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Abstract: In this work, we investigate the *human quiet stance regulation problem* using a single-link inverted pendulum model in the sagittal plane via the ankle joint's passive/active torques' actions. The active torque consists of ankle muscle contractions that are activated by the delayed action of the Central Nervous System (neural controller). The passive torque is related to the intrinsic mechanical properties of the muscle-tendon-ligament component. The failure of the human quiet stance is then directly related to the failure of one or both types of torques. We propose to model the neural controller as a delayed Proportional-Derivative-Acceleration controller acting on the ankle joint's angular position. By using the multiplicity-induced-dominancy property, the critical time delay of the motor control and the critical ankle-joint stiffness are both investigated.

Keywords: Human quiet stance regulation, Time-delay systems, stability and stabilization, multiplicity-induced-dominancy.

1. INTRODUCTION

The study of the control mechanism in charge of the equilibrium of human bipedal posture has attracted the attention of numerous researchers Winter (1995), Vette et al. (2010b), Fok et al. (2021a). Noting that the studies about postural stability are essential to improve the understanding of self-balance mechanisms of the human body. For instance, abnormal movements caused by neuromuscular diseases, such as Parkinson's disease, paroxysmal positional vertigo or sclerosis, result from abnormal muscle tone. Furthermore, maintaining the balance is a vital ability for humans: falls are the leading causes of accidental death and morbidity in the elderly, a fact which provides a strong motivation to understand the functioning of the quiet stance stability (Stepan, 2009), (Mihelj and Munih, 2004).

The degeneration of the balance control system in the elderly and their multiple pathologies have encouraged researchers to enhance the comprehension of the workings of the musculoskeletal system and the humans' management and organization of their stability in vertical stance, and their ability to maintain the human body. In this context, several analyses are based on the assumption that standing posture can be simplified as an inverted pendulum structure with one rotating degree of freedom about the ankle joint's axis in the sagittal plane (Morasso et al., 2019), (Sieber and Krauskopf, 2005), (Gage et al., 2004), (Winter et al., 1998). Hence, one of the main purposes of the control system during quiet stance is to afford the ankle joint's torque required to resist the gravity effect of the body and to guarantee that its center of mass remains close to the equilibrium position. For this purpose, one needs not only to establish a good model of the musculoskeletal system but also to analyze the control circuit, including the structural form of our *Central Nervous System* (CNS) instruction that controls the human motricity (Winters and Stark, 1987).
Actually, in a general way, the CNS generates neural commands to activate the muscles. The intern muscles' force combined with inertia and external forces, generates observable movements. The position, velocity and acceleration of the musculoskeletal system are measured and transmitted to CNS to close the loop with the required information to produce appropriate control decisions (Bortoletto et al., 2014), (El-Ati et al., 2020). However, there is a substantial time delay caused by the finite speed of signal propagation, and the performance of motor tasks is affected by the presence of time-delayed sensory feedback (Milton, 2011), (Begg and Palaniswami, 2006). Moreover, the intrinsic CNS functioning is complex. It is inherently a generator of high-dimensional and nonlinear dynamics. A substantial time delay of signal propagation in the nervous system should therefore be considered in the input signals. One intuitive but easy way to model such a CNS response is to identify it as a system of propagation, which is justified by the necessary lag-time for information to get through the neuronal axon. We refer the reader to Campbell (2007) for a summary of the different kinds of delays occurring in neural systems.

In the case of quiet stance, many approaches have been investigated for modeling the structure of the controller, as in (Asai et al., 2009) and (Tanabe et al., 2016). The most simple solution adopted by a number of researchers was a basic linear continuous-time feedback controller, based on proportional and derivative feedback (Fok et al., 2021b), (Fotuhi et al., 2020). In (Masani et al., 2006) and (Vette et al., 2010a), the authors consider that the ankle joint's torque required to control the body during a quiet posture can be generated in both passive and active ways. The passive torque component results from an intrinsic mechanical property: the stiffness and viscosity produced by the joint's viscoelasticity of the muscle-tendon ligament. On the other hand, the active torque component is generated by muscle contraction. This additional torque is controlled by the CNS, which stimulates contractions of the required muscles depending on the human body's sensory information about its kinematical and dynamical state, which are fed back to the CNS.

