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Finite-Time Controllability and Set Controllability of Impulsive Probabilistic Boolean Control Networks

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ABSTRACT This paper addresses the finite-time controllability and set controllability of impulsive probabilistic Boolean control networks (IPBCNs). Firstly, using the algebraic state space representation (ASSR) method, the transition probability matrix of IPBCNs is established. Secondly, a kind of finite step reachability matrix with probability one is constructed, based on which, several effective criteria are proposed for the finite-time controllability with probability one of IPBCNs. Thirdly, a necessary and sufficient condition is presented for the finite-time set controllability with probability one of IPBCNs by constructing the set controllability probability distribution vector. Finally, the obtained results are extended to switching topology case.

INDEX TERMS Probabilistic Boolean control network, impulsive effect, controllability, set controllability, algebraic state space representation.

I. INTRODUCTION

As one of the most significant issues in modern control theory, the concept of controllability was initiated for linear systems in 1960s [16]. In the following decades, stochastic nonlinear systems have drawn many scholars' interest and a sequence of results have been obtained [47], [48], [54]. Under this trend, controllability has been introduced into nonlinear systems [44], [53] and stochastic systems [1], [12]. Compared with the controllability of deterministic systems which shows the ability to steer any initial state to any target state under specific control inputs, the controllability of stochastic systems is more complicated, which includes complete controllability, approximate controllability and stochastic controllability [35]. All these concepts generalize the classical controllability of deterministic systems. When impulsive effects were considered, the controllability of stochastic impulsive systems was discussed in [17]. As a natural generalization of controllability, Cheng and Hu [6] developed the concept of set controllability for switched linear systems. Set controllability has many applications, such as set stabilization, synchronization, output tracking and so on [4], [19], [23], [29], [34], [51], [55].

As a special kind of nonlinear stochastic systems, probabilistic Boolean networks were firstly proposed by Shmulevich in 2002 [41] to accurately describe the regulatory process among genes. The steady state of probabilistic Boolean networks was analyzed in [42]. Pal *et al.* [38] investigated the infinite-horizon optimal control problem of probabilistic Boolean control networks (PBCNs). Qian and Dougherty [39] considered the steady-state distributions for structurally perturbed probabilistic Boolean networks. On the other hand, many evolutionary processes in biological networks are likely to be influenced by sudden changes in internal or external environment, which is often modeled in the form of impulses [37], [43]. Therefore, it is meaningful to consider PBCNs with impulsive effects. However, due to the lack of suitable mathematical tools, it is very hard to systematically study the controllability and control design problems of PBCNs and impulsive PBCNs (IPBCNs).

Recently, Cheng *et al.* [8] has pioneered an algebraic state space representation (ASSR) method to study Boolean networks [9], [24], [26], [52], [58], probabilistic Boolean networks [10], [14], [31], [36] and evolutionary games [11]. Using this effective mathematical tool, several fundamental issues have been properly solved for Boolean networks and probabilistic Boolean networks, including state estimation [3], controllability and observability [13], stability and

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stabilization [27], [30], optimal control [18], [46], and so on. Since then, the study of controllability has made rapid progress and achieved fruitful results. Li and Sun [20] considered the controllability of PBCNs via open-loop control and closed-loop control, respectively. In [32], Liu *et al.* further analyzed the controllability problem of PBCNs and gave some more general conclusions. In [57], several concepts of set reachability has been introduced to PBCNs, including finite-time set reachability with probability one and asymptotically reachability in distribution.

As is well known to us all, impulsive effect is a common phenomenon, and some recent novel works have been developed for impulsive systems [22], [45]. Especially, considering the sudden changes in gene regulation, the impulsive effect on genetic regulatory networks attracts many scholars' interest [40], [56]. It is noted that the ASSR method was also applied to Boolean networks with impulsive effects [5], [49]–[51]. Li and Sun [21] firstly introduced impulsive effects into Boolean networks. Liu *et al.* [33] studied the controllability of Boolean networks with impulsive effects avoiding certain forbidden states. As was revealed in [2], impulsive phenomenon may change the stability and controllability of Boolean networks. For probabilistic Boolean networks, Li *et al.* [25] showed that impulses may change the finite-time stability. Hu *et al.* [15] solved the problem of stabilizing PBCNs by designing a state feedback controller and impulsive strategies. However, to our best knowledge, there are no results available on the finite-time controllability and set controllability [7] of impulsive PBCNs (IPBCNs), which motivates the study of this paper.

