Recursive dynamics interconnection framework applied to angular velocity control of drilling systems

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***Abstract*— In this paper, we apply a recursive dynamics interconnection framework to achieve reliable angular velocity control of a multi-sectional drilling device with the bit off-bottom. The proposed model describing the torsional motion of the drill string takes into account the Coulomb friction between the drill string and the borehole. It has been field validated. Compensating the effects of these Coulomb side forces is necessary to avoid *stick-slip oscillations*. More precisely, the torsional motion of each section of the drilling pipe is modeled by a wave equation (with some non-linear terms corresponding to the Coulomb friction), while the dynamics of the top-drive and Bottom Hole Assembly are modeled by ordinary differential equations. The reference trajectory tracking problem for the drill string is solved by considering each section independently. The Coulomb side forces are seen as disturbances acting on each section of the drill string. These disturbances do not depend on time, providing there is enough energy in the system (i.e., that we have overcome the static friction). We propose here a switching-mode controller: using first a classical PI controller (with poor performance), we break the static friction. Then, the disturbance terms that correspond to the Coulomb friction terms can be estimated and compensated. The trajectory tracking is guaranteed using a *recursive control* procedure. The performances of the controller are illustrated through simulations using various field scenarios.**

I. INTRODUCTION

Due to the depletion of raw materials, extraction and drilling of oil or gas have become more and more challenging. To extract resources in deep hydrocarbon reservoirs, petroleum companies use narrow boreholes with often complex deviating well paths. The drill strings with extreme aspect ratios are usually made of many steel drill pipe sections, ending with a stiffer drill collar, often represented as a lumped element. Drilling involves many dynamic phenomena, in particular vibrations with many negative impacts. It has been pointed out that some unwanted oscillations were caused by the drill string interaction with the boreholes [16]. Among them, the most prevalent and destructive are torsional vibrations (*stick slip*) characterized by a sequence of *stick* (when the bit stops rotating) and *slip* (with a sudden release of energy) [17].

In order to design appropriate controllers to avoid this phenomenon, much research effort has been made to understand it better. It appeared it might be caused by down-hole conditions (such as significant drag, tight or formation characteristics [20]). However, since stick-slip also appears with the bit off-bottom, i.e., when there is no such bit-rock interaction, it is assumed that it may also be caused by *along-string* Coulomb-type frictions [12], [15], [23], which are not taken into account in most models. This is though of significant importance in the current context of long well-bores with high inclination.

In this paper, we consider the distributed model proposed in [4]. Unlike simpler models, [14], the torsional drill string dynamics are here modeled by a set of hyperbolic Partial Differential Equations (PDE), coupled with an Ordinary Differential Equation (ODE) representing the dynamics of the Bottom Hole Assembly (BHA). The Coulomb friction is modeled by a transition between dynamic and static torques and is represented by a differential inclusion that can be seen as a disturbance term acting on each section.

Regarding industrial applications, it is crucial, when starting up drill string rotation, to control the down-hole velocity of the drill string while avoiding undesired oscillations with a high Rate Of Penetration (ROP). Using the former understanding of stick-slip phenomenon, many controllers have been developed through the last decades. A wide variety of stick-slip mitigation controllers can therefore be found, Mostly correspond to high gains PI control laws, following the SoftSpeed and SoftTorque approaches. Even though they are easy to analyze and implement with low computational effort, they sometimes fail to compensate for the stick-slip oscillations efficiently. Some approaches proposed to add new compensating terms to the existing PI controllers [3], using the differential flatness of the system. More precisely, the reduction from static to dynamic friction is seen as a disturbance that is compensated, thus conforming to the canonical 3-DOF controller design for tracking and disturbance rejection. However, this type of controller has only been designed for uni-sectional drilling devices with a lumped BHA.

More recently, a new approach known as *recursive dynamics interconnection framework* has been introduced in [6]. This framework is particularly appealing for stabilizing the different sections of the drilling device. It is based on an iterative construction of the output feedback controller, using estimation and prediction of the boundary states. Following this approach, we can independently consider each section of the drilling device, leading to a more realistic representation of the wave propagation along the system. We can then apply this new framework to design a control law for the torsional motion of the drilling system.

More precisely, the contribution of this paper is to propose an innovative alternative control strategy and a simple estimator of the state for multi-sectional drilling devices. The Coulomb friction terms are taken into account for each section. Unlike existing designs (as the one proposed in [3]) that consider uni-sectional drill strings (the collar part being usually lumped with the BHA), our framework can be applied to a drilling device composed of an indefinite number of sections. The performances of the controller resulting from this approach are compared with controllers traditionally used by field engineers (Soft-Torque, Z-Torque).

