Supplementary Note for

"Efficient Algorithms for Densest Subgraph Discovery on Large Directed Graphs"

Here is the fix for KS-Approx provided by Prof. Barna Saha:

Instead of choosing between the min-indegree and min-outdegree node, we will do the following.
(1) Fix all the nodes in S (out degree nodes) Call $G_0^0 = G$
(2) Keep deleting min-indegree nodes
(3) Remove any nodes in S and T that has zero degree.
Call the configurations that you obtained as $G_1^0, G_2^0, \dots \dots, G_n^0$.
Now, go back to G_0^0. Delete the minimum out-degree node. Call this graph G_0^1 and repeat steps 2 and 3.
This gives configurations $G_1^1, G_2^1, \dots \dots, G_n^1$.
Go back to G_0^1. Delete the minimum out-degree node. Call this graph G_0^2, and repeat steps 2 and 3.
Continue.
This way, you will get n^2 configurations.
Now
(1') fix the indegree nodes (nodes in T) Call $H_0^0 = G$.
(2') Keep deleting min-outdegree nodes
(3') Remove any nodes in S and T that has zero degree.
Call the configurations that you obtained as $H_1^0, H_2^0, \dots \dots, H_n^0$.
Now, go back to H_0^0. Delete the minimum in-degree node. Call this graph H_0^1. Continue like above. Again you get another set of n^2 nodes.
Compute the max density of all these configurations and return the maximum.
Now Theorem 2 will go through, as there will be a configuration where out-degree will be greater than λ_o and indegree will be greater than λ_i simultaneously. (refer to [1])

Bibliography

[1] S. Khuller and B. Saha, "On finding dense subgraphs," in *International Colloquium on Automata, Languages, and Programming*, 2009.