In this paper, we investigate the finite-time controllability and set controllability with probability one of IPBCNs. In order to facilitate the analysis of IPBCNs, we convert the dynamics of IPBCNs into the ASSR framework. Then, we define the set controllability probability distribution vector and set controllability index for IPBCNs, based on which, we present several criteria for the finite-time controllability and set controllability with probability one of IPBCNs. These criteria are easily checked via MATLAB. Furthermore, as an extension, we also discuss the set controllability of IPBCNs with switching topology and establish some criteria. It is shown that impulsive effects play an important role in the finite-time controllability analysis of PBCNs (see Remark 4).

Notations: $\mathcal{D} := \{1, 0\}$. The set of all $m \times r$ real matrices is denoted by $\mathcal{M}_{m\times r}$. $Col_i(M)$, $Row_j(M)$ and $(M)_{i,j}$ represent the i-th column, j-th row and (i,j)-th entry of matrix M, respectively. The column set of M is denoted by $Col(M)$. $\Delta_n := Col(I_n)$, where I_n is the $n \times n$ identity matrix. $\delta_n^k := Col_k(I_n)$, $k = 1, \cdots, n$. $\mathcal{L}_{m\times r} := \{M \in \mathcal{M}_{m\times r} : Col(M) \subset \Delta_m\}$ denotes the set of all $m \times r$ logical matrices. $\mathcal{B}_{m\times r} := \{M \in \mathcal{M}_{m\times r} : (M)_{i,j} \in \mathcal{D}\}$ denotes the set of all $m \times r$ Boolean matrices. $\Gamma_{m\times r} := \{M \in \mathcal{M}_{m\times r} : (M)_{i,j} \geq 0, \ \sum_{i=1}^{m}(M)_{i,j} = 1, \ \forall j = 1, \cdots, r\}$ denotes the set of all $m \times r$ stochastic matrices. Denote the $m \times r$ matrix with all entries being 1 by $\mathbf{1}_{m,r}$. Especially, when $r = 1$, the m-dimensional column vector is denoted by $\mathbf{1}_m$. For an $m \times r$ matrix $M := (m_{i,j})$, define $\lfloor M \rfloor := (\lfloor m_{i,j} \rfloor)$, where $\lfloor \ \rfloor$ represents the floor function. Denote $\bigvee_{k=1}^{r} M_k := M_1 \vee M_2 \vee \cdots \vee M_r$, where M_i, $i = 1, \cdots, r$ are matrices with the same dimension. For two matrices $M, N \in \mathcal{M}_{m\times r}$, $M \geq N$ represents that $(M)_{i,j} \geq (N)_{i,j}$, $i = 1, \cdots, m, j = 1, \cdots, r$.

II. MAIN RESULTS

A. FINITE-TIME CONTROLLABILITY OF IPBCNS

The dynamics of IPBCN is given as follows:

$$\begin{cases} X(t+1) = f_1\big(X(t), U(t)\big), & t \in \mathbb{N} \setminus \Lambda; \\ X(t_k) = f_2\big(X(t_k - 1)\big), & k \in \mathbb{Z}_+, \end{cases} \tag{1}$$

where $X(t) = \big(x_1(t), \cdots, x_n(t)\big) \in \mathcal{D}^n$, $U(t) = \big(u_1(t), \cdots, u_m(t)\big) \in \mathcal{D}^m$ are state and control input of system (1), respectively. The impulsive effects of system (1) occur at time $t = t_k$, where the impulsive time sequence $\{t_k : k \in \mathbb{Z}_+\} \subseteq \mathbb{Z}_+$ satisfies $0 := t_0 < 1 < t_1 < \cdots < t_k < \cdots$. The non-impulsive time sequence is expressed as $\{t : t-1 \in \mathbb{N} \setminus \Lambda\} \subseteq \mathbb{Z}_+$, where $\Lambda := \{t_i - 1 : i \in \mathbb{Z}_+\}$. $f_1 : \mathcal{D}^{n+m} \to \mathcal{D}^n$ and $f_2 : \mathcal{D}^n \to \mathcal{D}^n$ are both Boolean mappings. At each time, f_1 and f_2 are chosen from $\{f_1^1, \cdots, f_1^l\}$ and $\{f_2^1, \cdots, f_2^r\}$, respectively, with $\mathbb{P}\{f_1 = f_1^i\} = p_{1,i}$, $i = 1, \cdots, l$ and $\mathbb{P}\{f_2 = f_2^j\} = p_{2,j}$, $j = 1, \cdots, r$. Obviously, $\sum_{i=1}^{l} p_{1,i} = 1$, $\sum_{j=1}^{r} p_{2,j} = 1$.