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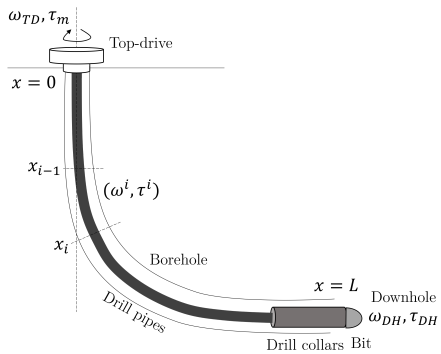

Fig. 1. Schematic drill string in a deviated borehole.

The layout of this paper is as follows. First, the model we use for our drilling system is detailed in Section II. We then design in Section III a new type of controller relying on a recursive interconnected dynamics framework. We present in Section IV an estimation and prediction method for the boundary states based on the same iterative technique. The performance of this approach is illustrated on a validated test case scenario in Section V. Finally, some concluding remarks end the paper in Section VI.

II. Torsional vibrations model

A distributed model representing the torsional motion of the multi-sectional drilling device is detailed below.

A. Distributed torsional dynamics

As illustrated in Fig.1, we consider a deviating drilling device of length L. The curvilinear abscissa is denoted $x \in [0, L]$. The torsional dynamics are represented using the popular distributed wave model [4], [13], and validated against field data.

1) Structural assumptions: We neglect the axial motion of the drill string, and only consider the torsional dynamics. The bit is off-bottom (no bit rock interaction). We neglect the effects of pressure differential along the drill string and the Stribeck curve, such that the transition from static to dynamic Coulomb friction is not continuous.

2) Multi-section drill string model: The discontinuities between the different sections of the drilling device, and, in particular, the junctions between the drill pipes and the drill collars are represented by a discontinuity in impedance. For practical reasons, the drill string is usually made up of interconnected pipes with different material properties (lengths, density, inertia or Young's modulus). These changes in the characteristic line impedance cause reflections in the traveling waves [1] that may be difficult to compensate. For any $N \in \mathbb{N}$, $i \in [\![0, N-1]\!]$, the spatial coordinate x_i corresponds to the junction between the $(i)^{\text{th}}$-section and the $(i+1)^{\text{th}}$-section. By convention, we have $x_0 = 0$, $x_N = L$. We use the superscript $\cdot^i$ to denote a variable or physical parameter related to section[1] i.

3) Angular motion: Denote by $\Phi(t, x)$ the angular displacement of the drill string. Define for $(t, x) \in [0, \infty) \times [0, L]$, $\omega(t, x) = \frac{\partial \Phi(t,x)}{\partial t}$ the angular velocity, and $\tau(t, x)$ the angular torque derived from the strain. It is given as the local relative compression $\tau(t, x) = JG\frac{(\Phi(t,x)-\Phi(t,x+dx))}{dx}$, with J the polar moment of inertia, and G the shear modulus.
Using the adequate Euler-Bernoulli model, we can derive the torsional dynamics under the assumption of elastic deformations and by using equations of continuity and state. For each section i of density ρ^i, we have

$$\frac{\partial \tau^i(t, x)}{\partial t} + J^i G^i \frac{\partial \omega^i(t, x)}{\partial x} = 0, \qquad (1)$$

$$J^i \rho^i \frac{\partial \omega^i(t, x)}{\partial t} + \frac{\partial \tau^i(t, x)}{\partial x} = S^i(t, x). \qquad (2)$$

The source term $S^i(t, x)$ models the frictional contact with the borehole (see below for a proper definition). Continuity of the angular velocity and torque are imposed at the boundary, such that $\tau^i(t, x_i) = \tau^{i+1}(t, x_i)$, $\quad \omega^i(t, x_i) = \omega^{i+1}(t, x_i)$.

4) Coulomb friction model: Following the approach of [4], the interaction with the borehole along section i is modeled by $S^i(t, x) \doteq -\mathcal{F}^i(t, x) - k_t \rho^i J^i \omega^i(t, x)$, where k_t is the viscous shear stress, and $\mathcal{F}^i(t, x)$ the along-side Coulomb friction between the drill string and the borehole. In practice, we can consider that the viscous shear stresses are negligible, such that for all sections $k_t \approx 0$. The function $\mathcal{F}$ is modeled by using the following inclusion

$$\begin{cases} \mathcal{F}(t, x) = \text{sign}(\omega(t, x)) F_d(x), & |\omega(t, x)| > \omega_c, \\ \mathcal{F}(t, x) \in [-F_s(x), F_s(x)], & |\omega(t, x)| < \omega_c, \end{cases}$$

