility function.

Visibility Function: Set $(d_c)_r = 0$, then evaluate v_{SMSR} as follows:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_5 = \begin{cases} v_{\text{SMSR}}(d_c, d_{on}, p)_9 & \text{if } (d_c)_g = 0.5, \\ v_{\text{SMSR}}(d_c, d_{on}, p)_{10} & \text{otherwise.} \end{cases}$$

Case 6: $(d_c)_r > 0$ and $(d_c)_g = 0.5$ and no dominant direction and $(d_{on})_b = 0$ and $(d_{on})_a = 0$ and $(d_c)_r \neq 0.75$

Illustration:



Description:

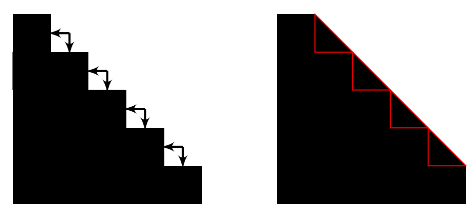
In this case, the edge discontinuity has the size of a shadow map sample and there is a single discontinuity direction in x and y axes. The relative coordinate p_y and the oriented normalized discontinuity $(d_{on})_r$ are used to evaluate v_{SMSR}.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_6 = \begin{cases} 0 & \text{if } 1 - (d_{on})_r < p_y, \\ 1 & \text{otherwise.} \end{cases}$$

Case 7: $(d_c)_r > 0$ and $(d_c)_g = 0.25$ and no dominant direction and $(d_{on})_b = 0$ and $(d_{on})_a = 0$ and $(d_c)_r \neq 0.75$

Illustration:



Description:

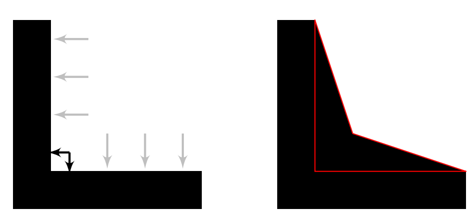
In this case, the edge discontinuity has the size of a shadow map sample and there is a single discontinuity direction in x and y axes. The relative coordinate p_y and the oriented normalized discontinuity $(d_{on})_r$ are used to evaluate v_{SMSR}.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_7 = \begin{cases} 0 & \text{if } 1 - (d_{on})_r < 1 - p_y, \\ 1 & \text{otherwise.} \end{cases}$$

Case 8: Dominant $(d_c)_r > 0$ and dominant $(d_c)_g > 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$

Illustration:



Description:

For a fragment located at the intersection of two edge discontinuities, we must evaluate two visibility functions and take the minimum value.

Visibility Function:

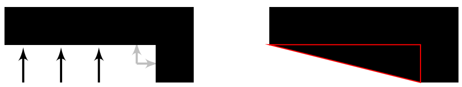
$$a = \begin{cases} v_{\text{SMSR}}(d_c, d_{on}, p)_9 & \text{if } (d_c)_g = 0.5, \\ v_{\text{SMSR}}(d_c, d_{on}, p)_{10} & \text{otherwise.} \end{cases}$$

$$b = \begin{cases} v_{\text{SMSR}}(d_c, d_{on}, p)_{11} & \text{if } (d_c)_r = 0.5, \\ v_{\text{SMSR}}(d_c, d_{on}, p)_{12} & \text{otherwise.} \end{cases}$$

$$v_{\text{SMSR}}(d_c, d_{on}, p)_8 = \min(a, b)$$

Case 9: $(d_c)_r = 0$ and $(d_c)_g = 0.5$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:



Description:

When we have a discontinuity only to the bottom direction, the revectorization depends on the oriented normalized discontinuity $(d_{on})_r$ and the relative coordinate p_y.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_9 = \begin{cases} 0 & \text{if } 1 - p_y < (d_{on})_r, \\ 1 & \text{otherwise.} \end{cases}$$

Case 10: $(d_c)_r = 0$ and $(d_c)_g = 0.25$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:

Description:

When we have a discontinuity only to the top direction, the revectorization depends on the oriented normalized discontinuity $(d_{on})_r$ and the relative coordinate p_y.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_{10} = \begin{cases} 0 & \text{if } p_y < (d_{on})_r, \\ 1 & \text{otherwise.} \end{cases}$$

Case 11: $(d_c)_r = 0.5$ and $(d_c)_g = 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:

Description:

When we have a discontinuity only to the left direction, the revectorization depends on the oriented normalized discontinuity $(d_{on})_g$ and the relative coordinate p_x.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_{11} = \begin{cases} 0 & \text{if } p_x < (d_{on})_g, \\ 1 & \text{otherwise.} \end{cases}$$

Case 12: $(d_c)_r = 0.25$ and $(d_c)_g = 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:

Description:

When we have a discontinuity only to the right direction, the revectorization depends on the oriented normalized discontinuity $(d_{on})_g$ and the relative coordinate p_x.

Visibility Function:

$$v_{\text{SMSR}}(d_c, d_{on}, p)_{12} = \begin{cases} 0 & \text{if } 1 - p_x < (d_{on})_g, \\ 1 & \text{otherwise.} \end{cases}$$

2.1 Tuning

The SMSR technique consists of a set of 12 cases. In the shader, each one of these cases is tested by the algorithm until one of the case expressions is true. The default value is used to handle unexpected cases. Instead of ordering each one of these cases arbitrarily, we can take advantage of the frequency of occurrence of the SMSR cases to prioritize the test of the most frequent cases.

We estimated the average frequency of occurrence for each one of the SMSR cases shown in this supplementary document, as can be seen in Figure 1. We have measured this frequency of occurrence for three different models used in the paper. Here, we want to provide a general view of the SMSR cases which are most likely to be present in shadow mapping.

