Some remarks on the fragility of Smith predictors used in haptics.

Bogdan Liacu, Irinel-Constantin Morarescu, Claude Andriot, Silviu-Iulian Niculescu, Didier Dumur, Patrick Boucher, Frédéric Colledani

2011 11th International Conference on Control, Automation and Systems
Oct. 26-29, 2011 in KINTEX, Gyeonggi-do, Korea

Some Remarks on the Fragility of Smith Predictors used in Haptics

Bogdan Liacu[1,3,4], Constantin-Irinel Morarescu[2], Claude Andriot[3], Silviu Niculescu[1], Didier Dumur[4], Patrick Boucher[4] and Frédéric Colledani[3]

[1]Laboratoire des Signaux et Systèmes (LSS), CNRS-SUPELEC, 3 rue Joliot Curie, 91192 Gif-sur-Yvette Cedex, France.
E-mail: {bogdan.liacu, silviu.niculescu}@lss.supelec.fr
[2]Institut National Polytechnique de Lorraine,2, avenue de la fort de Haye, 54500 Vandoeuvre-ls-Nancy, France.
E-mail: constantin.morarescu@ensem.inpl-nancy.fr
[3]CEA LIST, Laboratoire de Simulation Interactive, 18 Route du Panorama, BP6 Fontenay-aux-Roses, F-92265 France.
E-mail: claude.andriot, frederic.colledani@cea.fr
[4]SUPELEC E3S, Control Department, 3 rue Joliot Curie, 91192 Gif sur Yvette Cedex, France.
E-mail: {didier.dumur, patrick.boucher}@supelec.fr

Abstract: In this paper we propose a method to study the fragility of Smith predictor controllers used in haptics. In order to develop controllers for real environments, a careful analysis must be taken into account for the variation of the parameters. Generally, real systems present parameter variations which often lead the system to an unstable behavior. Using a geometric approach, we derive a simple method to study the fragility of Smith predictors for two cases - constant and uncertain delays. Illustrative examples complete the presentation.

Keywords: Smith predictor, delay, haptics.

1. INTRODUCTION

Virtual environments have become very popular and are used in many domains, like prototyping (figure 1.a example of prototyping using haptic interfaces and virtual environment [7]), trainings for different devices and assistance in completing difficult tasks (figure 1.b virtual environment used for task assistance/supervision [2], [4]).

In figure 2 we present the general scheme of a haptic system. The ideal haptic system must have:

- position tracking error as small as possible between the haptic interface and the virtual object,
- high degree of transparency, i.e. in free motion, the force feedback felt at the haptic interface end must be as small as possible and in case of hard contact, a stiff response is desired.

The main problems of such systems are linked to the delays and their effects on stability and transparency. For complex virtual environments, the processing time can increase substantially and can introduce unwanted effects and behaviors. More precisely, in free motion the delay effect can be felt by the viscosity phenomenon (high force feedback felt at the haptic interface end), in the case of a hard contact with the environment, the impact effect will not be stiff, or the most unwanted situation is to loose the system stability due to the delays. The delays must be taken into account and included in the control laws. However, a trade-off between stability, position tracking error and transparency must be always made.

A *classic* solution for time delays problems is the Smith predictor control which can predict the objects response and compensate time delays.

In this paper we propose a method to study the fragility of Smith predictor controllers used in haptics. In order to develop controllers for real environments, a careful analysis must be taken into account for the variation of the

Fig. 1 a.Virtual Prototyping. b.Virtual Supervision.

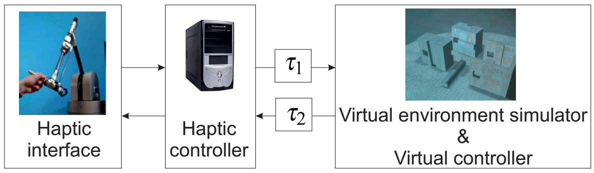

Fig. 2 General Scheme of a Haptic System

parameters. Generally, real systems present parameter variations which often lead the system to an unstable behavior. In our opinion, the notion of *controller fragility* is more appropriate for such a study, see, for instance, [1], [6], [9]. Roughly speaking, the fragility describes the deterioration of closed-loop stability due to small variations of the controller parameters. Our intention is to detect non-fragile controllers by appropriate construction of the closed-loop stability regions in the corresponding controller parameter-space. A more in depth discussion on the effects induced by the system's parameters on the (closed-loop) stability of delay systems can be found in [18], [16]. A simple geometric argument, inspired by the ideas suggested in [10], will allow us to conclude on the best controller's choice.

