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Integration of Train-Set Circulation and Adding Train Paths Problem Based on an Existing Cyclic Timetable

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ABSTRACT With the aim of supporting the process of scheduling a hybrid cyclic timetable and adapting traffic needs, this paper is concerned with inserting additional train paths problem for tactical and short-term planning applications with multiple objectives. Focusing on an existing cyclic timetable on a high-speed passenger rail line, the problem is to minimize both the total adjustments for initial trains and the number of required train-sets. The timetable scheduling, train-set planning and rescheduling are three complex optimization problems respectively and usually solved in a sequential manner. In this paper, these phases are integrated in a mixed integer programming model, and the multi-objective adding train paths (ATP) model is proposed, which decides simultaneously initial trains' modifications, additional trains' schedules and train-set circulation. The tolerance of disruption for initial trains including allowed adjustment and periodic structure is taken into account in light of the practical concerns. The difficulty of this integrated problem is that train-sets circulation is usually determined after all of the train lines and timetable have been fixed. However, in the adding train paths problem, the additional trains do not exist in the initial timetable. In order to solve the problem in a reasonable time, a relaxation approach to the adding paths problem is presented. We start from fixed train-set route, and then apply flexible train-set route that provides possible alternative turning activities to decrease the number of required train-sets. A case study based on Shanghai-Hangzhou high-speed rail line in China illustrates the methodology and compares the performance of various settings of perturbation tolerance, time window and train-set applications.

INDEX TERMS Timetable, additional train, train-set circulation, tolerance of disruption, periodic structure.

I. INTRODUCTION

This paper considers how to alter an existing cyclic train timetable to include additional train services. The primary motivation of the adding train paths (ATP) problem occurs as a result of current operational problems in China Highspeed Railway (HSR). In response to releasing the dense traffic on exiting railway network and increasing competition with other transportation modes, such as auto, truck and air, high-speed railway is developed rapidly in China over the past few years. Train timetable as the basis for performing train, train-set and crew scheduling, its optimization becomes an essential approach for railway industries to increase revenues and reduce the operation costs.

Since the introduction of cyclic timetables in the Netherlands, many other European countries have adopted the concept due to its several obvious advantages both in transport marketing and train operation planning. In a cyclic timetable, a train for a certain destination leaves a certain station at the same time every cycle time, say every one hour usually. It means after each hour, same pattern of train traffic will repeat itself. During the timetable planning process, three types of timetables are constructed: the Basic One-hour

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Timetable (BOT), the Weekly Timetable, and the Daily Timetable.

In China, the cyclic timetable is also applicable to some relatively short high-speed railway lines, such as Beijing-Tianjin line which are featured by single type of trains and its passenger flows are majored by commuting passengers. However, for some long-distance high-speed railway lines, such as Beijing-Guangzhou line with 2,105km, it is not suitable to apply a pure cyclic timetable. The reasons mainly be observed in the following aspects:

(1) In China HSR, two types of trains, namely high-speed trains (250-300 km/h) and medium-speed trains (160-200 km/h), are designed to operate. Moreover, in order to serve the large volume of traffic passing through this corridor, some medium-speed trains are planned to run on both high-speed line and adjacent regular rail lines as interline trains (Zhou and Zhong (2005)). Timetables in regular rail lines are non-cyclic, the interline trains in China HSR consequently can not be scheduled periodically in the train diagram neither.
(2) In the long-distance high-speed lines, the distribution of passenger flow is non-equilibrium both on space and time. A number of low frequency trains are necessary to meet the demand of passenger flow. In addition, sunset-departure and sunrise-arrival trains are also expected to meet the requirement of long time trip. However, if these trains of low frequency are scheduled as cyclic trains, it will lead to a waste of capacity.
(3) Furthermore, though with significant advantages, the cyclic timetable also has some disadvantages. Firstly, the ODs of some trains may be adjusted for cyclic scheduling, and thus some passengers may need to transfer as some trains with low service frequency may be cancelled. On the other hand, cyclic timetable may yield higher costs than non-cyclic ones. The occupation degree of the late evening trains is much less than during the rest of the day, but a cyclic timetable offers the same train service. Therefore, the cyclic timetable may be considered only if these disadvantages are not prominent in China HSR.

In fact, the above mixed operation policy with various types of passenger trains in China HSR calls for more sophisticated timetable planning methodologies and techniques. Moreover, a hybrid timetable concept named *"cyclic + non-cyclic" timetable* is possible, with a cyclic core timetable, in which non-cyclic trains such as low frequency and interline trains are inserted as extra trains (Xie and Nie (2009) and Yang *et al.* (2010)). This generic timetable consists of both cyclic trains and non-cyclic trains and be scheduled in tactical planning. It is created by first constructing a pure cyclic timetable, then removing a number of services in off peak hours and finally inserting non-cyclic trains. Nowadays, the cyclic timetable modes have been well developed, but the timetable-based train insertion technique without breaking the initial cyclic structure and minimise the number of required train-sets is still a significant demand for research.

Moreover, the technique of inserting new trains also can be applied in short-term planning that concerns the redevelopment of a generic timetable in order to adapt to the demands of the individual weeks or days, such as national holidays or major sports events that attract a lot of people, that generally require an increase of train services. The additional trains are inserted while taking the structure of the planned timetable into account. This is done to perturb the according existing train services as little as possible or similarly within acceptable levels.

This problem is of considerable difficulty and must be performed in practice. The ATP problem, as shown in Figure 1, firstly is an integration of scheduling and rescheduling problem. Train dispatcher both modifies the given timetable to manage the disruptions in existing operations and establishes schedules for extra trains. More importantly, many additional trains may not be inserted because of a shortage of train-set capacity. A train-set is the physical unit of rolling stock to cover a train trip, and composed of a set of passenger cars and power units(s). It means, to obtain a match between the requested additional trains and the available number of train-sets becomes an essential criterion for the adding paths problem. Consequently, how to cover the entire trains with minimum train-sets must be also taken into account in the adding path problem.

In contrast to the conventional train-set planning problem, the train-set planning in adding train paths problem is more complex. On one hand, train-set costs are principal components of the costs of a passenger railway operator, which requires minimal number of train-set. On the other hand, disturbances for scheduled train-set circulation may occur during the insertion, which may cause large disruptions for initial service. For example, in the phase of tactical planning, changes to the scheduled cyclical utilization of train-set might decrease the efficiency and accessibility for maintenance of train units. In the phase of short-term planning, changes to the initial train-set circulation may require coordination with the local traffic controllers of the infrastructure managers to ensure that proposed shunting operations and other local issues are possible (Maróti (2006)). Consequently, the deviation to initial train-set circulation should also be taken into account during minimising the number of required train-sets in the ATP problem.

In conclusion, this paper will give an account of how to reconstruct an existing cyclic train schedule by inserting additional train services. The ATP problem occurs not only in the tactical planning phase when scheduling a generic "cyclic + non-cyclic" timetable, but also in the short-term planning phase to adapt the increase of passenger flow. Focusing on a high-speed passenger rail line in an existing network, the problem is to minimize both:

1) the total adjustments for initial trains to keep the advantages of cyclic timetables, and
2) the number of required train-sets for entire trains to decrease costs.

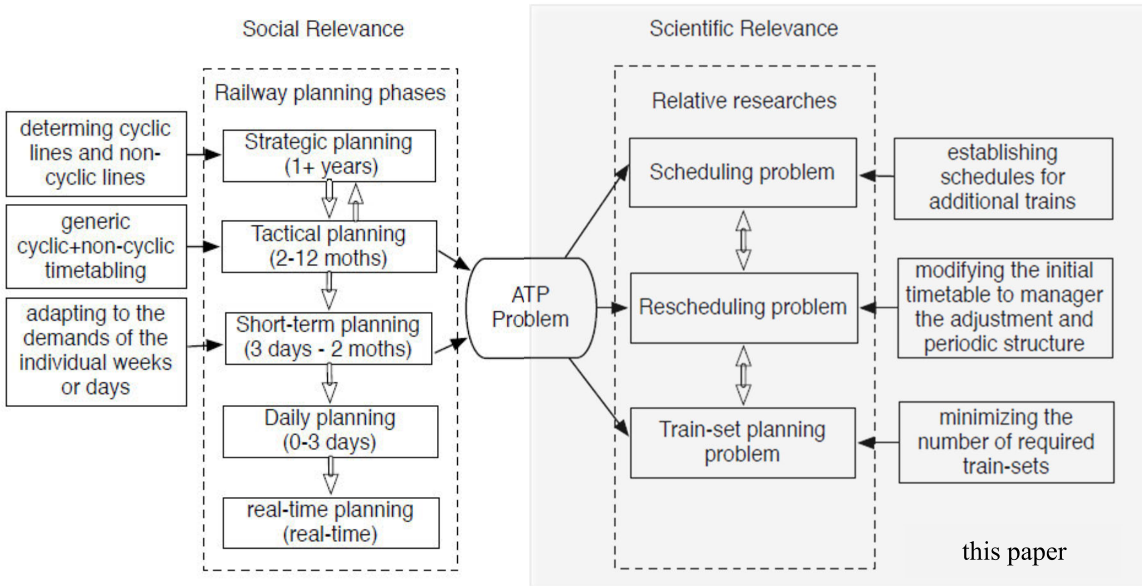


FIGURE 1. **Adding train paths problem.**

In this paper, we have developed an integrated model to insert additional train paths efficiently in an existing cyclic timetable. The timetable scheduling, train-set planning and rescheduling are three complex optimization problem respectively and usually solved in a sequential manner. In our work, we integrate these phases into the adding train paths problem in a model that decides simultaneously initial trains' modifications, additional trains' schedules and train-set circulation. We present a mixed integer programming model for the adding paths problem. The tolerance of disruption for initial trains including allowed adjustment and periodic structure is taken into account. A relaxation approach to the adding paths problem is presented. From numerical investigations based on highspeed railway in China, the proposed framework and associated techniques are tested and shown to be effective.

The organization of this paper is as follows. A summary of related work is presented in Section 2. In Section 3 we firstly formulate a general ATP problem. Furthermore, the tolerance of disruptions are considered in order to keep the advantages of the initial cyclic timetable. Section 4 deals with the train-set circulation in ATP problem, and proposes an approach to solve the nonlinear integrated model. This approach starts from fixed train-set route to linearize the model and then further constructs a reformulated model based on flexible train-set route to get minimum number of required train-sets. Experiments where the formulation and the approaches are applied based on China HSR in Section 5. Discussions including future work and conclusions will finally follow in Section 6.

II. REVIWE OF RELATED LITERATURE

A. LITERATURE REVIEW ON TRAIN SCHEDULING AND RESCHEDULING

As analyzed in Section 1, the ATP problem is an integration of rescheduling and scheduling. There are indications that some of the previous models and techniques could be modified and adapted to solve the adding paths problem. In recent years, train scheduling and rescheduling problems have a great deal of attentions. For example, Törnquist (2006) presents a list of foremost papers published on the area of rail timetable optimization between 1980 and 2006. There are varied models are used to formulate timetable scheduling and rescheduling problem.

Job shop scheduling problem, alternative graph models and event-activity graph model are extended and transferred to describe and formulate train operation problem. Since then we have observed the following papers: D'Ariano *et al.* (2007), Corman *et al.* (2012), Palgunadi *et al.* (2016), Xu *et al.* (2017), Yan and Goverde (2017), Julius (2019), Cacchiani *et al.* (2010a), Sels *et al.* (2016), Anita *et al.* (2019), Dollevoet *et al.* (2012), and Veelenturf *et al.* (2014).