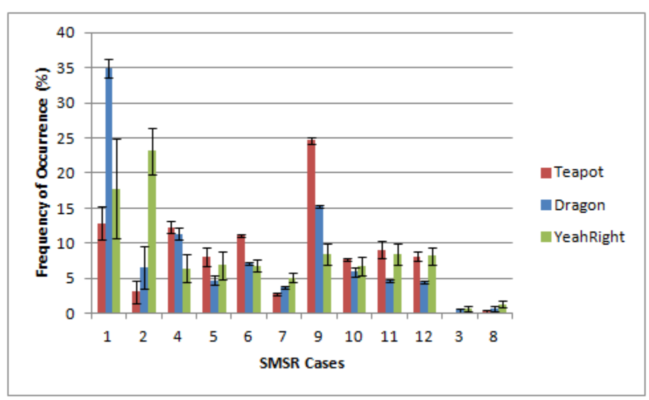


Figure 1: Frequency of occurrence (in %) measured for each case handled by the single-pass SMSR technique. These results were measured for three different models used in the paper and these results were obtained for several shadow map resolutions and light source positions. Error bars indicate standard deviation.

As shown in Figure 1, the scenarios where the discontinuity is to the left and right or top and bottom directions (**Case 3**) and the scenario of intersection between two edge discontinuities (**Case 8**) represent, on average, only 2% of the total usage of the SMSR technique. Nevertheless, these two cases must be handled by the revectorization technique to avoid the presence of artifacts in the final rendering. The occurrence of dual negative (**Case 1**) and dual positive (**Case 2**) edge discontinuities vary considerably according to the model used. The occurrence of the other cases remains stable for different models, shadow map resolutions and light source positions.

On the basis of the Figure 1, we state that an optimized implementation of the SMSR technique should test **Case 3** and **Case 8** only after testing all the other SMSR cases. The test order for the other cases may be implemented arbitrarily or adjusted to a specific model.

3 RSMSS Visibility Function

In RSMSS, a special care must be taken to obtain a coherent filtering for the scene. Moreover, the RSMSS technique deals with entering and exiting discontinuities, which increases the number of scenarios to be dealt with. In this document, we define v_{RSMSS} such that it can handle 31 different scenarios. For the cases not included here (e.g., $d_c = 0$), we assume $v_{\text{RSMSS}}(d_c, d_{on}, p) = 1 - (d_c)_b$ by default. The terms $v_{\text{RSMSS}}(d_c, d_{on}, p)_C$ and $v_{\text{RSMSS-C}}$ are used interchangeably only for convenience.

In this subsection, we present and discuss each one of the 31 cases in a similar fashion of the Section 2. The RSMSS technique can be efficiently implemented in a GLSL shader by the use of *step*, *mix* and *clamp* functions.

In some cases, the RSMSS technique needs additional information about the discontinuity space to compute the visibility function. Let us define the following additional terms:

- $n(d_c)$ is the compressed discontinuity evaluation for a 4-connected neighbourhood:

  $$n(d_c) = (d_c(x-o,y), d_c(x+o,y), d_c(x,y+o), d_c(x,y-o))$$

  , where o is the offset equivalent to one shadow map sample.

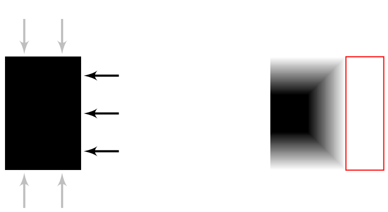
- d_{cb} is the edge discontinuity break. In other words, it is the fragment discontinuity measured during the traversal of the edge discontinuity which indicates that one of the ends of the edge was found.

- o_{rg} is a two-channel vector which stores, for a given shadow map sample located in an edge discontinuity, the offset needed to reach the neighbour shadow map sample in the opposite axis of the discontinuity direction axis, formally:

  $$o_{rg} = (8(0.5 - d_{rg}) - 1)o$$

  For the discontinuities $(d_c)_{rg} = 0.25$ or $(d_c)_{rg} = 0.5$, this vector lies in the interval $o_{rg} \in [-o, o]$. Therefore, we step only one shadow map sample to reach the neighbour.

Case 1: $(d_{on})_b = -1$ or $(d_{on})_a = -1$ and $(d_c)_b = 0$
Illustration:



Description:
A dual negative entering edge discontinuity does not end in a shadowed fragment. In this case, the edge discontinuity does not consist of a jagged shadow edge. As can be seen in the red rectangle, the dual negative entering edge discontinuity does not contribute to the filtering. Then, the filtering takes place for the exiting edge discontinuity only.
Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_1 = 1$$

Case 2: $(d_c)_r = 0.75$ and $(d_c)_g = 0.75$ and $(d_c)_b = 0$
Illustration:

Description:
If a shadow map sample has an entering discontinuity in all the four directions, the shadow map sample is closed to enforce the shadow consistency.
Visibility Function:

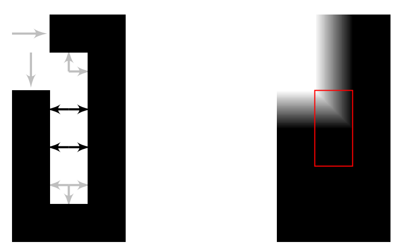
$$v_{\text{RSMSS}}(d_c, d_{on}, p)_2 = 0$$

Case 3: $(d_c)_r = 0.75$ and $(d_c)_g = 0.75$ and $(d_c)_b = 1$
Illustration:

Description:
In the same spirit of the **Case 2**, if a shadow map sample has an exiting discontinuity in all the four directions, we make it lit to enforce the illumination consistency.
Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_3 = 1$$

Case 4: $(d_{on})_b = 1$ and $(d_{on})_a = 1$ and $(d_c)_r = 0.75$ and $(d_c)_g = 0$ and $(d_c)_b = 0$
Illustration:



Description:
If the entering discontinuity is dual positive for the vertical and horizontal axes and the discontinuity is to the left and right directions only, different visibility functions may be used according to the position of the shadow map sample relative to its neighbours. In the illustration above, there are two shadow map samples which are situated in this case. However, as can be seen in the final rendering, one sample is filtered by the RSMSS technique, while the other sample is shadowed. To detect the difference between them, we check if there is an entering discontinuity on the left and right

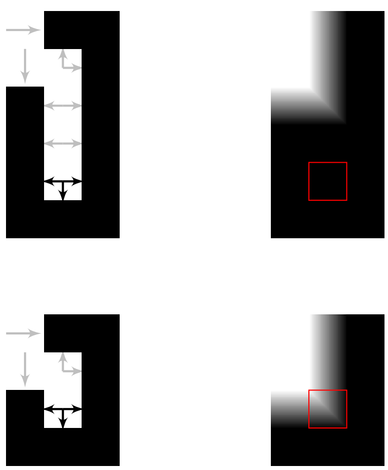
directions for the top and bottom neighbours. If the entering discontinuity to the left and right directions persists in these two neighbours, the current shadow map sample is shadowed. Otherwise, it is filtered. To filter the shadow map sample, we use the horizontal discontinuity of the neighbour shadow map sample in which $d_c \neq 0.75$.