The remaining paper is organized as follows: in section 2 the control scheme is presented. Next, the fragility algorithm is described in Section 3. Illustrative examples are considered in Section 4. Finally, some concluding remarks end the paper.

2. CONTROL SCHEME

In this section we will present the proposed control scheme using Smith predictor.

We will start from the *classical* dynamic (nonlinear) equations of motion for two similar robots in the framework of haptic systems:

$$M_1(x_1)\ddot{x}_1(t) + C_1(x_1, \dot{x}_1)\dot{x}_1 = -F_1(t) + F_h(t), \quad (1)$$

$$M_2(x_2)\ddot{x}_2(t) + C_2(x_2, \dot{x}_2)\dot{x}_2 = F_2(t) - F_e(t), \quad (2)$$

where x_1, x_2 are the haptic interface/virtual object position, F_h, F_e are the human/environmental forces, F_1, F_2 are the force control signals, M_1, M_2 are the symmetric and positive-definite inertia matrices, and C_1, C_2 are the Coriolis matrices of the haptic interface and virtual object systems, respectively.

Figure 3 presents the general control scheme of a haptic interface and a virtual environment including control.

The main idea is to use two similar PD controllers, one to control the haptic interface and another one for the virtual object. The controller equations are there given as follows:

$$F_1(t) = \underbrace{K_d(\dot{x}_1(t) - \dot{x}_2(t-\tau_2))}_{\text{delayed D-action}} \underbrace{+K_p(x_1(t) - x_2(t-\tau_2))}_{\text{delayed P-action}}, \quad (3)$$

$$F_2(t) = \underbrace{-K_d(\dot{x}_2(t) - \dot{x}_1(t-\tau_1))}_{\text{delayed D-action}} \underbrace{-K_p(x_2(t) - x_1(t-\tau_1))}_{\text{delayed P-action}}, \quad (4)$$

where τ_1, τ_2 are the forward and backward finite constant delays and K_p, K_d are the PD control gains.

In order to minimize the delays effects felt by the human operator we will add a Smith predictor in the control scheme of the haptic interface, figure 4.

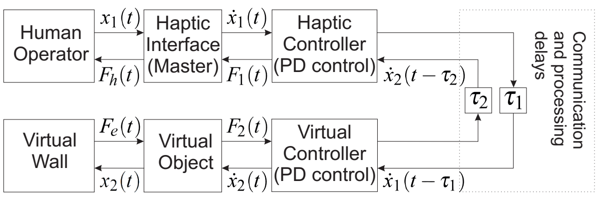

Fig. 3 General PD control scheme for haptic systems.

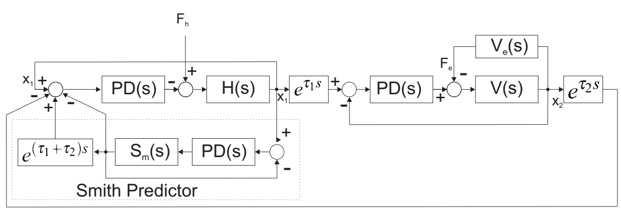

Fig. 4 Haptic control scheme including Smith predictor.

Considering these modifications, equation 3 becomes:

$$F_1(t) = \underbrace{K_d(\dot{x}_1(t) - \dot{x}_2(t-\tau_2) + \hat{\dot{x}}_2(t-(\tau_1+\tau_2)) - \hat{\dot{x}}_2(t)}_{\text{delayed D-action}} \underbrace{+K_p(x_1(t) - x_2(t-\tau_2) + \hat{x}_2(t-(\tau_1+\tau_2)) - \hat{x}_2(t))}_{\text{delayed P-action}}, \quad (5)$$

where $\hat{\dot{x}}_2$, $\hat{x}_2$ represent the estimated velocity and position for virtual object.

We are proposing this asymmetric control scheme because in this context is more important to achieved the desired behavior for the human operator. A second Smith predictor inserted in the virtual controller will introduce new uncertainties which will make the system more vulnerable to instability without adding additional improvements to the human operator's perception.