where $F_d(x) \doteq r_o(x)\mu_k F_N(x)$ (resp. $F_s(x) \doteq r_o(x)\mu_s F_N(x)$) corresponds to the dynamic (resp. static) Coulomb torque, and where ω_c is the angular velocity threshold where the Coulomb friction transits from dynamic to static. They depend on the outer drill string radius $r_o(x)$, the static (kinetic) friction coefficient μ_s (μ_k) and the normal force acting between the borehole wall and the drill string $F_N(x)$. The function $\mathcal{F}(t, x) \in [-F_s(x), F_s(x)]$ denotes the inclusion where $\mathcal{F}(t, x) = -\frac{\partial \tau(t,x)}{\partial x} - k_t \rho J \omega(t, x) \in [-F_s(x), F_s(x)]$.
The expression of $F_N(x)$ can be deduced from the torque model presented in [22], [4], and, in our case study, is supposed to be known.Note that when $|\omega(t, x)| > \omega_c$, the alongside friction term $\mathcal{F}$ only depends on the space. It is therefore easier to estimate and reject the disturbance caused by the side-forces in this case.

B. Underactuated network of interconnected hyperbolic PDE–ODE systems

1) Actuated boundary (top-drive): In the sequel, subscript $\cdot_{TD}$ denotes variables in $x = 0$. The top-drive is actuated by an electrical motor and travels vertically up and down to impart torque to the drill string. The motor torque corresponds to the control input $U(t) = \tau_m(t)$. Its dynamics satisfy

$$\frac{d}{dt}\omega_{TD}(t) = \frac{1}{I_{TD}}(\tau_m(t) - \tau_{TD}(t)), \qquad (3)$$

where I_{TD} corresponds to the top-drive inertia.

2) Coupling with the lumped BHA (downhole): Similarly, we use subscript $\cdot_{DH}$ to denote variables in $x = L$. By assumption, the bit is off-bottom such that $\tau_{DH}(t) = 0$. Here, similarly to many drill string models, the lower part of the drill string or downhole assembly (made of the collars and the bit) is approximated as a single lumped element (due to its relative shortness and heaviness), with average inertia

[1] When referring to general variables, it may be omitted.

$I_{DH} = \rho_{DH} L_{DH} J_{DH}$. It has a major impact on the drill string dynamics. In addition, in view of future extensions, it is particularly interesting to consider such a PDE-ODE coupling at the downhole (as such kind of coupling naturally appears when adding a bit-rock interaction in the model, for instance). A force balance on the lumped BHA yields

$$\frac{d}{dt}\omega_{DH}(t) = \frac{1}{I_{DH}}(\tau_{DH}(t) - D(t)). \quad (4)$$

The term D accounts for the now lumped effect of the distributed source terms acting on the collars. We have $D(t) = \int_{\text{collar}} S(t,x)dx$.

3) Riemann invariants: We now rewrite the dynamics (1)-(2) in the form of a chain of hyperbolic PDEs. Define the standard *Riemann invariants* [19] by

$$\alpha^i = \omega^i + \frac{c_t^i}{J^i G^i}\tau^i, \quad \beta^i = \omega^i - \frac{c_t^i}{J^i G^i}\tau^i, \quad (5)$$

with $c_t^i = \sqrt{\frac{\rho^i}{J^i}}$ the velocity of the torsional wave on the i^{th}-section. These new variables satisfy

$$\frac{\partial \alpha^i(t,x)}{\partial t} + c_t^i \frac{\partial \alpha^i(t,x)}{\partial x} = \frac{S^i(t,x)}{J^i \rho^i}, \quad (6)$$

$$\frac{\partial \beta^i(t,x)}{\partial t} - c_t^i \frac{\partial \beta^i(t,x)}{\partial x} = \frac{S^i(t,x)}{J^i \rho^i}. \quad (7)$$

The continuity conditions now read as

$$\alpha^{i+1}(t,x_i) = a_1^i \alpha^i(t,x_i) + a_2^i \beta^{i+1}(t,x_i), \quad (8)$$

$$\beta^i(t,x_i) = a_3^i \alpha^i(t,x_i) + a_4^i \beta^{i+1}(t,x_i), \quad (9)$$

where $a_1^i = \frac{2}{1+Z^i}$, $a_2^i = \frac{Z^i-1}{1+Z^i}$, $a_3^i = \frac{1-Z^i}{1+Z^i}$, $a_4^i = \frac{2Z^i}{1+Z^i}$, and where $Z^i = \frac{c_t^i}{J^i G^i} / \frac{c_t^{i+1}}{J^{i+1} G^{i+1}}$ is the relative magnitude of the impedance. It corresponds to reflections of incoming waves from both sides. At the two ends, we have

$$\alpha^1(t,0) = -\beta^1(t,0) + 2\omega_{TD}(t), \quad (10)$$

$$\beta^N(t,L) = 2\omega_{DH}(t) - \alpha^N(t,L). \quad (11)$$

The boundary conditions (3)-(4) read

$$\dot{\omega}_{TD}(t) = \frac{1}{I_{TD}} U(t) - \frac{J^1 G^1}{c_t^1 I_{TD}}(\omega_{TD}(t) - \beta^1(t,0)), \quad (12)$$

$$\dot{\omega}_{DH}(t) = \frac{J^N G^N}{c_t^N I_{DH}}(\alpha^N(t,L) - \omega_{DH}(t)) - \frac{D(t)}{I_{DH}}. \quad (13)$$