The mathematical models that consider the delay effects in the CNS are delay functional differential equations of infinite dimension. These delay effects may exhibit a complex dynamical behavior. It has been recently shown (see, e.g., Boussaada and Niculescu (2016a,b); Boussaada et al. (2018, 2019); Mazanti et al. (2020b,a)) that, for some quasipolynomials occurring in systems with time-delays, multiple real roots are often dominant, a property usually referred to as *Multiplicity-Induced-Dominancy* (MID for short). If, in addition, this multiple dominant root is negative, exponential stability is guaranteed. Moreover, a control-oriented MID approach was first proposed in Boussaada et al. (2019) for second-order delay equations and extended in Balogh et al. (2020) for general n^{th}-order linear time-invariant dynamical systems with a single delay. Indeed, it has been shown that under appropriate conditions, the MID property may assess the critical delay established in previous works (Boussaada et al., 2015; Boussaada and Niculescu, 2018; Boussaada et al., 2018). Inspired by recent results in Balogh et al. (2022b), Molnar et al. (2021) and Insperger et al. (2013) where the authors have illustrated that the feedback of acceleration, in addition to position and velocity feedback, allows for better regulate the human balancing, we consider in this paper a CNS modeled as a delayed Proportional-Derivative-Acceleration controller (PDA-controller for short). Moreover, it has been emphasized experimentally the beneficial effect of the acceleration in the feedback in describing the human balance Nataraj et al. (2012). Our main contribution relies on the use of the MID property to first control the active torque dynamic that stabilizes the quiet stance and then to quantify the critical values of two physiological parameters leading to a failure of the equilibrium: the critical time delay of the motor control system and the critical ankle-joint stiffness.

The paper is organized as follows. Section 2 introduces the model structure used to study the quiet stance body control. Section 3 presents the main results, that is, the stability of the system is analysed owing to the fulfilled MID property. Also, two cases of human balance failure related to a defect in the active torque and in the passive torque, respectively, are considered. Section 4 concludes this contribution.

2. COMPUTATIONAL MODEL OF THE QUIET STANCE CONTROL CONCEPT:

As shown in Fig. 1, the diagram of the inverted pendulum represents an approximation of the entire human body as a single rigid segment, excluding the feet that can rotate about the ankle joints. Let T_a be the torque acting at ankle joints produced by body muscles and θ be the absolute sway angle with respect to a fixed vertical reference. Thus, (1) is the motion equation of the inverted pendulum that models the movements of the human body around the quiet stance equilibrium.

$$J\,\ddot{\theta} = m\,g\,L\,sin(\theta) + T_a + \rho. \tag{1}$$

In (1), m and J stand for the mass and the moment of inertia for the human body above the ankle, respectively; L is the distance between the human body center of mass and the ankle; g is the acceleration due to gravity; ρ is the torque disturbance, which is assumed sufficiently small with respect to the other torques, as in Vette et al. (2010b).

Notice that the ankle joint's torque can be considered as the addition of a passive and an active component in the following way:

$$T_a = K\theta + B\dot{\theta} + f_A(\theta_\tau, \dot{\theta}_\tau, \ddot{\theta}_\tau), \tag{2}$$

where K and B are the passive stiffness and the passive viscosity parameters, respectively. The first two terms in the equation represent the passive feedback torques, with no time delay, related to the intrinsic mechanical impedance of the ankle joint. The active torque component, denoted by f_A, is generated by the contractile elements of the ankle muscles. This torque is regulated by the CNS via the feedback of the kinematics' and dynamics' neural information. It is determined as functions of delay affected tilt angle $\theta_\tau(t) := \theta(t-\tau)$, the angular velocity $\dot{\theta}_\tau(t) := \dot{\theta}(t-\tau)$ and the acceleration $\ddot{\theta}_\tau(t) := \ddot{\theta}(t-\tau)$ respectively. It is worth mentioning that there is a substantial time delay caused by the finite speed of neural signal propagation. Thus, the performances of motor tasks are affected by the presence of delayed sensory feedback (Milton, 2011). This time delay represents the time accrued to transmute information from the ankle's somatosensory system to the brain. It is estimated to be about 40 *ms* for people without any physiological abnormalities.