In this case the resulting controller on the haptic side has the form below:

$$\overline{C} = \frac{K_p + K_d s}{1 + (K_p + K_d s) SM(s)(1 - e^{-s\tau})}, \quad (6)$$

where $\tau = \tau_1 + \tau_2$ and $S_m(s)$ represents the closed loop transfer function of the virtual object control and model used in the Smith predictor:

$$SM(s) = \frac{(K_p + K_d s) S_m(s)}{1 + (K_p + K_d s) S_m(s)}, \quad (7)$$

and $S_m(s) = V(s)$ (the virtual object model).

Considering the controller above on the haptic side, and a classic PD controller on the virtual side, the overall closed loop transfer function of the system in the case of fix delays $H_{x1/F_h} : \mathbb{C} \times \mathbb{R}^2 \times \mathbb{R}_+ \mapsto \mathbb{C}$ is given by:

$$H_{x1/F_h}(s, K_p, K_d, \tau) = \frac{Q(s, K_p, K_d)}{P(s, K_p, K_d, \tau)}. \quad (8)$$

Due to the system variations some uncertainty Δ on the *nominal delay* value τ_0 may be taken into account. The uncertainty is considered bounded and it satisfy the constraint:

$$|\Delta| < \delta, \quad \delta > 0.$$

The delay τ can be written as $\tau = \tau_0 + \Delta$ and equation (6) rewrites as follows:

$$\overline{C_\Delta} = \frac{K_p + K_d s}{1 + (K_p + K_d s) SM(s)(1 - e^{-s\tau_0} + e^{-s(\tau_0 + \Delta)})}, \quad (9)$$

and the overall closed loop transfer function is described by the following equation (see [11], [17]):

$$H_{x1/F_h}(s,K_p,K_d,\tau,\Delta)=\frac{Q(s,K_p,K_d)}{P(s,K_p,K_d,\tau,\Delta)}, \quad (10)$$

where the characteristic equation is defined as follows:

$$\begin{aligned}P(s,K_p,K_d,\tau,\Delta) &= P_1(s,K_p,K_d)\\ &+ P_2(s,K_p,K_d)(e^{-\tau s}-e^{-(\tau+\Delta)s}).\end{aligned} \quad (11)$$

For more details regarding the stability regions for Smith predictors subject to delay uncertainty, please refer to [14].

In the rest of the paper, we will use $P(s,K_p,K_d,\tau,\Delta)$, considering $\Delta=0$ for the case with fix and known delays.

3. STABILITY ANALYSIS

The stability analysis of the system described by the characteristic equation (10) will be performed in two steps:

- **Step 1:** firstly, we consider that the delay value is perfectly known and constant (i.e. $\tau=\tau_0, \Delta=0$);
- **Step 2:** secondly, we consider the controller gains are fixed at step 1 and we derive the stability regions in the delay parameter space (τ_0,Δ) (i.e. both the nominal delay value and the uncertainty may vary).

For the brevity of the paper and without any loss of generality, we make the following:

Assumption 1: The polynomials P and Q are such that $\deg(\mathrm{Q})\leq\deg(\mathrm{P})$.

Assumption 2: The polynomial P does not have any roots at the origin, that is $P(0)\neq 0$.

Assumption 3: The polynomials P and Q do not have common zeros.

Assumption 4: The polynomials P and Q satisfy the following condition:

$$\lim_{s\to\infty}\left|\frac{Q(s,K_p,K_d)}{P(s,K_p,K_d,\tau,\Delta)}\right|<\frac{1}{2}.$$

For discussions on the implications of these assumptions the readers are referred to [8], [14], [5].

3.1 Stability in controller gains space

In the sequel, we recall some geometric results that enable us to generate the stability crossing curves in the space defined by the controller's parameters (K_p, K_d) (similar results for different types of dynamics can be found in [5] - delay parameters space and [12], [15] - some particular class of distributed delays). These curves represent the collection of all pairs (K_p, K_d) for which the characteristic equation has at least one root on the imaginary axis of the complex plain.

