A Scalable Approach for Data-Driven Taxi Ride-Sharing Simulation

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***Abstract*—As urban population grows, cities face many challenges related to transportation, resource consumption, and the environment. Ride sharing has been proposed as an effective approach to reduce traffic congestion, gasoline consumption, and pollution. Despite great promise, researchers and policy makers lack adequate tools to assess tradeoffs and benefits of various ride-sharing strategies. Existing approaches either make unrealistic modeling assumptions or do not scale to the sizes of existing data sets. In this paper, we propose a real-time, data-driven simulation framework that supports the efficient analysis of taxi ride sharing. By modeling taxis and trips as distinct entities, our framework is able to simulate a rich set of realistic scenarios. At the same time, by providing a comprehensive set of parameters, we are able to study the taxi ride-sharing problem from different angles, considering different stakeholders' interests and constraints. To address the computational complexity of the model, we describe a new optimization algorithm that is linear in the number of trips and makes use of an efficient indexing scheme, which combined with parallelization, makes our approach scalable. We evaluate our framework and algorithm using real data – 360 million trips taken by 13,000 taxis in New York City during 2011 and 2012. The results demonstrate that our framework is effective and can provide insights into strategies for implementing city-wide ride-sharing solutions. We describe the findings of the study as well as a performance analysis of the model.**

***Keywords*-taxi ride sharing; simulation; shortest-path index; scalability;**

I. Introduction

With the steady growth in urban population [1], cities face huge challenges around transportation, resource consumption, and pollution. Ride sharing has been proposed as a strategy to decrease road traffic and gasoline consumption [2] while at the same time serving the transportation needs of city dwellers. In large cities, there is substantial unused capacity that can be filled by ride-sharing services. Consider, for example New York City (NYC): each day, taxi cabs make 500 thousand trips and serve 600 thousand passengers; this translates into an average occupancy rate of only 1.2 passengers per trip [3]. While private companies such as Uber, Lift, Via, Bandwagon and Cab With Me already provide ride-sharing services, they represent a small percentage of the market.

A wide deployment of ride sharing requires a better understanding of its tradeoffs. This is challenging since there are multiple stakeholders with different and sometimes conflicting interests. Governments want less traffic and pollution; taxi companies want to maximize their profits; and passengers would like to reach their destination quickly and cheaply. To design an effective policy, these interests need to be considered. Early approaches to this problem have been primarily devised on the basis of survey data [4] and analysis of psychological incentives [5]–[7]. Ride sharing has also been modeled as an optimization problem whose objective is to identify optimal ride-sharing schedules [8]–[13]. However, these approaches focus on small-scale problems, such as sharing at airports, since large-scale optimization is often computationally infeasible.

The availability of large volumes of taxi trip data creates new opportunities to apply data-driven approaches to this problem. Santi et al. [14] proposed a graph-based approach to the taxi ride-sharing problem and reported on results using NYC taxi data. Their model computes optimal sharing strategies for trips and contains two key parameters: the maximum number of trips that can be shared and the maximum delay customers can tolerate. While this allows the study of sharing benefits as a function of passenger inconvenience, the model has important limitations. Notably, it is intractable for scenarios that consider the sharing of three or more trips and it assumes that all trips are known in advance. Ma et al. [15] introduced T-Share, a ride-sharing dispatch system that can serve real-time requests issued by passengers and generates schedules that reduce the total travel distance. This system, however, was not designed to support simulations: it neither provides the necessary parameters to simulate different scenarios, nor does it scale to very large data sets.

We propose a data-driven simulation framework that enables the analysis of a wide range of ride-sharing scenarios. In contrast to the model used in [14], in our model, trips need not be known in advance – the framework supports the simulation of real-time ride sharing which serves unplanned trips, and fits well the models using different vendors, such as Yellow cabs and Uber. Furthermore, the simulation model represents taxis and trips as distinct entities, and provide variables that enable the study of realistic scenarios by taking into account the needs and constraints of multiple stakeholders. These include different customer preferences (e.g., maximum number of additional stops and wait time), and taxi-specific constraints often dictated by ride-sharing vendors, for example, the number of passengers on a per-taxi basis and maximum number of shared

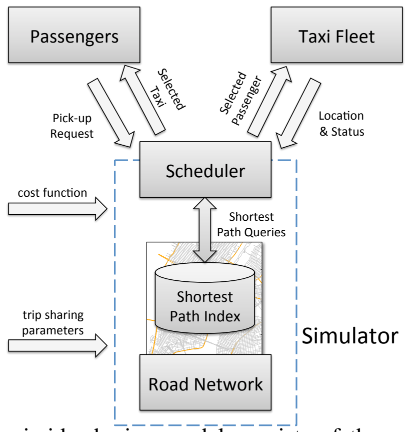

Fig. 1: Our taxi ride-sharing model consists of three main entities: taxis, passengers and a scheduler. Given a trip request issued in real-time, our model assigns this trip to a taxi in such a way to optimize a given cost function while at the same time respecting a set of predefined constraints.

trips. The flexibility of the model comes at a cost: assigning trips to taxis in real-time is computationally expensive. We describe a new optimization algorithm that is linear in the number of trips and makes use of an efficient indexing scheme, which combined with parallelization, makes our approach scalable. We evaluate the efficiency and effectiveness of our model using taxi data from NYC, which contains information about over 360 million trips taken by the NYC's 13,000 taxis in 2011 and 2012. The results demonstrate that our approach is efficient: one simulation using over 150 million trips can be run in under 10 minutes using a 1200-core cluster, allowing multiple scenarios to be studied in a timely manner. We also show that the framework is effective and can provide insights into strategies for implementing city-wide ride-sharing solutions. We experimentally compare our approach against [14] and argue that their model can underestimate the benefits of the taxi ride sharing.

II. Ride-Sharing Simulation Model

Given a trip request issued in real-time, our model assigns this trip to a taxi to optimize a pre-defined cost function while at the same time respecting a set of constraints.

A. Simulation Components

The main components of our simulation model are illustrated in Fig. 1 and we describe them below.

Taxi Fleet. The taxi fleet refers to the set of taxis that are involved in the simulation. In contrast to previous works, where taxis are considered as homogeneous objects, to support a multi-vendor environment (e.g., yellow cabs, green cabs, and black car services)[1] and different types of cabs, we consider each taxi as a distinct object with its own ride-sharing specifications, which include: passenger capacity, maximum number of shared trips (often dictated by the vendors), maximum wait time for pick-up, and extra time for drop-off. In addition to these sharing constraints, each taxi also maintains information about its current speed, occupancy and the list of stops it has to make to serve its scheduled riders. The list of variables associated with a taxi is given in Table I.