Denote $\mathbb{P}\{X(s; X_0, U) = X_d\}$ by the maximum transition probability from the initial state $X_0 \in \mathcal{D}^n$ to the target state $X_d \in \mathcal{D}^n$ in s steps under a given control sequence $\big\{U(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\big\}$. Then, we give the concept of finite-time controllability with probability one for system (1).

Definition 1: Consider system (1).

(i) $X_d \in \mathcal{D}^n$ is said to be reachable from $X_0 \in \mathcal{D}^n$ with probability one at time s, if there exists a control sequence $\big\{U(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\big\} \subseteq \mathcal{D}^m$ such that $\mathbb{P}\{X(s; X_0, U) = X_d\} = 1$.

(ii) $X_d \in \mathcal{D}^n$ is said to be reachable from $X_0 \in \mathcal{D}^n$ with probability one, if there exists a positive integer s such that X_d is reachable from X_0 with probability one at time s.

(iii) System (1) is said to be finite-time controllable with probability one at $X_0 \in \mathcal{D}^n$, if for any $X_d \in \mathcal{D}^n$, there exist a positive integer s and a control sequence $\{U(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\} \subseteq \mathcal{D}^m$ such that $\mathbb{P}\{X(s; X_0, U) = X_d\} = 1$.

(iv) System (1) is said to be finite-time controllable with probability one, if for any $X_0 \in \mathcal{D}^n$, system (1) is finite-time controllable with probability one at X_0.

Using the semi-tensor product method [8], we now establish the ASSR of IPBCN (1).

Taking the vector form of logical variables and setting $x(t) = \ltimes_{i=1}^{n} x_i(t) \in \Delta_{2^n}$, $u(t) = \ltimes_{i=1}^{m} u_i(t) \in \Delta_{2^m}$, the algebraic representation of IPBCN (1) can be expressed as follows:

$$\begin{cases} x(t+1) = L_1 u(t) x(t), & t \in \mathbb{N} \setminus \Lambda; \\ x(t_k) = L_2 x(t_k - 1), & k \in \mathbb{Z}_+, \end{cases} \tag{2}$$

where $L_1 \in \mathcal{L}_{2^n \times 2^{n+m}}$, $L_2 \in \mathcal{L}_{2^n \times 2^n}$, $\mathbb{P}\{L_1 = L_{1,i}\} = p_{1,i}$, $i = 1, \cdots, l$, $\mathbb{P}\{L_2 = L_{2,j}\} = p_{2,j}$, $j = 1, \cdots, r$, $L_{1,i}$ and $L_{2,j}$ are structural matrices of f_1^i and f_2^j, respectively.

Denote the expectation of $x(t+1)$ by $\mathbb{E}x(t+1)$. Then one can obtain the following expected system:

$$\begin{cases} \mathbb{E}x(t+1) = M_1 u(t) \mathbb{E}x(t), & t \in \mathbb{N} \setminus \Lambda; \\ \mathbb{E}x(t_k) = M_2 \mathbb{E}x(t_k - 1), & k \in \mathbb{Z}_+, \end{cases} \tag{3}$$

where $M_1 = \sum_{i=1}^{l} p_{1,i} L_{1,i} \in \Gamma_{2^n \times 2^{n+m}}$ and $M_2 = \sum_{j=1}^{r} p_{2,j} L_{2,j} \in \Gamma_{2^n \times 2^n}$ are transition matrices.

In the following, we use the above ASSR framework to consider the finite-time controllability of IPBCNs.