As marked in Fig. 1, we consider that the motor control time delay can be different than the feedback time delay. It is then placed within the neural controller, which also contains the CNS's action. Therefore, the neural controller is modeled as a delayed PDA controller. The delay affecting the motor control represents the time accrued to the CNS decision-making process and neural transmission from the CNS to the plantar flexor muscles. It is then considered a variable. This is a suitable way to study the effect of the CNS decision-making time delay caused by some diseases on the system's stability. Also, the estimation of the motor control delay is a difficult task and the exact value is not known (Masani et al., 2006). In the literature, the feedback time delay was estimated to be within the range of 35 to 40 *ms* (Applegate et al., 1988) and some researchers have used the same value for the motor control delay as a minimum physiological value, as in Masani et al. (2008).

Concerning the neuro-muscular contraction dynamic, it has been shown that the plantar flexor muscles generate the central part of the active ankle torque as they produce a continuous activity during a quiet stance; see (Winters and Crago, 2000) for more details. Since the muscle length change is very small during quiet stance, the models proposed to capture the isometric activation-force relationship have been used, specifically via a second-order low-pass system. It has been demonstrated in Masani et al. (2008) that a critically damped second-order model can successfully capture the ankle torque generation process during stance. The transfer function for the muscle contraction system is thus given by

$$G(s) = \frac{K_g\omega_n^2}{(s+\omega_n)^2} = \frac{K_g}{(T^2s^2+2Ts+1)}, \tag{3}$$

where K_g and ω_n are the gain and natural frequency of the second-order system respectively, and $T := 1/\omega_n$ is the twitch contraction time. Note that the twitch contraction time rep-

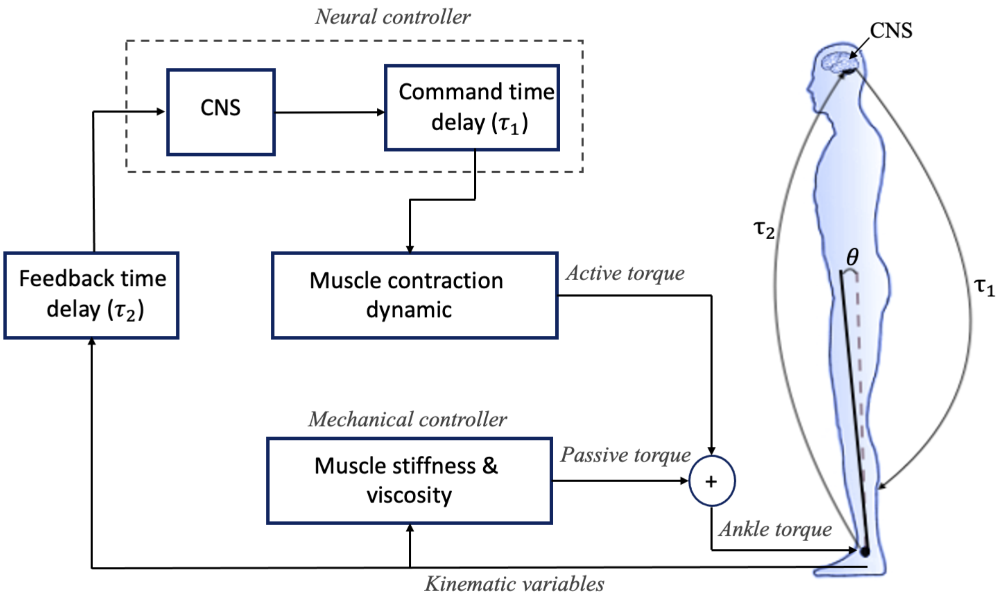

Fig. 1. Quiet stance control concept. The active torque is regulated by the neural controller via the body kinematics feedback and generated by the plantar flexor muscles' contractions, whereas the passive torque depends on the muscle-tendon mechanical properties (stiffness and viscosity). The command time delay is placed within the neural controller with the CNS's action. An inverted pendulum is used to model the quiet human body stance.