[1]http://www.nyc.gov/html/tlc/html/industry/current_licensees.shtml

Taxi Ride-Sharing Constraints	Description
C	capacity of taxi expressed as the number of passengers
n_{share}	maximum number of trips that can be shared by each rider (or -1 if the taxi only limits this as per customer preferences)
t_{delay} or d_{delay}	maximum time/distance the taxi is permitted to let its customers wait for their pick-up.
t_{extra} or d_{extra}	maximum additional time/distance incurred by sharing a ride.

Vehicle States	Description
o	current occupancy, i.e., number of passengers on-board
id	taxi identification number
v	current speed
$s_0, \ldots, s_k$	list of stops the taxi has to make; s_0 is the location of the last stop location.
$d_{odometer}$	current odometer reading
d_{driven}	distance from s_0

Stop Info	Description
p	the location of the stop (could be expressed as the intersection number in the road network)
$o_\triangle$	the number of passengers associated with this stop, where $o_\triangle > 0$: a pick-up $o_\triangle < 0$: a drop-off $o_\triangle = 0$: a waypoint (e.g., to look for riders)

TABLE I: Taxi specifications: ride-sharing variables and vehicle state information associated to each taxi and stop event.

Passenger Ride-Sharing Constraints	Description
o_p	number of passengers
t_{pick}	time requested for pick-up
p_{pick}	pick-up location
p_{drop}	drop-off location
n_{share}	maximum number of trips to be shared
t_{delay}	maximum time the customer group is willing to wait for pick-up
t_{extra}	maximum time the customer group is willing to tolerate in addition to the actual trip time

TABLE II: Customer group ride-sharing parameters.

A taxi in our model is a dynamic object and is always on the move. Even without any passenger on-board, the taxi still has a destination where it drives to, for example, to look for new passengers. To capture this, we allow each taxi to define favorite locations at certain times of the day that it would drive to when it is not occupied.

Passengers. We assume that passengers ride in groups of size greater than or equal to one. Each group is associated with a drop-off location and a set of ride-sharing constraints (e.g., how many other groups they are willing to share a ride with and how much additional time/distance they can tolerate). We assume this information is available when a customer initiates a taxi request, either by calling a control center, using a mobile app, or communicating this directly to a driver in case of street hailing. Table II lists the variables associated to passengers in our model.

Scheduler. The scheduler receives pick-up requests from passengers and finds the most appropriate taxi for each of them based on pre-defined metrics. To do so, the scheduler must be aware of all taxi locations along with their current states at all times (see Table I). We describe the details of the scheduling algorithm in Section III.

Trip Data. The proposed simulation framework is data-driven, allowing historical trip data to be used to study the tradeoffs of different ride-sharing strategies. We assume the availability

Input Parameters	**Description**
m	number of taxis
C	default taxi capacity. Optionally, the capacity for each taxi may be customized by providing an additional array $\{c_0, c_1, ..., c_{m-1}\}$.
n_{share}	maximum number of trips to be shared by default. This number initially populates both of the n_{share} values of each taxi and passenger ride-sharing constraints (see Table I, II), unless specified otherwise. The motivation for having this parameter defined globally was to allow agencies/taxi vendors to define their fleet policy in addition to what described by the drivers and riders.
$t_{delay}(d_{delay})$, $t_{extra}(d_{extra})$	maximum wait and additional time(or distance) each customer could spend by default. Similar to n_{share} above, the motivation for having these parameters defined globally was to give additional flexibility to the users.
$\mathbf{f}(r, c)$	a cost (metric) function that given a taxi c and a pick-up request r return the cost of accommodating r with c. By default, this function evaluates the additional distance that c must drive to accommodate r.

TABLE III: Simulation input parameters.

of trip data which include: taxi ID, pick-up and drop-off time, latitude and longitude for both pick-up and drop-off, travel distance, and number of passengers. We model trips and taxis as separate objects, and track the state of each taxi during the simulation. This is equivalent to assuming that the data set is a sample of trips from an unknown distribution and ignoring that each trip was originally associated with a particular taxi. ***Road Network.*** The underlying road network of a city is represented as a directed graph $G(V, E)$. All taxis travel along this road network. Each directed edge $e \in E$ represents a road segment, and each node $v \in V$ represents the intersection of two or more roads. When a road allows traffic flow in both directions, there are two directed edges for that road. Given a segment e_i, t_i is the distance (or time) a vehicle must travel from one intersection point to another along e_i. Note that traffic conditions can be easily incorporated in the model by introducing weights on edges of the graph as explained in Section III. We assume that the origin and destination of a trip correspond to nodes in this graph. If the trip begins or ends in the middle of a road segment, we approximate the location to the nearest intersection node.

B. Data-Driven Simulation

The simulation engine aims to derive the *best* ride-sharing scenario based on a set of input parameters (shown in Table III) in a *data-driven fashion*, where pick-up requests are derived from historical data. It operates in an event-driven manner and updates its state when a pick-up request is issued. When a customer group requests a taxi, the scheduler receives the information and requests all taxis to report their status (i.e., position and sharing status). The scheduler then computes the additional cost for each taxi to accommodate this trip based on the cost function f, and selects the taxi with the minimal cost that satisfies all ride-sharing constraints. If no appropriate taxi is found, the request is denied. As we discuss in Section III, the simulation engine allows different scheduling strategies.

Since taxi ride-sharing typically occurs in real-time, our approach needs to support online simulations, that is, simulations where trip requests are issued dynamically. Thus, the scheduler must evaluate the current conditions and respond to the customer immediately. If the request is accepted, the current state need to be updated as well. The need for high throughput and immediate responses separates us from previous work. For instance, [14] assumes that all the trips are known in advance which can substantially reduce computational requirements but does not lead to a realistic ride-sharing solution.

III. Simulation Algorithm

In the taxi ride-sharing problem, the goal is to minimize the total cost or maximize the total utility of sharing while meeting a set of constraints. Examples of costs include travel distance, CO_2 emissions, gasoline consumption, time-to-pick-up, idle time, or weighted combinations of these basic cost functions. We use the following formulation to design our optimization algorithm. Let $f(r_i, c_j)$ be the additional cost (e.g., distance, CO_2 emissions) a cab c_j incurs to share its current trips with a new trip r_i. Let n be the number of trips and m be the number of taxis. The minimum total travel cost for the first i trips is:

$$T(i) = \begin{cases} T(i-1) + \min_{1 \le j \le m}\{f(r_i, c_j)\} \\ T(i-1) \text{ if there is no available cab.} \end{cases}$$

We can use a similar formulation to maximize the utility (e.g., revenue). Henceforth, to describe our algorithm, we use the additional travel distance for a cab c_j to accommodate a trip r_i as cost function $f(r_i, c_j)$ and try to minimize the total travel distance $T(n)$.