D'Ariano *et al.* (2007) propose a fixed speed model and variable speed model for find a conflict-free timetable in real time after train operations are perturbed. Simple dispatching rules, a greedy heuristic based on the alternative graph formulation, and a branch-and-bound algorithm are evaluated in this paper. Cacchiani *et al.* (2010a) deal with the problem of timetabling non-cyclic trains. A Mixed-Integer Programming model (MIP) is proposed to look for the maximum-weight path in a comparability graph. Palgunadi *et al.* (2016) presents the scheduling model for the double-track railway to minimize the delay time with First Come First Serve and priority queue as the dispatching rules. Corman *et al.* (2012) deal with the bi-objective problem of minimizing delays and missed connections to provide a set of feasible non-dominated schedules. They formulate the Bi-objective Conflict Detection and Resolution(BCDR) problem as an alternative graph and two heuristic algorithms are developed to compute the Pareto front of non-dominated schedules.

Xu *et al.* (2017) proposed is a Mixed-Integer Linear Programming (MILP) model to reduce the delay, considering speed management, when high-speed trains run under a quasi-moving block signaling system. Yan and Goverde (2017) presented several extended Periodic Event Scheduling Problem (PESP) models with objectives of minimal train journey time, headway deviation of half the cycle time, and the number of dwell time stretches. Julius (2019) introduce a periodic delay management model capable of evaluating periodic timetables with respect to delay resistance. Cutting Plane Approach and Iterative Improvement Heuristic are proposed to solve two models respectively. Sels *et al.* (2016) develop a MILP model to minimize the total passenger travel time, including their ride, dwell and transfer activities, as well as the typical primary delays and their consequential knock-on delays in practice. Dollevoet *et al.* (2012) presents an iterative optimization framework macroscopic Delay Management model and the microscopic Train Scheduling model. They propose a disposition timetable and validate microscopically for a bottleneck station of the network, which reduces delay propagation and minimizes passenger delays. Veelenturf *et al.* (2014) proposing an Integer Linear Programming (ILP) formulation for solving the timetable rescheduling problem to deal with large disruptions on a real-world railway network, which takes into account constraints that allow to partially integrate it with the rolling stock rescheduling phase.

These approaches and models could in theory be adapted for the adding paths problem too. Regarding the objectives applied, for initial timetable no real dominating objective function could be found but there is a tendency towards minimising the total modifications and weighted modifications. Minimising delay costs, travel time, passenger annoyance are some other examples. The main drawback of these approaches is that it does not explicitly differentiate between initial trains and additional trains. Compared with scheduling problem, in adding paths problem not all trains have to be added to the schedule, some are already there. It may however be necessary for existing services to be rescheduled in later passes of that approach as changes are forced upon them from the insertion of new services; this needs to be further investigated. In addition, compared with dispatching problem, in ATP problem it may not be allowable for the timings of certain trains to be altered; initial trains may be not only forwards (later start) but also backwards (earlier start) rescheduled; the violation of cyclic structure should also be taken into account.

B. LITERATURE REVIEW ON ROLLING STOCK PLANNING

Our second goal is to cover the new timetable with a minimal level train-sets as will be specified in detail later. This problem is known as the *rolling stock planning problem* which, in its general sense, consists of operations that uncoupling units from or coupling units to trains due to efficiency reasons, such as described in Nielsen (2011), Maróti (2006), Fioole *et al.* (2006), Budai *et al.* (2009), Alfieri *et al.* (2002) and Peeters and Kroon (2008). The China HSR is, however, currently operated with homogeneous rolling stock. There are two various train-set types composed of 8 passenger and one power cars or 16 passenger and two power cars. Moreover, due to the requirement of organization, combining and splitting of trains is not adopted in China HSR. That is, each trip specifies a type of train-set beforehand and is performed in such a way that the assignment of train-set to trains can not be changed during a trip. It simplifies our problem into the train-set routing problem, see Anderegg *et al.* (2002). A train-set routing is determined by the number of train-sets and the route, namely the sequence of trains, that each train-set traverses in a time horizon, such as Hong *et al.* (2009) present a two-phased train-set routing algorithm to cover a weekly train timetable with minimal working days of a minimal number of train-sets.

Timetable scheduling, rescheduling and rolling stock routing are three complex optimization problem respectively. Some papers have already done many researches on an combined model, such as Cacchiani *et al.* (2012), D'Ariano *et al.* (2008), Flier *et al.* (2008) and Nielsen *et al.* (2012) integrate rolling stock circulation into rescheduling problem, and Leon (2003) and Cadarso and Marín (2011) integrate rolling stock planning problem into timetable scheduling problem. Cacchiani *et al.* (2012) present an optimization model based on recoverability when faced with different disruption scenarios. A large MILP is formulated and Benders decomposition is used to solve the LP-relaxation of this model. D'Ariano *et al.* (2008) consider the problem of managing disturbance in real time. The rolling stock circulation are assumed to be fixed as many other literatures. They model the constraints of rolling stock and passenger connections in a same way. Flier *et al.* (2008) relax the assumption of fixed rolling stock circulation, and allow changes in the vehicle schedules if they lead to a better disposition timetable. A model for delay management in which the decisions dealing with the circulations are integrated. This problem is proved to be NP-hard even in very simple networks. A polynomial case is identified and different properties and approaches are suggested in this paper. Leon (2003) integrate rolling stock planning into cyclic timetable scheduling problem. Starting with the most basic case, in which the rolling stock circulation is fixed beforehand, a model of more freedom in choosing the rolling stock circulation is formulated. However, for the above approach to be applicable, we do need to decide beforehand which pairs of trains are allowed to turn on one another.

The strength of our problem formulation is the ability to integrate train-set circulation into adding paths problem, which is composed of both train-set rescheduling and routing problem.

C. LITERATURE REVIEW ON ADDING TRAIN PATHS PROBLEM

Although the adding paths technology is very important, there has been few direct related discussions about adding

TABLE 1. Characteristics of adding paths problem and solution approaches.

Publication	Background	Model	Constraint	Objective	Solution	Infrastructure and problem size evaluated for
Cacchiani et al. (2010b)	(F)	(ILP)	(FS)	Maximize the number of additional trains and minimize the violations to the ideal insertion	(HA)	(D,S),(N), (679/202/1440/24-48-96,520/202/1440/24-48-96, 0/202/1440/188338-554-64)
Ingolotti et al. (2004)	(F/P)	(CSM)	(FS), (TW), (PC)	Minimize the average traversal time of new train	(DP), (PR)	(S), (N), (81/65/-/20)
Flier et al. (2009)	(F/P)	(LPM), (CSM)	(VS), (TW)	Supports railway planners by computing a set of Pareto optimal solutions with respect to travel time and expected delay to additional trains.	(SP)	(D), (L), (-/-/-/1)
Burdett and Kozan (2009)	(P)	(MIP)	(FS), (TW), (SC)	Minimize the total weighted time window violations and the makespan.	(CA), (HA)	(D,S), (N,L), (6/3/48/1-5, 6/5/279/1-5 ,10/10/88/1-5, 24/10/127/1-5,15/5/196/1-5 ,54/30/494/1-5, 20/20/202/1-5, 20/12/220/1-5, 20/24/403/1-5)
Tan et al. (2015)	(P)	(MIP)	(FS), (CN)	Minimizes consecutive delay to existing timetable	(BB), (AG), (SP), (PR)	(D),(L),(36/60/240/1-15)
Tan et al. (2017)	(F/P)	(MIP)	(VS), (TW), (CN)	Minimum disruptions to original timetable	(SM)	(D),(L),(24/9/180/-)
Tan et al. (2020)	(P)	(MIP)	(VS), (TW), (PC), (PS)	Minimize the total adjustments for initial trains	(BB)	(D), (L), (79/9/420/10-12-14-16)
Our paper	(P)	(MIP)	(VS), (TW), (PC), (PS), (TS)	Minimize the total adjustments for initial trains and minimize the number of required trainsets	(BB)	(D), (L), (79/9/420/10-12-14-16)

paths problem. The only papers to our knowledge are presented in Table 1 which summarizes the studies, like ours, dealing with inserting passenger or freight trains into an existing timetable.

Cacchiani *et al.* (2010b), Burdett and Kozan(2009)and Ingolotti *et al.* (2004) solve the problem of inserting freights trains with assumption that all of the initial trains can not be changed. In Cacchiani *et al.* (2010b), the additional trains are inserted with predefined ideal departure/arrival time and minimum stopping time at each station that must visit, meanwhile, alternative routes are taken into account. In Ingolotti *et al.* (2004), additional trains are inserted at a randomly fixed time belonging to the time window at each iteration and priority rule is predefined for each overtaking and meeting. Burdett and Kozan (2009) proposes an inserting process that consists of 3 phases by fixing or unfixing some scheduled services. Flier *et al.* (2009) compute a set of Pareto optimal train schedules with respect to risk and travel time. Their method aims at finding robust train paths in the sense that the additional train has a low risk of delay upon arrival at its final station and supporting railway planners by computing a set of recommended train paths for a given train request. Tan *et al.* (2015) deal with the problem of inserting passenger trains based on an Alternative Graph Model. The ATP problem is decomposed into three sub-problems. They first find the optimal insertion for a fixed order timetable and then add retiming and reordering phases on that basis. The three sub-problems are solved iteratively until no improvement is possible within a time limit of computation. Tan *et al.* (2017) and Tan *et al.* (2020) propose MIP models to minimum disruptions to original timetable with variable travel time. In Tan *et al.* (2017), additional trains are inserted by finding feasible paths on each route path. Safety headway constraints and connections are considered with predefined variable running and dwell time. In Tan *et al.* (2020), priority for overtaking, reasonable time window and periodic structure are simultaneously taken into account.

Our study is different from the previous ones in two things. Firstly, we constraint the violation of periodic structure to the initial cyclic timetable. Secondly, we consider bicriteria

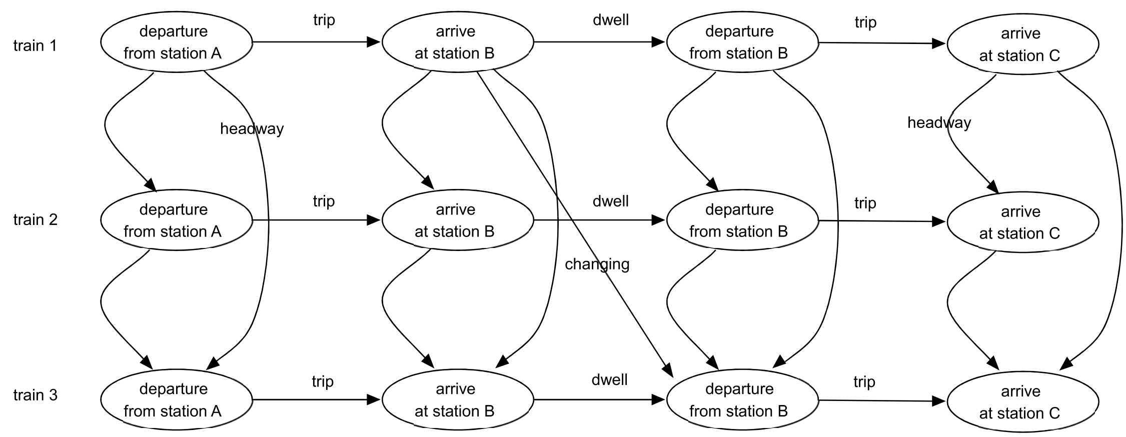


FIGURE 2. Event-activity graph.

objectives, minimizing the total adjustment and at the same time minimizing the number of required train-sets, to the adding train paths problem. The difficulty in this paper is that train-sets circulation are usually determined in the tactical planning phase *after* all of the train lines and timetable have been fixed. However, in the adding paths problem, the additional trains do *not* exist in the initial timetable, and even the number of additional trains depends on the number of instantly available train-set at the right place.

III. GENERAL ADDING TRAIN PATHS PROBLEM FORMULATION

This section is intended to illustrate the general ATP problem without considering the train-set planning problem. In this work, we focus our attention on the case in which all trains travel in the same direction along the tracks, which is almost always the case in HSR in China.

Let $\mathcal{T}$ denote the set of trains. A general railway network is represented by a graph $\mathcal{G} = (\mathcal{S}, \mathcal{B})$,where each node $s \in \mathcal{S}$ and segment $b \in \mathcal{B}$ in the network represents respectively a station, and a collection of one or multiple tracks between stations, with no intermediate station in between.