3.1.1 Stability regions

According to the continuity of zeros with respect to the system's parameters (see, for instance, [3] for the continuity with respect to delays), the number of roots in the right half plane (RHP) can change only when some zeros appear and cross the imaginary axis. Therefore, a useful concept is the frequency crossing set Ω defined as the set of all real positive ω for which there exist at least a pair (K_p, K_d) such that:

$$P(j\omega;K_p,K_d,\tau,\Delta)=0. \quad (12)$$

We only need to consider positive frequencies ω, that is $\Omega\subset(0,\infty)$ since obviously,

$$P(j\omega;K_p,K_d,\tau,\Delta)=0 \iff \overline{P(-j\omega;K_p,K_d,\tau,\Delta)}=0. \quad (13)$$

Proposition 1: For a given $\tau\in\mathbb{R}_+$ and $\omega\in\Omega\subset\mathbb{R}_+^*$ a corresponding crossing point (K_p, K_d) is given by the solutions of the following system:

$$\begin{cases}\Re\left(P(j\omega;K_p,K_d,\tau,\Delta)/_{s=j\omega}\right) &= 0,\\ \Im\left(P(j\omega;K_p,K_d,\tau,\Delta)/_{s=j\omega}\right) &= 0,\end{cases} \quad (14)$$

Remark 1: It is easy to see that $\forall\ \omega\in\Omega$ we have $Q(j\omega)\neq 0$. Otherwise, $Q(j\omega)$= 0, that contradicts Assumption 1.

Let $\Omega_{K_p^*,K_d^*}$ denotes the set of all frequencies $\omega>0$ satisfying (14) for at least one pair of (K_p, K_d) in the rectangle $\mid K_p\mid\leq\bar{K}_p, \mid K_d\mid\leq\bar{K}_d$. Then, when ω varies within some interval Ω_l satisfying (14) define a continuous curve. Denote $\mathcal{T}_l$ the curve corresponding to Ω_l, $\forall l\in 1,\ldots,\mathcal{N}$ and consider the following decompositions:

$$R_0+jI_0=j\frac{\partial H(s;K_p,K_d,\tau)}{\partial s}\Big|_{s=j\omega}, \quad (15)$$

$$R_1+jI_1=-\frac{\partial H(s;K_p,K_d,\tau)}{\partial K_d}\Big|_{s=j\omega}, \quad (16)$$

$$R_2+jI_2=-\frac{\partial H(s;K_p,K_d,\tau)}{\partial K_p}\Big|_{s=j\omega}. \quad (17)$$

The implicit function theorem indicates that the tangent of $\mathcal{T}_l$ can be expressed as follows:

$$\begin{pmatrix}\frac{dK_p}{d\omega}\\ \frac{dK_d}{d\omega}\end{pmatrix}=\begin{pmatrix}R_2 & R_1\\ I_2 & I_1\end{pmatrix}^{-1}\begin{pmatrix}R_0\\ I_0\end{pmatrix} =\frac{1}{R_2I_1-R_1I_2}\begin{pmatrix}R_1I_0-R_0I_1\\ R_0I_2-R_2I_0\end{pmatrix}, \quad (18)$$

provided that:

$$R_1I_2-R_2I_1\neq 0. \quad (19)$$

In order to derive the stability region of the system given by (8), [13] characterized the smoothness of the crossing curves and the corresponding direction of crossing.

Proposition 2: The curve $\mathcal{T}_l$ is smooth everywhere except possibly at the point corresponding to $s=j\omega$ is a multiple solution of (8).

3.1.2 Direction of Crossing

The next paragraph focuses on the characterization of the crossing direction corresponding to the curves defined by (14). We will call the direction of the curve that corresponds to increasing ω the *positive direction*. We will also call the region on the left hand side as we head in the positive direction of the curve *the region on the left*.

Proposition 3: Assume $\omega \in \Omega_l$, K_p, K_d satisfy (14), and ω is a simple solution of (12) and:

$$P(j\omega'; K_p, K_d, \tau, \Delta) \neq 0, \forall \omega' \neq \omega, \tag{20}$$

(i.e. (K_p, K_d) is not an intersection point of two curves or different section of a single curve). Then, as (K_p, K_d) moves from the region on the right to the region on the left of the corresponding crossing curve, a pair of solution of (8) crosses the imaginary axis to the right (through $s = j\omega$) if

$$R_1 I_2 - R_2 I_1 > 0. \tag{21}$$

The crossing is to the left if the inequality is reversed. Any given direction, (d_1, d_2), is to the left-hand side of the curve if its inner product with the left-hand side normal $\left(-\frac{\partial K_d}{\partial \omega}, \frac{\partial K_p}{\partial \omega}\right)$ is positive, i.e.,

$$-d_1 \frac{\partial K_d}{\partial \omega} + d_2 \frac{\partial K_p}{\partial \omega} > 0, \tag{22}$$

from which we have the following result.