The algorithm considers all trips in chronological order. It attempts to mimic real-time dispatching by minimizing $T(n)$ in an online fashion. For each trip r_i, the state of a cab c_j is updated based on the time elapsed since the last trip r_{i-1} and the computed additional distance $f(r_i, c_j)$. Trip r_i is assigned to the cab with the minimum additional distance, and the total cost $T(i-1)$ is updated. Note that this is not a globally optimal solution: we do not consider all the possible combinations of trips to be shared. However, this matches a more realistic scenario in which we are not able to foresee the future trips or make changes to the past trips.

A. Preprocessing Phase

Before running the simulation, trips are sorted in chronological order (by pick-up time), and as described in Section IV, erroneous data are eliminated. We use a graph representation of the road network. We obtain this representation from the data in [16], which contains the longitude and latitude of each intersection as well as the distances between any two adjacent intersections. We apply Dijkstra's algorithm to compute shortest paths and distances between any two given intersections. In this step, we also make sure that the shortest paths are indexed in a cache-coherent layout to facilitate our in-simulation queries (see Section III-D). The running time of the preprocessing phase is $O(k^3 + n \log n)$, where k is the number of intersections and n is the number of trips.

Note that traffic conditions can be easily integrated in the simulation. One can replace a single shortest path matrix with a different shortest path matrix for each hour of the day. In each of these matrices, distance between any two intersections is adjusted according to the traffic conditions that can be

Algorithm 1 Taxi ride-sharing simulation

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1:  Parameters: $m$: the number of cabs, $C$: the capacities of cabs, $n_{share}$, $d_{delay}$, $d_{extra}$, $f(r, c)$
2:  Inputs: $R$: a set of trips
3:  $R \leftarrow \mathrm{Sort(R)}$
4:  $\mathrm{InitializeCabs(m, C)}$
5:  for $i = 1$ to $n$ do
6:      $\mathrm{ElapsedTime} \leftarrow \mathrm{PickUpTime}(r_i) - \mathrm{PickUpTime}(r_{i-1})$
7:      $f^* \leftarrow \infty$
8:      for $j = 1$ to $m$ do
9:          $\mathrm{UpdateState}(c_j)$
10:         $f_{ij} \leftarrow f(r_i, c_j)$
11:         if $f^* > f_{ij}$ then
12:             $f^* \leftarrow f_{ij}$
13:             $c^* \leftarrow c_j$
14:         end if
15:     end for
16:     $\mathrm{Assign}(r_i, c^*)$
17:     $T(i) \leftarrow T(i-1) + f^*$
18: end for
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Algorithm 2 Cost Function $f(r, c)$

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1:  Inputs: $r$: a trip, $c$: a cab
2:  $S \leftarrow$ a list of stops of $c$ including its current location $\{s_0, s_1, \ldots, s_k\}$
3:  $D_{drop} \leftarrow \mathrm{ShortestPathDistance}(s_k, p_{drop})$ {$s_k$: the last stop in $S$}
4:  $idx_{pick}, D^* \leftarrow \mathrm{FindPickUpOrder}(r, c, S, D_{drop})$
5:  if $idx_{pick} \leq k$ then
6:      $D_{pick} \leftarrow D^* - D_{drop}$
7:      $idx_{drop}, D^* \leftarrow \mathrm{FindDropOffOrder}(r, c, S, idx_{pick}, D^*, D_{pick})$
8:  end if
9:  return $D^*$
10: Output: $D^*$: an additional distance for cab $c$ to accommodate trip $r$
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inferred if additional traffic data is available. In particular, this can be achieved by scaling the weights of the edges in the road network by the ratio of the average speed on that edge at a given time and a base speed (for instance, speed limit).

Finally, observe that by separating the notion of a cab from a trip, we gain extra flexibility that is not present in [14]. For example, we are able to study the relationship between the number of cabs and shareability. We can also consider scenarios in which different cabs have different capacities. Moreover, we can select the initial locations for each individual cab.

B. Simulation Phase

As shown in Algorithm 1, we consider each of the n trips in chronological order. For each pick-up request r_i, we go through all the cabs in the fleet and update their states based on the time elapsed from the last trip's pick-up time. For each cab c_j, we also compute additional cost $f(r_i, c_j)$ and we assign r_i to the cab with the lowest additional cost.

To update the state of a cab (i.e., location, occupancy and stops that it is visiting to pick up and drop off passengers), we need its speed, which is defined to be the speed of the trip whose pick-up or drop-off takes place next. This can be estimated using the duration (drop-off time − pick-up time) and the distance of this trip. Once we have an estimation of the speed, the traveled distance and position of each cab can be interpolated from the time it spent from the beginning of the trip. At the same time, we also update the cab occupancy and planned stops if there was any scheduled drop off or pick up along its traveled itinerary.

The most straightforward way to compute additional cost $f(r_i, c_j)$ is to explicitly find an optimal route for c_j that includes the pick-up and drop-off locations (p_{pick} and p_{drop} respectively) of r_i and to compare its cost with the cost of the current route for c_j. However, computing the optimal path is known as the Sequential Ordering Problem (SOP) which is a version of the Traveling Salesman Problem and is NP-hard [17]. Thus, to make the computation tractable, we use a heuristic to find a best route for c_j to accommodate r_i. We first find a position to insert p_{pick} of r_i into the list of stops S assuming that the order to visit those stops stays the same and p_{drop} of r_i is added to the end of the route. After that, we adjust the order of p_{drop} so that we can find a route with the lower additional cost. Along with computing $f(r_i, c_j)$, we check if c_j has enough capacity for all the passengers of r_i and if the constraints given by n_{share}, d_{delay} and d_{extra} are satisfied. Otherwise, we set $f(r_i, c_j)=\infty$. After we assign r_i to the cab with the minimal additional cost, we update the stops S to reflect this assignment. The computation of $f(r_i, c_j)$ is described in Algorithm 2; we discuss it in detail below.

Let $S=\{s_0, s_1, \ldots s_k\}$ be a list of scheduled stops for cab c_j, and let p_{pick} and p_{drop} be the pick-up and drop-off locations of r_i, respectively. If c_j is vacant, the additional cost $f(r_i, c_j)$ is simply the sum of shortest path distances between its current location s_0 and p_{pick}, and p_{pick} and p_{drop}.