In order to profit from specific topological structures of each individual railway network, an event-activity graph $G = (V, E)$ as presented in Flier *et al.* (2008) is adopted in this paper, see Figure 2. It is a widely used mathematical model for scheduling events with time constraints, whose nodes V are called *events* and whose directed edges E are *called activities*.

The set V of events consists of all *arrival events* and departure events, i.e. $V = V_{arr} \cup V_{dep}$,

$$V_{arr} = \{(t, s, arrical) : train\ t \in \mathcal{T}\ arrives\ at\ station\ s \in \mathcal{S}\},$$

$$V_{dep} = \{(t, s, departure) : train\ t \in \mathcal{T}\ departs\ from\ station\ s \in \mathcal{S}\},$$

The events of set V are linked by directed edge set E, which are called activities and consists:

- *Trip activities* $E_{trip} \subset V_{dep} \times V_{arr}$ model the driving of a train between two consecutive stations.
- *Dwell activities* $E_{dwell} \subset V_{arr} \times V_{dep}$ model the stopping of a train at a station.
- *Changing activities* $E_{change} \subset V_{arr} \times V_{dep}$ model a transfer connection from one station to another.
- *Headway activities* $E_{headway} \subset V_{dep} \times V_{dep} \cup V_{arr} \times V_{arr}$ model the security headway between two consecutive departures and arrivals at the same station.

A. NOTATIONS

Tables 2 and 3 first list general subscripts and input parameters used to formulate the general ATP problem without considering the train-set using. All the parameters and variables are nonnegative integers, and the unit of time is one minute.

B. OPTIMIZATION MODEL FOR GENERAL ATP PROBLEM

The first objective of the ATP problem is to minimize the adjustments for all initial events,

$$(\mathbf{ATP})\ \text{Minimize} : N_{ad} = \sum\nolimits_{i \in V^{ini}} ad_i \tag{1}$$

The constraints used in the double-track adding paths model are presented as follows,

Reasonable Time Window:

$$x_i \geq tw_i^{min} \quad \forall i \in V^{add} \tag{2}$$
$$x_i \leq tw_i^{max} \quad \forall i \in V^{add} \tag{3}$$

Variable Trip Time on Segment:

$$x_j - x_i \geq trip_e^{min} + \rho_i * \epsilon^a + \rho_j * \epsilon^d \quad \forall_e = (i, j) \in E_{trip} \tag{4}$$

$$x_j - x_i \leq trip_e^{max} + \rho_i * \epsilon^a + \rho_j * \epsilon^d \quad \forall_e = (i, j) \in E_{trip} \tag{5}$$

Dwell Time at Station:

$$x_j - x_i \geq \rho_i \cdot dwell_e^{min} \quad \forall_e = (i, j) \in E_{dwell} \tag{6}$$
$$x_j - x_i \leq \rho_i \cdot dwell_e^{max} \quad \forall_e = (i, j) \in E_{dwell} \tag{7}$$
$$\rho_i = 1 \quad \forall_i \in V_{arr} : pls_i = 1 \tag{8}$$

Minimum Headway:

$$x_j - x_i \geq h_e \cdot \lambda_{ij} - M \cdot \left(1 - \lambda_{ij}\right) \quad \forall (i, j) \in E_{headway} \tag{9}$$

TABLE 2. General subscripts and parameters.

Symbol	Description
$\mathcal{G} = (\mathcal{S}, \mathcal{B})$	The railway network graph, where $\mathcal{S}$ is the set of station, and $\mathcal{B}$ is the set of segments
$G = (V, E)$	The event-activity graph, where V is the set of events (nodes)$V = V_{arr} \times V_{dep}$, and E is the set of activities(edges) $E = E_{trip} \cup E_{dwell} \cup E_{change} \cup E_{headway}$
$\mathcal{T}^{add}$	The set of additional trains, i.e. $\mathcal{T}^{add} \subset \mathcal{T}$, $\mathcal{T}$ is the set of all trains
$\mathcal{T}^{ini}$	The set of initial trains, i.e. $\mathcal{T}^{ini} \subset \mathcal{T}$
V^{add}	The set of additional events, i.e. $i \in V^{add}: t(i) \in \mathcal{T}^{add}$
V^{ini}	The set of initial events, i.e. $i \in V^{ini}$: $t(i) \in \mathcal{T}^{ini}$
i, j, i', j'	Event index, $i, j, i', j' \in V$
e	Activity index, $e = (i, j) \in E$
$t(i)$	Train index, which represents the corresponding train of event i
$s(i)$	Station index, which represents the station at which event i takes place
$b(i, j)$	segment index, which represents the segment on which activity $e = (i, j)$ takes place
M	A sufficiently large positive integer

TABLE 3. Input parameters for general ATP problem.

Symbol	Description
π_i	The time instant at which event $i \in V$ takes place in the initial timetable
pls_i	0 or 1, indicating if train $t(i)$can bypass/ must stop at station $s(i)$,respectively
ε^a	The required acceleration time
ε^d	The required deceleration time
tw_i^{min}	The lower bound of the time window at which event $i$takes place
tw_i^{max}	The upper bound of the time window at which event $i$takes place
h_e	The minimum headway time of headway activity e, $e \in E_{headway}$
$trip_e^{min}$	The minimum travel time of trip activity e, $e \in E_{trip}$
$trip_e^{max}$	The maximum travel time of trip activity e, $e \in E_{trip}$
$dwell_e^{min}$	The minimum stop time of dwell activity e, $e \in E_{dwell}$
$dwell_e^{max}$	The maximum stop time of trip activity e, $e \in E_{dwell}$
p_{ij}	0 or 1, indicating if train $t(i)$has lower or same /higher priority than $t(j)$
Δ_i	The maximum allowable adjustment of event i
T	Cyclic time, 1 hour in this paper
E_{peri}	The set of periodic activities,$E_{peri} \subset V_{dep} \times V_{dep} \times V_{arr} \times V_{arr}$model pairs of periodic trains
θ	The maximum allowable deviation to periodic structure

$$x_i - x_j \geq h_e \cdot (1 - \lambda_{ij}) - M \cdot \lambda_{ij} \quad \forall (i, j) \in E_{headway} \tag{10}$$

$$\lambda_{ij} = \lambda_{i'j'} \quad \forall (i, i'), (j, j') \in E_{trip} : b(i, i') = b(j, j') \tag{11}$$

Priority for Overtaking:

$$\lambda_{ij} - \lambda_{i'j'} \leq 0 \quad \forall (i, i'), (j, j') \in E_{dwell} : s(i) = s(j), p_{ij} = 1 \tag{12}$$

$$\lambda_{ij} - \lambda_{i'j'} > 0 \quad \forall (i, i'), (j, j') \in E_{dwell} : s(i) = s(j), p_{ij} = 0 \tag{13}$$

Adjustment of Initial Schedules:

$$x_i - \pi_i \geq ad_i \quad \forall i \in V^{ini} \tag{14}$$

$$\pi_i - x_i \geq ad_i \quad \forall i \in V^{ini} \tag{15}$$

TABLE 4. Decision variables for general ATP problem.

Symbol	Description
x_i	The time instant at which event $i \in V$ takes place in the initial timetable
λ_{ij}	0 or 1, indicating if event j takes place before/ after event i
ρ_i	0 or 1, indicating if train $t(i)$ bypasses/ stops the station $s(i)$ in the new timetable
ad_i	The adjustments of initial event$i$,$i \in V^{ini}$
N_{ad}	The total adjustments to initial trains.

Operator Preferences:

$$x_i, ad_i \geq 0 \quad \forall i \in V \tag{16}$$

$$\rho_i, \lambda_{ij} \in \{0, 1\} \quad \forall i \in V \tag{17}$$

Constraints (2-3) represent the reasonable departure time window for trains. For some trains, the freedom of selecting departure times from original station (arrival times at destination) is limited. This especially applies to international trains and interline trains. Time windows of departure (arrival) times are usually chosen on the board stations. When $tw_i^{min} = tw_i^{max}$, it ensures that the departure (arrival) time is fixed.

Constraints (4-5) relate the actual trip time on section. Taking speed variation dynamics into consideration, the trip time in section is flexible between the minimal $trip_e^{min}$ and the maximal $trip_e^{max}$. In addition, due to the safety and passenger comfort requirements, high-speed trains usually take at least several minutes to fully stop or reach a cruise speed even with highly efficient acceleration and deceleration performance (Zhou and Zhong (2005)). In this situation, when train stops, i.e. $p_i = 1$, the corresponding actual trip time has to exactly take into account the required acceleration time ε^a and deceleration time ε^d.

As shown in constraints (6-8), train must stop at all stations at which it calls such as for passengers, i.e. $pls_i = 1$, then $p_i = 1$. More precisely, extension of a scheduled stop or additional stops is also permitted for operational requirements, i.e. $pls_i = 0$, then $p_i = 0/1$. However, due to commercial and operating reasons, stopping time must be bounded. The actual dwell time should be no less than the planned minimum $dwell_e^{min}$ and no more than the maximum $dwell_e^{max}$ dwell time.

The headway constraints (9-10) describe the minimum headway requirements between the departure times and arrival times of consecutive trains at the same station. Constraint (11) implies the sequence of trains could not change at two adjacent stations, which guarantees trains do not overtake each other on a segment b.

Constraints (12-13) enforces that train of higher priority can not be overtaken by a lower train.

Constraints (14-15) record the magnitude of the right or left shifts ad_i of every initial event i. The parameters π_i specifies the scheduled time for event i in the initial timetable.

In practical applications the level of acceptable adjustment widely differs according to the train service type. It should also be mentioned that in some circumstances some train services must be strictly fixed and can suffer no disruption, such as interline and some high-speed trains. Similarly, the tolerance of other passenger trains in HSR to delays and alterations is quite limited. Consequently, constraints (18-19) imply that a certain amount of adjustments are allowed for initial trains with respect to the planned departure times. Clearly, if the corresponding schedule is fixed then $\Delta_i = 0$ holds.

Allowable adjustment to initial schedules:

$$x_i - \pi_i \leq \Delta_i \quad \forall i \in V^{ini} \tag{18}$$

$$\pi_i - x_i \leq \Delta_i \quad \forall i \in V^{ini} \tag{19}$$

Besides, the additional trains should be inserted with taking the periodic structure into account. During the process of inserting and rescheduling, one usually runs into problems that the periodicity of initial cyclic timetable might be ruined. In order to fully take the advantage of cyclic timetable, the periodic pattern of initial trains is desired to be guaranteed. Then the strategy of *periodic rescheduling* is essential to make up the deviations to the periodic structure.

Periodic rescheduling means that when a difference made to a cyclic train, the trains in other cyclic times which belong to the same train line with the disturbed trains should also be rescheduled to ensure the cyclic arrivals and departures in the entire timetable. In contrast to the conventional rescheduling strategy which is common in dispatching problem in previous researches, the periodic rescheduling takes the cyclic structure into consideration besides resolving conflicts. However, sometimes we do not want to fixed the reschedules too much beforehand, then a bandwidth θ is introduced to the periodic constraint.

The periodic structure of exiting cyclic timetable can be restricted by constraints (20 - 21).