Visibility Function:

$$v_{\text{RSMSS-4}} = \begin{cases} 0 & \text{if } (n(d_c))_b = (n(d_c))_a \\ & \text{and if } (n(d_c))_a = d_c, \\ p_x + p_y - 1 & \text{if } (n(d_c))_a \neq d_c \\ & \text{and if } ((n(d_c))_a)_r = 0.5, \\ p_y - p_x & \text{if } (n(d_c))_a \neq d_c \\ & \text{and if } ((n(d_c))_a)_r = 0.25, \\ p_x - p_y & \text{if } (n(d_c))_b \neq d_c \\ & \text{and if } ((n(d_c))_b)_r = 0.5, \\ 1 - p_x - p_y & \text{if } (n(d_c))_b \neq d_c \\ & \text{and if } ((n(d_c))_b)_r = 0.25 \end{cases}$$

Case 5: $(d_{on})_b = 1$ and $(d_{on})_a = 1$ and $(d_c)_r = 0.75$ and $(d_c)_g \neq 0$ and $(d_c)_b = 0$

Illustration:



Description:

Here, we have a complement of the **Case 4**. If the entering discontinuity is dual positive for the vertical and horizontal axes and the compressed discontinuity is to the left and right and some vertical direction, different visibility functions may be used according to the position of the shadow map sample relative to its neighbours. To detect the difference between them, we check if there is an entering discontinuity on the left and right directions for the neighbour in the opposite direction of the vertical discontinuity.

Visibility Function:

$$a = p_x - (d_{on})_g + 1$$

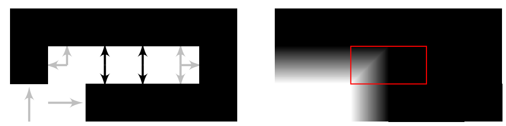
$$b = 2 - (d_{on})_g - p_x$$

$$g = \begin{cases} 1 - p_y & \text{if } (d_c)_g \neq 0.25, \\ p_y & \text{otherwise.} \end{cases}$$

$$v_{\text{RSMSS-5}} = \begin{cases} \min(a,b,g) & \text{if } d_c(x, y + o_g) = d_c \\ p_x + p_y - 1 & \text{if } (d_c(x, y + o_g))_r = 0.5 \\ & \text{and if } (d_c)_g = 0.25, \\ p_y - p_x & \text{if } (d_c(x, y + o_g))_r = 0.25 \\ & \text{and if } (d_c)_g = 0.25, \\ p_x - p_y & \text{if } (d_c(x, y + o_g))_r = 0.5 \\ & \text{and if } (d_c)_g = 0.5, \\ 1 - p_x - p_y & \text{if } (d_c(x, y + o_g))_r = 0.25 \\ & \text{and if } (d_c)_g = 0.5. \end{cases}$$

Case 6: $(d_{on})_b = 1$ and $(d_{on})_a = 1$ and $(d_c)_g = 0.75$ and $(d_c)_r = 0$ and $(d_c)_b = 0$

Illustration:



Description:

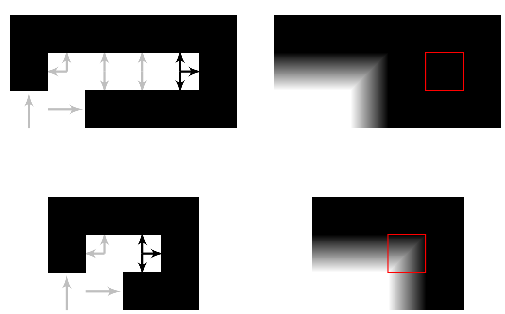
This is a scenario similar to the one presented in **Case 4**. The difference here is that the compressed discontinuity is to the top and bottom directions only.

Visibility Function:

$$v_{\text{RSMSS-6}} = \begin{cases} 0 & \text{if } (n(d_c))_r = (n(d_c))_g \\ & \text{and if } (n(d_c))_g = d_c, \\ p_x + p_y - 1 & \text{if } (n(d_c))_g \neq d_c \\ & \text{and if } ((n(d_c))_g)_g = 0.25, \\ p_y - p_x & \text{if } (n(d_c))_r \neq d_c \\ & \text{and if } ((n(d_c))_r)_g = 0.25, \\ p_x - p_y & \text{if } (n(d_c))_g \neq d_c \\ & \text{and if } ((n(d_c))_g)_g = 0.5, \\ 1 - p_x - p_y & \text{if } (n(d_c))_r \neq d_c \\ & \text{and if } ((n(d_c))_r)_g = 0.5 \end{cases}$$

Case 7: $(d_{on})_b = 1$ and $(d_{on})_a = 1$ and $(d_c)_g = 0.75$ and $(d_c)_r \neq 0$ and $(d_c)_b = 0$

Illustration:



Description:

This is a scenario similar to the one presented in **Case 5**. The difference here is that the compressed discontinuity is to the top and bottom directions and some discontinuity in the horizontal axis.