If c_j has passengers, we need to find the best positions to insert stops p_{pick} and p_{drop} into S. The challenge is to find the order for those with the smallest additional distance and at the same time satisfy all the constraints, in particular, making sure that each trip is shared with at most n_{share} other trips and the number of passengers on a cab does not exceed the capacity C at all stops. In addition, for each trip we ensure that the delay distance (the distance from the location where this trip was assigned to its pick-up location) and extra distance (the length of the route from p_{pick} to p_{drop} including other stops minus the shortest path distance between p_{pick} and p_{drop}) does not exceed d_{delay} and d_{extra} respectively. To do so, we aggregate information about the maximum number of shared trips, maximum occupancy, the delay and distance among the stops and each stop maintains such information. We use this information to check if all the constraints would be satisfied for a trip request and the trips that are already being served.

To compute the additional distance to accommodate r_i, we assume temporarily that the drop-off will happen after the last stop s_k. Let D_{drop} be the length of a shortest path between s_k and p_{drop}. As mentioned above, we first find a position for p_{pick} (Algorithm 2, line 4). Suppose $l' \in \{0, \ldots, k\}$ is such that each trip containing one of stops $s_{l'}, \ldots, s_k$ can be shared with at least one more trip and the occupancy o of c_j does not exceed the capacity C of c_j at any stop of $s_{l'}, \ldots, s_k$. For each $l \in \{l', \ldots, k-1\}$, we try to insert p_{pick} between s_{l-1} and s_l. If the lengths of shortest paths between s_{l-1} and p_{pick}, p_{pick} and s_l, and s_{l-1} and s_l are D_1, D_2 and D_3 respectively, then an additional distance is defined to be $D=D_1+D_2-D_3+D_{drop}$. Then, we determine the position of p_{pick} that minimizes this additional distance D and at the same time satisfies the delay and extra distance constraints given by d_{delay} and d_{extra} respectively for both r_i and all the trips in service. Let $f'(r_i, c_j)$ be the minimum of D over l and $s_p \in S$ be the first stop after p_{pick}.

Next, we search for a best position for p_{drop} (Algorithm 2, line 7). Similarly to the previous process, for each $s_l \in \{s_{p-1}{=}p_{pick}, s_p, s_{p+1}, \ldots, s_{k-1}\}$, we query for the shortest path distances between s_l and p_{drop}, p_{drop} and s_{l+1}, and s_l and s_{l+1}. Let D_4, D_5, D_6 denote these distances respectively. Then, an additional cost for each new route is defined as $D'{=}f'(r_i, c_j){-}D_{drop}{+}D_4{+}D_5{-}D_6$. We select the route that minimizes the additional cost, i.e., $\min\{D', f'(r_i, c_j)\}$, and set $f(r_i, c_j)$ to be this quantity. If a route is such that the extra distance or delay for r_i and trips in service exceed d_{extra} or d_{delay}, then the route is discarded. Note that the algorithm performs pruning. It stops considering a cab once the delay constraint given by d_{delay} is no longer satisfied. It also prunes stops (the underlying search space of the approach) – it only considers stops that satisfy capacity and sharing constraints.

Time complexity. The complexity of the simulation phase of our algorithm is $O(nm)$, which is linear in the number of trips. This allows us to scale dynamic pick-up and delivery tasks to large data sets. However, a large number of shortest-path queries is needed for each trip and cab (Algorithm 2, line 3, 4 and 7). Even though the algorithm is efficient, these queries become bottleneck for efficiency. Below, we propose two strategies that exploit parallelism and an efficient shortest-path indexing scheme to support large scale simulations.

Discussion. Approximation algorithms for SOP have been studied extensively [17], [18]. While our heuristic may appear to have a performance that is suboptimal compared to these algorithms when a trajectory for a particular cab is considered, the tradeoff of using a heuristic approach is substantially reduced in our study due to the large number of cabs involved. More precisely, suppose $\mathcal{T}$ is a set of all possible trajectories. If we assume that the set of optimal trajectories $\mathcal{T}^* \subseteq \mathcal{T}$ associated with each cab is an i.i.d sample of size l and $X_1, X_2, \ldots, X_l$ are costs of these trajectories, then the probability that the minimum cost found by our heuristic is more than ϵ away from the optimal cost Θ is:

$$P(\min(X_i) - \Theta > \epsilon) = \big[P(X_i - \Theta > \epsilon)\big]^l = \left[\tfrac{M-\epsilon}{M}\right]^l$$

where M is the maximum possible cost and the second equality follows from the definition of uniform distribution. For instance, suppose $l{=}5000$, $M{=}30$ miles and $\epsilon{=}0.2$ mile, the probability that the minimum cost derived by the heuristic would be more than 0.2 mile away from the optimal cost is $2.9857e{-}15$.

C. Exploring Parallelism

Though our simulation algorithm achieves linear scaling with the number of trips, running a simulation at a large scale, e.g., with one year of data, can be prohibitively expensive. Using one CPU core, our algorithm takes almost 15 minutes to complete a simulation with 11,500 taxis for a single day. To address this problem, we leverage parallel architectures using two strategies: *intra-request* for the cost computation and *inter-partition* for the temporal simulation.

Intra-Request Parallelism. Since each simulation step depends on the results of the previous step, it is not possible to achieve parallelism at this level, i.e., having each thread

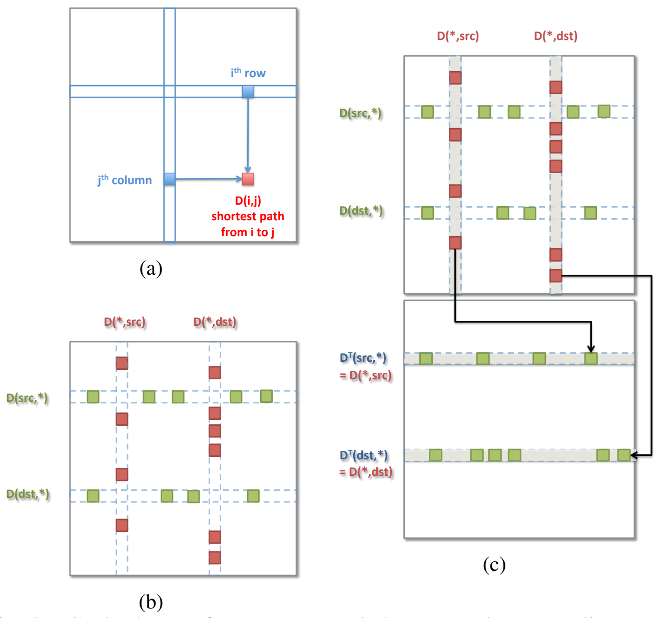


Fig. 2: Cache layout for precomputed shortest paths: (a) a distance matrix as a result of running Dijkstra's algorithm on all intersection pairs; (b) memory access patterns from computing $f(r, c)$ for a pick-up request going from *src* to *dst* – red indicates a high chance of cache misses; (c) storing the transpose of the distance matrix to avoid cache misses in backward lookups.