Allowable deviation to periodic structure:

$$x_j - x_i \leq (\pi_j - \pi_i) + \theta \quad \forall (i, j) \in E_{peri} \tag{20}$$

$$x_j - x_i \geq (\pi_j - \pi_i) - \theta \quad \forall (i, j) \in E_{peri} \tag{21}$$

IV. TRAIN-SET CIRCULATION IN ADDING TRAIN PATHS PROBLEM

A. DECOMPOSITION

As analyzed in Section 1, differ from the previous researches on integrated timetable scheduling and train-sets planning, the train-set circulation in ATP problem requires both minimum number of train-sets and small deviation to scheduled

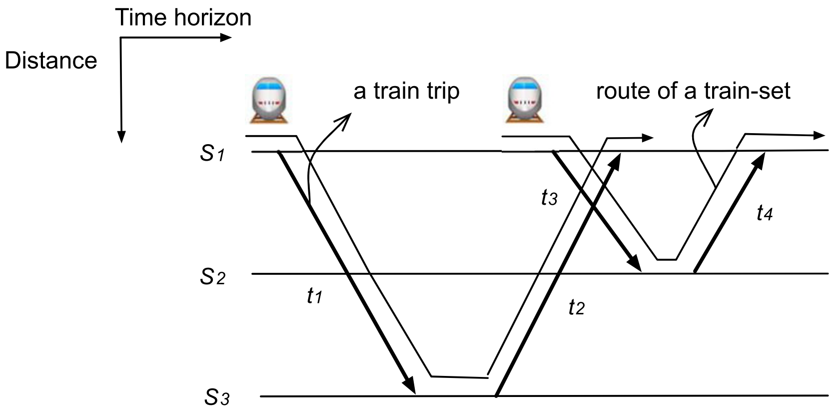


FIGURE 3. A simple train network.

TABLE 5. Input parameters for train-set circulation in ATP problem.

Symbol	Description
V_{start}	The set of departure activities for trains at their origins
V_{end}	The set of arrival activities for trains at their destinations
V_{start}^{add}	The set of departure activities for the additional trains at their origins, i.e.,$V_{start}^{add} \subset V_{start}$
V_{end}^{add}	The set of arrival activities for the additional trains at their destinations, i.e., $V_{end}^{add} \subset V_{end}$
E_{circ}	The set of circulation activities, $E_{circ} \subset V_{arr} \times V_{dep}$model the turn around activities between two trains
L_e	The minimum connecting time for train t(i) turning around to $t(j)$, where$e = (i,j) \in E_{circ}$
st_{ij}	1 or 0, indicating if event i and j take place at the same station
T_{hor}	The time horizon for a closed circulation of train-set
E_{circ}^{ini}	The set of current circulation activities for initial trains
E_{circ}^{add}	The set of circulation activities for additional trains
E_{circ}^{fix}	An arbitrary train-set circulation which is fixed beforehand, and$E_{circ}^{fix} \subseteq E_{circ}^{add}$
E_{circ}^{alt}	The set of alternative turn around activities to the arbitrary fixed train-set circulation, i.e.$E_{circ}^{alt} \subseteq E_{circ}^{add} - E_{circ}^{fix}$
$E_{circ}^{alt^*}$	The set of alternative turn around activities which respect the minimal turn around time, i.e.$E_{circ}^{alt^*} := \{e = (i,j) \in E_{circ}^{alt} : x_j - x_i \geq L_e\}$

train-set plan. Hence, for the sake of avoiding large disruptions to the scheduled services and solving adding paths problem within a reasonable computational force, the train-set planning in adding train paths problem is decomposed into two sub-problem in this paper,,

(1) for the initial timetable, the current train-set circulation is assumed to be fixed beforehand. It is solved as a rescheduling problem with a tight constraint, that a train-set operates the same existing trains in the same sequence as it is scheduled in the initial timetable.
(2) for the additional trains, the train-set circulation problem is equivalent to covering all the additional trains with minimal number of train-sets.

In China, there exist two different approaches on the application of train-set; a train-set is applied in *certain* or *uncertain* railroad region respectively. The first approach is that a train-set first carries out the journey from terminus A to terminus B, and subsequently a reverse journey from B to A. In the second approach, circulation of a train-set is not limited to a specific railroad region between terminus A and terminus B. After the journey from A to B, it may carry out an arbitrary journey, for example to C, as long as it originates from B. We used innovative operations research to devise efficient schedules for this resource. For sake of clarity, we describe the model of train-set circulation based on the approach under uncertain railroad region. If desired, it is also can be adopted easily for the approach under certain railroad region.

The following notations, shown in Tables 5 and 6, are used to formulate the train-set planning the ATP problem.

B. THE FORMULATION OF TRAIN-SET CIRCULATION IN ATP PROBLEM

A concept of *rotation* is introduced to solve the train-set circulation problem in ATP problem. The term *rotation* is

TABLE 6. Decision variables for train-set circulation in ATP problem.

Symbol	Description
x_i	The time instant at which event $i \in V$takes place in the new timetable
le_{ij}	The actual turn around time for event $e \ inE_{circ}$
q_{ij}	1 or 0, indicating if $e = (i,j) \in E_{circ}$ is chosen or otherwise
u_{ij}	1 or 0, indicating if minimal turn around time between event i and j is respected, i.e.$x_j - x_i \geq L_e$ or otherwise
l_{ij}	1 or 0, indicating if $e = (i,j) \in E_{circ}^{alt^*}$ or otherwise
k_{ij}	1 or 0, indicating if$e = (i,j) \in E_{circ}^{alt^*}$is chosen or not
N_{ts}	The number of required train-sets
N_{ts}^{fix}	The number of required train-sets for fixed train-set circulation

widely used in the airline industry (Lloyd *et al.* (1997)). The aircraft rotation problem is to determine the routes flown by each aircraft in a given fleet. It is also can be adopted as well in railway system. By a rotation, we mean a circulation in which, as the time horizon repeats, a single train-set covers all the trains. This means every train- set, in the long run, covers the same set of routes. A rotation is a desirable practice in that it maintains train-sets and rails in a homogeneous condition.

To explain a rotation, we consider the example in Figure 3. This simple train network shows 4 trains assigned to a line group. The train-set rotation problem orders these trains. For the trains in Figure 3, one possible route is $t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_1$ to cover train t_1, t_2, t_3 and t_4, then repeat the sequence. We represent the corresponding rotation simply by $t_1 - t_2 \rightsquigarrow t_3 - t_4 \rightsquigarrow t_1$.

The train-set circulation is scheduled on a daily basis in this paper. Therefore, the train trips that are shown in Figure 3 are repeated on subsequent days. For rotation $t_1 - t_2 \rightsquigarrow t_3 - t_4 \rightsquigarrow t_1$, on day 1 train-set c_1 takes t_1, t_2 and then spends the night in station s_1. On day 2, train-set c_1 takes train trips t_3 and t_4 and train-set c_2 takes trains t_1 and t_2. This completes the rotation. This rotation covers the 4 train trips with two train-sets. The symbol $\rightsquigarrow$ indicates an overnight stay between trains, and the number of overnight implies the number of train-sets required to complete the rotation.

The train-set circulation in ATP problem is to cover all of the additional trains with minimum number of required train-sets. Using the definition of rotation, we can now define feasible and optimal train-set circulation for the new timetable. The objective and constraints are presented as follows:

$$(\textbf{TSR})\ \text{Minimize} : N_{ts} = \left[\sum_{(i,j)\in E_{circ}} le_{ij}q_{ij} + \sum_{(i,j)\in E_{train}} (x_j - x_i) \right] / T_{hor} \tag{22}$$

$$\text{subject to:}\ x_j - x_i \geq L_e \quad \forall_e = (i,j) \in E_{circ}^{ini} \tag{23}$$

$$\sum_{i \in V_{end}^{add}} q_{ij} = 1 \quad \forall i \in V_{end}^{add},\ \forall j \in V_{start}^{add} \tag{24}$$

$$\sum_{j \in V_{start}^{add}} q_{ij} = 1 \quad \forall i \in V_{end}^{add},\ \forall j \in V_{start}^{add} \tag{25}$$

$$x_j - x_i \geq L_e - \left(1 - u_{ij}\right) M \quad \forall e = (i,j) \in E_{circ}^{add} \tag{26}$$

$$x_j - x_i < L_e - u_{ij} M \quad \forall e = (i,j) \in E_{circ}^{add} \tag{27}$$

$$le_{ij} \geq M(1 - st_{ij}) + T_{hor}(1 - u_{ij}) + (x_j - x_i) \quad \forall e = (i,j) \in E_{circ}^{add} \tag{28}$$

$$le_{ij} \geq L_e \quad \forall e = (i,j) \in E_{circ}^{add} \tag{29}$$

$$x_i, le_{ij} \geq 0 \quad \forall i \in V, \forall (i,j) \in E_{circ}^{add} \tag{30}$$

$$u_{ij}, q_{ij} \in \{0,1\} \quad \forall i \in V_{end}^{add}, j \in V_{end}^{ini} \tag{31}$$

Then the number of required train-sets to finish the train rotation in a time horizon is formulated as (22). In China HSR, each trip usually has a closed circulation of train-set on a daily basis, therefore the time horizon $T_{hor} = 1440$(min). Constraint (23) will be used in keeping the initial train-set circulation to the existing trains. Constraints(24-25) imply that for each $i \in V_{end}^{add}$ or $j \in V_{start}^{add}$,exactly one circulation activity starting from i or ending at j has to be respected. Constraints (26-27) indicate that $u_{ij} = 0$ if $x_j - x_i < L_e$, otherwise $u_{ij} = 1$.

As shown in constraint (28), the actual turn around time le_{ij} is stated as follows,

$$le_{ij} \geq \begin{cases} x_j - x_i & \text{if } st_{ij} = 1,\ u_{ij} = 1 \\ T_{hor} + x_j - x_i & \text{if } st_{ij} = 1,\ u_{ij} = 0 \\ M & \text{if } st_{ij} = 0 \end{cases} \tag{32}$$

if the time difference between arrival event i and departure event j is respected to the minimal turn around time oftrain-set, i.e. $x_j - x_i \geq L_e$, then $le_{ij} \geq x_j - x_i$. Else, if $x_j - x_i < L_e$, after the end of i, the train-set has to make antime horizon (such as overnight) stop at station $s(i)$ and then it can turn to j. Consequently, the turn around time of train-set in this circumstance respects $le_{ij} \geq T_{hor} + x_j - x_i$. If i and j do not occur at the same station, to connect i and j a train-set

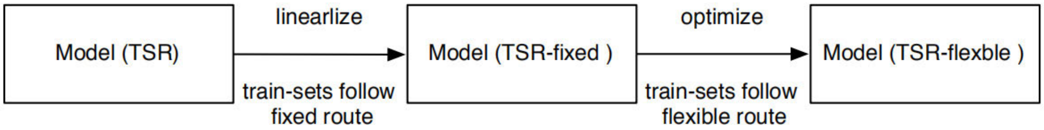


FIGURE 4. Flow chart of the approach.

goes from one station to another as an empty train. Empty trains are to be avoided as much as possible, and note that empty trains occur in tactical rolling stock circulations quite rarely. That is, we ran the model without empty trains by setting $le_{ij} = M$ when $s(i) \neq s(j)$, where M is a large enough constant as defined previous. Constraints (29) specify that the minimal turn around time must be respected for each circulation activity.

The classical train-set circulation problem is to determine the specific route flown by each train-set based on a given schedule, that the timetable specifies the departure and arrival times of the trips as well as the actual turn around time of each potential circulation activity. However, in adding paths problem, the additional trains do not exist, and even the number of additional trains depends on the number of instantly available train-sets at the right place. It means in the function (22), besides the route of train-set q_{ij}, the departure time x_j, the arrival time x_i and the actual turn around time le_{ij} are all decision variables. Consequently, the objective function (22) is non-linear, which would make the train-set routing problem computationally.

C. APPROACH TO SOLVE THE TRAIN-SET CIRCULATION IN ATP PROBLEM

In order to solve the nonlinear integrated model (TSR) within acceptable computation time, this section describes an approach which starts from preselected fixed train-set route to linearize objective function in model (TSR), and then flexible route is further applied by choosing alternative turn around connections to get optimal solution. The flow chart of the propose approach is shown in Figure 4.

1) FIXED TRAIN-SET ROUTE

We start with a description of the most basic case, in which the route of train-set is fixed beforehand. This fixation is such that the route of train-set, namely the sequence of train trips that each train-set traverses is predefined arbitrarily, and then we can find the optimal insert solutions with minimal number of train-sets based on the fixed train-set route pattern. See the example in Figure 3, if train-set route $t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_1$ is chosen beforehand, we denote these fixed turn around activities as E^{fix}_{circ}, and $E^{fix}_{circ} \subseteq E^{add}_{circ}$. Then

$$\{(t_1, t_2), (t_2, t_3), (t_3, t_4), (t_4, t_1)\} \subseteq E^{fix}_{circ}$$

is preselected to indicate the route of train-set and their corresponding q_{ij} are set to be 1, else $q_{ij} = 0$.