Visibility Function:

$$a = p_y - (d_{on})_r + 1$$

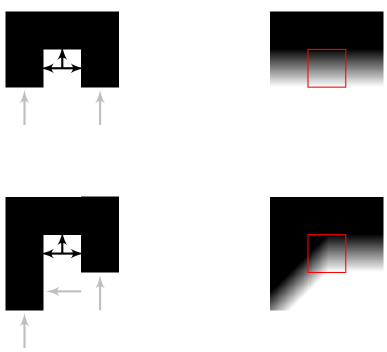
$$b = 2 - (d_{on})_r - p_y$$

$$g = \begin{cases} 1 - p_x & \text{if } (d_c)_r \neq 0.25, \\ p_x & \text{otherwise.} \end{cases}$$

$$v_{\text{RSMSS-7}} = \begin{cases} \min(a,b,g) & \text{if } d_c(x - o_r, y) = d_c \\ p_x + p_y - 1 & \text{if } (d_c(x - o_r, y))_g = 0.25 \\ & \text{and if } (d_c)_r = 0.5, \\ p_y - p_x & \text{if } (d_c(x - o_r, y))_g = 0.25 \\ & \text{and if } (d_c)_r = 0.25, \\ p_x - p_y & \text{if } (d_c(x - o_r, y))_g = 0.5 \\ & \text{and if } (d_c)_r = 0.5, \\ 1 - p_x - p_y & \text{if } (d_c(x - o_r, y))_g = 0.5 \\ & \text{and if } (d_c)_r = 0.25 \end{cases}$$

Case 8: $(d_c)_r = 0.75$ and $(d_c)_g \neq 0$ and $(d_c)_b = 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:



Description:

For a positive-negative entering edge discontinuity, if we have a discontinuity to the left and right direction and some discontinuity in the vertical direction, we must analyze the position of the shadow map sample in relation to its neighbours to determine the filtering. In practice, we must traverse the shadow map samples in the opposite direction to the vertical discontinuity direction until we find a discontinuity break.

Visibility Function:

$$g = \begin{cases} 1 - p_y & \text{if } (d_c)_g \neq 0.25, \\ p_y & \text{otherwise.} \end{cases}$$

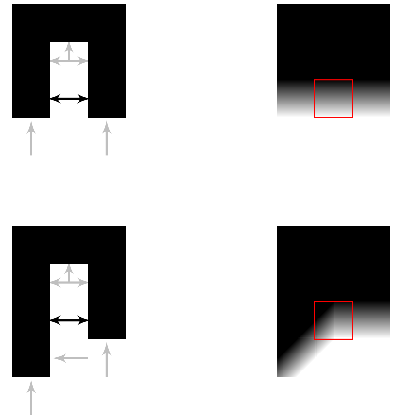
$$a = \begin{cases} g & \text{if } (d_{cb})_r \neq 0.25 \\ 2 - p_x - (d_{on})_g & \text{otherwise.} \end{cases}$$

$$b = \begin{cases} g & \text{if } (d_{cb})_r \neq 0.5 \\ 1 + p_x - (d_{on})_g & \text{otherwise.} \end{cases}$$

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_8 = \begin{cases} 1 - (d_{on})_g & \text{if } (d_{cb})_r = 0, \\ \min(a,b) & \text{otherwise.} \end{cases}$$

Case 9: $(d_c)_r = 0.75$ and $(d_c)_g = 0$ and $(d_c)_b = 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:



Description:

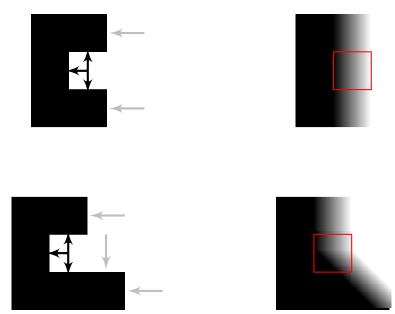
For a positive-negative entering edge discontinuity, if we have a discontinuity only to the left and right directions, we must analyze the position of the shadow map sample in relation to its neighbours to compute the filtering. In practice, we traverse the shadow map samples in top and bottom directions until we find a discontinuity break for at least one of them.

Visibility Function:

$$v_{\text{RSMSS-9}} = \begin{cases} 1 - (d_{on})_g & \text{if } (d_{cb}(x, y+1))_r = 0 \\ & \text{or } (d_{cb}(x, y-1))_r = 0 \\ 1 + p_x - (d_{on})_g & \text{if } (d_{cb}(x, y+1))_r = 0.5 \\ & \text{or } (d_{cb}(x, y-1))_r = 0.5 \\ 2 - p_x - (d_{on})_g & \text{if } (d_{cb}(x, y+1))_r = 0.25 \\ & \text{or } (d_{cb}(x, y-1))_r = 0.25 \end{cases}$$

Case 10: $(d_c)_r \neq 0$ and $(d_c)_g = 0.75$ and $(d_c)_b = 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:



Description:

This is a scenario similar to the one presented in **Case 8**. The difference here is that the discontinuity is to the top and bottom directions and there is some discontinuity to the left or right direction. In this case, we traverse the discontinuity space in the opposite direction of the horizontal discontinuity direction until we find a discontinuity break.

Visibility Function:

$$g = \begin{cases} p_x & \text{if } (d_c)_r \neq 0.25, \\ 1 - p_x & \text{otherwise.} \end{cases}$$

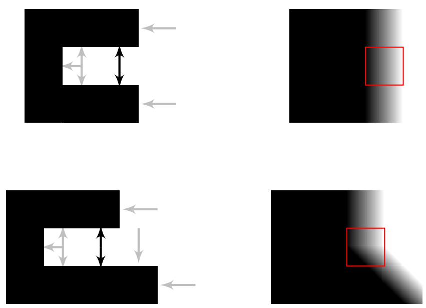
$$a = \begin{cases} g & \text{if } (d_{cb})_g \neq 0.25 \\ 1 + p_y - (d_{on})_r & \text{otherwise.} \end{cases}$$

$$b = \begin{cases} g & \text{if } (d_{cb})_g \neq 0.5 \\ 2 - p_y - (d_{on})_r & \text{otherwise.} \end{cases}$$

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{10} = \begin{cases} 1 - (d_{on})_r & \text{if } (d_{cb})_g = 0, \\ \min(a, b) & \text{otherwise.} \end{cases}$$

Case 11: $(d_c)_r = 0$ and $(d_c)_g = 0.75$ and $(d_c)_b = 0$ and $(d_{on})_b = 0$ and $(d_{on})_a = 0$

Illustration:



Description:

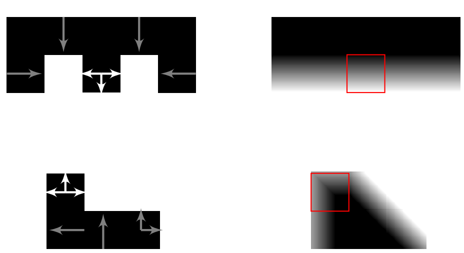
This is a scenario similar to the one presented in **Case 9**. The difference here is that the discontinuity is to the top and bottom directions only. In this case, we traverse the discontinuity space at the top and bottom directions until we find a discontinuity break for at least one of them.