execute one step. However, computing the sharing cost of each taxi with respect to a pick-up request can be done in parallel since this evaluation for each taxi is independent from each other (Algorithm 1, lines 9–14). In our implementation, we use a thread pool model to distribute works, i.e., a set of taxis, among machine cores for this computation. To minimize the inter-thread communication, each worker thread processes a number of taxis at time.

Nonetheless, all workers still need to synchronize with each other at the end of each pick-up request (Algorithm 1, line 16-17) to assign the best solution for the trip request. Generally, a lock has to be used to avoid race conditions. However, using locks on hundreds of millions of iterations would be a bottleneck itself. Thus, to work around this issue, we make use of atomic operations that are available on modern x86 architectures to construct our work queue in a lock-free manner [19]. Precisely, using 8 threads, our simulator was able to finish a one-day run in just under four minutes, with the lock-free queue giving us a 20% boost in performance.

Inter-Partition Parallelism. The intra-request strategy parallelizes tasks based on the number of taxis, which is relatively small. Thus, it is not able to leverage larger systems with thousands of cores. In order to utilize larger resources, we need to parallelize tasks based on the number of pick-up requests. As stated above, running simulations in parallel on separate pick-up requests is not possible due to the dependency of steps. Fortunately, there were almost no trips sharing during the early morning hours, 4AM, which coincides with the AM shift change of NYC taxis [3]. Using this insight, we can safely divide our data into independent simulations of one-day in size

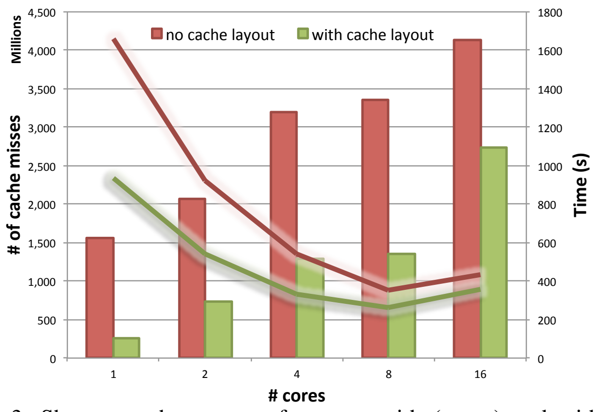


Fig. 3: Shortest path query performance with (green) and without (red) cache-coherent layout. The cache-coherent layout reduced the number of cache misses substantially (3x-6x less); thus, improve the overall simulation performance. The experiments were run on an entire day worth of taxi trip records (∼300k requests), using n_{share}=4.

without sacrificing the correctness of our simulation. This way, an entire year simulation could be divided into 365 sub-tasks that can be run in parallel. Nevertheless, a final reduction step is still needed for reporting the result of the entire simulation. This phenomena is not specific to NYC and one can expect that periods of low taxi activity that occur naturally due to human diurnal cycle can always be used to parallelize the simulation.

Spawning processes for the above sub-tasks using a traditional framework, such as MPI, should serve our need; however, it offers little aid in performing data analysis on the results (such as generating plots of shared trips). We have extended our system to support the MapReduce framework and allowing the integration of analysis tasks using MapReduce jobs. In our setup, each mapper is a simulator program that can process independently a set of pick-up requests. Data filtering may be applied at this stage to limit data requests to some constraints, e.g., a spatio-temporal condition. Most analysis tasks happen in the reduce phase of our framework. Depending on the analysis, users can specify an appropriate output for the map phase. The results reported in Section IV were obtained using the MapReduce framework.

D. Cache-Coherent Shortest Path Index

Our simulation algorithm uses shortest path queries extensively (Algorithm 2). This is where our computation spends the most time. In particular, each computation of $f(r_i, c_j)$ makes a series of shortest path queries to all stops of c_j to exhaustively find the minimal solution. Since the complexity of our algorithm is $O(mn)$, this could result in a very large number of queries. For example, performing a simulation of 11500 taxis with n_{share}=4 on a day worth of taxi trip records (∼300k requests) would require over **3 billion** shortest path queries. Thus, it is of utmost importance that we build an efficient shortest path indexing scheme to support such queries.

Our initial approach was to precompute and cache the shortest distances (and their predicates) for all possible intersection pairs in NYC. The storage size of this matrix is fairly small (about 500MB for roughly 10,000 intersections), and would fit completely on commodity PCs. Therefore, our shortest path queries are now reduced to just memory accesses. The data structure for this caching scheme is depicted in Fig. 2a.

Nevertheless, our experiments still show under utilization of CPUs when running large experiments. Inspecting further, we noticed that there was a large number of L2/L3 cache misses for these queries (>50% of the memory access). This can be explained by the memory access pattern which is described in Fig. 2b. Each time we compute the cost to accommodate a request, we have to issue shortest path queries originating from as well as going to its pick-up location *src* and drop-off location *dst*. In fact, over 95% of the queries involve *src* and *dst*. Among these queries, forward lookups, i.e., finding shortest paths originating "from" an intersection, would present a cache-coherent memory access patterns (depicted in green). However, backward lookups, i.e., finding shortest paths going "to" an intersection, is likely to incur cache misses most (if not all) of the time. This is because elements inside a column could be tens or hundreds of megabytes apart from each other. In this case, the entire shortest path computation would need be in cache to have all requests served without any penalty.