To avoid useless case distinctions, we use the convention:

$$\alpha^0(t,0) = \omega_{TD}, \quad \beta^{N+1}(t,L) = \omega_{DH}, \quad (14)$$

$$a_1^0 = 2,\ a_2^0 = -1,\ a_3^0 = 0,\ a_4^0 = \frac{J^1 G^1}{c_t^1 I_{TD}}, \quad (15)$$

$$a_1^N = \frac{J^N G^N}{c_t^N I_{DH}},\ a_2^N = 0,\ a_3^N = -1,\ a_4^N = 2. \quad (16)$$

Using the method of characteristics, we obtain

$$\alpha^i(t,x_i) = \alpha^i(t - \frac{(x_i - x_{i-1})}{c_t^i}, x_{i-1}) + d^i(t), \quad (17)$$

$$\beta^i(t,x_{i-1}) = \beta^i(t - \frac{(x_i - x_{i-1})}{c_t^i}, x_i) + d^i(t), \quad (18)$$

where $d^i(t) = \int_{x_{i-1}}^{x_i} \frac{1}{J^i \rho^i c_t^i} S^i(t - \frac{s - x_{i-1}}{c_t^i}, s + x_i - x_{i-1})dx$. Thus, the effect of the Coulomb friction terms can be seen as disturbances acting at the different junctions. Note that if $|\omega| < \omega_c$ all over the drilling device, the terms d^i are constant.

C. Control objective and specifications

The objective of this paper is to design a torque control input $\tau_m(t)$ that regulates the downhole angular velocity $\omega_{DH}(t)$ at the beginning of a drilling operation while avoiding entering a stick-slip limit cycle. It can be seen as a classical linear disturbance rejection and tracking problem. Following the approach from [3], we can construct a reference trajectory $\omega_{ref}(t)$ using smooth functions, for instance mollified *bump* functions.

As previously mentioned, when the drilling device is in a slipping mode ($|\omega(t,x)| > \omega_c$), the estimation and rejection of the disturbance terms is facilitated. It motivates a *switching mode* control law, in which we first increase the torque to break the static friction. Then, we apply the recursive control procedure presented in this paper. The reference trajectory must ensure that the angular velocity stays in the controllable zone $|\omega(t)| > \omega_c$, such that the disturbance terms $d^i(t)$ are constant. Even though we consider herein that they are known, these terms can be estimated using the soft-sensor described in [2]. After the release of the BHA from the stick phase, the control objective consists in tracking the reference trajectory, i.e. we want $\omega_{DH} \to \omega_{ref}$.

III. Recursive dynamics interconnection framework

To reach our control objective, we use the recursive dynamics interconnection framework introduced in [6] and completed in [21] to design output-feedback controllers. In this approach, we consider each section of the drill pipe as an independent subsystem, for which we solve a stabilization and output tracking problem. Roughly speaking, the proposed control law is recursively obtained by considering for each subsystem a virtual input that guarantees that its output converges to the virtual input of the next subsystem. In the end, we ensure that the virtual input of the last subsystem guarantee $\omega_{DH}(t) \longrightarrow \omega_{ref}(t)$. More precisely, considering a section i, let us define the *virtual input* $\hat{V}_i(t)$, corresponding to the action of the $(i-1)^{th}$(upstream) section on this section.

A. Reference trajectory tracking for the downhole ODE

First, let us determine the virtual input $\hat{V}_{N+1} \equiv \alpha^N(t,L)$ guaranteeing the convergence of ω_{DH} to the reference trajectory ω_{ref}. With (16), equation (13) rewrites $\dot{\omega}_{DH}(t) = a_1^N(\hat{V}_{N+1}(t) - \omega_{DH}(t)) - \frac{D(t)}{I_{DH}}$. Then, we can define $\hat{V}_{N+1}(t)$ as

$$\hat{V}_{N+1}(t) \doteq \frac{1}{a_1^N}(\dot{\omega}_{ref}(t) + a_1^N \omega_{ref}(t) - K_D(\omega_{DH}(t) - \omega_{ref}(t))) + \frac{1}{a_1^N I_{DH}} D(t), \quad (19)$$

with $K_D > 0$ allowing the first term to stabilize the system without perturbation, while the second term cancels the effect of disturbance. We therefore ensure that $(\omega_{DH}(t) - \omega_{ref}(t))$ exponentially converges to zero.