In this section, we presuppose $t_1 \geq 2$. Otherwise, when $t_1 = 1$, the following conclusions can be similarly drawn. We split M_1 into 2^m equal blocks as $M_1 = [M_{1,1}\ M_{1,2}\ \cdots\ M_{1,2^m}]$. Given an initial state $x(0) \in \Delta_{2^n}$ and an open-loop control sequence $\{u(t) : t \in \mathbb{N} \setminus \Lambda\} = \{u(0) = \delta_{2^m}^{i_0}, \cdots, u(t_1 - 2) = \delta_{2^m}^{i_{t_1-2}}, u(t_1) = \delta_{2^m}^{i_{t_1-1}}, \cdots\}$, for any given positive integer s, by iteration, one can summarize

$$\mathbb{E}x(s) = \bar{M}_s x(0),$$

where

$$\bar{M}_s = \begin{cases} \prod_{q=t_{k-1}-k+j}^{t_{k-1}-k+1} M_{1,i_q} M_2 \prod_{q=t_{k-1}-k}^{t_{k-2}-k+2} M_{1,i_q} \cdots \\ M_2 \prod_{q=t_2-3}^{t_1-1} M_{1,i_q} M_2 \prod_{q=t_1-2}^{0} M_{1,i_q}, \\ \text{when } s = t_{k-1} + j, \\ 1 \leq j \leq t_k - t_{k-1} - 1; \\ M_2 \prod_{q=t_k-k-1}^{t_{k-1}-k+1} M_{1,i_q} \cdots M_2 \prod_{q=t_2-3}^{t_1-1} M_{1,i_q} \\ M_2 \prod_{q=t_1-2}^{0} M_{1,i_q}, \text{ when } s = t_k, \end{cases} \tag{4}$$

and $k \in \mathbb{Z}_+$.

Lemma 1: Consider system (1). Given $x_0 = \delta_{2^n}^i \in \Delta_{2^n}$, $x_d = \delta_{2^n}^j \in \Delta_{2^n}$ and $\{u(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\}$, it holds that

$$\mathbb{P}\{x(s; x_0, u) = x_d\} = (\bar{M}_s)_{j,i}. \tag{5}$$

For any $k \in \mathbb{Z}_+$, set

$$\bar{Q}_s = \begin{cases} \bigvee_{i_{s-k}, \cdots, i_0 = 1}^{2^m} \bar{M}_s, & \text{when } s = t_{k-1} + j, \\ 1 \leq j \leq t_k - t_{k-1} - 1; \\ \bigvee_{i_{s-k-1}, \cdots, i_0 = 1}^{2^m} \bar{M}_s, & \text{when } s = t_k. \end{cases}$$

It is evident that for any given open-loop control sequence $\{u(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\}$, we have $\bar{Q}_s \geq \bar{M}_s$. From the above construction, all the controllability information is contained in $\bar{Q}_s$. Since we are concerned about only the reachability with probability one, using the floor function, we obtain $\lfloor \bar{Q}_s \rfloor \in \mathcal{B}_{2^n \times 2^n}$, which is called the s-step reachability matrix with probability one.

Based on Definition 1, Lemma 1 and the construction of $\lfloor \bar{Q}_s \rfloor$, we have the following result.

Theorem 1: Consider system (1). Given $x_0 = \delta_{2^n}^i$ and $x_d = \delta_{2^n}^j$.

(i) x_d is reachable with probability one from x_0 at the s-th step, if and only if $(\lfloor \bar{Q}_s \rfloor)_{j,i} = 1$.

(ii) x_d is reachable with probability one from x_0, if and only if there exists a positive integer s such that $(\lfloor \bar{Q}_s \rfloor)_{j,i} = 1$.

(iii) System (1) is finite-time controllable with probability one at x_0, if and only if there exists a positive integer s such that

$$\bigvee_{\tau=1}^{s} Col_i(\lfloor \bar{Q}_\tau \rfloor) = \mathbf{1}_{2^n}. \tag{6}$$

(iv) System (1) is finite-time controllable with probability one, if and only if there exists a positive integer s such that

$$\bigvee_{\tau=1}^{s} \lfloor \bar{Q}_\tau \rfloor = \mathbf{1}_{2^n \times 2^n}. \tag{7}$$

Proof: We firstly prove conclusion (i). Conclusion (ii) can be directly obtained from conclusion (i).

From Definition 1, x_d is reachable with probability one from x_0 at the s-th step, if and only if there exists an open-loop control sequence $\{u(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\} \subseteq \Delta_{2^m}$ such that

$$\mathbb{P}\{x(s; x_0, u) = x_d\} = 1,$$

which together with Lemma 1 implies that

$$1 = \mathbb{P}\{x(s; x_0, u) = x_d\} = (\bar{M}_s)_{j,i} \leq (\bar{Q}_s)_{j,i}. \tag{8}$$

Thus, $(\bar{Q}_s)_{j,i} = 1$, that is, $(\lfloor \bar{Q}_s \rfloor)_{j,i} = 1$.