We propose a simple, yet efficient, cache-coherent layout to increase the cache coherence of shortest path lookups. By transposing the shortest path matrix, backward lookups become forward lookups and vice versa. Therefore, we elected to store an additional transposed matrix in our shortest path cache to convert all backward lookups to forward lookups. As illustrated in Fig. 2c, each CPU only needs to keep 4 rows of data in its L2/L3 cache to serve all shortest path queries related to a trip request. This comes at a cost of doubling the shortest path data structure; however, it significantly improves the simulation performance. Fig. 3 shows the shortest path query performance with (green) and without (red) cache-coherent layout. The cache-coherent layout was able to reduce the number of cache misses up to 6 times on a single core, thus, resulting in a 2x speedup of the overall performance. The experiments were run on an entire day worth of taxi trip records (∼300k requests), using n_{share}=4. However, the speed-up of using multiple cores was not demonstrated after 4 cores, especially at 16 cores, due to the saturation of memory bandwidth that overshadowed the computation cost. We found that using 4 cores yields the best performance ratio; hence, we used this setup for our experiments, unless noted otherwise.

E. Complexity Analysis

As mentioned above, the *serial complexity* of our simulation is $O(mn)$, asymptotically. A tighter bound is, however:

$$O(n \cdot (mC_f + C_a)) \tag{1}$$

where C_f and C_a are the complexity of our cost function (Algorithm 2) and taxi assignment (Algorithm 1, line 16), respectively. Both C_f and C_a are $O(|S|)$, where $|S|$ is the maximum number of stops each taxi maintains. Since $O(|S|)$=$O(n_{share})$, the complexity of our algorithm is indeed $O(n_{share}mn)$. Given that n_{share} is usually much smaller than m and n (e.g., 4 or 5 vs. roughly ∼10k and ∼500k, respectively), we can consider n_{share} as just a constant.

For the *parallel complexity analysis*, we show that our implementation is *cost optimal*, i.e., its asymptotic running time multiplied by the number of parallel processors involved in the computation is comparable to the running time of the best serial implementation [20]. Since the inter-partition parallelism is a direct share-nothing computation and is expected to achieve cost optimality through the map phase of MapReduce, our focus is on the intra-request parallelism. For a thread pool of size p, the sharing cost computation of m taxis is evenly distributed to p threads, resulting in the time complexity:

$$O(n \cdot (\tfrac{m}{p} C_f + C_s + C_a)) \tag{2}$$

where C_s is the synchronization time of all threads at the end of each request. In our case, $C_s{=}O(p)$ since we only need to compare solutions of p threads to select the optimal one. By definition, our framework can achieve cost optimality iff:

$$\begin{aligned} p \cdot (n \cdot (\tfrac{m}{p} C_f + C_s + C_a)) &= O(nm) \\ n \cdot (mC_f + p^2 + p \cdot C_a) &= O(nm) \end{aligned} \tag{3}$$

In order to satisfy Equation 3, the following must be true: $C_f{=}O(1)$, $p^2{=}O(m)$ (or $p \leq \sqrt{m}$), and $C_a \leq \frac{m}{p}$. In our case where p is always set to at most 8 and n_{share} is a constant, all of the above conditions are always true. Thus, the parallelism in our framework is cost optimal.

The above constraints also define the class of algorithms that can be plugged into our framework without loss of scalability. In particular, the algorithm that finds a route for each cab (C_f) should have a time complexity that is independent from m and n while selecting which cab should service a given trip (C_a) must be done in linear time. These guarantee the asymptotic complexity of the system to be $O(mn)$. However, if a more complex algorithm is desired, it can still be integrated into our framework, possibly with an additional cost. For example, employing a trip selection algorithm that runs in $O(m)$ time or a taxi selection algorithm that runs in $O(m^2)$ time would result in an overall complexity of $O(m^2n)$ for our system. Nevertheless, the inter-partition parallelism would always be in place regardless of the scheduling algorithms chosen.

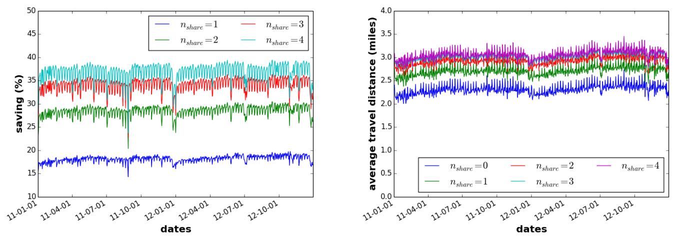
IV. Experimental Evaluation

In this section, we demonstrate the scalability of our approach using the NYC taxi trip data set described below. To run our experiments, we used the open source Apache Hadoop software library on a 1200-core cluster. We also describe findings of our study which indicate that our approach is effective and derives information that may be useful to policy makers, the taxi industry and riders.

A. Data

For our study, we used the NYC taxi trip data set of 2011 and 2012 which was provided to us by Taxi & Limousine Commission (TLC) through a FOIL request.[2] This represents a superset of the data set used in [14], where only 2011 trips were considered. The data contains information about 360 million trips taken by the 13,237 taxis in NYC. Each trip is represented by a vector with the following fields: taxi ID (medallion ID), pick-up and drop-off times, latitude and longitude for both pick-up and drop-off, travel distance in miles, and number of passengers. All IDs are anonymized. We considered all trips that occurred within Manhattan, and between Manhattan and the two international airports: John F. Kennedy and LaGuardia. These trips constitute the majority of all taxi trips in NYC [21]. We also eliminated data that appeared to be erroneous, including trips without passengers, with the average speed less than 3 mph, and that start or end at invalid locations. The 260 million trips that remained after selection and cleaning were used in our experiments.

[2]An open data set containing one year of data is available at http://chriswhong.com/open-data/foil_nyc_taxi.



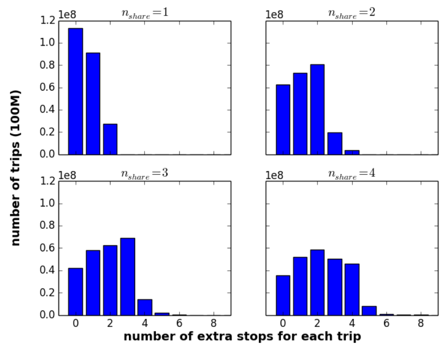
(a) (b)

Fig. 4: (a) The percentage of saved total travel distance through ride-sharing for $n_{share}{=}1, 2, 3, 4$ (b) The average travel distance for $n_{share}{=}0, 1, 2, 3, 4$.

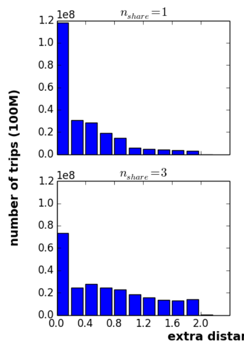
B. Taxi Ride Sharing in NYC: A Case Study

Recall that taxi ride sharing involves three major entities: the city, taxi companies (drivers) and passengers. Each of them aims to minimize different costs: the city is interested in reducing pollution and traffic, the taxi companies want minimize the operating costs while maximizing revenues, and passengers want to get their destinations as fast and cheaply as possible. Thus, a solution to the taxi ride-sharing problem has to take into account tradeoffs between these competing objectives.