Corollary 1: Let ω, K_p and K_d satisfy the same condition as Proposition 3. Then as (K_p, K_d) crosses the curve along the direction (d_1, d_2), a pair of solutions of (16) crosses the imaginary axis to the right if

$$d_1(R_2 I_0 - R_0 I_2) + d_2(R_1 I_0 - R_0 I_1) > 0. \tag{23}$$

The crossing is in the opposite direction if the inequality is reversed.

3.2 Stability in delay parameters space

Let us consider now that the controller gains are fixed $K_p = K_p^*$, $K_d = K_d^*$ and discuss the influence of delay parameters on the stability of the system. The following results are presented in [14]:

Proposition 1: The crossing set Ω consists of a finite number of intervals of finite length and it is determined by solving

$$\left|\frac{Q(j\omega, K_p^*, K_d^*)}{P(j\omega, K_p^*, K_d^*, \tau, \Delta)}\right| \geq \frac{1}{2}. \tag{24}$$

In what follows we use the notation $h(j\omega) = \frac{P(j\omega, K_p^*, K_d^*, \tau, \Delta)}{Q(j\omega, K_p^*, K_d^*)}$, $\tau_1 \triangleq \tau_0$, $\tau_2 \triangleq \tau_0 + \Delta$. For a given $\omega \in \Omega$ we may find the set $\mathcal{T}_\omega$ consisting of all the pairs (τ_1, τ_2) satisfying $H(j\omega, K_p^*, K_d^*, \tau_1, \tau_2) = 0$ as follows:

$$\tau_1 = \tau_1^{u\pm}(\omega) = \frac{\angle h(j\omega) + (2u-1)\pi \pm q}{\omega}, \tag{25}$$
$$u = u_0^{\pm}, u_0^{\pm} + 1, u_0^{\pm} + 2, ...$$

$$\tau_2 = \tau_2^{v\pm}(\omega) = \frac{\angle h(j\omega) + 2v\pi \mp q}{\omega}, \tag{26}$$
$$v = v_0^{\pm}(u), v_0^{\pm}(u) + 1, v_0^{\pm}(u) + 2, ...$$

where $q \in [o, \pi]$ is given by:

$$q(j\omega) = \cos^{-1}\left(\frac{1}{2|h(\omega)|}\right) \tag{27}$$

and u_0^+, u_0^- are the smallest integers (may be dependent on ω) such that the corresponding values $\tau_1^{u_0^+ +}$, $\tau_1^{u_0^- -}$ are nonnegative, and v_0^+ and v_0^- are integers dependent on u such that $\tau_2^{v_0^+ +} \geq \tau_1^{u+}$, $\tau_2^{v_0^- -} > \tau_1^{u-}$ are satisfied. The position in Figure 5 corresponds to $(\tau_1^{u+}, \tau_2^{v+})$ and the mirror image about the real axis corresponds to $(\tau_1^{u-}, \tau_2^{v-})$.

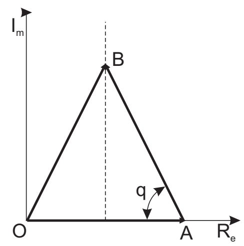

Fig. 5 Triangle formed by 1, $h(s)e^{-\tau_1 s}$ and $h(s)e^{-\tau_2 s}$.

If we define $\mathcal{T}^+_{\omega,u,v}$ and $\mathcal{T}^-_{\omega,u,v}$ as the singletons $(\tau_1^{u+}(\omega), \tau_2^{v+}(\omega))$ and $(\tau_1^{u-}(\omega), \tau_2^{v-}(\omega))$ respectively, then we can characterize $\mathcal{T}_\omega$ as follows:

$$\mathcal{T}_\omega = \left(\bigcup_{u \geq u_0^+, v \geq v_0^+} \mathcal{T}^+_{\omega,u,v}\right) \bigcup \left(\bigcup_{u \geq u_0^-, v \geq v_0^-} \mathcal{T}^-_{\omega,u,v}\right)$$

The set of stability crossing curves in delay parameter space is defined by:

$$\mathcal{T} = \bigcup_{k=1}^{N} \mathcal{T}^k, \quad \mathcal{T}^k = \bigcup_{\omega \in \Omega_k} \mathcal{T}_\omega \tag{28}$$

Remark 2: The distance between (τ_0, τ_0) and $\mathcal{T}$ is a measure of fragility of the controller (K_p^*, K_d^*) w.r.t. delay uncertainty.