Visibility Function:

$$v_{\text{RSMSS-11}} = \begin{cases} 1 - (d_{on})_r & \text{if } (d_{cb}(x-1,y))_g = 0 \\ & \text{or } (d_{cb}(x+1,y))_g = 0 \\ 1 + p_y - (d_{on})_r & \text{if } (d_{cb}(x-1,y))_g = 0.25 \\ & \text{or } (d_{cb}(x+1,y))_g = 1 \\ 2 - p_y - (d_{on})_r & \text{if } (d_{cb}(x-1,y))_g = 0.5 \\ & \text{or } (d_{cb}(x+1,y))_g = 0.5 \end{cases}$$

Case 12: $(d_c)_r = 0.75$ and $(d_c)_g \neq 0$ and $(d_c)_b = 1$

Illustration:



Description:

This scenario is similar to the **Case 8**, but now for an exiting edge discontinuity.

Visibility Function:

$$g = \begin{cases} p_y & \text{if } (d_c)_g \neq 0.25, \\ 1 - p_y & \text{otherwise.} \end{cases}$$

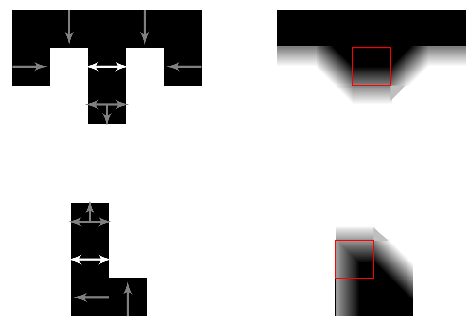
$$a = \begin{cases} 1 - p_x & \text{if } (d_{on})_a = 1, \\ (d_{on})_g - p_x & \text{otherwise.} \end{cases}$$

$$b = \begin{cases} p_x & \text{if } (d_{on})_a = 1, \\ (d_{on})_g + p_x - 1 & \text{otherwise.} \end{cases}$$

$$v_{\text{RSMSS-12}} = \begin{cases} (d_{on})_g & \text{if } (d_c(x, y+o_g))_r = 0, \\ \max(g, b) & \text{else if } (d_c(x, y+o_g))_r = 0.25, \\ \max(g, a) & \text{else if } (d_c(x, y+o_g))_r = 0.5, \\ (d_{on})_g & \text{else if } (d_{cb})_r = 0, \\ \max(g, 1-p_x) & \text{else if } (d_{cb})_r = 0.5, \\ \max(g, p_x) & \text{else if } (d_{cb})_r = 0.25 \end{cases}$$

Case 13: $(d_c)_r = 0.75$ and $(d_c)_g = 0$ and $(d_c)_b = 1$

Illustration:



Description:

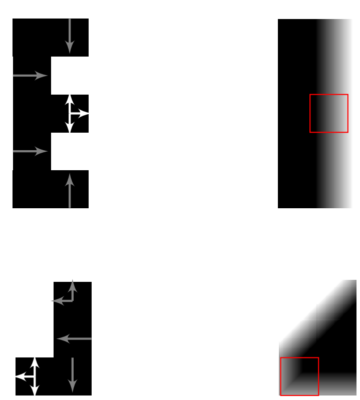
This scenario is similar to the **Case 9**, but now for an exiting edge discontinuity.

Visibility Function:

$$v_{\text{RSMSS-13}} = \begin{cases} (d_{on})_g & \text{if } (d_{cb})_r = 0, \\ 1 - p_x & \text{else if } (d_{cb})_r = 0.5 \\ & \text{and if } (d_{on})_a = 1, \\ (d_{on})_g - p_x & \text{else if } (d_{cb})_r = 0.5 \\ & \text{and if } (d_{on})_a \neq 1, \\ p_x & \text{else if } (d_{cb})_r = 0.25 \\ & \text{and if } (d_{on})_a = 1, \\ (d_{on})_g + p_x - 1 & \text{else if } (d_{cb})_r = 0.25 \\ & \text{and if } (d_{on})_a \neq 1 \end{cases}$$

Case 14: $(d_c)_r \neq 0$ and $(d_c)_g = 0.75$ and $(d_c)_b = 1$

Illustration:



Description:

This scenario is similar to the **Case 10**, but now for an exiting edge discontinuity.

