Control, optimization and games for networked dynamical systems

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par

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I would also like to thank the PhD students and Master students with whom I had the opportunity to work with Jomphop Veetaseveera, Tiago Goncalves, Bikash Adhikari, Emmanuel Kravitzch, Olivier L. de Silva, Amine Hazzami, Etienne Gorski, Sylvie Chadad, Yeh Yu, Hassan Mohamad and Thomas Mongaillard.

It's a pleasure to work at CRAN with people such as Jamal Daafouz, Jérôme Lohéac, Jeremie Kreiss, Marc Jungers, Pierre Riedinger, Gilles Millérioux, Christine Pierson, Didier Maquin etc.

Resumé

Ce manuscrit est divisé en quatre chapitres. Le premier est un résumé détaillé. Il commence par présenter brièvement mon CV et une courte biographie, suivi d'un résumé de mes activités de recherche, de mes collaborations, de l'encadrement d'étudiants, de mes autres responsabilités, de mes projets de recherche et enfin d'une liste complète de mes publications.

Le deuxième chapitre est un résumé de mes travaux dans le domaine des systèmes de contrôle en réseau. La principale motivation du travail que j'ai effectué dans ce chapitre est d'étudier l'impact des effets tels que les pertes de paquets stochastiques et d'autres limitations du réseau de communication sur un système de contrôle, c'est-à-dire lorsque les mesures sur les entrées de contrôle sont communiquées à l'aide d'un réseau sans fil peu fiable. . De nombreux travaux sur la théorie du contrôle supposent des modèles idéaux et souvent déterministes pour le canal sans fil. Par exemple, même en considérant les abandons de paquets, des hypothèses sont faites sur le nombre maximum d'abandons de paquets consécutifs. L'une des principales contributions de mon travail a été de considérer des modèles de canaux sans fil réalistes, puis de fournir des conditions de stabilité et des garanties de performances pour le système de contrôle en réseau. D'autres orientations incluent des modèles de communication réalistes pour le peloton de véhicules, le contrôle multi-agents avec une structure de communication spécifique et le contrôle de robots avec des objectifs de communication.

Le troisième chapitre est une synthèse de mes recherches sur l'analyse et le contrôle des systèmes dynamiques interconnectés. Chaque section de ce chapitre est consacrée à un axe de recherche spécifique que j'ai effectué sur les modèles de dynamique d'opinion, les stratégies de marketing viral, le contrôle des épidémies et l'étude des processus de financement participatif. J'ai utilisé les chaînes de Markov, l'optimisation, la théorie des jeux et les processus de décision de Markov pour les recherches de ce chapitre.

Le dernier chapitre offrira quelques perspectives sur mes recherches futures.

Abstract

This manuscript is partitioned into four chapters. The first one is an extended resume. It starts by briefly presenting my CV and short biography, followed by a summary of my research activities, my collaborations, supervision of students, other responsibilities, research projects, and finally a full list of my publications.

The second chapter is a summary of my work in the domain of networked control systems. The main motivation for the work I have done in this chapter is to study the impact of effects like stochastic packet drops and other limitations in the communication network on a control system, i.e., when the measurements on control inputs are communicated using an unreliable wireless network. Many works in the state of art on control theory assume ideal and often deterministic models for the wireless channel. For example, even when considering packet drops, assumptions are made on the maximum number of consecutive packet drops. One of the main contributions of my work was to consider realistic wireless channel models and then provide conditions for stability and performance guarantees for the networked control system. Other directions include realistic communication models for vehicular platooning, multi-agent control with a specific communication structure, and the control of robots with communication objectives.

The third chapter is a summary of my research on the analysis and control of inter-connected dynamical systems. Each section in this chapter is dedicated to a specific line of research I have done on opinion dynamics models, viral marketing strategies, control of epidemics, and a study of crowd-funding processes. I have used Markov chains, optimization, game theory, and Markov decision processes for the research in this chapter.

The final chapter will provide some perspectives on my future research.

Contents

Chapter 1

Introduction

1.1 Organization and summary of scientific activities

My scientific activity ever since I joined CRAN as a CNRS researcher has been centered around dynamical networks and can be classified into two themes:

- Networked control of systems (NCS) with constrained or uncertain communication
- Analysis, control and games on interconnected dynamical systems

Some of the work that I have done may not fit into either of these themes and will not be presented in detail, but the related papers can still be read online on HAL. The first chapter of this thesis will be a long CV and a presentation of my scientific career.

In the theme of NCS, I combine the expertise I had during my PhD, i.e., on the design of energy-efficient wireless communication systems, with the expertise of CRAN on control systems. In typical communication systems, the objective is to maximize or satisfy a certain target data rate while minimizing the energy consumed by adjusting the signal transmission power. However, in control systems, the communication has a very specific purpose, which is to aid the controller.

Therefore, we solve energy optimization problems with a constraint imposed by the control objectives. The challenge here was to account for the stochastic nature of communications and identify relevant metrics of control performance that can be mathematically linked with the communication parameters. Further directions and details on networked control systems are provided in Chapter 2.

In the context of opinion dynamics, we have proposed new models that can capture several features ignored in the literature. This model, although based on consensus as a driving force, under certain network structures results in opinion polarization, which is also observed in reality. We have also studied the problem of marketing resource allocation when a certain company (or two competing companies) tries to sell their product by marketing over a digital social platform. Further directions and details on this theme are provided in Chapter 3.

The fourth chapter will give my perspectives on future research and conclude this HDR thesis.

1.2 Curriculum vitae

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Google scholar: https://tinyurl.com/yckrxxwy

Professional experience

- **Oct.2016 – present**, CNRS researcher (CR) at Centre de Recherche en Automatique de Nancy.
- **Nov.2015 –Sep. 2016**, Post-doctoral research fellow at Centre de Recherche en Automatique de Nancy.
- **Aug.2014 –Oct.2015**, Post-doctoral research fellow at Singapore University of Technology and Design.

Education

- **Dec.2010-Jun.2014**, PhD in Physics from Laboratoire des Signaux et Systèmes, Supélec, Gif-sur-Yvette. "On the energy-efficiency optimization of wireless networks" which was funded by Orange Labs (CIFRE). Defended on June 20, 2014. My PhD advisors were Prof. Samson Lasaulce and Prof. Merouane Debbah.
- **2009-2010**, Master of Science (Optics) from Warsaw University of Technology,Poland.
- **2008-2009**, Master of Science (Photonics) from Friedrich-Schiller-Universitat, Germany.
- **2005–2008**, Bachelor of Physics, from Chennai Mathematical Institute, India.

Publication record: 31 peer-reviewed journals articles, over 40 international (peer-reviewed) conference papers, 1 book chapter and 5 patents.

Supervision: 7 ongoing PhD student supervision and 2 PhD students who defended their thesis, and 4 Master's internship (M2).

Distinctions: 1 Best student paper award for the paper presented in the 6th International Conference on Performance Evaluation Methodologies and Tools (2012).

Other professional activity: Reviewer and participant of several top journals and conferences in the fields of automatic control and telecommunications like IEEE Trans. on Automatic Control, IEEE Conference on Decision and Control, IEEE Control System Letters, IEEE Trans. on Control of Network Systems, IEEE Transactions on Wireless Communications, Automatica, European Control Conference, International Journal of Control, Automation and Systems, American Control Conference and IEEE Transactions on Vehicular Technology.

1.3 Collaborations

1.3.1 National collaborations

1. Prof. Salah Eddine Elayoubi (L2S, CentraleSupelec). He was involved during my PhD since 2011 and we have been collaborating since. Now, we work on control of vehicle platoons with imperfect communications and have co-supervised a PhD student and applied for an agence nationale de la recherche (ANR) grant. (2 journals and many conference papers)

2. Prof. Antoinne Berthet (L2S, CentraleSupelec). We started working together on co-supervising a PhD student in the framework of an ANR project.

3. Prof. Elena Panteley (L2S, CentraleSupelec). We started collaborating on the CEFIPRA project we are a part of and have published a conference paper and we have a journal paper to be submitted soon.

4. Dr. Yezekael Hayel (University of Avignon). We work on opinion dynamics modelling and control using continuous Markov chains. (2 conferences since 2017)

1.3.2 International collaborations

1. Prof. Dragan Nesic (University of Melbourne) since 2017 on networked control systems. He visits CRAN every year and I have also visited him in Melbourne in December 2017 (before CDC) and in 2019. We have 1 journal paper and 2 conference papers together with ongoing works under review.

2. Prof. Lucian Busoniu (Technical University of Cluj-Napoca, Romania) since 2017 on multi-agent control and learning. I visited his lab in 2017, 2018, and 2019 and we have 2 conference papers and 2 journals published in common.

3. Dr. Andre Oliveira and Prof. O.L.V Costa (University of Sao Paolo, Brazil) on Markov jump linear systems. Dr. Olivera visited CRAN for 1 year(2017-2018) during which we worked together and have published 1 conference paper and 2 journal papers.

4. Prof. Srikant Sukumar and Ravi Banavar (Indian Institute of Technology - Bombay, India). We have started collaborating in the framework of a new CEFIPRA project. I visited IIT-Bombay in this framework in March 2019 but further collaboration was paused due to the Covid restrictions on travel.

5. Prof. Daniel Quevedo (University of Queensland), since 2019 on stochastic non-linear control systems. We invited him to CRAN in 2019 and since then started a fruitful collaboration which has resulted in a CDC 2020 paper, an IEEE TAC paper and an IEEE L-CSS paper.

6. Prof. Maurice Heemels (Eindhoven University of Technology), since 2019 on noisy control systems. He visited CRAN in 2019 and in 2020 where we managed to start a collaboration that resulted in an ECC conference paper.

1.4 Student supervision and other responsibilities

1.4.1 PhD supervision as co-directeur de thèse

I am currently co-supervising four PhD students.

1. Emmanuel Kravitzch, with Yezekayel Hayel from the University of Avignon and Antoinne Berthet from Centrale-Supelec.
   Title: Opinion dynamics over adaptive networks
   Networks in general, both in science and nature, are generated through evolutionary processes, and are, therefore, fundamentally dynamic. Two types of dynamics drive the evolution of individual states. First, network topology changes as nodes and edges appear and disappear. Second, each node state is influenced by flows of information, energy, matter, or opinions received from their neighbors. Often, local processes confined to the individual nodes drive both types of dynamics and global network structure emerges from the combination of many local interactions. One important question is to understand the relationships between the dynamics of processes over the network, the network structure, and its function. He worked on building and studying a voter model in which agents may break and/or form links with other agents depending on their opinions, and on bounded confidence models.

2. Amine Hazzami, with Nabil-Kazi Tani from IECL (University of Lorraine).
   Title: Stochastic games over networks with application to cyber-insurance
   Over the past ten years, the growing number of cyber-related issues and incidents have made cybersecurity one of the main concerns in insurance. Cyber risk has thus been described as the second major risk of the decade to come in the Axa Future Risks report of 2021. The main guarantees of cyber-insurance contracts currently on the market are the identification of the problem and its extent, the implementation of corrective actions, additional protections, preventions, and the payment of certain costs associated with the claim. The prices of this type of contract are very variable and depend on several macroeconomic factors such as the size of the company, its turnover

as well as its exposure to IT risk (typically via the number of computers or servers). However, one aspect that is often ignored is the impact of the graph structure on the safety and robustness of the network. In this PhD, we aim to mathematically characterize the effect of the underlying network topology on prices, cyber risk assessment, and management. To do so, we model the interactions between attackers and defenders as a stochastic game. We will build upon the well-known framework of Colonel Blotto games which studies an adversarial zero-sum resource allocation game.

3. Etienne Gorski, with Prof. Constantin Morarescu (CRAN).
   Title: Robustness of specific switching control laws for platooning
   Vehicular platoon control can effectively achieve group consensus, reduce fuel consumption, improve vehicular running safety, and increase road capacity. However, this emerging technology faces several challenges unreliable vehicle-to-vehicle communications, sensitivity to road traffic, and additions/reductions in platoon size. A small gap results in a short inter-vehicular distance, which is fuel efficient when the vehicles are moving at constant speeds due to air drag reductions, but when the vehicles accelerate and brake a lot, a longer time gap is more fuel efficient. Similarly, a poor communication network motivates a bigger inter-vehicular gap for safety, while a smaller gap can be maintained with reliable communications. All these issues motivate platoons that switch the control strategy according to the scenario [1,2]. However, while the string stability of a given control scheme is well studied [3], guarantees on a switching control are rarely examined.

4. Thomas Mongaillard, with Samson Lasaulce (CRAN).
   Title: Individual and collective decarbonization strategies: a game theory and neuroscience based approach
   The state-of-the-art decarbonization strategies by individuals and the mechanisms for encouraging individuals to follow a global decarbonization plan appear to be incomplete on a formal level. As a result, the decarbonization strategies proposed are typically sub-optimal in the sense of an overall performance metric given for the incentive and do not benefit from the support of the actors involved in decarbonizing. To resolve this problem, one of the

objectives of the thesis is to make significant advances in terms of modeling a set of individuals (emitting CO_2) both under the influence of other individuals via a social network and under the influence of an external inciting or regulating entity (e.g., a government). We will design decarbonization strategies and incentive mechanisms that guarantee properties of strategic stability (and therefore individual adhesion) and that exploit the dynamic attributes of the social network graph. The synergy of game theory, dynamic systems, and neuroscience constitutes a promising new point of view to achieve the objectives of this project.

5. Hassan Mohamad, with Samson Lasaulce (CRAN).
   Title: "Interactions between intelligent generative agents: a point of view at the interface of game theory and machine learning"
   This PhD is focused on strategic information transmission between AIs or agents that solve problems with the help of AI. A classical example is that of federated learning where users exchange the parameters of their neural network or other machine learning schemes rather than the data they collect. The assumption in classical federated learning is that all agents have the same utility and sample a common source, but want to maximize their utility while maintaining the privacy of the data collected. However, if their utilities are not the same, they may communicate strategically and this interaction will be modeled as a game and studied during this thesis.

6. Sylvie Chadad, with Yezekael Hayel (University of Avignon).
   Title: Stochastic control for optimizing crowdfunding projects dynamics
   Crowdfunding is popular among entrepreneurs who aim to bypass classical funding channels and directly address the crowd via Internet-based Crowdfunding platforms. In this PhD, we use Markov models, dynamic programming, and game theory to model the fund collection process. We will also propose strategies that can potentially improve the success probability of a project in terms of collecting funds. We will look at this problem from the perspective of funders, the project managers as well as the platform.

I have co-supervised two PhD students who successfully defended their thesis in

2021.

1. Dr. Jomphop Veetaseveera co-supervised with Prof Constantin Morarescu (CRAN)
Title: Communication aware control for multi-agent systems", defended in December 2021.
We look at multi-agent systems and consider the problem of global consensus or synchronization. In contrast to existing works that focus on minimizing a global cost, we are interested in the costs incurred by each agent for both communication and mechanical movements. This is highly relevant in applications like automatic driving where the resources available to a given agent are limited and minimizing the sum cost is not desired as it can result in unfair allocations. We use control theory and game theory to study and propose solutions in this context.

2. Dr. Tiago Rocha Goncalves, co-supervised with Dr. Salah Eddine Elayoubi (L2S-CentraleSupelec)
Title: Robust control of vehicles in platoons over imperfect wireless channels, defended in November 2021.
We look at the application of platooning on roads where vehicles have to be controlled with wireless communication enabled between the vehicles. Specifically, we consider the situation where the wireless channel is imperfect and noisy, resulting in packet drops which can lead to safety issues. The main goal of this thesis is to study and propose some robust control techniques that can adapt to noise in the wireless channel.

I have also had the pleasure of working with some other PhD students: Bikash Adhikari and Olivier L. de Silva and being involved in their research.

1.4.2 Master and undergraduate supervision

I supervised Mr. Etienne Gorski in the 2023 on distributed vehicular control. Specifically, we studied the design of cooperative cruise control under communication delays.

I supervised Mr. Rachid Guedjali (Master 2) in the beginning of 2021. The topic was consensus dynamics coupled with its graph dynamics.

I supervised Mr. Jomphop Veetaseveera (Master 2) in the beginning of 2018. The topic was "Distributed control of multi-agent systems with guaranteed costs", which lead to his thesis.

I also supervised Mme Fatma Torjmen (Bacc 2) in 2017. The topic was "Control of multiple robots with communication objectives".

1.4.3 Responsibilities in CRAN

- Member of 'Conseil de laboratoire du CRAN' since 2017.
- I was also responsible for organizing seminars in the Vosges 2018 and 2019 as part of the CRAN CO2 team activity (2 day event).

1.5 Research projects and funding

Research contracts

Project 1. EU Innovative training network (ITN) project Touch-enabled Tactile Internet Training Network and Open Source Testbed (TOAST): This project is lead by Prof. Qi Zhang from Aarhus University (Denmark).

Scientific summary: The objectives of TOAST are twofold. First, TOAST will train the next generation of excellent early-stage researchers (ESRs) in the emerging and interdisciplinary field of the touch-enabled Tactile Internet (TI) by connecting researchers with complementary expertise from academia and industry to develop holistic solutions for TI, thereby contributing to the success of TI in Europe. Second, TOAST will develop an Open Source Tactile Internet Testbed. This testbed will, on one hand, be used to demonstrate, evaluate, and validate the research results of the ESRs and become one of our core instruments to do remote training and experiments using teleoperation, disseminate TOAST's results, and crystallize public engagement. On the other hand, the testbed will be made available to the public accompanied by educational material with the purpose of establishing a standard platform for TI research and education. It will be designed to support instructors around the world in running TI experiments and integrating the experiments and results into their lectures or laboratories in a plug-and-play manner. For TI researchers, the testbed will provide well-defined interfaces that facilitate future extensions and improvements. In this way, TOAST will have an impact on the training of young researchers in the area of TI and the field in general far beyond the duration of the project. TOAST's impact is assured by a streamlined research training program based on Open Science principles. It will feature the active participation of industrial partners, joint academia-industry supervision, experimentation on the shared TI testbed, and training of transferable skills.

Project 2. ANR project on Understanding, Modeling and Improving the outcome of Crowdfunding campaigns (UMICrowd): This project is led by Prof. Salah Eddine Elayoubi from CentraleSupelec. I was the person responsible for this project from the side of CRAN.

Scientific summary UMICrowd project explores CF from economical and sociological perspectives, using advanced mathematical modeling tools, Artificial Intelligence (AI), and empirical analysis. It aims to propose decision-making tools that help entrepreneurs in designing their campaigns and CFP managers in selecting, classifying, and promoting projects. A quantitative empirical analysis of the CF solution will complement the qualitative analysis. We will analyze the dynamics of CF, including those of the Crowd (fund collection), of the entrepreneurs (pace of project creation, interactions), and of the platform (project selection, classification, and promotion processes). We will observe these dynamics on large datasets collected from CFPs, using tools from data science and AI. Mathematical modeling based on game theory will then explain the behavior of CF players observed empirically. While projects that are requesting funding simultaneously are in competition over the potential funders, attracting funders to the CFP is a common objective for entrepreneurs and CFP managers. Both static and repeated games will be used to study the behavior of entrepreneurs and the interaction between them on a common CFP.

Project 3. ANR project Opinion dynamics in social networks in presence of multiple decision-makers (NICETWEET): This project is led by Prof. Constantin Morarescu from CRAN and also involves BETA (an economic laboratory in Nancy), University of Avignon and Centrale-Supelec.

Scientific summary In the NICETWEET project, we develop a new methodology to address an important and generic problem that appears in economics, telecommunications, energy markets, and politics. Several decision-makers (DMs) are in competition for propagating ideas or selling goods, services, etc. to a large number of agents who are connected through a physical or digital social network (SN). The agents are thus under the (endogenous) influence of their neighbors in the SN graph, but also under the (exogenous) influence of the DMs. To address this problem, we have formed an interdisciplinary team that possesses the required expertise: control theory and more specifically, opinion dynamics, game theory, information theory, complex networks analysis, and economics. In contrast with the closest works, several DMs are present and the SN models have new features that make them more realistic. Some of these features are the time-varying number of agents; the presence of extremists; the DMs play a repeated marketing game under partial information. In this technically challenging setting, we will characterize the equilibrium states and utilities, and design practical space-time marketing strategies.

Project 4. CEFIPRA project 2019-2021 "High-performance formation control in the presence of uncertainties and communication constraints" between CRAN, L2S and IIT-B (India). The grant supports movement between the partner institutions and 3 PhD grants (one in France and two in India). I am involved as a member.

Scientific summary Cooperative control of mechanical systems, and in particular formation control, has recently seen a proliferation of research and applications, primarily due to its low cost, robustness, and specific application requirements. The target of the current proposal is to establish formation control while minimizing communication and control efforts under a variety of uncertainties due to modeling and measurement errors. The specific application domain for the current proposal will be mechanical systems interacting through a variety of communication channels (optical, wireless, ultrasound etc.) with the objective being that individual agents achieve and maintain a relative pose (position and orientation) with respect to their neighbors. The current proposal extends the recent research activity of the proposers in three fundamental directions: a) formation under sensor bias and graph uncertainties, b) formation under time-varying and state-dependent interconnections, and c) event-triggered control/communication for formation attainment.

Project 5. PEPS project 2017 Impact of Opinion Dynamics In social NEtwork Campaigns. I was the PI of this project.

Scientific summary Social networks are now some of the most preferred platforms through which marketing campaigns are launched. The existing studies propose marketing strategies that do not take into account the opinion dynamics in social networks. The motivation of IODINE is to study investment strategies for campaigning over a social network, firms must not only look at the immediate effect generated by the campaign, but also at the spreading of the advertising or opinion over the social network. The objective of the present project is to combine opinion dynamics models and game theory models to study the strategy for campaigns by competing firms, that are trying to maximize their profit. The opinion of any agent considered in this study corresponds to its preference towards one of the firms over the other, and each firm must allocate marketing resources based on both agent social connections (centrality) and their current opinion. Our goal is to characterize the strategies of competing firms, based on well-known concepts in game theory like the Nash equilibrium.

Project 6. PEPS project 2018 Dynamique d'opinion dans un reseau social en presence de marketers en competition (YPSOC) between L2S, CRAN and BETA. I was a member of this project.

Scientific summary Social media marketing is a rising trend. Typically, opinion dynamics literature looks at the interactions of social agents within a social network, i.e. the influence of opinion is internal. The motivation of YPSOC was to study the opinion evolution caused by external influencers such as companies or political entities on the network. Mathematically, tools from control theory and game theory will be used with application to economics.

Project 7. INS2I and CNRS 80'PRIME call 2019 project DECIDE between CRAN and BETA. I am a member of this project.

Scientific summary The emergence of digital tools has enabled online platforms to provide consumers with structured information on goods and services, which is based on not just the recommendations from experts but also the experience of other consumers. The resulting opinion dynamics on the goods influence the decisions of consumers, which in turn impact consumers' welfare, firms' profits, and the platform's profit. The main goal of this interdisciplinary project was to develop a theoretical framework using the tools of game theory and multi-agent systems to analyze the behavior of online platforms while taking into account the opinion dynamics of consumers. One important challenge of the DECIDE project was to propose socially optimal platform strategies, i.e., strategies that maximize consumers' welfare. The project's originality is to combine the strong expertise of CRAN in networked dynamic systems control and analysis, as well as in opinion dynamics, with the expertise of BETA in economic modeling and analysis.

1.6 List of publications

Journal articles:

1. O. L. de Silva, S. Lasaulce, I-C. Morarescu, V.S. Varma, A game theory analysis of decentralized epidemic management with opinion dynamics, IEEE Transactions on Control of Network Systems 2024 (provisionally accepted).

2. A.I. Maass, D. Nesic, R. Postoyan, V.S. Varma, S. Lasaulce, D.A. Carpintero, Transmit power policies for stochastic stabilisation of multi-link wireless networked control systems, Automatica 2024 (provisionally accepted).

3. B. Adhikari, J. Veetaseveera, V.S. Varma, C. Morarescu, E. Panteley, Computationally efficient guaranteed cost control design for clustered networks, Automatica, Vol. 163, 111588, 2024.

4. O. L. de Silva, V.S. Varma, M. Cao, I-C. Morarescu, S. Lasaulce, A Stackelberg Viral Marketing Design for Two Competing Players, in IEEE Control Systems Letters, vol. 7, pp. 2922-2927, 2023.

5. V.S. Varma, R. Postoyan, D. E. Quevedo and I-C. Morarescu, Event-Triggered Transmission Policies for Nonlinear Control Systems Over Erasure Channels, in IEEE Control Systems Letters, vol. 7, pp. 2113-2118, 2023.

6. E. Kravitzch, Y. Hayel, V.S. Varma, A. Berthet, Analysis of a Continuous-Time Adaptive Voter Model, Physical Review E, 107(5), 054307, 2023.

7. T.R. Gonçalves, R.F. Cunha, V.S. Varma, and S. E. Elayoubi, Fuel-efficient switching control for platooning systems with a deep reinforcement learning approach, IEEE Transactions on Intelligent Transportation Systems, vol. 24, no. 12, pp. 13989-13999, Dec. 2023.

8. T.R. Goncalves, V.S. Varma, and Salah E. Elayoubi, Relay-assisted platooning in wireless networks: a joint communication and control approach, IEEE Transactions on Vehicular Technology, 72(6), 7810-7826, 2023.

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10. L. Salahaldin, V.S. Varma, S.E. Elayoubi, When and how to intervene for saving an entrepreneur's crowdfunding campaign, Finance Research Letters, 50, 103189, 2022.

11. J. Loheac, V.S. Varma, I.-C. Morarescu, Time optimal control for a mobile robot with a communication objective, Mathematics and Computers in Simulation, 201, 96-120, 2022.

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14. S. Lasaulce, C. Zhang, V.S. Varma, I.-C. Morarescu, Analysis of the tradeoff between health and economic impacts of the Covid-19 epidemic, Frontiers in Public Health, Vol. 9, 620770, 2021.

15. A. M. Oliveira, V.S. Varma, R. Postoyan, I-C Morarescu, J Daafouz and O.L.V. Costa, Network-aware Controller Design with Performance Guarantees for Linear Wireless Systems, IEEE Transactions on Automatic Control, 66(9), 4297-4302, 2020.

16. V.S. Varma, I.C. Morãrescu, M. Ayouni - Analysis of opinion dynamics under binary exogenous and endogenous signals, Nonlinear Analysis: Hybrid Systems, Vol. 38, 2020.

17. L. Busoniu, V.S. Varma, J. Loheac, A. Codreanu, O. Stefan, I.C. Morarescu, S. Lasaulce - Learning control for transmission and navigation with a mobile robot under unknown communication rates. Control Engineering Practice, 100: 104460, 2020.

18. I.C. Morarescu, V.S. Varma, L. Busoniu, S. Lasaulce, Space-time budget allocation policy design for viral marketing. Nonlinear Analysis: Hybrid

Systems, Vol. 37, 100899, 2020.

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Chapter 2

Networked control of systems with constrained or uncertain communication

As someone with a background (PhD thesis) in telecommunications who joined a primarily automatic control-oriented group at CRAN, the most natural way of working together with the researchers at CRAN was to combine our expertise and work on the domain of networked control systems. I have explored four main directions of research in this theme, which are organized into four sections of this chapter.

It became apparent that while control engineers study similar issues of reducing the communication frequency or improving efficiency as those working on communication, the existing literature rarely considers stochastic packet drops, which are a common assumption in wireless literature. This motivated the first direction of my research in this theme, i.e., energy-efficient time-triggered communication strategies in networked control systems (NCS). In this direction, we have proposed a novel framework for studying the stochastic stability of both linear and non-linear systems and consequently improving communication efficiency.

Another direction I have studied can be seen as a specialized application of networked control systems. This direction is in the communication and control design for vehicular platoons. We study this multi-agent synchronization problem under imperfect wireless communication and the consequences of this imperfection (characterized by packet loss and delay).

Next, I have studied the synchronization of multi-agent systems with guaranteed individual costs. In this direction, we look at the problem of multi-agent synchronization, where every agent must rely on local information to achieve global synchronization. In contrast with existing works, we also look at the individual control costs incurred by each agent while achieving the global objective.

The last direction I have explored in this theme is slightly different from the rest and involves the control of a robot with communication objectives. Here, the objective of the robot is to perform a communication task before reaching a destination, which is in contrast with the previous directions where the control was constrained by the communication network. Specifically, we look at a robot that must empty its data buffer by communicating with a given communication access point (base station or antenna) before reaching a certain destination. The challenge arises from the non-linear dependence of the expected communication rate on the distance between the robot and the access point. In this context, the problem is to optimize the trajectory of the robot so that the mechanical energy or time taken is minimized.

2.1 Energy-efficient time-triggered communication strategies in NCS

This work was in collaboration with A.M. Oliveira (Uni. Sao Paolo, Brazil), Prof. Romain Postoyan, Prof. Constantin Morarescu and Prof. Daniel Quevedo from (Uni. Queensland, Australia).

State of the art and context

Energy efficiency in communication systems has gained a rising amount of interest in recent years [1, 2]. For devices such as cellular phones, unplugged laptops, wireless sensors, and mobile robots, smart energy management is essential due to the limited supply of energy available. Various studies have investigated the design of energy efficient communication systems, i.e., minimizing energy while maintaining a certain quality of service parameter, or maximizing the ratio of data rate to energy consumed, see [2] for an extensive survey. When wireless communication is done with single antennas, one of the most relevant techniques to improve energy efficiency is *transmission power control*. In works like [3, 4], transmission power is optimized so that the ratio between the number of packets transmitted successfully and the power consumed is maximized.

In the context of control systems, wireless networks offer appealing features as they allow remote control and exhibit many advantages over traditional wired point-to-point set-ups in terms of flexibility, ease of maintenance, reduced weight, and volume. On the other hand, the communication constraints induced by the network need to be appropriately handled to guarantee the desired performance for the closed-loop system, which motivates research on so-called *networked control systems* (NCS), see, e.g., [5] and the references therein. Often, the energy expenditure due to communications is ignored in works on NCS, or it is treated in an adhoc way, in the sense that the less often we transmit (or the smaller the packet length), the less energy we spend to communicate, for instance. In wireless communication, transmitting less often does not always imply improving the energy efficiency of communication, as the probability of successful communication depends on several factors like the power of the transmission signal, the quality of the wireless channel, noise levels, etc. [1, 6]. There is, therefore, a need for suitable energy-efficient transmission policies for communication devices in wireless NCS (WNCS).

Contributions: Linear case

We develop a new notion for discrete-time networked control systems, referred to as, the *stochastic allowable transmission interval* (SATI). The SATI is described by a certain interval length N, a cumulative probability η of a transmission to occur before N steps have elapsed since the previous successful transmission, and δ the instantaneous transmission success probability after N steps have passed. Here, δ corresponds to the success probability when using the maximum possible transmission power. This approach can be seen as a stochastic version of time-triggered control instead of independently distributed probabilities of the inter-transmissions intervals, as in [7]. During the N steps, packet drop probabilities can be designed in a flexible manner by adapting the transmission power as long as the constraint on η is satisfied. This flexibility can be exploited in order to design energy-efficient communication policies. After N steps have passed, the maximum power is used until a successful transmission occurs. In our work, (i) we design time-triggered policies, as opposed to event-triggered policies, which is a different implementation paradigm with its associated pros and cons as explained above, (ii) we consider output feedback laws, and not only state-feedbacks, (iii) we allow for varying holding strategies, not only model-based ones, and (iv) our approach allows minimization of the communication cost while ensuring a given control performance. We started to address this problem in [6] using a deterministic constraint on the maximum allowed time between two transmissions, which resulted in very strong assumptions about the channel state, in particular, we assumed that a successful transmission is possible at any given time. This condition is no longer needed in this work. In [8], we introduce the notion of SATI and present results on stability. We have also accounted for control performance and optimized communication energy in this framework (work published in IEEE TAC).

Additionally, we have also studied the co-design problem in the framework of SATI, i.e. to also design the controller gain matrix in addition to the network parameters. This was started in [9] where we look at state feedback controller design for stability in the SATI framework. We have also published an extension version of this result in a much more general setting.

Contributions: Non-linear case and time-varying channels

We have also recently worked on finding a similar framework for non-linear control systems. While mean-square stability can no longer be ensured in this context, we use Lyapunov-based techniques and define a notion of stochastic stability based on the expectation of the Lyapunov function. Additionally, we also allow and treat the case of time-varying channel fading, which modifies the packet failure probability, i.e., a good channel can result in the packet being successful even with a low transmission power, while a bad channel might imply that even at max power, the packet is almost always dropped. This work has been published in IEEE TAC (2023) and has been presented in IEEE CDC 2021. We have also published recently in IEEE LCSS (2023) an event-triggered strategy to further improve communication efficiency in this context.

2.2 Communication and control design for vehicular platoons

This work was led by Tiago R. Goncalves, the PhD student I co-supervised with Prof. Salah Eddine Elayoubi in L2S, CentraleSupelec.

State of the art and context

A vehicle platoon is a particular formation of a group of coordinated vehicles where a short inter-vehicle distance is maintained by automation and Vechicle-to-Vehicle (V2V) communication technologies [10]. Vehicle platooning is expected to improve fuel efficiency [11] and reduce traffic congestion [12] by gathering vehicles close together, thus reducing the air resistance of the platoon's members. Experimental analysis has shown that with such a convoy formation, a bus, following another one, is able to reduce the aerodynamic drag by about 40% when the speed is 80km/h and 10m of inter-vehicular distance is maintained [13]. However, the feasibility and deployment of platoons require a reliable and fast exchange of information between vehicles so that control actions are based on the most up-to-date information about the road and traffic status. Nevertheless, such an exchange of information occurs over unreliable wireless communication channels subject to inherent characteristics such as delay and packet loss. A careful design of the communication schemes for platoons is thus essential, but the focus in this design must shift from the objective Quality of Service (QoS) measures (delay, packet loss) to the application-layer performance, essentially the minimum achievable inter-vehicle distance without any risk of vehicle collisions.

Contributions

Since all vehicles exploit the information broadcasted by the leader, the reliability of this communication link is crucial to platoon control performance and safety. We have tackled several important problems in this context.

1. We adapt the parameter that weighs the leader information as a function of the packet success rate of the leader broadcast link. Extensive numerical simulations highlight the performance of the proposed methodology and the advantages of adapting this parameter in real time. Published as a conference

paper.

2. We propose a novel V2V relaying scheme that carries the leader message by appending it along with the neighbor-to-neighbor messages. Then, we study the performance of this scheme in terms of packet loss, delay, and control performance. Published as a conference paper and in IEEE Transactions on Vehicular Technology.

3. We propose a novel switching controller that switches between two well known and string-stable controllers to minimize the average fuel consumption depending on the road traffic conditions (and the jammer behavior). Due to reductions in air drag, maintaining a small distance from the preceding vehicle is efficient in the ideal case, in which all vehicles move at near constant speeds. However, when the traffic is perturbed, maintaining this short gap can result in a higher fuel cost due to the high control effort. This work has been published in IEEE Transactions on Intelligent Transport Systems.

2.3 Synchronization of multi-agent systems with guaranteed individual costs

This work was lead by M Jomphop Veetaseveera, who was a PhD student at CRAN under the supervision of myself and Prof. Constantin Morarescu.

State of the art and context

The main goal of this work was to design a decentralized control strategy that allows multi-agent systems to achieve synchronization with local or individual performance guarantees. Decentralized coordination control of multi-agent systems has attracted a lot of attention during the past decade. In the decentralized control design paradigm, each system is able to implement and design its own control law without the help of a central entity. The multi-agent formalism enables treating problems that arise in many application domains, such as engineering [14] or sociology [15]. Consensus and synchronization were mainly studied for agents with linear dynamics interacting through a graph, see [16] for instance.

In several existing works, the control cost related to synchronization is not studied, and the objective is related to stability. In [17], the authors explain how optimal control with centralized design, i.e., when the dynamics are perfectly known for all agents and the initial state is known, is NP-hard in the multi-agent framework due to the information structure imposed by the graph. The problem of distributed control design while trying to reduce a common or individual cost remains largely open, and most of the existing works focus on stabilization or on achieving synchronization asymptotically. Recently, in [18], the authors present the design of a decentralized control strategy that allows singularly perturbed multi-agent systems to achieve synchronization with global performance guarantees. Additionally, [18] assumes that all agents utilize the same gain, which can be restrictive in some cases.

Contributions

In contrast with existing results, our main objective was to guarantee that the cost incurred by each agent does not exceed a given bound during the process of synchronization. This can be of practical relevance in applications such as automatic cruise control on highways, where each vehicle wants to follow the vehicle in front of it (a common objective of synchronization), but also wants to ensure that its fuel consumption is not too high (individual cost). In such applications, considering a global cost might not be fair to each individual vehicle.

In [19], we study the problem of static-output feedback synchronization in a multi-agent system, which guarantees individual performance bounds inspired by the concept of satisfaction equilibria in game theory. We look for solutions that are belong to the set of satisfaction equilibria, i.e., the cost associated with each agent is upper-bounded by a given γ. In this context, we provide conditions in the form of LMIs that can verify if a given set of gains is a satisfaction equilibrium. We provide a method to generate the gain for a certain agent when the gains for the other agents are known, and this is used in an iterative algorithm inspired by the best response algorithm, which can synthesize a satisfaction equilibrium.

Next (in a paper published in Automatica), we provide an alternate design procedure that is suitable for large-scale networks. The first method proposed had a complexity that scaled polynomially (to the power of five) with the number of agents due to the LMI's being scaled with this number. The novel design procedure is suitable for large-scale clustered networks, i.e., networks in which agents are grouped in densely connected clusters that are weakly connected to other clusters. These types of networks are common in social networks (communities), power grids (geographical clustering), etc. In this context, we are able to use time-scale separation techniques to design an internal and external control (for synchronization within the cluster and to other clusters, respectively) at a much lower complexity, which scales with the number of clusters rather than the total number of agents.

2.4 Control of a robot with communication objectives

This work was led by Dr. Daniel B. Licea in collaboration with myself, Prof. Samson Lasaulce (L2S), Prof. Jamal Daafouz (CRAN), and Prof. Mounir Ghogho (Uni. Rabat). The work related to learning was led by Prof. Lucian Busoniu (TU Cluj-Napoca).

State of the art and context

In wireless networks, it is generally assumed that the trajectory of a mobile terminal is chosen by the user. Under this assumption, the communication and motion aspects are obviously independent. However, there are more and more applications for which this classical paradigm needs to be questioned. For instance, when an unmanned aerial vehicle or a robot has to collect data from a field of wireless sensors, it typically has to optimize its trajectory to receive the data correctly (see, e.g., [20][21]). The problem under consideration is as follows: A mobile terminal has to move from a starting point to a target point while transmitting a certain amount of data along its trajectory. The data has to be sent to a wireless base station, which receives the signal with a signal-to-noise ratio (SNR) that depends on the distance between the mobile and the base. Therefore, the mobile has to choose a trajectory that allows the data to be uploaded successfully (which is made possible by having a sufficiently large SNR) and to manage the energy consumed for moving. To the authors' knowledge, the closest work to the work we conducted was given by [22]. Therein, a robot has to optimize its trajectory to minimize a cost, which consists of the sum of the energy consumed by the transmitter and the motion energy.

Contributions

In [23], we have studied the problem where a mobile robot (MR) must download (or upload) a given amount of data from an access point and also reach a certain destination within a given time period. We assumed that the wireless signal strength was determined purely based on the path loss. In [24], we relax these strong assumptions and account for small-scale fading and shadowing effects. We show how to design offline a robust reference trajectory with a limited amount of information and high uncertainty about the wireless channel. This trajectory will allow the MR to reach the goal point and completely transmit the content of its buffer to the access point (AP) with a sufficiently high probability.

The main contributions of these papers are as follows:.

- Trajectory planning of a MR starts from an arbitrary point, which must reach a certain target point and download a certain number of bits from a nearby access point.
- Optimization of the trajectory to minimize a cost function that depends on the amount of data left in the buffer to be downloaded and the energy consumed.
- Considering a robust cost function that accounts for the random fluctuations of the wireless channel due to small-scale fading and shadowing effects,.

In [25], we consider the problem where the rate map is completely unknown, i.e., the location of the radio access points and their strength are unknown. The objective of the robot is to empty its data buffer as soon as possible. Exploring the map will lead to a better communication rate, but the exploration itself will cost time. We use tools from learning (supervised learning) to improve the tradeoff between exploration and exploitation.

Chapter 3

Analysis, control and games on inter-connected dynamical systems

Another theme that I have explored during my time as a CNRS researcher was the analysis and control of multi-agent and other interconnected dynamical systems. Compartmental modeling traditionally describes the movement of a substance from one compartment to another, with origins based on the metabolism of tracer-labeled compound studies in the 1920s. However, this can be generalized and applied to various domains, and it is often used to describe the spread of viruses, both biological and computer-based, over a large number of agents interacting over a graph. We may also use this framework to describe the evolution of opinions held by a population. My passion for applied stochastic processes and game theory has motivated me to study several interesting problems that arise in this theme.

One of the first directions explored was opinion dynamics modeling. Specifically, we have developed a novel model of opinion dynamics inspired by the CODA (continuous opinion discrete action) model by [26] and the classical voter model using Markov chains. This analysis of such models is especially interesting in the fluid limit, i.e., when the population grows large and approaches infinity. In this regime, several unexpected and counter-intuitive results, when compared to the finite number of agent cases, may be observed.

Another direction we have studied is that of marketing strategies over social networks. In this direction, we study how opinion dynamics can be influenced and controlled by external agencies (such as an advertising campaign), which may be singular or in competition. We use optimization, control theory, and game theory to model the influence of the external entity(ies).

The next direction I have looked at is that of controlling epidemics. In this direction, we study how a central entity or several decision-makers decide to control an epidemic with a cost that has a health component and a term that relates to the economic loss due to the control.

Finally, we have also looked at an exciting new control of crowdfunding processes: In this direction, we study the interactions between a crowdfunding platform, entrepreneurs, and the crowd (funders). Specifically, we model the crowd as rational decision-makers (or players) that decide which project to invest in as a function of project attractiveness and the probability of the project successfully collecting funds using game theory. At the next level, the crowdfunding platform may intervene to promote certain projects to improve its chances of success, and we formulate this scenario as an optimization problem.

In the following, I will detail the state of the art and the contributions I have made in these directions.

3.1 Opinion dynamics modeling: discrete opinions and binary actions

This work was lead by me and was done in collaboration with Prof. Constantin Morarescu (CRAN), Dr. Yezekayel Hayel (Univ. Avignon) and Prof. Dragan Nesic (Uni. Melbourne).

State of the art and context

Recent social science experiments [27, 28] have shown that information propagated via social media impacts not only the opinions and decisions of the users who directly observe this information but also their friends. The understanding of different features related to opinion dynamics in social networks has gained more and more importance in politics and economics. One such feature is observational learning, which refers to the fact that individuals extract information from others' actions. This type of learning has been studied in economics [29, 30] in contexts where agents make decisions – for example, buy product A or product B – sequentially so that agent 2 observes the action of agent 1, agent 3 observes the actions of agents 1 and 2, and so on. Assuming that product A is better than product B but that each agent observes an imperfect private signal about the relative qualities of these products, rational agents can reach a consensus on the wrong decision, i.e., there is a positive probability that all agents buy product B. Such an outcome is called an information cascade: if enough agents at the beginning of the sequence have a bad private signal indicating that B is better than A, this information is aggregated in a way that makes all subsequent agents ignore their private information and follow the decisions of previous agents by choosing product B. Interestingly, information cascades can also occur in networks where agents engage in observational learning [31, 32]. This motivated several researchers from various disciplines, such as sociology, mathematics, physics, computer science, and engineering, to study the dynamics of opinions in social networks.

In early studies of opinion dynamics, the heterogeneity is neglected and the dynamics of the network, as well as the update rule, are simplified. For instance, DeGroot [33] considered a fixed network in which individuals repeatedly update their opinions by averaging the opinions of their neighbors. Under some mild assumptions about the network structure, this dynamics always leads to consensus. After two decades, this model was modified in different ways. On one hand, Friedkin-Johnsen [34] proposed a model in which the initial condition (representing initial beliefs, social class membership, etc.) plays an important role since it weights at each step the individual opinion evolution. On the other hand, bounded confidence dynamics on the network topology were added to the simple updating rule proposed by DeGroot. This results in the Deffuant [35] and the Hegselmann-Krause [36] models which consider that each individual can be influenced only by those having opinions close to their own. While in the models above, the opinion can take any real number, there are models in which the opinions are restricted to a discrete set of values. These models generally come from statistical physics, and the most employed are the Ising [37], voter [38], and Sznajd [39] models. They are used to model processes in which individuals have to choose one among a certain number of decisions. It is worth noting that all the models presented above assume that each individual has access to the exact value of the neighbor's state. To model a more realistic behavior, a mix of discrete actions, and continuous opinion was proposed in [40]. This model, a type of social learning, considers the opinion as a measure of confidence in the binary choices or actions, and only these actions are accessible to the neighbors.

Contributions

Our work was inspired by [40], but in contrast, we provide a theoretical analysis in order to support our numerical results. In [41], a deterministic version of [40] was studied, and it was shown that this deterministic model leads to a variety of asymptotic behaviors including consensus, oscillation of opinions, or clustering according to local agreements. We discretize the opinion levels in [40] and with some modifications, are able to model the opinion dynamics as a Markov process. We use $X_n(t) \in \{\theta_1, \ldots, \theta_N\}$, with N even to represent the opinion with $\theta_n \in (0, 1)$, $\theta_n < \theta_{n+1}$, $\theta_n = \theta_{N+1-n}$ and $\theta_{N/2} < 0.5$. The action of an agent n is 0 if $X_n(t) < 0.5$ and 1 otherwise. Basically, the opinion levels are symmetric around 0.5 and represent the confidence in actions 0 or 1, the action 1 is taken if opinion is greater than 0.5. Agents interact over a social graph and observe only the actions of their neighbors. If the neighbor action is 1, the opinion of an agent shifts to the right with a certain probability and to the left if the neighbor action is 0.

The main results and contributions are as follows

1. In the finite (and small) number of agents case, there are exactly two absorbing states, one with all agents having opinion θ_1 (action 0) and other with all agents having action 1, implying global consensus. The Markov process will almost surely converge to one of these absorbing states assymtotically.

2. However, when the number of agents is arbitrarily large and the graph has some specific structures, the time taken to converge to the absorbing states can be arbitrarily large, and the system stays in a "metastable" equilibrium, which persists for long durations in the transient regime. These results are presented in [42] and [8] for discrete time systems; and in [43] and [44] for a continuous time Markov chain.

3. We have also considered open systems, where agents may activate or deactivate from the network with a certain probability. In this case, we assume that the "new" agents who are activated enter the network with a random opinion. This results in no consensus, as a new agent with a contradicting opinion may emerge at any time. However, when the number of agents is large and the entry/exit rate is sufficiently small, we show that a form of practical consensus can be maintained for a long duration. This result is presented in [45].

3.2 Marketing strategies over social networks

This work was primarily led by me and was done in collaboration with Prof. Samson Lasaulce (CRAN), Prof. Constantin Morarescu (CRAN), and Prof. Lucian Busoniu (TU Cluj-Napoca).

State of the art and context

In many domains, such as economics and politics, people (e.g., consumers or voters) are both influenced by their acquaintances, friends, or relatives and by external entities (e.g., marketers or candidates); these influencers are called, in a generic manner, *marketers*. These external entities are currently better realizing the potential of acquiring and exploiting some knowledge about the corresponding dynamics of a digital social network to design good strategies. Targeted and viral marketing constitute good examples illustrating this tendency [46]. To provide a specific example, quite recently, some firms have been starting to remunerate popular bloggers or YouTubers to promote some goods in their videos. The main purpose of the this waork was to study the evolution of people's opinions when they are under the combined influence of their "neighbors" (who may have different degrees of influence) and marketers (who typically have diverging interests). Whereas opinion dynamics (OD) has been attracting a lot of attention from researchers, in the control community, in particular, the problem of controlling opinion dynamics has been left almost unexplored. Additionally, if one considers the problem in the presence of multiple controllers instead of one, then only a couple of formal works seem to be available.

Among relevant works on controlled OD, we find [47, 48] in which the authors look at the role of controlling (from a single controller) a small number of agents of the network to enforce consensus. We also find recent attempts to control the discrete-time dynamics of opinions such that as many agents as possible reach a certain set after a finite number of influence instances [49]. The classical literature on non-cooperative games between marketers assumes a homogeneous population of consumers [50, 51, 52]. For the scenario that directly concerns the present work, namely the scenario that involves multiple controllers or marketers influencing consumers over social media, the closest works are given by [53] and [54]. In [53], the authors consider multiple influential entities competing to control the opinion of consumers in a game-theoretical setting.

Contributions

We consider a social graph with N agents who are interested in picking firm 1 or 2 (the firms are political, social, or economic entities). In this work, we take $x_n(t) \in [0,1]$ to be the opinion of an agent n at time t which represents the probability of agent n purchasing or voting for the firm 1, while $1 - x_n(t)$ is the probability of the agent picking firm 2. The firms influence the opinions through campaigns, while the opinions evolve internally based on the social graph at other times. This is modeled as follows:

$$\begin{cases} \dot{x}(t) & = & -\mathbf{L}x(t) & \forall t \in \mathbb{R} \setminus \mathcal{T} \\ x(t_k^+) & = & \Phi(x(t_k), a_1(k), a_2(k)) & \forall t_k \in \mathcal{T}, k \in \mathcal{K}, \end{cases} \tag{3.1}$$

where $\mathbf{L}$ is the graph Laplacian matrix, t_k are campaign instances,

$$\Phi(x(t_k), a_1(k), a_2(k)) = (\phi(x_1(t_k), a_{1,1}(k), a_{2,1}(k)), .., \phi(x_N(t_k), a_{1,N}(k), a_{2,N}(k)))^\top$$

and

$$\phi(x_n(t_k), a_{1,n}(k), a_{2,n}(k)) = \frac{x_n(t_k) + a_{1,n}(k)}{1 + a_{1,n}(k) + a_{2,n}(k)}. \tag{3.2}$$

which gives how the campaigns impact the opinion of consumers based on the actions $a_{i,n}(k)$ made by player i at stage k on the consumer n. Each firm wants to maximize its market share $\int_{t_k}^{t_{k+1}} \sum_n x_n(t)dt$ for firm 1 and $\int_{t_k}^{t_{k+1}} \sum_n (1 - x_n(t))dt$ for firm 2, while also considering the cost for advertising/marketing ($a_{i,n}$).

In [54], we study the single campaign problem, i.e., the campaign happens only once, and use the node centrality to define the agent influence power and show how the marketers can exploit this quantity to allocate their marketing budget over the agents and therefore optimize their return on investment in terms of market share. We use a static or one-shot game model and conduct the corresponding Nash equilibrium (NE) analysis. The obtained results clearly show the benefit of designing target marketing strategies by using the available knowledge about the graph of the network of agents. We prove the existence and uniqueness of the NE and provide an algorithm to reach it. However, this interesting analysis is incomplete as it is assumed that there is only one campaign. Moreover, when the marketers implement the derived one-shot NE strategies, one does not know about the long-term behavior of the marketers.

Remarkably, a long-term analysis conducted in [55] reveals that the marketers may have an interest in stopping to invest and therefore influence the consumers and accept to operate at a network equilibrium point in terms of market shares. Exploiting the key results in [54], we propose a coopetition marketing strategy that combines one-shot Nash equilibrium actions with no advertising. Basically, the two firms play the one-shot NE for the first few campaigns. Once the opinions of all the agents converge sufficiently close to a certain equilibrium (which we prove exists and is reached asymptotically), the two firms can agree to stop advertising, thereby saving the cost of advertising. Under reasonable sufficient conditions, it is proven that the proposed coopetition strategy profile Pareto-dominates the solution of [54].

We have also looked at optimal resource allocation when there is only one external entity (firm) who wants to maximize the number of agents with a favorable opinion and when the firm has a limited budget. We look at both temporal (over campaigns) and spatial (over agents) allocation of the budget in this scenario and propose a dynamic programming based approach to solve the problem under certain assumptions [56].

3.3 Control of epidemics

This was done in collaboration with Prof. Samson Lasaulce (DR, CRAN), Prof. Constantin Morarescu (Prof, CRAN), Dr. Olivier Lindamulage (PhD student), and Chao Zhang (Ast. Prof, Wuhan, China).

State of the art and context

Severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) (in short, COVID-19) was officially declared on March 11, 2020, by the World Health Organization (WHO). In the absence of an effective pharmaceutical treatment (or vaccine), the best strategy to control the spread of COVID-19 was to reduce interactions between susceptible and infected individuals, for example, through early detection and social distancing rules [57]. In 2020, China, France, Italy, Spain, and many other countries adopted uniform centralized social distancing policies in order to reduce the spread of the COVID-19 epidemic. However, uniform policy management caused inadequacy between the measure severity level and the local situation [58]. Several consequences of this mismatch were observed, such as avoidable local economic losses, potentially avoidable psychological damages, and frustration. Decentralizing decision-making in a federal system has several potential advantages: it is closer to the citizen; it accesses more accurate information and takes into account local needs and circumstances; and it should improve economic performance and public sector efficiency at the local level (see [59]). The experiences acquired during the COVID-19 pandemic, determined a change of strategy allowing regions (e.g., provinces in China, states in the USA, Länder in Germany, or regions in France) to locally adapt the decision-making process. Consequently, in 2021, within a given country, the public health guidance implemented was different for regions depending on pre-existing socio-economic characteristics of the geographic area, baseline healthcare capacity, and other health-related population features (i.e., smoking prevalence, obesity rate, global health indices), and by prioritizing aspects such as education, social welfare, economy, or health.

It is also important to analyze the efficiency of the strategies implemented by policymakers, as they do not always have the desired results. For example, in France, there was a significant "gap" between public health recommendations to control the COVID-19 threat and the activities of a minority of the population who, despite warnings, continued to engage in potentially dangerous activities for public health (e.g., attending social gatherings). Motivated by these observations, we propose a mathematical model to evaluate the effects of decentralization on epidemic management that takes into account perturbing dynamics such as social behaviors or vaccination.

Contributions

As a first step, we have studied the simple two-step control of initiating a lockdown and then releasing the population with an SIR model in order to determine the efficiency of the lockdown measures. Specifically, we look at the trade-off between health (measured by the infected population) and the economy (depending on the duration of the lockdown). This work was published in 2021 [60].

Later, we consider a networked SIR model coupled to a time-varying opinion dynamics model. We propose a one-shot game (played over a finite time horizon) based on an epidemic model that includes behavioral drift. This is in contrast to existing works that consider infinite-time games with constant actions without behavioral changes from the population. We set up a static and strategic form game, and we give a complete analysis of the Nash equilibrium strategy (existence, uniqueness, efficiency, determination). We have submitted a paper studying this game in early 2023 [61].

3.4 Control of crowdfunding processes

This was done in collaboration with Prof. Salah E. Elayoubi (Prof., CentraleSupelec) and Prof. Linda Salahaldin (ESCE International Business School, Paris).

State of the art and context Crowdfunding is becoming a popular funding vehicle for entrepreneurs as it allows them to address the crowd directly via Internet-based crowdfunding platforms (CFPs). This popularity has increased the competition between funding campaigns that attract a crowd of people with the same interests and are usually classified by the CFP within the same category (e.g., arts, innovation, gaming, etc.). While CFPs act as neutral facilitators both for the entrepreneur and the crowdfunders, there is continuous interaction within the crowd and with the entrepreneurs. Several works try to model the behavior of the crowd, the platform, and the entrepreneurs. [62] compares two forms of CF, reward-based and equity-based, and shows that the choice made by the entrepreneur to select between these two forms depends essentially on the amount of required capital and that equity-based CF is more suitable for large projects. [63] models investors' choices when facing asymmetric information about the project's quality (good or bad) and shows that higher proportions of informed investors do not always lead to more good projects being funded, while a higher proportion of bad projects can paradoxically increase the number of good projects that end up funded. Other literature gives an overview of the key factors that contribute to the success of the project.

Our objective in this context is to model these interactions during the lifetime of a campaign and to derive levers for entrepreneurs to influence the course of their campaign.

Contributions: First, we propose modeling the interaction of the crowdfunders as a non-cooperative game, which allows us to develop an understanding of the interaction between them and the main parameters that influence their funding decisions. The crowd determines which projects get funded as a function of their attractiveness and their current funding status. We study this game in [64] and provide insights on which projects may get funded when certain thresholds on collected funds are met.

Secondly, we have studied the correlation between a platform recommending a certain project and its chance of success. Using this study, we study the optimization problem of which projects to recommend at what times in order to maximize the utility of the platform (social or economic) [65].

Chapter 4

Conclusion and perspectives for future research

My research over the last few years had the central theme of networked dynamical systems and had two distinct branches, as detailed in the two preceding chapters. While working on these topics, some of the limitations and challenges I have faced, as well as the emergence of artificial intelligence as the go-to way to solve complex problems, have motivated me to include this aspect in my future research. Therefore, my research for the next several years will continue around networked dynamical systems, with additional themes involving machine learning and AI. Rather than simply applying AI, my expertise in game theory and control motivates me to study the interactions between AIs (and humans) using game theory. These interests perfectly align with the theme of the research group that I am a member of in CRAN, which is CO2 (Control, Optimization, and Communication).

4.1 Game theory and AI

4.1.1 Distributed learning

Scientific context: Distributed gradient descent (DGD) methods were introduced in [66] and have been well studied in the literature. This involves a multi-objective optimization problem in which the optimization variable is shared across all agents and a consensus-type dynamic is proposed in order to achieve a common solution. These algorithms have been used to solve various optimization problems, such as large-scale matrix factorization and linear programs [67]. Recently, distributed *synchronous* gradient-based methods and convergence properties for differential NE were studied in [68]. Many of these results apply synchronous algorithms, which typically result in a shorter computation time as all agents compute local solutions in parallel, but will require more frequent communications as agents must share information after each step of the gradient descent. When all the agents share a communication medium, this requirement can lead to poor communication performance and, consequently, slower updates in the algorithm.

On the other hand, methods like the best-response algorithm typically used in game theoretical problems have a smaller number of communications or iterations at the cost of a lot of computation being performed by each player at each iteration and a longer total running time. These issues motivate the development of an algorithm that can reach the Nash equilibrium or another desired operating point much faster than the asynchronous best-response dynamics while requiring fewer computations than synchronous algorithms like the synchronous gradient descent.

Objectives: The main idea of this direction of research is to combine methodology from event-triggered control and game theory to produce distributed algorithms that are more efficient in terms of the communications-computations trade-off. In preliminary work, we have looked at the convergence of distributed gradient descent algorithms [69] and how to improve the trade-off between communication and computation with a fixed round-robin-like protocol for action updates. Future work will look at an event-based mechanism for selecting which players update when and how our results can be applied to more general algorithms.

4.1.2 Interactions between intelligent generative agents

Scientific context: The enormous potential of machine learning algorithms is beginning to be better understood by the general public. For instance, the general public is starting to discover open artificial intelligence (AI) platforms for language (e.g., Bert, ChatGPT, or PaLM) and for images (e.g., DALL-E and Midjourney). Technically, these platforms are in part based on generative artificial intelligence. A fundamental question appears to be to what extent such intelligent agents may interact with each other (e.g., cooperate or compete). This question naturally leads us to use synergy tools from game theory and machine learning.

Objectives: First, we will study the case of generative adversarial networks (GANs) to understand the game in which a content generator (image, text) is in conflict with a content discriminator whose role is to know whether the content has been artificially generated. The Nash equilibrium strategies of this game will be analyzed and determined in the case of small games (that is, for small neural networks, by adapting techniques used in continuous game analysis) and for very large games (by resorting to approximations often used in random matrix theory and mean-field games). Second, we will move from this two-player zero-sum game to a general setting with an arbitrary number of players and payoff functions. Multi-agent reinforcement learning will be exploited to treat the latter general case. This direction will be led by a PhD student, whom I co-advise with Prof. Samson Lasaulce at CRAN.

4.1.3 Networked control of haptic devices

Scientific context: The control of robots is a topic that has been attracting a lot of interest from both researchers and engineers in recent years. The applications of such robotic systems include: service robotics and physical human-robot collaboration, assistive and rehabilitation robotics, semi-autonomous cars, drones, etc. A particular area of interest is the area of network-controlled robots with humans in the loop and haptic feedback. Applications include remote surgery and teleoperation of machines in industrial areas. Such use cases have very stringent requirements in terms of latency, reliability, and throughput. However, two issues arise that need to be solved. First, such remote control applications may span over a wide area network, and, in any end-to-end network, losses or bursts of packet drops are bound to occur. Second, URLLC service in 5G has been designed for short, intermittent packets, while haptic feedback requires at the same time a high throughput and a low delay.

Objectives: The above-cited works look at continuous time systems and study idealized and well-established systems. However, for many practical applications like remote surgery, in which humans control the robot to do a certain task over a network, the system and the objective functions are not easy to model mathematically. In this situation, machine learning can be used by the robot to continue performing the task or at least maintain a desired level of system performance when the network is down. Another problem that may be solved using machine learning or classical optimization is to determine the allocation of radio resources among the three streams of video feedback, haptic feedback, and haptic control. The interactions between humans and local AI can be modeled within a cooperative game framework. This direction of research will be led by a PhD student I co-advise with Prof. Salah E. Elayoubi at CentraleSupelec and is part of the ITN project TOAST.

4.2 Adversarial games over networks

4.2.1 Colonel Blotto games for cyber-security

Scientific context: Over the past ten years, the growing number of cyber-related issues and incidents has made cybersecurity one of the main concerns in insurance. The COVID-19 pandemic has considerably increased the exposure of professionals and individuals to computer attacks. Cyber risk has thus been described as the second major risk of the decade to come in the Axa Future Risks report of 2021 [70]. It is even considered to be the main risk in France, according to the Allianz Risk Barometer ranking of 2021 [71]. Thus, large companies are subject to large-scale claims amounting to tens of billions of euros, making reinsurers and insurers vulnerable to the risk of accumulation. Reinsurers saw their loss ratio (claim to premium ratio) jump from 84% in 2019 to 167% in 2020 (see [72]), which led to an increase in premiums and a decrease in coverage on most of the offered contracts. The main guarantees of cyber insurance contracts currently on the market are the identification of the problem and its extent, the implementation of corrective actions (additional protections and programs of prevention), and the payment of certain costs associated with the claim. However, one aspect that is often ignored is the impact of the graph structure on the safety and robustness of the network. **Objectives:** In this context, my goal is to mathematically characterize the effect of the underlying network topology on prices, cyber risk assessment, and management. To do so, we model the interactions between attackers and defenders as a stochastic game. Dynamic and stochastic games pose various challenges and difficulties, and it is very hard to find theoretical solutions to characterize the Nash equilibrium. This direction will be explored in collaboration with Prof. Nabil Kazi Tani from IECL and our PhD student, Mr. Amine Hazzami, funded by the region of Grand-Est and by Fédération Charles Hermite (FCH). We will begin by adapting the well-known Blotto game to apply it to scenarios of cyberattacks. Specifically, we will extend the framework of Blotto games to the mean-field case and to the case of a stochastic differential game. Algorithms to predict the Nash equilibrium for a given graph and, thus, recommendations for a more robust network design are the expected results of this direction.

4.2.2 Control of opinion dynamics over adaptive networks

Scientific context: This task is devoted to the study of multi-agent systems coupled with topology dynamics. Basically, the network (graph) evolves in time, and the dynamics of the graph depend on the states of the agents, while the state of each agent evolves as a function of its neighboring states. This key feature has been studied in certain contexts [36, 73, 41] which study deterministic coupled dynamics models and on voter models [74] for opinion dynamics and epidemics [75]. In a previous work [76], I studied discrete opinion states evolving based on random gossiping. The work on this direction will be focused on studying the coupled dynamical processes of the network and the opinions with stochastic features using mean field techniques. The model under consideration will **mix stochastic dynamics of the opinion with deterministic dynamics of the interaction network.** Basically, the **graph will evolve** according to the rules in [36] while the opinions will be updated stochastically as in our preliminary work [76]. Although the combination of temporal networks with voter models is more recent, it has gained a growing deal of attention, often referred to as the "Adaptive Voter Model" (AVM) [74]. Most of the existing papers study a global linkage where the agents create new links by picking *uniformly at random* (u.r) among the whole population (rewire-to-random), sometimes with a homophilic refinement: rewiring is global u.r but only with agreeing agents (rewire-to-same) [77]. This u.r linkage has also been analyzed in the context of epidemics [75]. In contrast, local linkage has been far less studied.

Objectives: Our objective in this context is to study the interaction between two external (opposing) entities that try to manipulate the opinions of agents interacting over an adaptive network. This direction is led by Emmanuel Kravitzch, a PhD student recruited as part of the ANR project NICETWEET, and will be done in collaboration with Prof. Yezekayel Hayel from the University of Avignon and Prof. Antoine Berthet from Centrale-Supelec. He has already made significant progress on theoretically modeling and analyzing the behavior of the AVM we proposed, and the next step is to study the control of such systems.

4.2.3 Individual de-carbonization strategies

Scientific context: Global warming is a major environmental concern, and total carbon emissions keep increasing despite the active socio-political movement to curb it. De-carbonization strategies are of great importance both at the macro (at the level of big companies, and countries) and the micro (individuals) level. The state of the art on this subject contains both geophysical-type and economic-type studies, mainly based on empirical or ad-hoc strategies. While the literature on climate change is quite rich, to our knowledge, very few works apply formal game theory to mathematically model both the geophysical and strategic aspects. As a result, the de-carbonization strategies proposed are typically suboptimal in the sense of an overall performance metric given for the incentive and do not benefit from the support of the actors involved in de-carbonizing.

Objectives: To resolve this problem, our goal in this direction of research is to make significant advances in terms of modeling a set of individuals emitting CO2 both under the influence of other individuals via a social network and under the influence of an external inciting or regulating entity (e.g., a government). To do this, we will exploit and develop the most recent results in neuroscience, economics, control, and game theory to propose a realistic behavioral model of the individual in terms of decarbonization. We will construct a graph model of the social network that characterizes the mutual influence between the transmitting individuals and aggregates the individual behavioral model developed, similar to our previous work in [54]. We will study the interaction between two opposing players who try to control climate change and those who act in favor of increasing carbon emissions. The synergy of game theory, dynamic systems, and neuroscience constitutes a promising new point of view to achieve the research objectives, and will be led by Thomas Mongaillard, a newly recruited PhD student co-advised with Prof. Samson Lasaulce at CRAN.

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