4. FRAGILITY OF SMITH PREDICTORS

Based on [8], the main goal of this paper is to derive the biggest positive value d such that for a stabilizing controller with the Smith predictor built-in (K_p^*, K_d^*), the system is also stabilized by any pair K_p, K_d as long as:

$$\sqrt{(K_p - K_p^*)^2 + (K_d - K_d^*)^2} < d. \tag{29}$$

This problem can be more generally reformulated as: *find the maximum controller gains deviation* d *such that the number of unstable roots of (16) remains unchanged*.

First, let us introduce some notation:

$$\mathcal{T} = \bigcup_{l=1}^{\mathcal{N}} \mathcal{T}_l, \qquad \mathcal{T}_l = \{(K_p, K_d) | \omega \in \Omega_l\}, \tag{30}$$

$$\overrightarrow{k(\omega)} = (K_p(\omega), K_d(\omega))^T, \qquad \overrightarrow{k^*} = (K_p^*, K_d^*)^T, \tag{31}$$

where K_p^*, K_d^* are fixed.

Let us also denote $d_{\mathcal{T}} = \min_{l \in \{1,\dots,N\}} d_l$, where:

$$d_l = \min\left\{\sqrt{(K_p - K_p^*)^2 + (K_d - K_d^*)^2} | (K_p, K_d) \in \mathcal{T}_l\right\}. \tag{32}$$

With the notation and the results above, we have:

Proposition 4: The maximum parameter deviation from (K_p^*, K_d^*), without changing the number of unstable roots of the closed-loop equation (12) can be expressed as:

$$d = min\left\{K_{d\infty}, |K_p^* - K_p(0)|, \min_{\omega \in \Omega_f}\left\{\left\|\overrightarrow{k(\omega)} - \overrightarrow{k^*}\right\|\right\}\right\}, \tag{33}$$

where

$$K_{d\infty} := \begin{cases} \min\left\{\left|K_d^* - \left|\frac{q_n}{p_m}\right|\right|, \left|K_d^* + \left|\frac{q_n}{p_m}\right|\right|\right\} & \text{if} \quad m = n-1 \\ 0 & \text{if} \quad m < n-1 \end{cases}$$

where m, n represent the order of the polynomials P, Q and Ω_f is the set of roots of the function $f : \mathbb{R}_+ \mapsto \mathbb{R}$,

$$f(\omega) \triangleq \left\langle \left(\overrightarrow{k(\omega)} - \overrightarrow{k^*}\right), \frac{d\overrightarrow{k(\omega)}}{d\omega} \right\rangle \tag{34}$$

The explicit computation of the maximum parameter deviation d can be summarized by the following algorithm:

Step 1: First, compute the "degenerate" points of each curve $\mathcal{T}_l$ (i.e. the roots of $R_1 I_2 - R_2 I_1 = 0$ and the multiple solutions of (8)).

Step 2: Second, compute the set Ω_f defined by *Proposition* 4 (i.e. the roots of equation $f(\omega) = 0$, where f is given by (34)).

Step 3: Finally, the corresponding maximum parameter deviation d_l is defined by (32).

Remark 3: *(On the gains' optimization):* It is worth mentioning that the geometric argument above can be easily used for solving other robustness problems. Thus, for instance, if one of the controller's parameters is fixed (prescribed), we can also explicitly compute the maximum interval guaranteeing closed-loop stability with respect to the other parameter. In particular if K_d ("derivative") is fixed, we can derive the corresponding stabilizing maximum gain interval.

5. NUMERICAL EXAMPLES

We will consider a virtual environment and a haptic interface, figure 6, consists of one direct-drive motor and an optical quadrature encoder with 2000 pts/rev (with a gear ratio of 1/10). The controllers and the virtual simulation are running in real time mode (on RTAI Linux) with a sampling time of 1 ms.