resents the time duration from the contraction initiation via a stimulus (or impulse) that reaches the muscle body to the peak of the twitch. In Masani et al. (2008), the authors give more details concerning the physiological interpretation and the system parameters identification.

Fig. 2 shows a block diagram of the model used in the theoretical study. The model consisted of a neural controller using a delayed PDA-controller with gains k_p, k_d and k_a. After this controller, we have inserted the critically damped second-order model to account for the ankle torque generation process that provides the active torque component. After this controller, we inserted the critically damped second-order model to account for the ankle torque generation process, that provides the active torque component. Then, the passive torque component is due to the effects of the stiffness K and the viscosity B of the muscle-tendon-ligament joint's viscoelasticity. Both parameters are related and connected to the constitutive mechanical impedance without any time delay. In the current study, we propose investigating the effect of muscle weakness in keeping standing balance by decreasing the stiffness gain K. Finally, we introduce the transfer function of an inverted pendulum, which is used as a model for the quiet body stance.

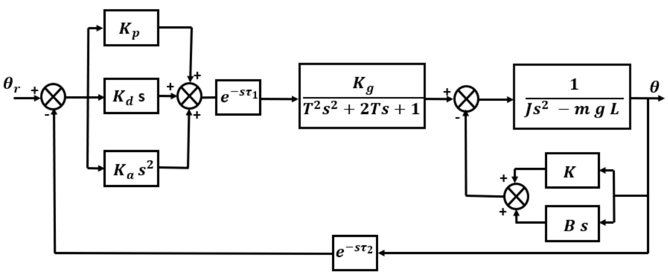

Fig. 2. The block diagram shows the computational model used in the theoretical study.

3. MAIN RESULTS

We use a parametric bio-mechanical model for the ankle joint system. The parameters are set to typical values of an adult, as reported in the literature (Masani et al., 2008), (Vette et al., 2010a), (Fok et al., 2021b). The corresponding numerical parameters are gathered in Table 1. The corresponding closed-

Parameters	value	Parameters	value
T	167 ms	J	55 $kg\,m^2$
K_g	58 Nm/V	L	0.846 m
B	5 Nms/rad	τ_2	40 ms
m	75 kg	g	9.8 m/s^2

Table 1. Numerical setting for a single inverted pendulum model and the muscle parameters

loop transfer function is given by:

$$TF(s,\tau) = \frac{\left(s^2k_a + sk_d + k_p\right) K_g\, \mathrm{e}^{-\tau_1 s}}{\left(s^2k_a + sk_d + k_p\right) K_g\, \mathrm{e}^{-s\tau} + \sum_{l=0}^{4} a_l\, s^l}, \tag{4}$$

where $\tau := \tau_1 + \tau_2$ and

$$\begin{cases} a_0 = K - mgL, \\ a_1 = 2\,(K - mgL)\,T + B \\ a_2 = (K - mgL)\;T^2 + 2BT + J \\ a_3 = (BT + 2J)\,T \\ a_4 = T^2 J. \end{cases}$$

To perform the stability analysis of the corresponding trivial solution, one investigates the characteristic function's Δ zero distribution;

$$\begin{aligned} \Delta(s,\tau) =& P_0(s) + P_1(s)\,\mathrm{e}^{-s\tau}, \\ =& \sum_{l=0}^{4} a_l\, s^l + \left(s^2k_a + sk_d + k_p\right) K_g\, \mathrm{e}^{-s\tau}. \end{aligned} \tag{5}$$

3.1 Nominal case: perfect running

In the first stage, we consider that both active and passive torques are perfectly running, guaranteeing thus an ideal human body's quiet stance. We assume that such a configuration is due to the fulfillment of the MID property.