We emphasize that an arbitrary train-set route would not have an effect on insertion pattern and the rotation when additional trains could be scheduled randomly in a time horizon. That is, in the adding paths problem, it turns out that it is usually not a problem if a 'wrong' train-set route is fixed, there seems to be a lot of flexibility in finding inserting solutions for additional trains and the rotations to minimize number of train-sets. For example, Figure 5 illustrates various insertion solutions and rotation patterns according to the same train-set route of $t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_1$. All of rotations require at least two train-sets to cover the same 4 trains in the time horizon.

Then by fixed the route, where qij is predefined as a set of constants, the model (TSR) can be linearized to

$$(\textbf{TSR}-\textbf{fixed})\ \text{Minimize}: N^{fix}_{ts} = \Big[\sum\nolimits_{(i,j)\in E^{fix}_{circ}} le_{ij}q_{ij} + \sum\nolimits_{(i,j)\in E_{train}} (x_j - x_i)\Big] / T_{hor} \quad (33)$$

$$\text{subject to: } x_j - x_i \geq L_e \quad \forall e = (i,j) \in E^{ini}_{circ} \quad (34)$$

$$x_j - x_i \geq L_e - (1 - u_{ij})M \quad \forall e = (i,j) \in E^{ini}_{circ} \quad (35)$$

$$x_j - x_i < L_e - u_{ij}M \quad \forall e = (i,j) \in E^{fix}_{circ} \quad (36)$$

$$le_{ij} \geq T_{hor}(1 - u_{ij}) + (x_j - x_i) \quad \forall e = (i,j) \in E^{fix}_{circ} \quad (37)$$

$$le_{ij} \geq L_e \quad \forall e = (i,j) \in E^{fix}_{circ} \quad (38)$$

$$x_i \geq 0 \quad \forall i \in V \quad (39)$$

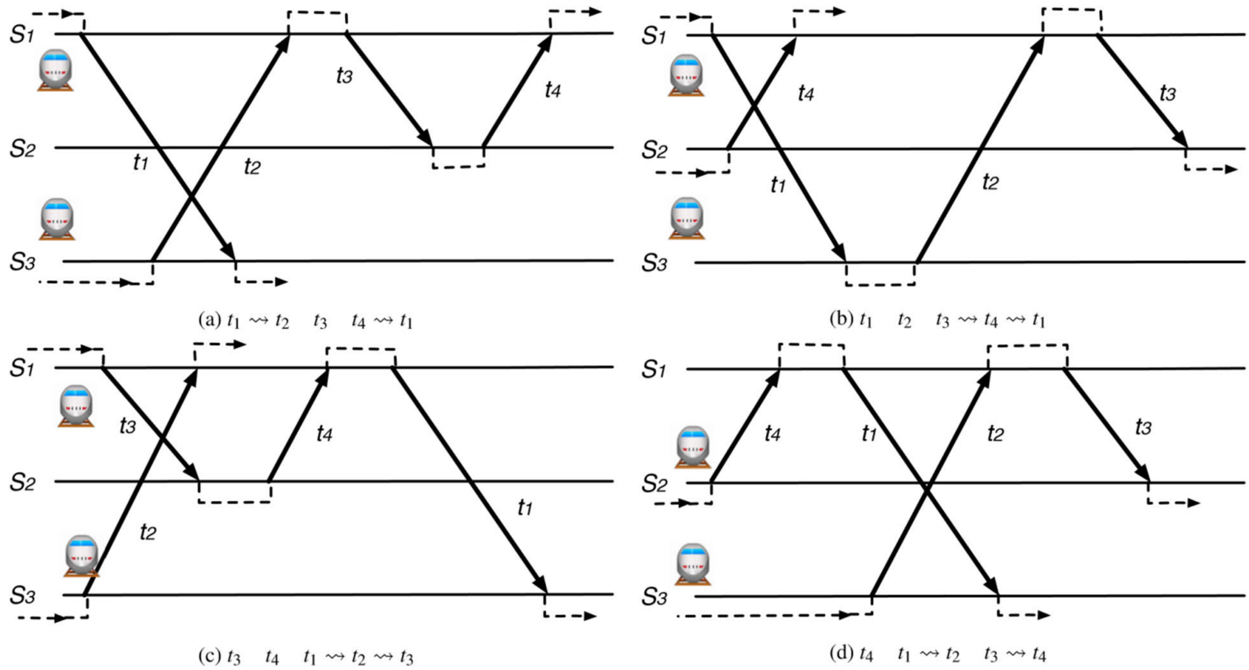
$$le_{ij} \geq 0 \quad V(i,j) \in E^{fix}_{circ} \quad (40)$$

$$u_{ij} \in \{0, 1\} \quad V(i,j) \in E^{fix}_{circ} \quad (41)$$

The minimal number of required train-sets with a fixed train-set route is formulated as the objective function (33). Same with Model (TSR), constraints (34-35) are used to enforce the initial train-set circulation and indicate whether $u_{ij} = 1$ or 0, respectively. Constraint (37) is similar to constraint (28) and calculates the actual turn around time. Constraint (38) enforces that the minimum turn around time is guaranteed.

2) FLEXIBLE TRAIN-SET ROUTE

Unlike the simple example presented in Figure 5, the insertion of additional trains is influenced by various constraints, such as existing trains in the timetable and reasonable departure and arrival time domain for passenger trains, which tends to make the additional trains can not be inserted randomly in



(a) $t_1 \rightsquigarrow t_2 \quad t_3 \quad t_4 \rightsquigarrow t_1$ (b) $t_1 \quad t_2 \quad t_3 \rightsquigarrow t_4 \rightsquigarrow t_1$

(c) $t_3 \quad t_4 \quad t_1 \rightsquigarrow t_2 \rightsquigarrow t_3$ (d) $t_4 \quad t_1 \rightsquigarrow t_2 \quad t_3 \rightsquigarrow t_4$

FIGURE 5. **Examples of various insertion solutions and rotation patterns to the same train-set route of $t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_1$.**

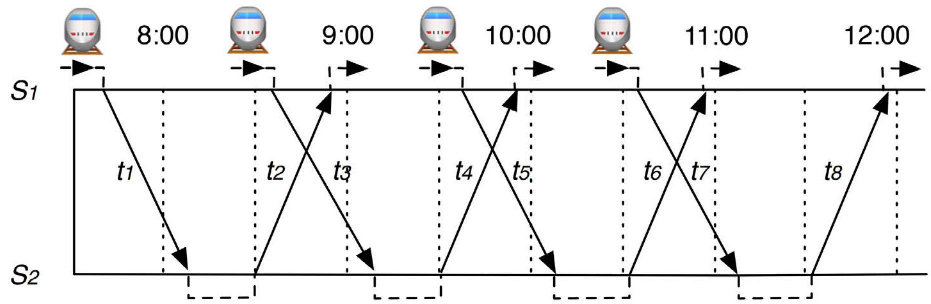


FIGURE 6. **An example with fixed train-set route.**

a time horizon. This will lead to many overnight stops and result in extra train-sets within a fixed route.

To explain this circumstance, we present the example in Figure 6. Here, for the sake of clarity, existing trains and intermediate stations are omitted. 8 additional trains denoted from t_1 to t_8 are needed to be inserted. t_1, t_3, t_5 and t_7 are trips from terminus s_1 to terminus s_2, and others the reverse journeys from s_2 to s_1. (t_2, t_3), (t_4, t_5), and (t_6, t_7) are supposed to start from the origin station in the time range of [8: 00,9: 00], [9: 00,10: 00], and [10: 00,11: 00] respectively due to the constraints of existing trains and predefined reasonable departure domain. If train-set flows the fixed route

$$t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_5 \rightarrow t_6 \rightarrow t_7 \rightarrow t_8 \rightarrow t_1$$

applying Model (TSR-fixed), we can get the insertion as showed in Figure 6, and the corresponding train-set rotation

$$t_1 \rightarrow t_2 \rightsquigarrow t_3 \rightarrow t_4 \rightsquigarrow t_5 \rightarrow t_6 \rightsquigarrow t_7 \rightarrow t_8 \rightsquigarrow t_1$$

It indicates that the route requires $N_{ts} = 4$ train-sets to cover additional trains.

However, obviously only 2 train-sets can cover the same insertion pattern, if train-set travels the route

$$t_1 \rightarrow t_2 \rightarrow t_5 \rightarrow t_6 \rightarrow t_3 \rightarrow t_4 \rightarrow t_7 \rightarrow t_8 \rightarrow t_1$$

and the corresponding rotation is

$$t_1 \rightarrow t_2 \rightarrow t_5 \rightarrow t_6 \rightsquigarrow t_3 \rightarrow t_4 \rightarrow t_7 \rightarrow t_8 \rightsquigarrow t_1$$

The increase of required train-set in Model (TSR-fixed) resulting from the fixed route omits the potential turn around connections. Practically, the route of a train-set is more likely to cross with the route of another at the same stations within the time intervals that are large enough. Then the sub-routes of arbitrary pairs of train-sets with overnight stop can be interchanged to yield a new route employing the same level of train-set. This implies that there are many alternative solutions with the same objective value, or even gain a better objective value.

In Figure 6, with the fixed route, train-set for $t_2 \rightsquigarrow t_3$, $t_4 \rightsquigarrow t_5$, $t_6 \rightsquigarrow t_7$ and $t_8 \rightsquigarrow t_1$ have overnight stops at s_1. There are opportunities for arbitrary pairs of train-sets to turn

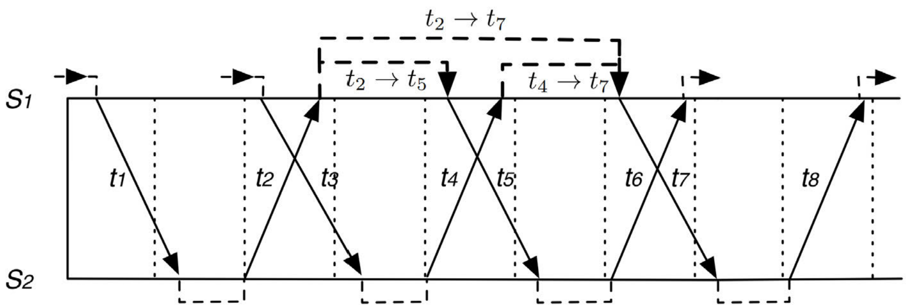


FIGURE 7. Alternative turn around connections on overnight stops.

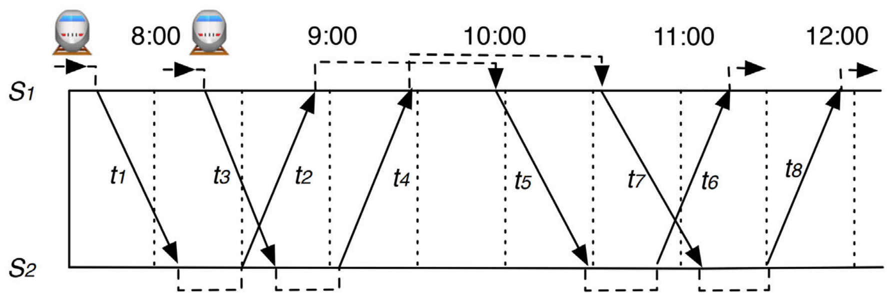


FIGURE 8. Insertion solution based on flexible train-set route.

around each other to obtain a new route. Here, t_2 can turn to t_5, t_7 or t_1, t_4 can turn to t_7, t_1 or t_3, and t_8 can turn to t_3, t_5 or t_7, see Figure 7. We denote these alternative turn around activities as E_{circ}^{alt}, then

$$\{(t_2, t_5), (t_2, t_7), (t_2, t_1), (t_4, t_7), (t_4, t_1), (t_4, ,t_3), (t_8, t_3), (t_8, t_5), (t_8, t_7)\} \subseteq E_{circ}^{alt}$$

Clearly, $E_{circ}^{alt} \subseteq E_{circ}^{add} - \mathrm{E}_{circ}^{fix}$.