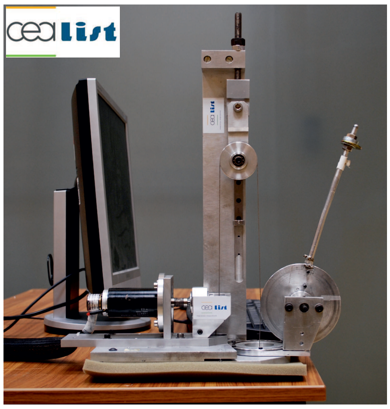

Fig. 6 Experimental Platform

The virtual object is modeled to be similar to haptic interface. The virtual wall which results in force environment F_e is defined by the following equation:

$$F_e = V_e = K_{wall}(x_2 - x_{wall}) + B_{wall}\dot{x}_2, \tag{35}$$

where $K_{wall} = 20000$ and $B_{wall} = 10$ represent the stiffness and damping used to compute the virtual force environment, x_{wall} is the virtual wall position and x_2, $\dot{x}_2$ are the virtual object position and velocity.

In figure 7 we present the stability region Γ for (K_p, K_d) with a fix time delay, $\tau = \tau_1 + \tau_2 = 100ms$. The stability zones correspond to frequency $w \in [0, 100]$.

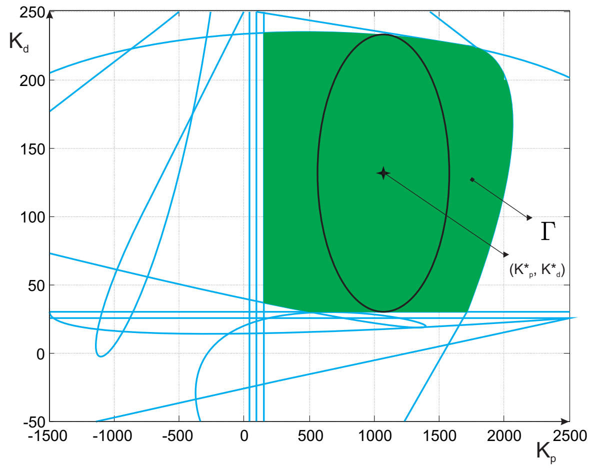

Fig. 7 Stability area for K_p and K_d - K_p^* = 1228 , K_d^* = 139 , d = 102.

Remark 4: According to the literature only the positive gains K_p and K_d must be considered.

Remark 5: It appears that we have a large choice of non-fragile controllers and we have chosen to represent graphically only the *"best" non-fragile one*.

In figure 8 we present the stability region Γ for (K_p, K_d) with a nominal time delay, $\tau_0 = \tau_1 + \tau_2 = 100ms$

and an uncertainty $\Delta = 25ms$. The stability zones correspond to frequency $w \in [0, 100]$.

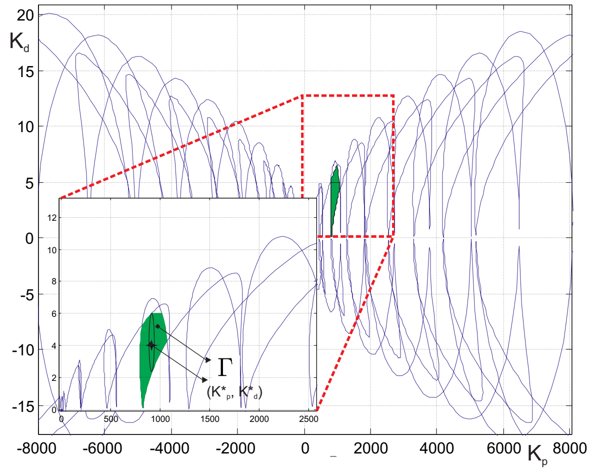

Fig. 8 Stability area for K_p and K_d - K_p^* = 915 , K_d^* = 4.08 , d = 1.98.

Remark 6: The choice of non-fragile controllers is smaller than the previous case, but still there is considerable interval of non-fragile gains. Similar to the previous case we have chosen to represent graphically only the *"best" non-fragile one*.

6. CONCLUSIONS

An ideal haptic system must have a *small* position tracking error in restricted motion and an *insignificant* force feedback (low viscosity i.e. high degree of transparency) in free motion.

In this paper, we have presented a simple method for analyzing the Smith predictors' fragility in the case of haptics. Furthermore, the choice of non-fragile controller is proposed by using some simple geometric arguments.

7. ACKNOWLEDGEMENTS

The work of Bogdan Liacu was financially supported by CEA LIST, Interactive Robotics Laboratory BP 6, 18 route du Panorama, F-92265 Fontenay-aux-Roses, France.

REFERENCES

[1] V. Alfaro. Pid controllers fragility. *ISA Transactions*, 46(4):555 – 559, 2007.

[2] O. David, Y.Measson, C. Bidard, C. Rotinat-Libersa, and F. X. Russotto. Maestro: a hydraulic manipulator for maintenance and decomissionning. *European Nuclear Conference (ENC), Bruxelles, Belgium*, 2007.

[3] L. El'sgol'ts and S. Norkin. *Introduction to Theory and Applications of Differential Equations with Deviating Arguments*. Academic Press: New York, 2006.

[4] F. Gosselin, C. Mgard, S. Bouchigny, F. Ferlay, F. Taha, P. Delcampe, and C. D'Hauthuille. A vr training platform for maxillo facial surgery. *Applied Human Factors and Ergonomics (AHFE) International Conference, Miami, Florida, USA, Advances in Cognitive Ergonomics*, 2010.

[5] K. Gu, S.-I. Niculescu, and J. Chen. On stability crossing curves for general systems with two delays. *Journal of Mathematical Analysis and Applications*, 311(1):231 – 253, 2005.

[6] L. Keel and S. Bhattacharyya. Robust, fragile or optimal? In *American Control Conference, 1997. Proceedings of the 1997*, volume 2, pages 1307 – 1313, jun 1997.

[7] A. Lecuyer, C. Andriot, and A. Crosnier. Interfaces haptiques et pseudo-haptiques. *Journees Nationales de la Recherche en Robotique*, 2003.

[8] B. Liacu, C. Mendez-Barrios, S.-I. Niculescu, and S. Olaru. Some remarks on the fragility of pd controllers for siso systems with i/o delays,. *4th International Conference on System Theory and Control, Sinaia, Romania*,, 2010.

[9] P. Makila, L. Keel, and S. Bhattacharyya. Comments on robust, fragile, or optimal? [and reply]. *Automatic Control, IEEE Transactions on*, 43(9):1265 –1268, sep 1998.

[10] C. Mendez-Barrios, S.-I. Niculescu, C.-I. Morarescu, and K. Gu. On the fragility of pi controllers for time-delay siso systems. In *Control and Automation, 2008 16th Mediterranean Conference on*, pages 529 –534, june 2008.

[11] W. Michiels and S.-I. Niculescu. On the delay sensitivity of smith predictors. *Int.J. of Systems Science*, 34(543-551), 2003.

[12] C. Morarescu. *Qualitative analysis of distributed delay systems: Methodology and algorithms*. Ph.D. thesis, University of Bucharest/Universite de Technologie de Compiegne, September, 2006.

[13] C. Morarescu, S. Niculescu, and K. Gu. On the geometry of pi controllers for siso systems with input delays,. *Proceedings of IFAC Time Delay Systems, Nantes, France*,, 2007.

[14] C. Morarescu, S. Niculescu, and K. Gu. On the geometry of stability regions of smith predictors subject to delay uncertainty,. *IMA Journal of Mathematical Control and Information*,, 24(3):411 – 423, 2007.

[15] C. Morarescu, S. Niculescu, and K. Gu. On the stability crossing curves of some distributed delay systems,. *SIAM J. Appl. Dyn. System*, 6:475 – 493, 2007.

[16] S.-I. Niculescu. *Delay effects on stability. A robust control approach*. Springer: Heidelberg, LNCIS, 2001.

[17] S.-I. Niculescu. *Delay effects on stability. A robust control approach*, volume 269. Springer: Heidelberg, Lecture Notes in Control and Information Sciences, 2001.

[18] R. Sipahi, S. Niculescu, C. Abdallah, W. Michiels, and K. Gu. Stability and stabilization of systems

with time delay. *Control Systems, IEEE*, 31(1):38
–65, feb. 2011.