Proposition 1. The following assertions hold:

i) For system parameters given in Table 1, $K, k_a, k_d, k_p \in \mathbb{R}$ and an arbitrary $\tau \ll 1$, the maximal multiplicity of roots of Δ is bounded by 5.

ii) A given complex number s_0 is a fifth-order root of Δ if, and only if, s_0 is real and it corresponds to a root of the elimination-produced-polynomial $\mathbb{P}$ with

$$\mathbb{P}(s) = \sum_{l=0}^{5} p_l\, s^l \tag{6}$$

such that

$$\begin{aligned}
p_0 =& B\tau^5 + 6(2BT+J)\tau^4 + 6(9BT+10J)T\tau^3 \\
&+ 24(4BT+9J)T^2\tau^2 + 72(BT+4J)T^3\tau \\
&+ 144JT^4, \\
p_1 =& 2(2BT+J)\tau^5 + 12(3BT+4J)T\tau^4 \\
&+ 36(3BT+8J)T^2\tau^3 + 48(2BT+13J)T^3\tau^2 \\
&+ 432JT^4\tau, \\
p_2 =& 2(3BT+4J)T\tau^5 + 36(BT+3J)T^2\tau^4 \\
&+ 18(3BT+22J)T^3\tau^3 + 408JT^4\tau^2, \\
p_3 =& 4(BT+3J)T^2\tau^5 + 12(BT+8J)T^3\tau^4 \\
&+ 168JT^4\tau^3, \\
p_4 =& (BT+8J)T^3\tau^5 + 30JT^4\tau^4, \\
p_5 =& 2JT^4\tau^5.
\end{aligned}$$

iii) If s_0 is real and is the rightmost root of $\mathbb{P}$ then s_0 is also the rightmost root of Δ. Furthermore, if s_0 is negative, then it corresponds to the exponential decay rate of the closed-loop system's trivial state solution.

Proof 1. i)-ii) First, observe the linear dependency of the quasipolynomial Δ in the left-free parameters K, k_a, k_d, $k_p \in \mathbb{R}$, considered as tuning parameters. So, solving the system of equations $\Delta(s) = 0$ and the first four derivatives $\Delta^{(1)}(s) = \ldots = \Delta^{(4)}(s) = 0$ for the variable s and parameters K, k_a, k_d, $k_p \in \mathbb{R}$ provides a coherent system. If additionally, one considers the fifth-order derivative then the corresponding delay is necessarily greater than one, which is inconsistent with delays corresponding to the motor control time delay. One exploits again the linear dependency of the quasipolynomial in the left-free parameters K, k_a, k_d, $k_p \in \mathbb{R}$. An elimination procedure is set allowing to transform the problem of solving a system of quasipolynomial functions into a polynomial system of equations. As a matter of fact, the vanishing of Δ for $s = s_0$ permits the elimination of the exponential term in this fashion

$$e^{-s_0\tau} = -\frac{P_0(s_0)}{P_1(s_0)}.$$

Substituting this last equality in the system of equations consisting of the first four derivatives $\Delta^{(1)}(s_0) = \ldots = \Delta^{(4)}(s_0) = 0$ results in the expressions of K, k_a, k_d, k_p as well as the corresponding elimination polynomial $\mathbb{P}$.

iii) One easily checks that the open-loop characteristic polynomial P_0 is real rooted for the parameters' values given in Table 1. So that, one uses the result from Balogh et al. (2022a) to show that s_0 corresponds to the exponential decay rate of the closed-loop system solution.

The nominal numerical values of the controller's gains are provided in Table 2. Fig. 3 illustrates the validity of the MID property in the case of a perfect behavior of the active and the passive torques. This nominal configuration gives the gains K^*, k_p^*, k_d^*, k_a^* and $\tau^* = 80$ *ms* which will be considered in the remaining analysis.