The new train-set route have influences both on the number of train-sets and insertion solution. Since only when the $e \in E_{circ}^{alt}$ which respects the minimal turn around time is chosen, the new route will avoid unwanted overnight stops and then improve efficiency of train-set consequently. On the example of Figure 7, only activities in

$$\{(t_2, t_5), (t_2, t_7), (t_4, t_7)\} \subseteq E_{circ}^{alt^*}$$

have the opportunity to decrease the number of required train-sets. Clearly, $E_{circ}^{alt^*} \subseteq E_{circ}^{alt}$. Although it is not necessary in a feasible solution, for each chosen $e \in E_{circ}^{alt^*}$ will reduce by one train-set in rotation. If the minimal turn around time at s_1 is set to be 1 hour, Figure 8 shows the inserting solution applying flexible train-set route, where t_2 turns to t_5 and t_4 turns to t_7. It illustrates that when $(t_2, t_5) \subseteq E_{circ}^{alt^*}$ and $(t_4, t_7) \subseteq E_{circ}^{alt^*}$ is chosen in the new route, the number of required train-sets decreases by two and then required train-sets decreases by two and then $N_{ts} = 4 - 2 = 2$. The above is summarized in the following lemmas.

Lemma 1: Consider a fixed train-set route with predefined E_{circ}^{fix}, any alternative turn around activity $e = (i, j) \in E_{circ}^{alt}$,can be given by the constraints

$$q_{ii'} = 1, \quad u_{ii'} = 0 \quad \forall \left(i, i'\right) \in E_{circ}^{fix} \tag{42}$$

$$q_{j'j} = 1, \quad u_{j'j} = 0 \quad \forall (j', j) \in E_{circ^*}^{fix} \tag{43}$$

Remark 1: Note that, (i, i') and (j', j) are arbitrary pairs of turn around activities in fixed train-set route. By assumption E_{circ}^{fix}, $q_{ii'} = 1$ illustrates that i turns to i' in preselected train-set route. $u_{ii'} = 0$ indicates $x_{i'} - x_i < L_e$, that means the time intervals between i and i' is smaller than minimal turn around time, and then $i \rightsquigarrow i'$ has an overnight stop. Similarly, $j' \Rightarrow j$ is an overnight stop too. Then $(i, j) \in E_{cic}^{alt}$ is an alternative turn around activity between an arbitrary pair of overnight stops.

Lemma 2: Consider an alternative turn around activity $=(i, j) \in E_{circ}^{alt}$, the set of alternative turn around activities without overnight stops $E_{circ}^{alt^*}$ can be given as follows.

$$E_{circ}^{alt^*} = \left\{(i, j) : u_{ij} = 1, \forall (i, j) \in E_{circ}^{alt}\right\}$$

Lemma 3: Consider a fixed train-set route with predefined E_{circ}^{fix}. If an alternative turn around activity $(i, j) \in E_{circ}^{alt^*}$ is chosen, the number of required train-sets will reduce by one.

Based on the above analysis, we can now formulate the train-set planning problem with flexible route in adding paths problem. Firstly, we introduce new binary variable l_{ij} to indicate that if $(i, j) \in E_{circ}^{alt^*}$, then $l_{ij} = 1$, else $l_{ij} = 0$.

Remark 2: According to lemmas (1-3), $(i, j) \in E_{circ}^{alt^*}$ if and only if constraints as follows.

$$q_{ii'} = 1, \quad u_{ii'} = 0 (i, i') \in E_{circ}^{fix} \tag{44}$$

$$q_{j'j} = 1, \quad u_{j'j} = 0 (j', j) \in E_{circ}^{fix} \tag{45}$$

$$u_{ij} = 1 (i, j) \in E_{circ}^{alt} \tag{46}$$

are satisfied. It is equivalent to that when $q_{ii'} + q_{j'j} - (u_{ii'} + u_{j'j} - u_{ij}) = 3$, then $(i, j) \in E_{circ}^{alt^*}$.

Hence, constraints l_{ij} can be rewritten by q and u

$$l_{ij} = \begin{cases} 1 & \text{if } q_{ii'} + q_{j'j} - (u_{ii'} + u_{j'j} - u_{ij}) = 3 \\ 0 & \text{otherwise} \end{cases} \tag{47}$$

Next, we introduce another binary variable k_{ij} to present circumstances that whether the alternative turn around activity $(i,j) \in E_{circ}^{alt*}$ is chosen in the new flexible route.

As analyzed in lemma (4.3), the number of required train-sets can be represented as

$$N_{ts} = N_{ts}^{fix} - \sum_{(i,j)\in E_{circ}^{alt*}} k_{ij} \tag{48}$$

Then we can now state the train-set circulation in the ATP problem as follows,

$$(\textbf{TSR}-\textbf{flexible}) \text{ Minimize}: N_{ts} = \Big[\sum\nolimits_{(i,j)\in E_{circ}^{fix}} le_{ij} q_{ij} + \sum\nolimits_{(i,j)\in E_{trip}} (x_j - x_i)\Big] / T_{hor} - \sum\nolimits_{(i,j)\in E_{circ}^{alt*}} k_{ij} \tag{49}$$

$$\text{subject to: } x_j - x_i \geq L_e \quad \forall_e = (i,j) \in E_{circ}^{ini} \tag{50}$$

$$x_j - x_i \geq L_e - (1 - u_{ij})M \quad \forall_e = (i,j) \in E_{circ}^{fix} \tag{51}$$

$$x_j - x_i < L_e - u_{ij}M \quad \forall_e = (i,j) \in E_{circ}^{add} \tag{52}$$

$$le_{ij} \geq T_{hor}(1 - u_{ij}) + (x_j - x_i) \quad \forall_e = (i,j) \in E_{circ}^{fix} \tag{53}$$

$$le_{ij} \geq L_e \quad \forall_e = (i,j) \in E_{circ}^{fix} \tag{54}$$

$$3 - \left[q_{ii'} + q_{j'j} - (u_{ii'} + u_{j'j} - u_{ij})\right] \geq (l_{ij} - 1)M \quad \forall_e = (i,j) \in E_{circ}^{add} \tag{55}$$

$$3 - \left[q_{ii'} + q_{j'j} - (u_{ii'} + u_{j'j} - u_{ij})\right] > -l_{ij}M \quad \forall_e = (i,j) \in E_{circ}^{add} \tag{56}$$

$$k_{ij} \leq l_{ij} \quad \forall i \in V_{end}^{add}, j \in V_{start}^{add} \tag{57}$$

$$\sum\nolimits_{i\in V_{end}^{add}} k_{ij} \leq 1 \quad \forall j \in V_{start}^{add} \tag{58}$$

$$\sum\nolimits_{j\in V_{start}^{add}} k_{ij} \leq 1 \quad \forall i \in V_{end}^{add} \tag{59}$$

$$\text{k}_{ij} \leq st_{ij} \quad \forall i \in V_{end}^{add}, j \in V_{start}^{add} \tag{60}$$

$$u_{ij}, k_{ij} l_{ij} \in \{0, 1\} \quad \forall i \in V_{end}^{add}, j \in V_{start}^{add} \tag{61}$$

$$x_i, le_{ij} \geq 0 \quad \forall i \in V, \ \forall (i,j) \in E_{circ}^{add} \tag{62}$$

Constraints (50-54) are the same as in Model (TSR-fixed). Constrains (57) imply that only the selection of an alternative turn around activity which holds $(i,j) \in E_{circ}^{alt*}$ can reduce the number of train-sets. Constraints (58-59) are similar to constraints (24-25) and enforce that for every train there is exactly one train-set connection to turn to and be turned by another train. Constraints 60) specify that the alternative turning around is also forbidden between two operations which are of different stations.

The objective function of ATP problem has two weighted terms and the relative importance of each objective dictates the values of the associated weights, i.e. w_1 and w_2. Then the multi-objective ATP model is presented as follows.

$$\begin{aligned} &\text{Minimize: } w_1 \cdot N_{ad} + w_2 \cdot N_{ts} \\ &\text{subject to: Constraints } (2-21) \\ &\qquad\qquad\quad \text{Constraints } (50-62) \end{aligned} \tag{63}$$

V. COMPUTATIONAL TESTS

This section provides details of comprehensive numerical investigations to identify whether good solutions can be obtained using the methodology and techniques proposed in this paper. The primary aim of the adding paths problem is to solve the problem that

How to operate additional trains most appropriately with minimum number of train-sets and without leading large disruption to initial timetable?

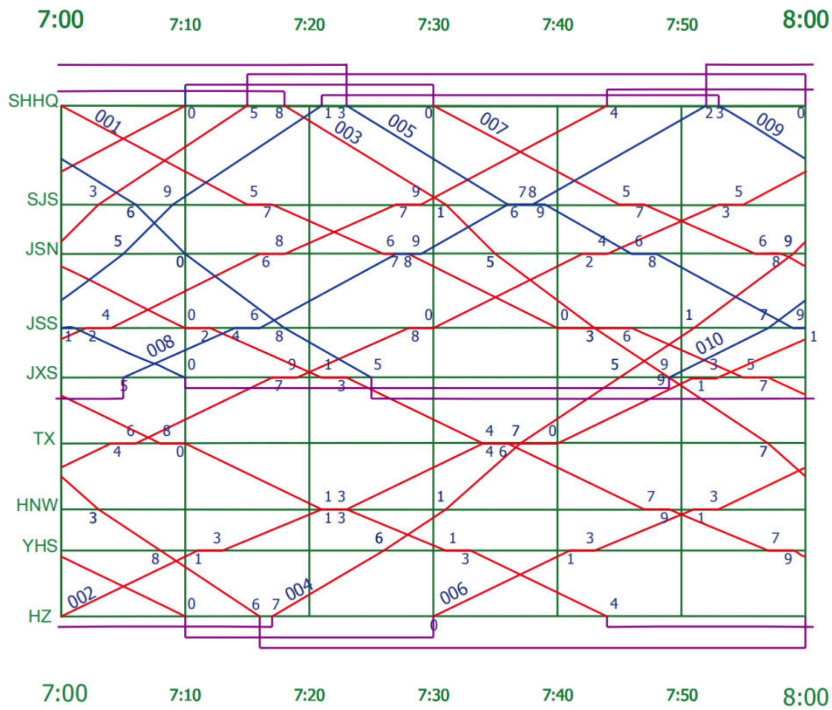
Meanwhile, we also would like to know the affecting factors to this problem. For example,

(1) *What effect the various level of accepted disruption have?*
(2) *What effect the introduction of time window constraints have?*
(3) *What effect the different application of train-sets have?*

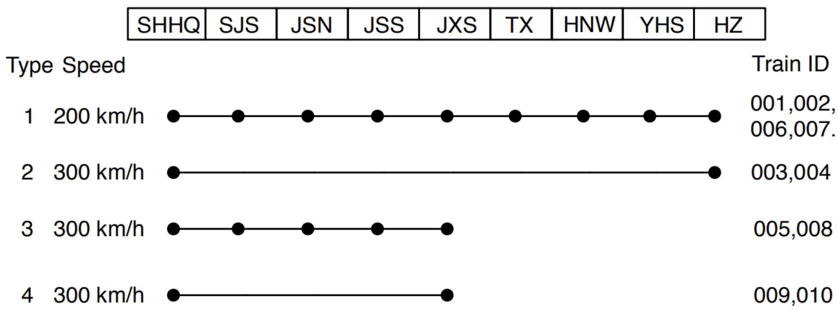
A. TEST PROBLEM

The formulation and the strategies have been applied to Shanghai-Hangzhou high-speed rail line, which consists of double-tracked high-speed railway lines that are the major links connection Shanghai Hongqiao (SHHQ), Songjian South (SJS), Jinshan North (JSN), Jiashan South (JSS), Jiaxing South (JXS), Tongxiang (TX), Haining West (HNW), Yuhang South (YHS) as well as Hangzhou (HZ). In this study, our focus is on a generic daily cyclic timetable in the time period of (6:00 am - 13:00 pm), and it includes 78 passenger trains in both down direction (from SHHQ to HZ) and up direction (from HZ to SHHQ). The cyclic nature of the timetable is illustrated in Figure 9a. Four Types of trains are used, see Figure 9b:

- type 1: medium-speed trains (200 km/h) which are composed of 2 trains (001,002,006 and 007) in each direction between the railroad region of SHHQ - HZ, and scheduled to stop at every intermediate stations,
- type 2: high-speed trains (300 km/h) which are composed of 1 train (003 and 004) in each direction between the railroad region of SHHQ - HZ, without any scheduled stop at intermediate stations,



(a) One hour time-space diagram for the track between Shanghai and Hangzhou



(b) Train types with various trip, speed and stop schedules in initial timetable

FIGURE 9. Time-space diagram for Shanghai-Hangzhou high-speed railway.