B. Recursive output tracking for the N sections

Denote $t_i = \frac{x_i - x_{i-1}}{c_t^i}$ the transport time along the i^{th} section. Using the boundary conditions (8) in (17), we obtain $\alpha^i(t,x_i) = a_1^{i-1}\alpha^{i-1}(t-t_i,x_{i-1}) + d^i(t) + a_2^{i-1}\beta^i(t-t_i,x_{i-1})$. To ensure that $\alpha^i(t,x_i)$ tracks $\hat{V}_{i+1}(t)$, we define

$$\hat{V}_i(t) \doteq \frac{1}{a_1^{i-1}}(\hat{V}_{i+1}(t+t_i) - d^i(t+t_i)) - \frac{a_2^{i-1}}{a_1^{i-1}}\beta^i(t,x_i).$$

This paves the way to a recursive definition of the virtual inputs. Note that each $\hat{V}_i$ requires future values of $\hat{V}_{i+1}$ (and consequently, future values of the downstream section states). The causality of the control law will be guaranteed using state-predictors (described in Section IV).

C. Output tracking for the top-drive ODE

Iterating the procedure on the N sections, we go back to the first section, whose state is interconnected with the top-drive ODE (12). To get ω_{TD} converge to $\hat{V}_1(t)$, we define the control input U as

$$U(t) = I_{TD}(\dot{\hat{V}}_1(t) + a_4^0(\omega_{TD}(t) - \beta^1(t,0)) - K_0(\omega_{TD} - \hat{V}_1(t)). \quad (20)$$

For any $K_0 > 0$, the output $(\omega_{TD}(t) - \hat{V}_1(t))$ exponentially converges to zero. Therefore, using the recursive definition of the virtual inputs $\hat{V}_i(t)$, starting from the downhole, up to $\hat{V}_1(t)$, we obtain a control input $\tau_m(t)$ guaranteeing the tracking of the reference downhole angular velocity.

D. Limitations

The control law (20) features several important drawbacks. First, as previously mentioned, the recursive definition of the virtual inputs requires the knowledge of future values of the PDE states and at the end of the downhole ODE state. Then, since the definition of the virtual inputs relies on the inversion of the ODE dynamics, it may result in a control law with zero-delay margins. It would then be unusable on the field due to the inevitable delays in the sensors and mechanical transport times. Finally, if the disturbance terms $d^i(t)$ are unknown, we would need to estimate them (and predict future values if they are not constant).

To overcome these limitations, we propose in the next section an observer-predictor design. It provides future values of the different states of the system. To obtain a strictly proper control law (thus guaranteeing acceptable robustness margins), it is possible to low-pass filter the control input τ_m (see [7, Chapter 7] or [10] for details).

IV. State Estimation and Prediction

For sake of simplicity, we assume here that the disturbance terms are known and constant. Indeed, considering the switching-mode strategy aforementioned (sending a sufficient amount of energy to break the static friction before stabilizing the system aroung the reference trajectory), we can guarantee that $|\omega(t)| > \omega_c$ (see [5] for more details) such that the d terms are constant. However, to guarantee that the transient satisfies the condition $|\omega(t)| > \omega_c$, *control Barrier Functions* [18] may be of interest. Unlike simple control algorithms (as PID controllers), the recursive control law we design requires a real-time estimation of the distributed states all over the drill string, as well as a prediction of their future values. In this section, we propose an state observer state on the recursive dynamics interconnection framework presented in [21]. It will be combined with a state-predictor to design an output-feedback control law. Though the resulting controller will be computationally more expensive than classical control laws, taking into account the delays and high-frequency content should lead to better performances.

A. Boundary state estimation

Consider the PDE system (6)-(7). We assume that the friction coefficients μ_k, μ_s are known (see [8]). When $|\omega(t)| > \omega_c$, the disturbance terms are constant. It is then sufficient to know the boundary states $\alpha^i(t,x_i), \beta^i(t,x_{i-1})$ to estimate the whole distributed states (α^i, β^i).

1) Estimation of the boundary terms at the surface: At the surface, we measure the torque and top-drive angular velocity at high speed (from 20Hz to 100Hz). We obtain $\beta^1(t,0)$ (or its delayed values) from (12): $\hat{\beta}^1(t,0) = \frac{c_t^1 I_{TD}}{J^1 G^1}(\dot{\omega}_{TD}(t) - \frac{1}{I_{TD}}U(t)) + \omega_{TD}(t)$.

Then, we can compute an estimation of $\alpha^1(t,0)$ using (10): $\hat{\alpha}^1(t,0) = -\hat{\beta}^1(t,0) + 2\omega_{TD}(t)$.

To compute an accurate value of the derivative of the measurement and prevent the amplification of noise, we can use an adequate low-pass filter $w(s)$.

2) Estimation of the distributed states: Injecting (8)-(9) into the time-delay equations (17)-(18), we obtain

$$\alpha^i(t,x_i) = a_1^{i-1}\alpha^{i-1}(t - \frac{(x_i - x_{i-1})}{c_t^i}, x_{i-1}) + a_2^{i-1}\beta^i(t - \frac{(x_i - x_{i-1})}{c_t^i}, x_{i-1}) + d^i, \quad (21)$$

$$\begin{aligned}\beta^{i+1}(t,x_i) &= \frac{1}{a_4^i}\beta^i(t,x_i) - \frac{a_3^i}{a_4^i}\alpha^i(t,x_i) \\ &= \frac{1}{a_4^i}\beta^i(t + \frac{x_i - x_{i-1}}{c_t^i}, x_{i-1}) - \frac{a_3^i}{a_4^i}\alpha^i(t,x_i) \\ &\quad - d^i. \end{aligned} \quad (22)$$

Consider $i \in [\![1,N]\!]$ and assume that $\alpha^i(t,x_{i-1}), \beta^i(t,x_{i-1})$ are known on the time interval $[t, t+t_i]$. Then, there exist $\mathcal{L}_{\alpha^i}$ and $\mathcal{L}_{\beta^i}$, such that

$$\alpha^i(t,x_i) = \mathcal{L}_{\alpha^i}(\alpha^{i-1}(\cdot,x_{i-1}), \beta^i(\cdot,x_{i-1})),$$
$$\beta^{i+1}(t,x_i) = \mathcal{L}_{\beta^i}(\alpha^{i-1}(\cdot,x_{i-1}), \beta^i(\cdot,x_{i-1})).$$

The expressions of the two linear operators directly derive from (21)-(22).

Consequently, it is possible to get an estimation of delayed values of the boundary states $\alpha^i(t,x_i), \beta^{i+1}(t,x_i)$, knowing $\alpha^{i-1}(t,x_{i-1}), \beta^i(t,x_{i-1})$. The corresponding delay $\delta_i \doteq \sum_{j=1}^i t_j$ depends on the section we consider. Let us denote t_{tot} as the largest delay ($t_{tot} = \delta_N$). Define the t_{tot}*-delay operator* $\bar{\cdot}$ such that for any function γ, we have $\bar{\gamma}(t) = \gamma(t - t_{tot})$. We can then define $\hat{\bar{\alpha}}^i(t,x_i), \hat{\bar{\beta}}^{i+1}(t,x_i)$ the estimations of the t_{tot}-delayed states $(\bar{\alpha}^i(t,x_i), \bar{\beta}^{i+1}(t,x_i)$. These estimations are available on a time horizon $[t, t+t_{tot}-\delta_i]$.

3) Estimation of the downhole state: We can now estimate t_{tot}-delayed values of the downhole ODE using these estimations. Indeed, assuming that we have estimates $\hat{\bar{\alpha}}^N(t,L), \hat{\bar{\beta}}^N(t,L)$, we can define an estimation of the downhole angular velocity as $\omega_{TD}^{\hat{\bar{}}} = \frac{\hat{\bar{\alpha}}^N(t,L) + \hat{\bar{\beta}}^N(t,L)}{2}$.

B. State-prediction

So far, we designed a state-observer that provides a real-time estimation of the delayed ODE-states and of the delayed boundary states. We now combine these estimations with state predictors to obtain a real-time estimation of the undelayed states. Moreover, as the virtual control inputs $\hat{V}_i$ requires the knowledge of future values of the states, the predictors of the boundary states $\bar{\alpha}^i(t,x_i), \bar{\beta}^i(t,x_i)$, will give $t_{tot}+\delta_i$ ahead of time values of these delayed boundary states. We denote $P_{\bar{\beta}_i}(t,s)$ (resp. $P_{\bar{\alpha}_i}(t,s)$), the state prediction of $\bar{\beta}_i(t,x_{i-1})$ (resp. $\bar{\alpha}_i(t,x_i)$) ahead a time $t_{tot}+\delta_{i-1}$ (resp. $t_{tot}+\delta_i$) [21]. We define them as follows

$$P_{\bar{\alpha}_i}(t,s) = \begin{cases} \hat{\bar{\alpha}}_i(s+t_{tot}+\delta_i, x_i) \\ \qquad \text{if } s \in [t-3t_{tot}-\delta_i, t-t_{tot}-\delta_i] \\ a_1^{i-1} P_{\bar{\alpha}_{i-1}}(t,s) + a_2^{i-1} P_{\bar{\beta}_i}(t,s) + d^i \\ \qquad \text{otherwise,} \end{cases} \tag{23}$$

$$P_{\bar{\beta}_i}(t,s) = \begin{cases} \hat{\bar{\beta}}_i(s+t_{tot}+\delta_{i-1}, x_{i-1}) \\ \qquad \text{if } s \in [t-3t_{tot}-\delta_{i-1}, t-t_{tot}-\delta_{i-1}] \\ a_4^{i} P_{\bar{\beta}_{i+1}}(t,s-2t_i) + a_3^{i} P_{\bar{\alpha}_i}(t,s-2t_i) + d^i \\ \qquad \text{otherwise.} \end{cases} \tag{24}$$

Finally, the state-prediction $P_{\bar{\omega}_{DH}}(t,s)$ of the downhole ODE $\bar{\omega}_{DH}$ ahead a time δ_N is classically (see [9], [11]) defined for $s \in [t-2t_{tot}, t]$ by

$$P_{\bar{\omega}_{DH}}(t,s) = \begin{cases} \hat{\bar{\omega}}_{DH}(s+2t_{tot}) \text{ if } s \in [t-3t_{tot}, t-2t_{tot}] \\ \mathrm{e}^{-\frac{J^N G^N}{c_t^N I_{DH}} t_{tot}} (\hat{\bar{\omega}}_{DH}(s) \\ + \int_s^{s+t_{tot}} \mathrm{e}^{-\frac{J^N G^N}{c_t^N I_{DH}}(s-\nu)} (\frac{J^N G^N}{c_t^N I_{DH}} P_{\bar{\alpha}_N}(t,\nu-2t_{tot}) \\ - \frac{D}{I_{DH}}) d\nu) \text{ otherwise.} \end{cases} \tag{25}$$

These predictors are well-defined and causal. From these definitions, we immediately have

$$P_{\bar{\alpha}_i}(t,s) = \hat{\bar{\alpha}}_i(s+t_{tot}+\delta_i, x_i), \ s \in [t-2t_{tot}-\delta_i, t],$$
$$P_{\bar{\beta}_i}(t,s) = \hat{\bar{\beta}}_i(s+t_{tot}+\delta_{i-1}, x_{i-1}), \ s \in [t-2t_{tot}-\delta_{i-1}, t],$$
$$P_{\bar{\omega}_{DH}}(t,s) = \hat{\bar{\omega}}_{DH}(s+2t_{tot}), \ s \in [t-3t_{tot}, t].$$

The numerical values of the predictors are updated at each time step, using past values stored in a buffer. They are initialized with the estimations.

V. Simulation results

In this section, we illustrate the performances of our approach using simulated data representing field scenarios. The test case we consider a 3000m long drilling device composed of a vertical section $[0, L_c]$, and a lateral section $[L_c, L_c+L_p]$ with a 60° deviation (denoted **Well A** in [4]). The drilling device is made of two sections whose parameters are given in Table V. The controller gain defined in (19) (resp.(20)) satisfies $K_D = 2$ (resp. $K_0 = 10$).

The transport equations are solved with Matlab using a first-order upwind scheme, to ensure numerical robustness and avoid spurious oscillations. Indeed, higher-order schemes perform poorly due to the temporal discontinuities introduced by the distributed Coulomb friction. Initially, we have a null angular velocity and neglect eventual residual torque. We use a spatial grid of 500 cells for the drill string. Here, the purpose of the simulation is to compare the performances of two algorithms on an idealized case. Before implementation on real systems, the computational effort should be taken into consideration. The time-step is chosen to satisfy Courant-Friedrichs-Lewy (CFL) condition. Initially, the drill string is at rest. After 20s, we first change the velocity setpoint to 120 RPM with a transition time of 10s. After 60s, we change it back to 60 RPM with the same transition time.

Param.	Value	Param.	Value
A^1	0.005 m^2	A^2	0.01 m^2
J^1	2.28×10^{-5} m^4	J^2	1.49×10^{-4} m^4
G^1	6.1×10^{10} m	G^2	6.7×10^{10} m
ρ^1	7850 kg/m^3	ρ^2	7850 kg/m^3
L_p	1700 m	L_c	1300 m
I_{TD}	2900 kg.m^2	I_{DH}	152.9 kg.m^2
ω_c	1.5 rad s^{-1}	τ_{max}	30 kNm

A. Free drill string

Following the simulation scenarios given in [3], we first consider the case of a drill string spinning freely, and neglect the kinematic and static friction terms $\mu_k = \mu_s \equiv 0$. This scenario may correspond to the vertical part of **Well A**. On Fig.2 is represented the evolution of the angular velocity at the bottom of the pipe $\omega_{DH}(t) = \omega(t, L_p + L_c)$, for the control input $U(t)$ designed with the recursive approach (20), and for the classical Z-torque controller (see [3]). The two algorithms present similar behaviors (there is a slight overshoot for the Z-torque controller).

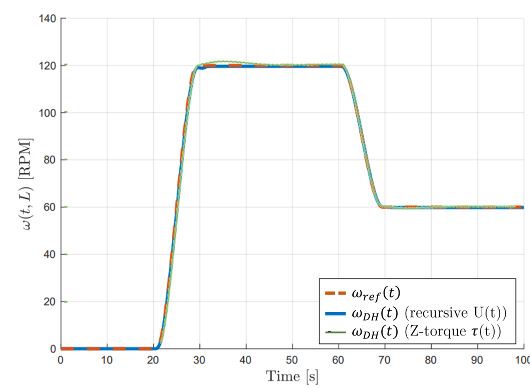

Fig. 2. Evolution of the downhole angular velocity.

B. Drill string with stick-slip

Next, we consider the real deviated **Well A**, where the side forces cannot be neglected anymore. They are assumed to be known without bias (using the estimation procedures given in [8] for instance). For sake of simplicity, we assume they are constant along the drill string $(\mu_s, \mu_k) = (0.45, 0.28)$. The controller we propose is however somehow robust to small discrepancies in the estimation, due to the effect of the filter. In practice, they may vary piecewise continuously (piecewise constant functions) with the type of rocks being drilled. This could be integrated in our model by considering more subsections. We apply the switching-mode controller [5], such that the torque is first increased to break the static friction before starting the control procedure. The time evolution of the downhole angular velocity is pictured in Fig.3 for both control strategies. With the Z-torque control law, $\omega_{DH}(t)$ exceeds the reference values during the transient and keeps oscillating. It fails taking into account the side forces (this is a consequence of the deviation of the well). Here, one can easily notice the importance of the feed-forward component and the disturbance cancellation term to reduce the oscillations induced by the Coulomb friction terms. With the new proposed controller, $\omega_{DH}(t)$ quickly converges to the reference trajectory as expected and has a smoother behaviour. A more detailed comparison of the controllers can be found in [5].

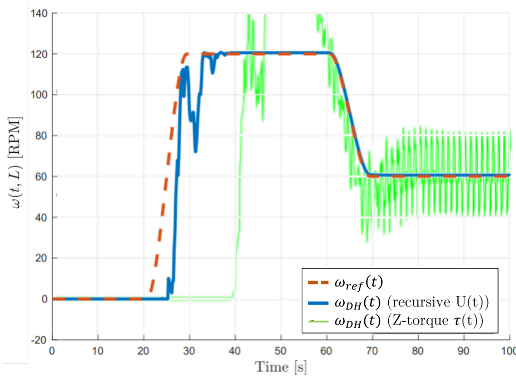

Fig. 3. Evolution of the downhole angular velocity.

VI. Concluding remarks

In this paper, we proposed an innovative controller design to stabilize the angular velocity of a multisectional drilling device. It is adapted for avoiding torsional stick-slip vibrations at the beginning of drilling operations. These stick-slip oscillations may occur due to the Coulomb side forces. The lower part of the drill string and the BHA are lumped into an ODE, and the resulting ODE-PDE-ODE system is rewritten in a time-delay form using Riemann invariants. The Coulomb friction terms can be seen as disturbances acting on this time-delay system. We proposed a switching mode control strategy. In the first mode, we break the static friction (using classical elementary control strategies and guaranteeing that the top drive velocity remains larger than ω_c) so that the disturbance induced by the side forces is now constant. Next, we use the recursive dynamics interconnection framework inspired by [21], associating estimations and predictions of the boundary states. The robustness of the resulting control law can be guaranteed using a low-pass filter.

The performances of our controller have been verified in simulations against a test case scenario. Compared to state-of-the-art PI controllers (soft-torque, Z-torque), the current control strategy shows better performance at the cost of an important computational effort. Indeed, the controller design requires computation of future values of the state (predictors), based on past values stored in buffer, and not direct integration of the output as for PI controllers. It would be of high interest to define several relevant specifications to compare the performances of the different controllers more precisely.

Moreover, the torsional motion of the drill string was assumed to be the dominating dynamic behavior. However, longitudinal or axial oscillations may also negatively impact the drilling device and should be taken into account to improve the quality of the model. Moreover, the parameters of the drilling device were assumed to be perfectly known (including the friction terms). Future works may focus on parameter estimation and the robustness of the proposed control law regarding errors in the system parameters.

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