Parameters	value	Parameters	value
k_p^*	$-107.65\ Nm/rad$	k_d^*	$-36.51\ Nms/rad$
k_a^*	$-3.08\ Nms^2/rad$	K^*	$9882.21\ Nm/rad$

Table 2. Numerical values of the controller's gains

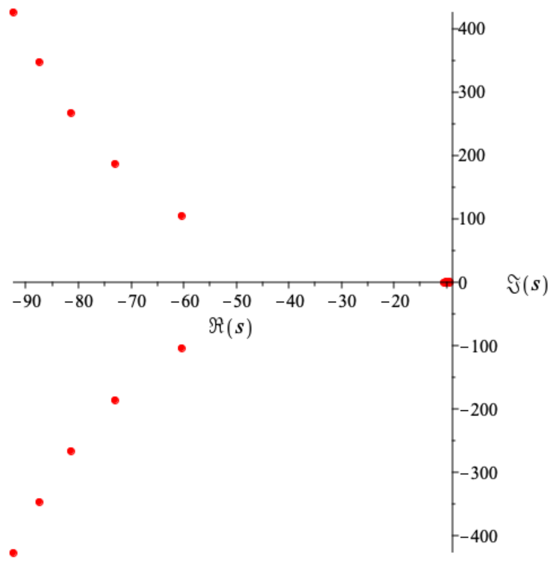

Fig. 3. Spectrum distribution for the closed-loop system in the perfect behavior case of both active and passive torques. The corresponding spectral abscissa is given by $s_0 \approx -9.85$.

3.2 Nerve impulse failure: the critical delay

This section considers the effect of the reaction-time increasing with respect to the nominal behavior on the human balance. More clearly, we investigate the impact of an increase of the delay $\tau := \tau^* + \sigma$ on the spectrum distribution of the closed-loop system, where σ is the amount of reaction-time increasing. Indeed, the analysis concerns the spectrum distribution of the parametric quasipolynomial given by

$$\begin{aligned}
\Delta(s,\sigma) =& \left(s^2 k_a^* + s k_d^* + k_p^*\right) K_g\, e^{-s(\tau^*+\sigma)} \\
&+ JT^2 s^4 + (BT+2J)Ts^3 \\
&+ \left((K^*-mgL)T^2 + 2BT + J\right)s^2 \\
&+ (2T(K^*-mgL)+B)\,s \\
&+ K^* - mgL,
\end{aligned} \tag{7}$$

with respect to the variation of the parameter σ. Fig. 4 illustrates the splitting mechanism enabling the spectrum branches to cross the imaginary axis. This corresponds to a Hopf point and the inception of a periodic orbit. The corresponding critical delay is $\tau_{crit} := \tau^* + \sigma_{crit} \approx 0.19\ s$.

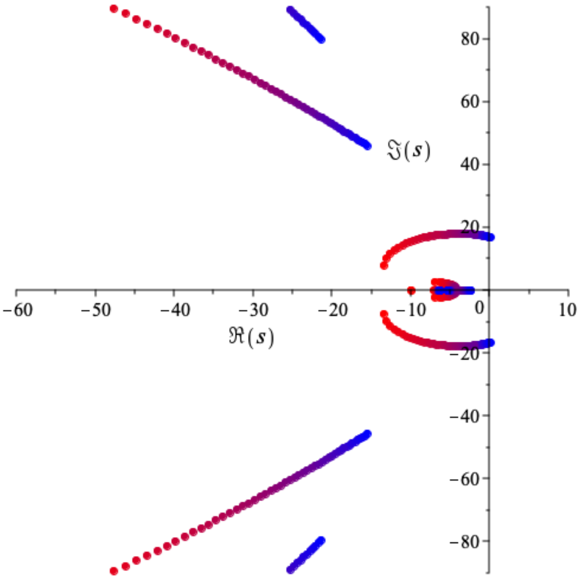

Fig. 4. Effect of the increase of the motor delay on the spectrum distribution of the closed-loop system. The splitting of the fifth-order root and crossing of the imaginary axis.

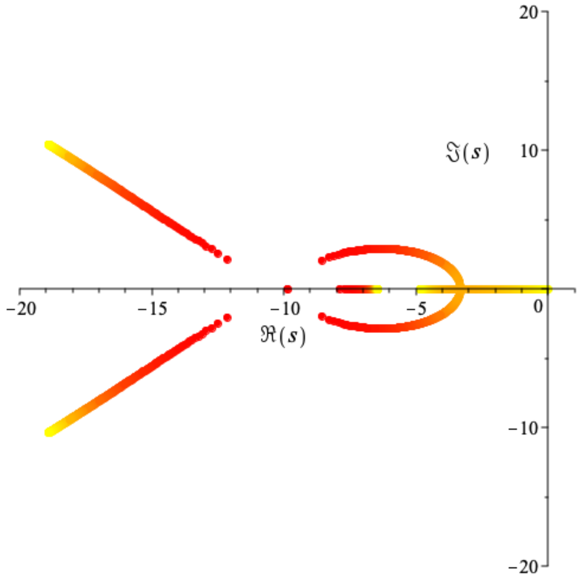

Fig. 5. Effect of the decrease of the ankle joint stiffness on the spectrum distribution of the closed-loop system. The splitting of the fifth-order root and the crossing of the imaginary axis.

3.3 Muscular-Skeleton failure: the critical ankle joint stiffness

This section deals with the effect of the ankle joint stiffness decreasing with respect to the nominal behavior on the human balance. More precisely, the analysis carries on the spectrum distribution of the parametric quasipolynomial given by

$$\begin{aligned}\Delta(s,\varepsilon) = &\left(s^2 k_a^* + s k_d^* + k_p^*\right) K_g \, \mathrm{e}^{-s\tau^*} \\ &+ J T^2 s^4 + (BT + 2J)\, T s^3 \\ &+ \left((K^* - \varepsilon - mgL)\, T^2 + 2BT + J\right) s^2 \\ &+ \left(2\,(K^* - \varepsilon - mgL)\, T + B\right) s \\ &+ (K^* - \varepsilon - mgL),\end{aligned} \tag{8}$$

with respect to the variation of the parameter ε. Unlike the crossing of the imaginary axis caused by nerve impulse failure, yielding periodic orbits, ankle-joint stiffness failure ensues the crossing through the origin, exhibiting some fold-bifurcation. Fig. 5 illustrates the splitting mechanism enabling the spectrum branches to cross the imaginary axis. This corresponds to a crossing at the origin of the complex plane. The corresponding critical stiffness is $K_{crit} := K^* - \varepsilon_{crit} \approx 6872\ Nm/rad$.

Remark 1. At critical values of the stiffness K_{crit}, the imaginary axis crossing occurs along the real axis. It corresponds to a loss of BIBO stability, and could be interpreted as a loss of stiffness due to muscular actions on the ankle, leading to a collapse of the quiet standing. On the other hand, the critical transmission delay τ_{crit} occurs along the imaginary axis crossing with a non vanishing crossing frequency, resulting in a periodic motion of the human body around the human stance equilibrium.

4. CONCLUDING REMARKS

A fourth-order model of the *human quiet stance regulation problem* based on the passive/active torques action on the ankle joint has been considered. We have assumed that the healthy functioning of the human quiet stance regulation is enabled by the fulfillment of the MID property. Its failure is then directly related to the defect in one or both types of torques. The critical motor control delay, *ie* the one inducing the limit of the stance's stability, as well as the critical ankle-joint stiffness have both been investigated and estimated. At these critical values, we have exhibited that the failure of the active torque induces a crossing through a Hopf-point and the inception of periodic orbit, while the failure of the passive torque results in a crossing at the origin of the complex plane generating a potential fold-bifurcation. The proposed modeling should be extended in the three-dimension case and completed with experimental validation in future works.

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