Next, we prove conclusion (iii). Conclusion (iv) can be easily obtained from Definition 1 and conclusion (iii).

According to Definition 1 and conclusion (ii), the sufficiency part of conclusion (iii) is straightforward. We only need to prove the necessity part. Assume that system (1) is finite-time controllable with probability one at $x_0 = \delta_{2^n}^i$. From conclusion (ii), for any $x_d = \delta_{2^n}^j$, one can find a positive integer s_j such that $(\lfloor \bar{Q}_{s_j} \rfloor)_{j,i} = 1$. Hence, there exists a positive integer $s = \max_{j \in \{1, \cdots, 2^n\}} s_j$ such that (6) holds. This completes the proof. □

Remark 1: Although system (1) has 2^n different states, the upper bound of s satisfying (6) may be greater than 2^n because of the influence of impulse and randomness.

B. FINITE-TIME SET CONTROLLABILITY OF IPBCNS

In this section, we investigate the finite-time set controllability with probability one of system (1).

Given a nonempty set $W \subseteq \Delta_{2^n}$, denote the transition probability from $x_0 \in \Delta_{2^n}$ to W in s steps by $\mathbb{P}\{x(s; x_0, u) \in W\}$. Before presenting the main result of this part, we firstly give the concept of finite-time set controllability with probability one.

Definition 2: Consider system (1). Given a nonempty initial set $\mathcal{A}_0 \subseteq \Delta_{2^n}$ and a nonempty target set $\mathcal{A}_d \subseteq \Delta_{2^n}$. System (1) is said to be finite-time set controllable from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one, if for any initial state $x_0 \in \mathcal{A}_0$, there exist a positive integer s and a control sequence $\{u(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\} \subseteq \Delta_{2^m}$ such that $\mathbb{P}\{x(s; x_0, u) \in \mathcal{A}_d\} = 1$.

Given the nonempty initial set

$$\mathcal{A}_0 = \{\delta_{2^n}^{\alpha_1}, \cdots, \delta_{2^n}^{\alpha_\phi}\} := \{\delta_{2^n}^{i} : i \in \Omega_0\} \tag{9}$$

and the nonempty target set

$$\mathcal{A}_d = \{\delta_{2^n}^{\beta_1}, \cdots, \delta_{2^n}^{\beta_\varphi}\} := \{\delta_{2^n}^{j} : j \in \Omega_d\}, \tag{10}$$

where $\phi = |\mathcal{A}_0|$, $\varphi = |\mathcal{A}_d|$, $\Omega_0 = \{\alpha_1, \cdots, \alpha_\phi\}$, $1 \le \alpha_1 < \cdots < \alpha_\phi \le 2^n$ and $\Omega_d = \{\beta_1, \cdots, \beta_\varphi\}$, $1 \le \beta_1 < \cdots < \beta_\varphi \le 2^n$.

In what follows, we study how to verify the finite-time set controllability from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one.

For the z-th control sequence $\{u^z(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\}$, based on (4), the set controllability probability distribution vector is defined as follows:

$$\bar{H}_s^z = \sum_{j \in \Omega_d} Row_j(\bar{M}_s^z), \tag{11}$$

where $\bar{M}_s^z$ is the probabilistic transition matrix under the control sequence $\{u^z(t)\}$.

From Lemma 1, given $x_0 = \delta_{2^n}^i \in \mathcal{A}_0$, one has

$$\begin{aligned}
&\mathbb{P}\{x(s; x_0, u) \in \mathcal{A}_d\} \\
&\quad = \sum_{j \in \Omega_d} \mathbb{P}\{x(s; x_0, u) = \delta_{2^n}^j\} \\
&\quad = \sum_{j \in \Omega_d} (\bar{M}_s^z)_{j,i} = (\bar{H}_s^z)_i,
\end{aligned} \tag{12}$$

where $(\bar{H}_s^z)_i$ represents the i-th entry of $\bar{H}_s^z$. Notice that $(\bar{H}_s^z)_i$ represents the probability from any initial state $\delta_{2^n}^i \in \mathcal{A}_0$ to the set $\mathcal{A}_d$. Thus, the set controllability information is contained in $\bar{H}_s^z$. We define the s-th step set controllability index as follows:

$$h_s = \begin{cases} \bigvee_{z=1}^{2^{m(s-k+1)}} h_s^z, & \text{when } s = t_{k-1} + j, \\ 1 \le j \le t_k - t_{k-1} - 1; & \\ \bigvee_{z=1}^{2^{m(s-k)}} h_s^z, & \text{when } s = t_k, \end{cases}$$

where $h_s^z = \prod_{i \in \Omega_0} (\lfloor \bar{H}_s^z \rfloor)_i$ and $k \in \mathbb{Z}_+$.