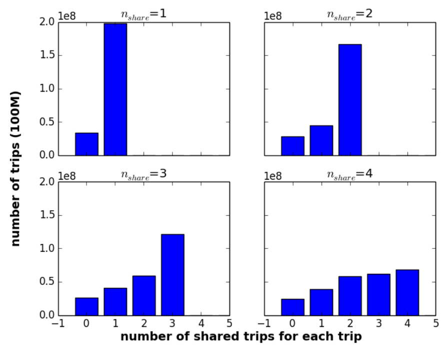
A natural metric that can be used to analyze the effects of ride sharing is the total distance traveled by all the cabs, which correlates with both the traffic volume and emissions. A decrease in the total traveled distance can also serve as an incentive for the taxi industry to engage in ride sharing, since such a decrease is likely to lead to a proportional decrease in the cost of running the business. However, ride sharing also imposes additional costs to the passengers in the form of extra travel distance and additional stops along the way. Therefore, for taxi ride sharing to be a practical solution for reducing emissions and traffic, one also needs to control the burden placed on customers. To account for this, we introduced two parameters in our simulation model: d_{extra}, the maximum extra distance for each trip, and n_{share}, the maximum number of trips that each trip can be shared with. In addition, we use another parameter, d_{delay}, which is a bound on the maximum distance a taxi is allowed to travel to pick up a customer. Note that a constraint on d_{delay} is naturally present in the scenario where no sharing occurs as well. Any constraints on d_{extra} and d_{delay} are equivalent to constraints on t_{extra} and t_{delay} which are expressed in terms of time. For our simulations, we



(a)



(b)



(c)

Fig. 5: (a) The distribution of number of extra stops for n_{share}=$1, 2, 3, 4$. Note that the number of additional stops for a trip is bounded by $2n_{share}$. (b) The distribution of extra distances for n_{share}=$1, 2, 3, 4$. Note that 0 mile extra distance for a trip means that between the pick-up and drop-off locations of this trip a cab traveled without picking up or dropping off other trips. (c) The distribution of number of shared trips for n_{share}=$1, 2, 3, 4$

express the constraints in terms of distance. We omit traffic conditions in the experiments because we did not have access to the appropriate traffic data.

By definition, each trip can be shared with at most n_{share} other trips. Note that the bound on n_{share} gives a control over the maximum number of extra stops that is at most $2n_{share}$. This parameter is also used in [14], where k=n_{share}+1 is used. However, the optimization proposed in [14] is NP-hard for k>2; thus, k=2 was chosen for the solution to be computationally feasible. Our approach scales for larger values of n_{share} as well, which allows us to study the effects of this parameter on the total savings and costs of the proposed ride-sharing solution.

To keep the waiting and service times within a reasonable interval, we set d_{delay}=1 mile and d_{extra}=2 miles (at most 5 minutes of waiting time and 10 minutes of extra service time if the average taxi speed is 12 mph). Note that our approach scales to include both of these parameters in the study. Finally, for simplicity, we set each taxi's capacity C=4. Table III describes the simulation parameters. Our simulation model can handle individual constraints by letting each customer set their own n_{share}, d_{delay}, and d_{extra}. This makes it possible to support more complex scenarios where riders have different ride-sharing preferences.

Varying the Degree of Sharing. We studied the effects of parameter n_{share}. The number of cabs used for the simulations was 9,500 cabs on Sundays and 11,500 cabs for the others days of the week. The simulation results for n_{share}=$1, 2, 3, 4$ are presented in Fig. 4 for each day between Jan 1st, 2011 and Dec 31st, 2012. Fig. 4a shows the ratio of the total travel distance saved by ride-sharing to the total travel distance of original trips (i.e., the sum of the shortest distances between pick-up and drop-off locations of the serviced trips). Fig. 4b shows the average travel distance for each n_{share}=$1, 2, 3, 4$ as well as n_{share}=0 (no ride-sharing). Fig. 5a shows the distribution of the number of additional stops for different values of n_{share} (1 to 4). For example, for n_{share}=1, 120×10^6 trips have no additional stops, and the maximum number of additional stops is 2. But when n_{share} increases to 2, fewer than 80×10^6 trips have 0 additional stops, and now, there are trips with up to 4 additional stops. Note that without ride-sharing, the number of extra stops for each trip is 0. Fig. 5b is similar to Fig. 5a, but shows the distribution of the extra travel distance for different sharing levels. We have also analyzed the distribution of the number of trips each particular trip was shared with. Fig. 5c suggests that as we increase n_{share} shareability increases.

Naturally, our results quantify a tradeoff between the savings in the total distance through ride-sharing and burden incurred by customers. As we increase n_{share}, the savings in the total distance increase. On the other hand, this also leads to an increase in the travel distance and number of extra stops for each trip. However, our results may also suggest that contrary to results in [14], n_{share}=2 or even 3 may be an optimal bound on the maximum number of trips to be shared, offering a better tradeoff between the savings and costs. In particular, for n_{share}=2 the total saving is 28.6% on average with the average extra distance of 0.57 miles, while for n_{share}=1 the saving is 18.2% with the average extra distance of 0.35 miles.

Other Simulations. We also conducted various other experiments. These included varying the number of cabs and using a different cost function whose objective is CO2 emissions. We omit these results due to space limitations. However, we must note that the efficiency of our framework makes it possible to experiment with many different scenarios – a simulation consisting of one year worth of data takes just under 10 minutes to complete.

C. Comparison of Different Approaches

We compare the simulation results obtained by our approach against the results reported for the Shareability Network (SN) in [14], which, to the best of our knowledge, is the only prior work that supports large-scale ride-sharing simulation. For direct comparison, we ran simulations with the same road network and data as in SN. The results are presented in Table IV. In columns 2 and 4, we report the ratio of the resulting number of trips through sharing to the original

	Our Framework		Shareability Network [14]	
Parameter Settings	Saved Trips	Saved Time	Saved Trips	Saved Time
n_{share}=1, d_{extra}=0.6 mi.	47%	18%	16%	8%
n_{share}=1, d_{extra}=1 mi.	46%	16%	27%	13%
n_{share}=2, d_{extra}=0.6 mi.	60%	31%	16%	8%
n_{share}=2, d_{extra}=1 mi.	61%	29%	27%	13%

TABLE IV: Comparison of our framework and Shareability Network approach [14]. Note that n_{share}=$1, 2$ are equivalent to k=2, 3 and d_{extra}=0.6, 1 mi. correspond to Δ=3, 5mins in [14]. Also, for direct comparison we took the results of [14] for δ=0 (δ: time window) since our framework is a real-time model.

