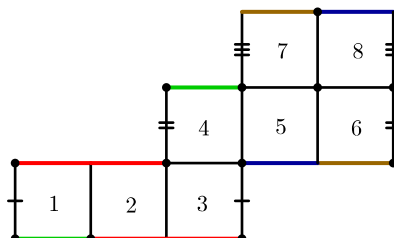# Combinatorics of flat surfaces

Exercises - Conference ALEA 2024

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**Exercise 1.** In the following picture, sides of same color are identified by translation.

(1) Compute the singularity degrees of this square-tiled surface. What is its genus ?
(2) How many horizontal and vertical cylinders are there ?
(3) Draw the map corresponding to the boundaries of the horizontal cylinders.
(4) Find a 1-cylinder square-tiled surface in the same stratum (one cylinder in the horizontal direction).

[*Indic: One can work either directly with the pattern or with the pair of associated permutations h, v.*]

Exercise 2. Recognize a (half-translation) square-tiled surface :
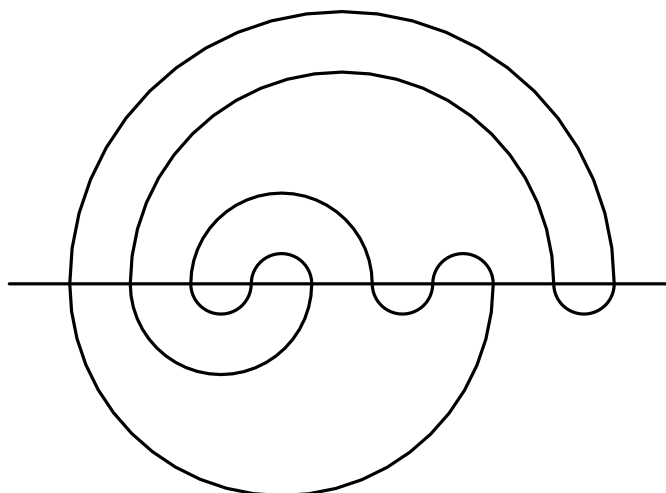
Exercise 3. (1) For any b_1 and b_2 positive integers, compute the number $N_\Gamma(b_1, b_2)$ of ways to put integer metrics on this map Γ such that the faces are of length b_1 and b_2.
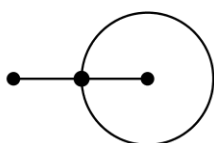
(2*) Compute the Masur-Veech volume of the stratum $Q(2, -1, -1)$.

References

- Package `flatsurf` of `Sage` (Vincent Delecroix):
  http://www.labri.fr/perso/vdelecro/surface-dynamics/0.3.2/origamis.html.
- Mini-course of Carlos Matheus on square-tiled surfaces:
  *https://if-summer2018.sciencesconf.org/data/pages/origamis_ Grenoble_ matheus_*
- Course of Jean-Christophe Yoccoz on translation surfaces and IETs:
- Reference paper on flat surfaces by Anton Zorich: