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An MID-based Control of a Vibrating Axisymmetric Membrane Using Piezoelectric Transducers

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Summary. In this paper, the problem of the active vibration control of a thin and flexible disc is addressed. The mechanical structure tackled here is equipped with two piezoelectric circular patches: one of them works as a sensor and the other is used as an actuator. Both are fixed on the disc, one on each side, and centered according to its axis of symmetry. The model of this system is obtained from a finite element analysis, leading to a linear state space model. The design of the proposed control scheme is based on delayed proportional actions. As a matter of fact, recent works emphasized the stabilizing effect of delayed feedback if a real multiple spectral occurs in the closed-loop called multiplicity-induced-dominancy (MID) property allowing to an assignment approach. The purpose of this work is to investigate the properties of the proposed MID-based output feedback controller in terms of vibration damping.

Introduction

This work focuses on the effect of multiplicity of spectral values on the exponential stability of a sixth-order retarded differential equation. An efficient way to study time-delay systems solution's stability is the frequency domain approach since in the Laplace domain, where a number of effective methods have been proposed, the stability analysis amounts to studying the distribution of the characteristic quasipolynomial function's roots, see for instance [10, 6, 7]. It is worth noting that the rightmost root for a quasipolynomial function corresponding to stable time-delay systems is actually the exponential decay rate of its time-domain solution, see for instance [9] for an estimate of the decay rate for stable linear delay systems. The dominancy induced from of a given multiple spectral value property called in the sequel *multiplicity-induced-dominancy* or MID for short is studied and analytically shown in scalar delay equations in [4], then in second-order systems controlled by a delayed proportional is proposed in [2] where its applicability in damping active vibrations for a piezo-actuated beam is proved. An extension to the delayed proportional-derivative controller case is studied in [3, 5] where the dominancy property is parametrically characterized and proven using the argument principle. Further, in [1] some sufficient conditions are established showing that such MID property holds for arbitrary-order dynamical system.

Problem statement

System description

The system considered here is described in Fig. 1. It is a composite membrane composed by a brass disc with a clamped circular edge. This disc is embedded into a mobile support moving only along the z axis. The moving support is subjected to an unknown acceleration, noted $w(t)$ in the sequel. This flexible membrane is equipped with two PZT-based piezoelectric patches: one used as an actuator and the other used as a sensor. The sensor's thickness is $0.7mm$. It is greater than the actuator's thickness which is $0.4mm$. An explanation about why should the thickness of a piezo-electric sensor must be greater than the actuator's one can be found in [11]. These circular patches are supposed to be rigidly bounded on the disc, one on each side, and centered according to the axis of symmetry. All the physical parameters of the materials used here can be found in [8].

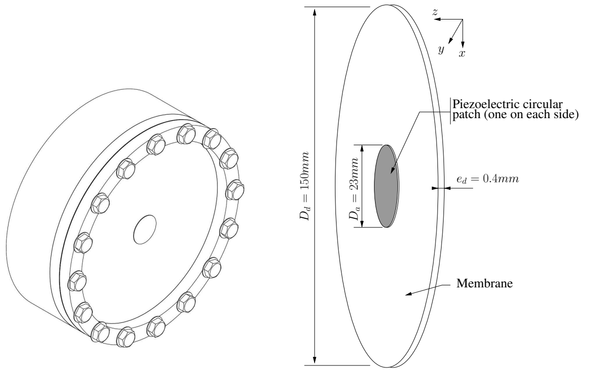

Figure 1: Axisymmetric composite membrane (dimensions on the right) inserted in the device which is subjected to vibrations (Computer Aided Design figure on the left)

The main dimensions of the composite membrane are given in Fig. 1. The controlled input noted $u(t)$ is the voltage applied across the piezoelectric actuator. The measured output noted $y(t)$ is the electric voltage delivered by the piezoelectric sensor. The disturbance input $w(t)$ is the total acceleration applied to the clamped circular edge of the structure. The controlled output that we consider, noted $z(t)$, is the z component of the acceleration of a point located at the center of the disc and on the upper side of the sensor.

The input-to-output transfer functions of finite order are derived from a Finite Element modelling of the axisymmetric membrane, following the steps described in [12].

Control approach

An interesting control objective is to damp the peaks of resonance of the first three bending modes, by using an *output feedback controller*, without affecting the vibrating modes that are neglected in the model considered here.

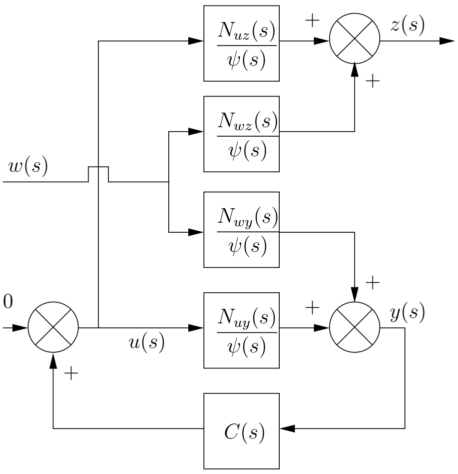

Figure 2: Feedback control structure.

By using the same notations as in [12], the piezo-actuated system is inserted in the output feedback control structure of Fig. 2, with a zero-reference signal and an input disturbance w corresponding to a rectangular impulse signal. The control problem consists in damping the vibrations due to the first three modes when the mobile support is subjected to a shock like disturbance. We define the output feedback control law $u(s) = C(s,\tau)\,y(s)$ involving the following Multiplicity-induced-dominancy (MID) controller given in Laplace domain by

$$C(s,\tau) := \frac{N(s,\tau)}{D(s,\tau)} \quad \text{where } N(s,\tau) := n_0 + n_{r_0}\, e^{-\tau s} \quad \text{and } D(s,\tau) := d_0 + d_{r_0}\, e^{-\tau s}. \tag{1}$$

By applying inverse Laplace transform, it can be easily shown that this control law is given in time domain by

$$u(t) = -\frac{d_{r_0}}{d_0} u(t-\tau) + \frac{n_0}{d_0} y(t) - \frac{n_{r_0}}{d_0} y(t-\tau) \tag{2}$$

which is an output feedback control law based on proportional actions plus delayed proportional actions.

Simulation results

The MID method gives the following numerical values for the parameters of the controller in (1) that assigns $\lambda_0 = -600$ as a rightmost root of multiplicity equal to 3: $n_0 \simeq 7.478025835$, $n_{r_0} \simeq 69.88393518$, $d_0 \simeq 1.626843813$, $d_{r_0} \simeq 5.858004955$ and $\tau \simeq 0.0001904171687$. To show the efficiency of the proposed MID-controller, we propose to compare, in Fig. 3, the time responses of both output signals in open-loop (blue) and in closed-loop (red) when the disturbance w is a rectangular impulse (black), say like a shock. We also put the time response of the control signal u that exhibits a peak of magnitude roughly equal to 12 V which is reasonable for this application.

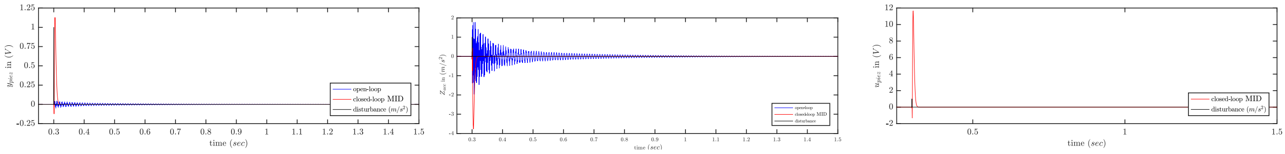

Figure 3: Time responses of the measured output y on the left, of the controlled output z on the middle and of the closed-loop control signal u on the right.

Conclusions

We have just shown a work dealing with the active vibration control of an axisymmetric membrane piezo-actuated, using closed-loop pole placement approach in order to design an efficient MID-based controller satisfying the control problem.

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