Visibility Function:

$$g = \begin{cases} 1 - p_y & \text{if } (d_c)_r \neq 0.25, \\ p_y & \text{otherwise.} \end{cases}$$

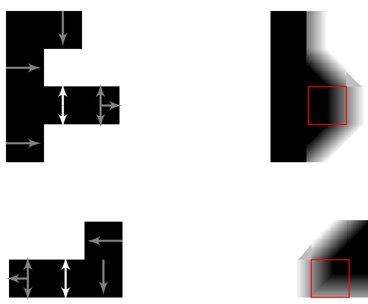
$$a = \begin{cases} 1 - p_y & \text{if } (d_{on})_b = 1, \\ (d_{on})_r - p_y & \text{otherwise.} \end{cases}$$

$$b = \begin{cases} p_y & \text{if } (d_{on})_b = 1, \\ (d_{on})_r + p_y - 1 & \text{otherwise.} \end{cases}$$

$$v_{\text{RSMSS-14}} = \begin{cases} (d_{on})_r & \text{if } (d_c(x - o_r, y)_g = 0, \\ \max(g, b) & \text{else if } (d_c(x - o_r, y)_g = 0.5, \\ \max(g, a) & \text{else if } (d_c(x - o_r, y)_g = 0.25, \\ (d_{on})_r & \text{else if } (d_{cb})_g = 0, \\ \max(g, 1 - p_y) & \text{else if } (d_{cb})_g = 0.25, \\ \max(g, p_y) & \text{else if } (d_{cb})_r = 0.5 \end{cases}$$

Case 15: $(d_c)_r = 0$ and $(d_c)_g = 0.75$ and $(d_c)_b = 1$

Illustration:



Description:

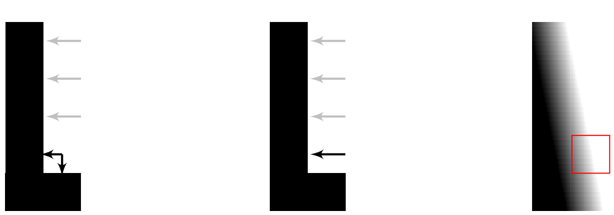
This scenario is similar to the **Case 11**, but now for an exiting edge discontinuity.

Visibility Function:

$$v_{\text{RSMSS-15}} = \begin{cases} (d_{on})_r & \text{if } (d_{cb})_g = 0, \\ 1 - p_y & \text{else if } (d_{cb})_g = 0.25 \\ & \text{and if } (d_{on})_b = 1, \\ (d_{on})_r - p_y & \text{else if } (d_{cb})_g = 0.25 \\ & \text{and if } (d_{on})_b \neq 1, \\ p_y & \text{else if } (d_{cb})_g = 0.5 \\ & \text{and if } (d_{on})_b = 1, \\ (d_{on})_r + p_y - 1 & \text{else if } (d_{cb})_g = 0.5 \\ & \text{and if } (d_{on})_b \neq 1 \end{cases}$$

Case 16: Dominant $(d_c)_r > 0$ and $(d_c)_g > 0$ and $(d_c)_b = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$

Illustration:



Description:

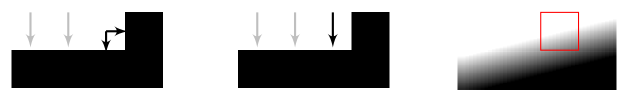
Let us assume the case where the shadow map sample is located at the corner of the jagged shadow edge and the dominant discontinuity axis of the entering edge discontinuity is the horizontal axis. In this case, we set $(d_c)_g = 0$ and determine the visibility function according to this new scenario.

Visibility Function: Set $(d_c)_g = 0$, then evaluate v_{RSMSS} as follows:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{16} = \begin{cases} v_{\text{RSMSS}}(d_c, d_{on}, p)_{20} & \text{if } (d_{on})_b = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{21} & \text{if } (d_{on})_a = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{23} & \text{otherwise.} \end{cases}$$

Case 17: $(d_c)_r > 0$ and dominant $(d_c)_g > 0$ and $(d_c)_b = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$

Illustration:



Description:

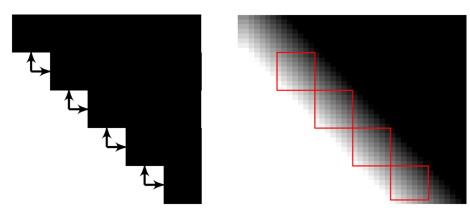
Let us assume the case where the shadow map sample is located at the corner of the jagged shadow edge and the dominant discontinuity axis of the entering edge discontinuity is the vertical axis. In this case, we set $(d_c)_g = 0$ and determine the visibility function according to this new scenario.

Visibility Function: Set $(d_c)_r = 0$, then evaluate v_{RSMSS} as follows:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{17} = \begin{cases} v_{\text{RSMSS}}(d_c, d_{on}, p)_{20} & \text{if } (d_{on})_b = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{21} & \text{if } (d_{on})_a = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{22} & \text{otherwise.} \end{cases}$$

Case 18: $(d_c)_r > 0$ and $(d_c)_g > 0$ and $(d_c)_b = 0$ and no dominant direction and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$

Illustration:



Description:

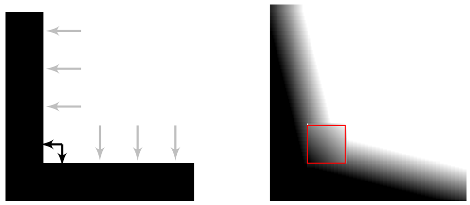
In this case, the entering edge discontinuity has the size of a shadow map sample and there is a single discontinuity direction in x and y axes. The relative coordinate p_y and the oriented normalized discontinuity $(d_{on})_r$ are used to evaluate v_{RSMSS}.

Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{18} = \begin{cases} 2 - (d_{on})_r - p_y & \text{if } (d_c)_g = 0.5, \\ 1 + p_y - (d_{on})_r & \text{if } (d_c)_g = 0.25. \end{cases}$$

Case 19: Dominant $(d_c)_r > 0$ and dominant $(d_c)_g > 0$ and $(d_c)_b = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$

Illustration:



Description:
For a fragment located at the intersection of two edge discontinuities, we must evaluate two visibility functions and take the minimum value.
Visibility Function:

$$a = \begin{cases} p_x & \text{if } (d_{on})_a = 1 \\ & \text{and if } (d_c)_r = 0.5 \\ 1 - p_x & \text{if } (d_{on})_a = 1 \\ & \text{and if } (d_c)_r = 0.25 \\ 1 + p_x - (d_{on})_g & \text{if } (d_{on})_a \neq 1 \\ & \text{and if } (d_c)_r = 0.5 \\ 2 - (d_{on})_g - p_x & \text{if } (d_{on})_a \neq 1 \\ & \text{and if } (d_c)_r = 0.25. \end{cases}$$

$$b = \begin{cases} 1 - p_y & \text{if } (d_{on})_b = 1 \\ & \text{and if } (d_c)_g = 0.5 \\ p_y & \text{if } (d_{on})_b = 1 \\ & \text{and if } (d_c)_g = 0.25 \\ 2 - (d_{on})_r - p_y & \text{if } (d_{on})_b \neq 1 \\ & \text{and if } (d_c)_g = 0.5 \\ 1 + p_y - (d_{on})_r & \text{if } (d_{on})_b \neq 1 \\ & \text{and if } (d_c)_g = 0.25. \end{cases}$$