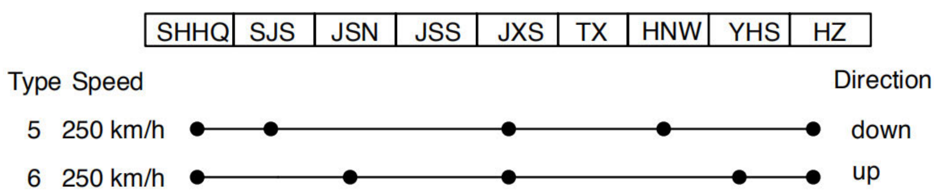


FIGURE 10. Speed and stops schedule for additional trains.

- type 3: high-speed trains (300km/h) which are composed of 1 train (005 and 008) in each direction between the railroad region of SHHQ - JXS, and scheduled to stop at every intermediate stations,
- type 4: high-speed trains (300km/h) which are composed of 1 train (009 and 010) in each direction between the railroad region of SHHQ - JXS, without any scheduled stop at intermediate stations.

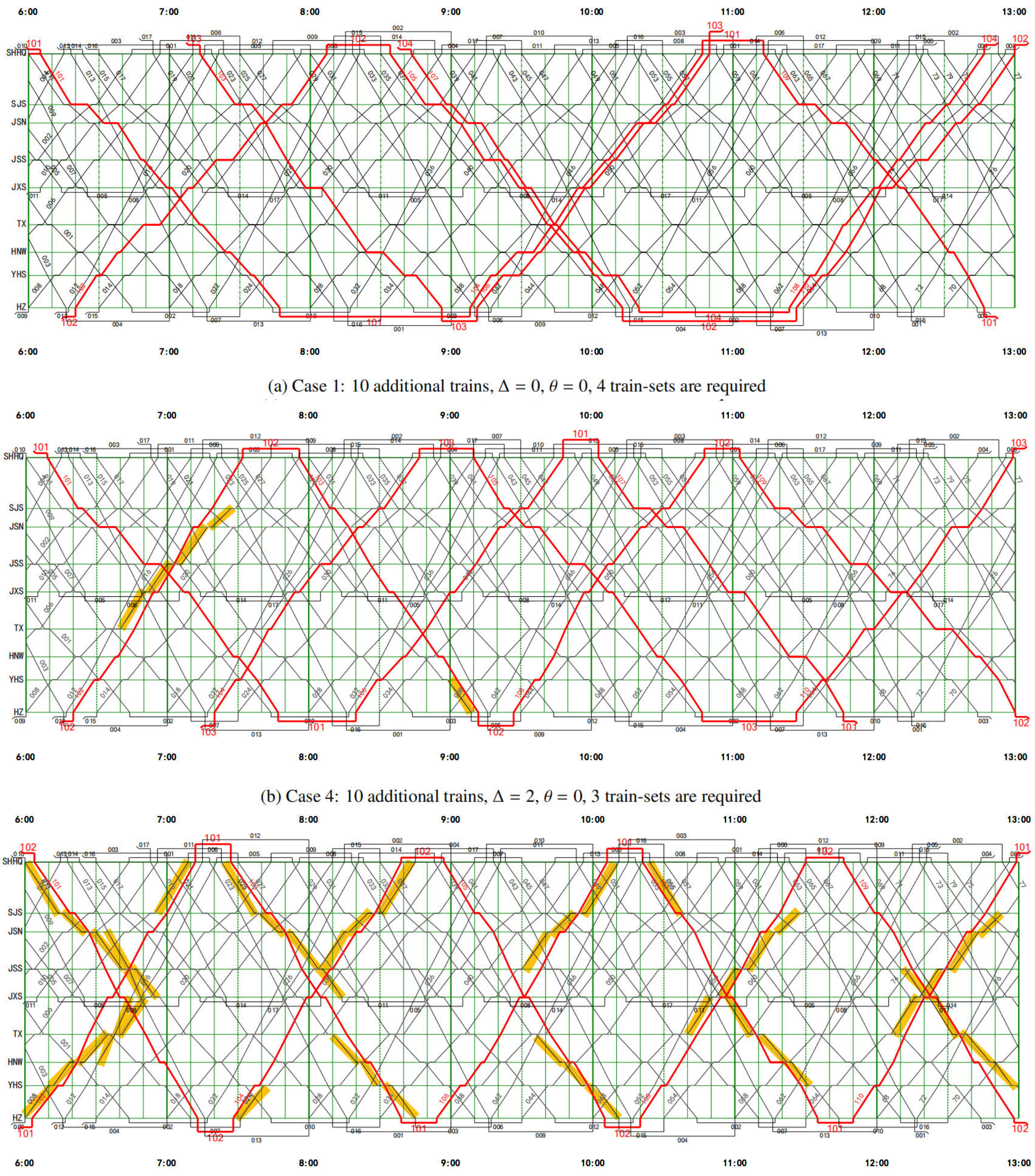
In the experiments, minimum headways are set to 3 minutes for both consecutive arrivals and departures. Acceleration and deceleration times are set to 2 and 1 minutes respectively for both high-speed and medium-speed trains. In addition, taking the variable velocity into consideration, maximum driving time is set to 110% (w.r.t. the minimum driving time). The train-set circulation for existing trains is constructed from the initial timetable.

TABLE 7. Results from experiments with different tolerance of disruption.

Case	Total number of additional trains	Tolerance		Results			
		Δ	θ	Objective value	Number of Train-sets	Adjustments (min)	$Time^a(sec)$
1	10	0	0	4000	4	0	10
2		1	0	4000	4	0	41
3		1	1	4000	4	0	49
4		2	0	3100	3	10	1822
5		2	1	3000	2	100	37
6		2	2	3000	2	100	155
7		3	0	3100	3	10	(0.64%)
8		3	1	3000	2	100	194
9		3	2	3000	2	100	172
10		3	3	3000	2	100	195
11	12	0	0	4000	4	0	23
12		1	0	4000	4	0	84
13		1	1	4000	4	0	109
14		2	0	3310	3	31	(5.04%)
15		2	1	3310	3	31	(4.53%)
16		2	2	3310	3	31	(3.92%)
17		3	0	3310	3	31	(6.04%)
18		3	1	3310	3	31	(4.53%)
19		3	2	3310	3	31	(5.74%)
20		3	3	3310	3	31	(6.94%)
21	14	0	0	5000	5	0	96
22		1	0	5000	5	0	188
23		1	1	5000	5	0	256
24		2	0	4200	4	20	(4.76%)
25		2	1	4100	3	110	(2.43%)
26		2	2	4100	3	110	(2.43%)
27		3	0	4210	4	21	(4.98%)
28		3	1	4110	3	111	(12.89%)
29		3	2	4100	3	110	(2.43%)
30		3	3	4100	3	110	(3.82%)
31	16	0	0	6000	6	0	348
32		1	0	6000	6	0	739
33		1	1	6000	6	0	644
34		2	0	4430	4	43	(9.71%)
35		2	1	4520	4	52	(11.50%)
36		2	2	4420	4	42	(9.28%)
37		3	0	4420	4	42	(9.50%)
38		3	1	4440	4	44	(9.91%)
39		3	2	4420	4	42	(9.50%)
40		3	3	4430	4	43	(9.71%)

[a] The numbers in the parentheses refer to the relative gap when the time limit of 1 h was exceeded without an optimalsolution being verified.

All the experiments are performed on a PC with Intel(R) Core(TM) i3 CPU 530 @ 2.93GHZ + 2.93GHZ and 8 GB of RAM, and all the algorithms are implemented in Visual Studio 2013 on the Windows 8.1, 64 bit. IBM ILOG Cplex 12.5 with default set is used as a solver. We have run all the instances with a time limit of 1 h, and all the results report the outcome when this time limit was reached (or when a proven optimal solution is found).



(a) Case 1: 10 additional trains, Δ = 0, θ = 0, 4 train-sets are required

(b) Case 4: 10 additional trains, Δ = 2, θ = 0, 3 train-sets are required

(c) Case 6: 10 additional trains, Δ = 2, θ = 2, 2 train-sets are required

FIGURE 11. Solutions with different level of tolerance.

B. RESULTS

1) PART 1: EXPERIMENTS WITH DIFFERENT TOLERANCE OF DISRUPTION

In this part, two new types of train without time window is planned to insert as extra trains in the initial timetable.

It is operated between the railroad region of SHHQ - HZ. The down train is scheduled to stop at intermediate stations SJS, JXS and HNW, while the up train stops at JSN, JXS and YHS at least 2 min, see Figure 10. For the sake of simplification, the maximum dwell time is set to 7 min at any

TABLE 8. Objective value of the cases 14-20 from Table 7 with different time limits.

Case	Time limit 3^a (min)			Time limit 6^a (min)			Time limit 9^a (min)		
	Objective value	Number of Train-sets	Relative gap (%)	Objective value	Number of Train-sets	Relative gap (%)	Objective value	Number of Train-sets	Relative gap (%)
14	3310 *	3	9.37	3310	3	9.37	3310	3	8.98
15	3310 *	3	9.37	3310	3	9.37	3310	3	9.37
16	3380	3	11.24	3310 *	3	9.65	3310	3	9.65
17	3820	3	21.47	3410	3	12.02	3330	3	11.76
18	3360	3	10.71	3360	3	10.71	3320	3	9.34
19	4610	4	34.92	3320	3	9.64	3310 *	3	9.37
20	3480	3	13.79	3330	3	9.91	3330	3	9.91

[a] The number with "*" refers to that no further improvements were found within 1 h.

TABLE 9. A comparison of inserting additional trains with and without time window.

Total number of additional trains	With time window					Without time window	
	6:00-8:00	8:00-10:00	10:00-13:00	Number of train-sets	Time (sec)	Number of train-sets	Time (sec)
10	4	4	2	4	0.15	4	10
12	4	6	2	6	0.18	4	23
14	4	6	2	6	0.29	5	96
16	6	8	2	8	0.43	6	348

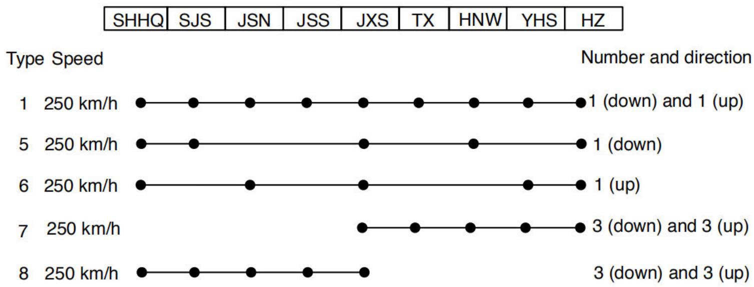


FIGURE 12. Information of additional trains.

arbitrary intermediate station in all experiments. The value of w1 and w2 in objective function (55) are set to 1000 and 10 respectively. All of the additional trains appear in pairs of reverse directions (i.e. one down train and one up train).