Data Size	Execution Time	Avg Execution Time per Day
1 day	2 mins, 18 sec	—
1 week	3 mins, 44 sec	32 sec
1 month	5 mins, 56 sec	12 sec
3 months	6 mins, 39 sec	4.4 sec
0.5 year	7 mins, 21 sec	2.4 sec
1 year	9 mins, 55 sec	1.6 sec
2 years	18 mins, 17 sec	1.5 sec
3 years	26 mins, 40 sec	1.5 sec

TABLE V: Execution time of the simulation on a 1200-core Hadoop cluster.

number of trips in the data set. Columns 3 and 5 contain the ratio of the simulated travel time to the original travel time. Observe that in our simulations, most of the trips are shared: 47% of saved trips with n_{share}=1, translating into 94% of trips being shared. On the other hand, the percentage of shared trips using SN is much lower; this is because with δ=0 (where δ is the time window; see Section V), the nodes are connected only if the start time of the two trips is very close and it leads to a much smaller network. This also results in significantly smaller savings in travel time.

D. Performance and Scalability

We conclude this section by reporting the execution time of ride-sharing simulation for different data sizes using a 1200-core Hadoop cluster. Table V summarizes the results. We observe that the average time it takes to process one day of data decreases as the size of data increases since we can use the parallelism more effectively. This shows that our framework scales to large data sets and enables us to study taxi ride sharing by exploring many different scenarios in timely manner.

V. Related Work

Ride sharing has been framed as an instance of the "dynamic pickup and delivery" problem [8], [9], and it has also been addressed via linear programming [10]–[12]. These optimization-based approaches are applicable to small-scale problems, for instance, sharing within airports, since large-scale optimization is often computationally infeasible. Heuristic-based solutions have been proposed for real-time dispatching of taxis [22], [23], but they also have limited scalability.

More recently, data-driven approaches have emerged. The work that is most closely related to ours is the simulation model proposed by Santi et al. [14]. They used a graph-based approach based on the notion of "shareability network", where nodes correspond to taxi trips, and two nodes are connected if those trips can be shared. Their model aims to maximize the number of shared trips or minimize the total time taken to accommodate all the trips. The structure of the shareability network depends crucially on two parameters: the maximum number of shared trips k per service and the maximum delay Δ that a customer can tolerate in a shared taxi service trip. These parameters control computational complexity of the problem. This solution is tractable only for k=2: the problem becomes NP-hard for larger values of k. Similarly, larger values of Δ translate into larger networks, requiring longer computation time. These restrictions limit the scenarios that one can explore with this approach. Moreover, their model can derive solutions that are not feasible in real life, since it does not explicitly take into account taxi positions and their capacity – it only examines whether it is beneficial to share a set of trips. For example, suppose that we decide that two particular trips t_1 and t_2 must be shared. Then, there must be a cab, say c, that will serve these two trips. Suppose also that in the original data set, c was serving trip t_1 and some other trip t_3 in this order. If t_1 and t_2 are assigned to c, it may not be possible for c to serve t_3 any more (or the cost may be too high). It is also unclear in this scenario what happens to the cab that was serving t_2 in the original data set.

Another limitation of this approach is that it assumes that trips are known in advance. While this assumption matches well car pooling scenarios where time and location of each trip are fixed in advance, it is not suitable for taxi ride-sharing, since trip requests arrive in real time. To address this issue, Santi et al. proposed a refinement of their model that prunes the shareability network to allow trips that start within a time window δ (e.g., five minutes) from each other to be shared. However, the model is real time only when δ=0, in which case our experiments suggest that this model tends to underestimate the benefits of the taxi ride-sharing. In Section IV, we present an experimental comparison between our framework and [14].

Ma et al. [15] proposed a real-time dispatch system for taxi ride-sharing. While related, our goal is different: we aim to support the simulation of a wide range of ride-sharing scenarios, and designed a model that can be parameterized accordingly. Like Ma et al., our model demands fast response times for queries that match trips to cabs. They do so by splitting a region into grid cells such that the distance between any two locations can be computed "heuristically" as the distance between the cells containing them. This allows their system to keep shortest path computations at a minimum, but at the cost of reduced accuracy. Moreover, the results are dependent on the selected grid size. In contrast, our system always use the "exact" shortest paths for optimizing ride-sharing schedules. We were able to achieve this with good performance and scalability using a cache-coherent indexing scheme (Section III-D). While our focus is on simulation, the experimental results indicate that our approach is promising for the dispatching scenario. [15] can serve 720K queries per hour (or 200 queries per second). Even without parallelism, our system can handle between 2K and 3K queries per second. Thus, exploring the use of our model for dispatching is a direction we would like to pursue in future work.

Huang et al. [24] proposed scheduling algorithms to dynamically match trip requests to vehicles with the minimum cost while trip waiting and service time constraints are satisfied. They showed that the kinetic tree algorithms outperform commonly used approaches, such as branch-and-bound and mixed-integer programming. As discussed in Section III, such algorithms can be integrated into our framework.

VI. Conclusion

In this paper, we presented a new framework that is both scalable and flexible to support the simulation of a rich set of realistic taxi ride-sharing scenarios. The scalability properties of the framework make it possible to run large-scale studies that explore a wide range of what-if scenarios through parameter sweeps. We have shown that this model attains a good balance between simplicity and expressiveness. Another important contribution of this work is the novel shortest path indexing scheme where we make use of cache-coherent layout to speed up shortest path queries substantially. The implementation of our simulation model is fully integrated with Hadoop's MapReduce, thus, enabling a variety of batch analysis tasks on taxi ride sharing. We applied the model to NYC taxi data and presented a case study that illustrates the capabilities and effectiveness of our system and design decisions.

There are several avenues we plan to pursue in future work. Our current shortest path indexing technique maintains a full distance matrix in memory. Though this could be mapped on disk, the storage size ($O(|V|^2)$) will not scale well for a large road network. We would like to experiment with a tiled caching strategy where we only keep the distance matrix for the most popular intersections and performing full shortest path computation for less popular nodes. In addition, we would like to implement a load balancer for the shortest path queries where the shortest path database could be located on a separate machine/cluster. This would allow us to make better use of the computing resources when having multiple simulator instances.

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