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{19} = \min(a, b)$$

Case 20: $(d_{on})_b = 1$ and $(d_{on})_a = 0$ and $(d_c)_b = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:

Description:
For a dual positive entering edge discontinuity only in the horizontal axis, the visibility function can be easily estimated by the relative coordinate p_y.
Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{20} = \begin{cases} 1 - p_y & \text{if } (d_c)_g = 0.5, \\ p_y & \text{else if } (d_c)_g = 0.25 \end{cases}$$

Case 21: $(d_{on})_b = 0$ and $(d_{on})_a = 1$ and $(d_c)_b = 0$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:

Description:
For a dual positive entering edge discontinuity only in the vertical axis, the visibility function can be easily estimated by the relative coordinate p_x.
Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{21} = \begin{cases} p_x & \text{if } (d_c)_r = 0.5, \\ 1 - p_x & \text{else if } (d_c)_r = 0.25 \end{cases}$$

Case 22: $(d_c)_r = 0$ and $(d_c)_g > 0$ and $(d_c)_b = 0$ and $(d_c)_g \neq 0.75$
Illustration:

Description:
When we have a discontinuity only in the vertical axis, revectorization depends on the oriented normalized discontinuity $(d_{on})_r$ and the relative coordinate p_y.
Visibility Function:

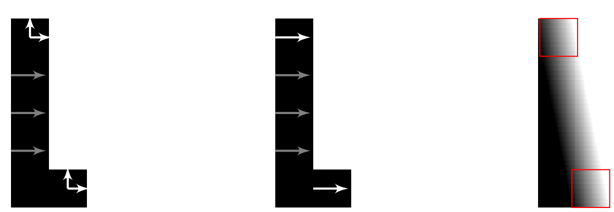
$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{22} = \begin{cases} 2 - p_y - (d_{on})_r & \text{if } (d_c)_g = 0.5, \\ 1 + p_y - (d_{on})_r & \text{if } (d_c)_g = 0.25 \end{cases}$$

Case 23: $(d_c)_r > 0$ and $(d_c)_g = 0$ and $(d_c)_b = 0$ and $(d_c)_r \neq 0.75$
Illustration:

Description:
When we have a discontinuity only in the horizontal axis, revectorization depends on the oriented normalized discontinuity $(d_{on})_g$ and the relative coordinate p_x.
Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{23} = \begin{cases} 1 + p_x - (d_{on})_g & \text{if } (d_c)_r = 0.5, \\ 2 - p_x - (d_{on})_g & \text{if } (d_c)_r = 0.25 \end{cases}$$

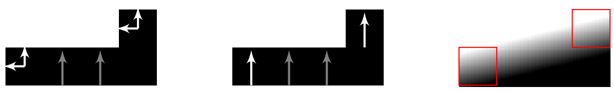
Case 24: Dominant $(d_c)_r > 0$ and $(d_c)_g > 0$ and $(d_c)_b = 1$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:



Description:
Let us assume the case where there is only one discontinuity for each axis and the dominant axis of the exiting edge discontinuity is the horizontal axis. In this case, we set $(d_c)_g = 0$ and determine the visibility function according to this new scenario.
Visibility Function: Set $(d_c)_g = 0$, then evaluate v_{RSMSS} as follows:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{16} = \begin{cases} v_{\text{RSMSS}}(d_c, d_{on}, p)_{28} & \text{if } (d_{on})_b = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{29} & \text{if } (d_{on})_a = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{31} & \text{otherwise.} \end{cases}$$

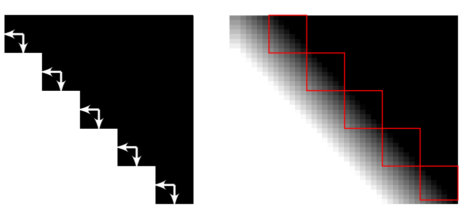
Case 25: $(d_c)_r > 0$ and dominant $(d_c)_g > 0$ and $(d_c)_b = 1$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:



Description:
Let us assume the case where there is only one discontinuity for each axis and the dominant axi of the entering edge discontinuity is the vertical axis. In this case, we set $(d_c)_r = 0$ and determine the visibility function according to this new scenario.
Visibility Function: Set $(d_c)_r = 0$, then evaluate v_{RSMSS} as follows:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{25} = \begin{cases} v_{\text{RSMSS}}(d_c, d_{on}, p)_{28} & \text{if } (d_{on})_b = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{29} & \text{if } (d_{on})_a = 1, \\ v_{\text{RSMSS}}(d_c, d_{on}, p)_{30} & \text{otherwise.} \end{cases}$$

Case 26: $(d_c)_r > 0$ and $(d_c)_g > 0$ and $(d_c)_b = 1$ and no dominant direction and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:



Description:
In this case, the exiting edge discontinuity has the size of a shadow map sample and there is a single discontinuity in x and y axes. The relative coordinate p_y and the oriented normalized discontinuity $(d_{on})_r$ are used to evaluate v_{RSMSS}.
Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{26} = \begin{cases} (d_{on})_r + p_y - 1 & \text{if } (d_c)_g = 0.5, \\ (d_{on})_r - p_y & \text{if } (d_c)_g = 0.25. \end{cases}$$

Case 27: Dominant $(d_c)_r > 0$ and dominant $(d_c)_g > 0$ and $(d_c)_b = 1$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:

Description:
For a fragment located at the intersection of two exiting edge discontinuities, we must evaluate two visibility functions and take the maximum value.
Visibility Function:

$$a = \begin{cases} 1 - p_x & \text{if } (d_{on})_a = 1 \\ & \text{and if } (d_c)_r = 0.5 \\ p_x & \text{if } (d_{on})_a = 1 \\ & \text{and if } (d_c)_r = 0.25 \\ (d_{on})_g - p_x & \text{if } (d_{on})_a \neq 1 \\ & \text{and if } (d_c)_r = 0.5 \\ (d_{on})_g + p_x - 1 & \text{if } (d_{on})_a \neq 1 \\ & \text{and if } (d_c)_r = 0.25. \end{cases}$$

$$b = \begin{cases} p_y & \text{if } (d_{on})_b = 1 \\ & \text{and if } (d_c)_g = 0.5 \\ 1 - p_y & \text{if } (d_{on})_b = 1 \\ & \text{and if } (d_c)_g = 0.25 \\ (d_{on})_r + p_y - 1 & \text{if } (d_{on})_b \neq 1 \\ & \text{and if } (d_c)_g = 0.5 \\ (d_{on})_r - p_y & \text{if } (d_{on})_b \neq 1 \\ & \text{and if } (d_c)_g = 0.25. \end{cases}$$

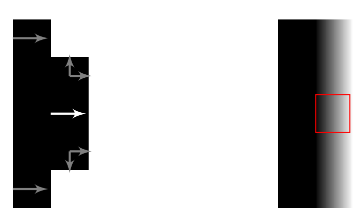
$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{27} = \max(a, b)$$

Case 28: $(d_{on})_b = 1$ and $(d_{on})_a = 0$ and $(d_c)_b = 1$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:

Description:
For a dual positive exiting edge discontinuity only in the horizontal axis, the visibility function can be easily estimated by the use of the relative coordinate p_y.
Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{28} = \begin{cases} p_y & \text{if } (d_c)_g = 0.5, \\ 1 - p_y & \text{otherwise.} \end{cases}$$

Case 29: $(d_{on})_b = 0$ and $(d_{on})_a = 1$ and $(d_c)_b = 1$ and $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$
Illustration:



Description:

For a dual positive exiting edge discontinuity only in the vertical axis, the visibility function can be easily estimated by the use of the relative coordinate p_x.

Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{29} = \begin{cases} 1 - p_x & \text{if } (d_c)_r = 0.5, \\ p_x & \text{otherwise.} \end{cases}$$

Case 30: $(d_c)_r = 0$ and $(d_c)_g > 0$ and $(d_c)_b = 1$ and $(d_c)_g \neq 0.75$

Illustration:

Description:

When we have an exiting discontinuity only in the vertical axis, revectorization depends on the oriented normalized discontinuity $(d_{on})_r$ and the relative coordinate p_y.

Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{30} = \begin{cases} p_y + (d_{on})_r - 1 & \text{if } (d_c)_g = 0.5, \\ (d_{on})_r - p_y & \text{if } (d_c)_g = 0.25 \end{cases}$$

Case 31: $(d_c)_r > 0$ and $(d_c)_g = 0$ and $(d_c)_b = 1$ and $(d_c)_r \neq 0.75$

Illustration:

Description:

When we have an exiting discontinuity only in the vertical axis, revectorization depends on the oriented normalized discontinuity $(d_{on})_g$ and the relative coordinate p_x.

Visibility Function:

$$v_{\text{RSMSS}}(d_c, d_{on}, p)_{31} = \begin{cases} (d_{on})_g - p_x & \text{if } (d_c)_r = 0.5, \\ p_x + (d_{on})_g - 1 & \text{if } (d_c)_r = 0.25 \end{cases}$$

3.1 Tuning

The implementation of the RSMSS technique is similar to the SMSR technique in the sense that one must test each one of the RSMSS cases to find the proper visibility function to filter the silhouette fragment. In this subsection, we analyze the frequency of occurrence of the RSMSS cases to prioritize the verification of the most frequent ones.

An estimated frequency of occurrence for each one of the RSMSS cases shown in this supplementary document can be seen in Figure 2.

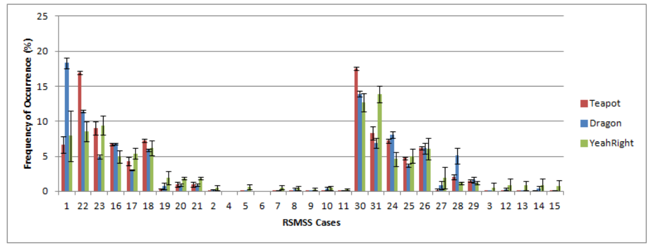


Figure 2: Frequency of occurrence (in %) measured for each case handled by the RSMSS technique. These results were measured for three different models used in the paper and these results were obtained for several shadow map resolutions and light source positions. Error bars indicate standard deviation.

As shown in Figure 2, the scenarios where we have discontinuity to the left and right or to the top and bottom directions (**Cases 2 to 15**) are rarer to occur. They represent only 1% of the total usage of the RSMSS technique. The usage of the other RSMSS cases is well distributed for entering and exiting discontinuities.

As depicted in Figure 2, the RSMSS cases must not be implemented in the order that they were described in Section 3. To implement the RSMSS technique efficiently in the shader, one must first branch the cases based on their discontinuity type (i.e., $(d_c)_b = 0$ or $(d_c)_b = 1$). Assuming the order presented by Figure 2, this is equivalent to choose between start checking **Case 1** ($(d_c)_b = 0$) or **Case 30** ($(d_c)_b = 1$). Then, for each one of the discontinuity types, the next branch must check if the discontinuity is to the left and right or to the top and bottom (i.e., $(d_c)_r \neq 0.75$ and $(d_c)_g \neq 0.75$). For entering discontinuities, this is equivalent to start checking **Case 22** or **Case 2**. For exiting discontinuities, the same is equivalent to start checking **Case 30** or **Case 3**. Finally, the cases which lie in such conditions are checked in the order of frequency of occurrence shown in Figure 2. Based on this distribution of conditional statements in the shader, we could save $20-30\%$ of the computational time needed by the RSMSS technique.