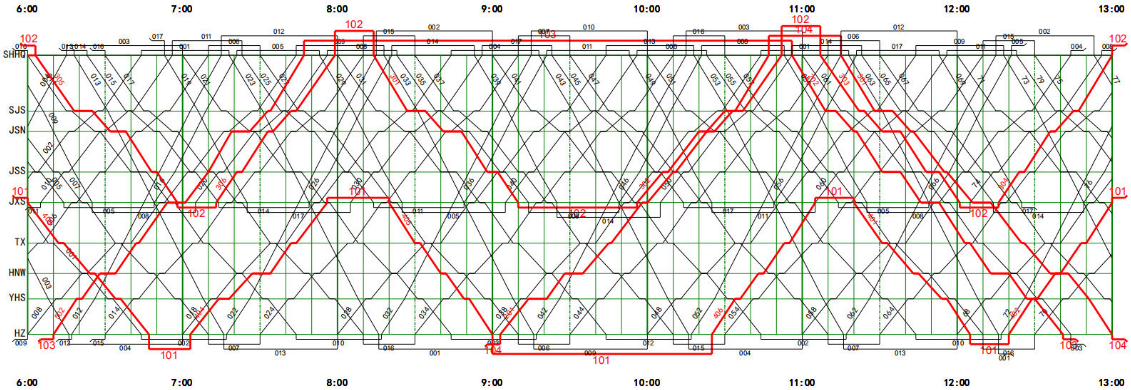
The results from the experiments with various tolerance of disruption and a number of 10, 12, 14 and 16 additional trains are tested respectively in Table 7. An increase both in the number of additional trains and the level of tolerance generate an increase in computational time predictably.

Besides, the two main aspects considered when analyzing the results in Table 7 are the differences between the tolerance of the initial timetable and quality of solutions generated. The increasing freedom (i.e. more options to modify the initial timetable) represented by bigger Δ and θ provides as good or better solutions, especially regarding the number of required train-sets for additional trains. The choice of high tolerance has an obvious effect on decreasing the number of train-sets.

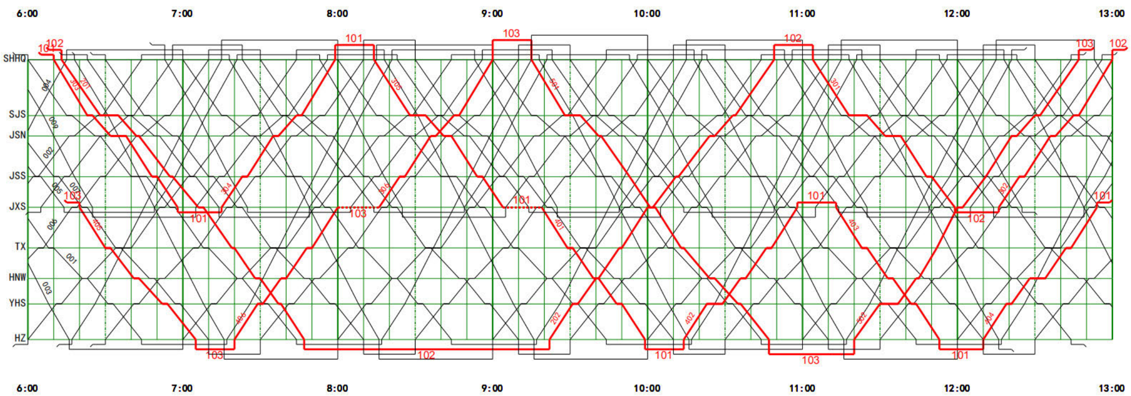
Figure 11 represents the solutions for inserting 10 trains with different level of tolerance.

(1) if the allowable adjustment $\Delta = 0$ and periodic structure $\theta = 0$, which implies the initial timetable is fixed, 4 train-sets are required to cover these 10 additional trains. The insertion and train-set circulation are shown in Figure 11a.
(2) if $\Delta = 2$ and $\theta = 0$, it constraints that the initial trains can be left or right shift at most 2 min but must departure from the corresponding original station at an exact periodic interval. Figure 11b demonstrates that 3 train-sets are required when the initial trains are modified 10 min (see the yellow rectangle area). The departures at original station attempt to keep same as initial schedules to prevent large adjustment.
(3) If $\Delta = 2$ and $\theta = 2$, the initial trains have a higher tolerance both in adjustment and periodic structure. By comparison, only 2 train-sets are sufficient for operating the same trains. The corresponding solution is illustrated in Figure 11c. The decrease in train-sets is at the cost of 100 min adjustment to initial timetable.

Even though high tolerance decrease the number of train-sets dramatically, on the other hand it impacts on problem size (i.e. the higher tolerance, the more initial trains can be rescheduled and the more options that additional trains can be inserted), and consequently may become more time consuming to solve the problem.



(a) Certain railroad region: 4 train-sets are required



(b) Uncertain railroad region: 3 train-sets are required

FIGURE 13. Experiments using different strategies of train-sets.

Table 7 only specify the best solutions found within a certain time while it also may be relevant to analyze the progress over time. In several of the scenarios presented in Table 7, for example when 12 trains are planned to insert, case (14-20) could not verify optimality of the found solutions (where optimal now refers to the optimal solution to the problem formulation of different tolerances) within 1 h, but a solution within an acceptable, relative gap was found. In Table 8, the solution progress in case (14, 15,16 and 19) is presented showing that the same solutions (and corresponding gap) were found within 10 min or less. In comparison with the results in Table 7, case (17, 18 and 20) provide similar solutions in 10 min shown in Table 8 that require the same train-sets but an extra 2 min, 1 min and 2 min adjustments in case (17), (18) and (19) respectively.

The relative gap in relation to the size of the objective value needs to be considered to provide an appropriate and effective stopping criterion. That is, in the case of $w_1 = 1000$ and $w_2 = 10$ in objective function (55), a relative gap of for example 33% may be tolerated for the problem with objective value 3000 that aims to get a solution with minimum train-sets,

2) PART 2: EXPERIMENTS OF INSERTING TRAINS WITH TIME WINDOW CONSTRAINTS

In practice, additional trains are usually supposed to insert in an specifical time period, namely time window constraints. For instance, in the application of increasing train services to meet the passenger flow, the additional trains are planned to departure in rush hours. Furthermore, when inserting interline trains, the options of time slot for departures and arrivals are very limited, even fixed at a precise time generally.

For sake of simplification, all of the initial trains are fixed and the considered time horizon is divided into 3 independent time period, i.e. 6:00-8:00, 8:00-10:00 and 10:00-13:00. The number of required new trains in each time period is shown in Table 9, and as well a comparison results of inserting trains with and without time window constraints. With time window constraints, the number of train-sets increases as expected due to imbalance utilization. However, the computational time dramatically decreases to less than 0.5 s even inserting 16 trains. The time window constraints not only narrow the search space of insertion, but also cut the option of circulation down.

3) PART 3: EXPERIMENTS USING DIFFERENT APPROACHES ON THE APPLICATION OF TRAIN-SET

Each approach has its own merits and demerits, independently or combinedly applied in different high-speed lines in China. For example, Beijing-Tianjin and Shanghai-Nanjing high-speed lines adopt an integrated strategy that only some of the train-sets are fixed in an certain region.

The adding path model proposed in this paper is adapted to both certain and uncertain railroad region approaches.

With the purpose of analyzing the impacts of different approaches on adding paths problem, 5 types of train with various railroad region are planned to insert in this experimental part. Figure 12 indicates the information of speed, stop schedule, running direction and railroad region, and number as well for each type of train.

The solution using different approaches of train-set application is illustrated in Figure 13. All of the initial trains are supposed to be fixed. Figure(16a) shows that using train-set in certain railroad region, 4 train-sets are required to cover all of the 16 additional trains. Train-set 101 carries train 302, 303, 304, 305, 306 and 301 successively in railroad region SHHQ-JXS; train-set 102 carries 403, 404, 405, 406, 401 and 402 in railroad region JXS-HZ; train-sets 103 and 104 carry in railroad region SHHQ-HZ.

However, when using train-set in uncertain railroad region, see Figure 13b, only 3 train-sets are required. In order to be distinguished from dwell activities, the turn around activities between trains of same direction are represented by red dotted line in Figure 13b, such as train-set 103 at station JXS where turning from train 406 to 306.

The uncertain region approach increases the utilization of train-set. As long as the requirements of connection time are met, a train-set runs under a number of lines to operate, which will enhances the flexibility of operating train-set.

VI. CONCLUSIONS AND FUTURE

The problem of inserting additional train services in an cyclic timetable was considered in this paper. An adding paths model considering the constraints of time window, variable trip time, acceleration and deceleration time, minimum headway, priority for overtaking, periodic structure and train-set circulation is proposed to minimize the total adjustments for initial trains and at the same time minimize the number of required train-sets for entire trains.

The adding paths problem is different from the usual timetable construction problem due to additional constraint of tolerance of disruptions for initial trains. These tolerance constraints may be viewed as the allowable adjustments and periodic structure. In this paper both settings were provided for in our techniques and investigated in our numerical investigations. In addition, the number of required train-sets is also taken into account as an important index. The train-set circulation in adding paths problem is decomposed to two sub-problem. For current train-set circulation, the initial train-set route is assumed to be fixed. For additional trains, different inserting patterns produce different train-set circulation and number of required train-set consequently. The first sub-problem can be simply dealt with as a rescheduling problem of a tight constraint to keep the current train-set circulation. The second is a train-set routing problem to cover all the additional trains with minimum number of train-sets. In adding paths problem it is nonlinear since the additional trains do not exist in the timetable, and even the number of additional trains may depend on the number of instantly available train-set at the right place. In order to linearize the adding paths problem, the concepts of rotation, fixed route and flexible route are introduced in this paper.

The numerical investigation consisted of three parts. In part 1 different control parameters for tolerance of disruption, composed of allowable adjustment and periodic structure, are investigated. In part 2 time window constraints are used to enforce adherence to the additional schedule. In part 3 two different strategies of train-set application are compared. From numerical investigations it is observed that the settings of perturbation tolerance, time window and strategy will effect the inserting solution and the number of require train-set; building the additional schedule from scratch using time windows is effective, since both the search space for insertion and the potential option for train-set route are decreased dramatically.

We have used standard software, CPLEX, with its branch-and-bound solution procedure and default settings of parameters. There may be more beneficial settings than the default settings (including branching strategies) for this particular problem. Hence, using more tailored solution software or parameter settings could potentially provide good solutions faster (Törnquist and Persson (2007)).

Obviously there are some aspects disregarded in the current formulation. In our model, all of the additional trains are forced to be added, which would lead to no result in practice. For example, when constructing a generic cyclic+non-cyclic timetable in tactical planning phase as shown in Figure 1, if the initial cyclic schedule is already tight, then only limited number of extra trains can be inserted. An integrated model is required to simultaneously determine the applicable proportion of cyclic lines and non-cyclic lines, and insert extra trains. Thus, additional research on this topic is in a significant demand but beyond the scope of this paper.

Another practical consideration is how to define and use suitable parameters in the objective function to represent the trade-off of the adjustments for initial timetable and the number of required train-set. In addition, all of the initial trains have the same value of penalty to be adjusted in this paper. Applying various penalties to high-speed and middle-speed trains for example may lead to middle-speed trains becoming less prioritised than high-speed trains.

Furthermore, ideally the inserting of traffic should be carried out with a network perspective and in a whole day time horizon, but the problem would become too large to solve within a reasonable time. Consequently the problem needs to be bounded somehow both in time and geographically. However, costs and gains that arise beyond the problem boundary should somehow be approximated and accounted for when considering a fragment of the overall inserting problem (Törnquist and Persson (2007)). In ongoing and future research, the development of algorithms, able to find near optimal solutions for large instances within acceptable computation time is worthwhile.

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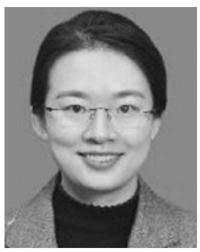

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