Based on Definition 2 and the construction of h_s, we can obtain the following result.

Theorem 2: Let $\mathcal{A}_0$ and $\mathcal{A}_d$ be given in (9) and (10), respectively. System (1) is finite-time set controllable from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one, if and only if there exists a positive integer s such that $h_s = 1$.

Proof: (Necessity) Assume that system (1) is finite-time set controllable from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one. From Definition 2 and (12), for any initial state $x_0 = \delta_{2^n}^i \in \mathcal{A}_0$, one can find a positive integer s and a control sequence $\{u^z(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\}$ such that

$$1 = \mathbb{P}\{x(s; x_0, u^z) \in \mathcal{A}_d\} = (\bar{H}_s^z)_i. \tag{13}$$

From the construction of h_s^z, we have $h_s^z = 1$, that is, $h_s = 1$.

(Sufficiency) Assume that $h_s = 1$ holds for some positive integer s. From the construction of h_s, there exists a control sequence $\{u^z(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\}$ such that $h_s^z = 1$. For any $x_0 = \delta_{2^n}^i \in \mathcal{A}_0$, from the construction of h_s^z, we have $(\lfloor \bar{H}_s^z \rfloor)_i = 1$, which together with (12) shows that

$$1 = (\bar{H}_s^z)_i = \mathbb{P}\{x(s; x_0 = \delta_{2^n}^i, u^z) \in \mathcal{A}_d\}.$$

Hence, from Definition 2, system (1) is finite-time set controllable from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one. □

Remark 2: One can use the finite-time set controllability with probability one to study the finite-time set stabilization, output tracking and synchronization of IPBCNs.

Remark 3: Especially, when $\phi = \varphi = 1$, the finite-time set controllability with probability one from $\mathcal{A}_0$ to $\mathcal{A}_d$ degenerates to the Part A: Finite-time controllability of IPBCNs.

C. SWITCHING TOPOLOGY CASE

In this part, as an extension, we consider the finite-time set controllability of IPBCN (1) with switching topology:

$$\begin{cases} x(t+1) = L_1^{\sigma(t)} u(t) x(t), & t \in \mathbb{N} \setminus \Lambda; \\ x(t_k) = L_2 x(t_k - 1), & k \in \mathbb{Z}_+, \end{cases} \tag{14}$$

where $\sigma : \mathbb{N} \to \{1, \cdots, v\}$ is the switching signal, $L_1^z \in \{L_{1,1}^z, \cdots, L_{1,l}^z\}$ with probability $\mathbb{P}\{L_1^z = L_{1,i}^z\} = p_{1,i}$, $i = 1, \cdots, l$, $z = 1, \cdots, v$, and $L_2 \in \{L_{2,1}, \cdots, L_{2,r}\}$ with probability $\mathbb{P}\{L_2 = L_{2,j}\} = p_{2,j}$, $j = 1, \cdots, r$.

Set $\sigma(t) = i \sim \delta_v^i$ and denote the switching control by $\bar{u}(t) = \sigma(t) \ltimes u(t) \in \Delta_{v2^m}$. We give the following concept of finite-time controllability and set controllability for system (14).

Definition 3: System (14) is said to be finite-time controllable with probability one, if for any $x_0, x_d \in \Delta_{2^n}$, there exist a positive integer s and a switching control sequence $\{\bar{u}(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\} \subseteq \Delta_{v2^m}$ such that $\mathbb{P}\{x(s; x_0, \bar{u}) = x_d\} = 1$.

Definition 4: Let $\mathcal{A}_0$ and $\mathcal{A}_d$ be given in (9) and (10), respectively. System (14) is said to be finite-time set controllable from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one, if for any $x_0 \in \mathcal{A}_0$, there exist a positive integer s and a switching control sequence $\{\bar{u}(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\} \subseteq \Delta_{v2^m}$ such that $\mathbb{P}\{x(s; x_0, \bar{u}) \in \mathcal{A}_d\} = 1$.

Based on the construction of $\bar{u}(t)$, one can get the following expected system:

$$\begin{cases} \mathbb{E}x(t+1) = F_1\bar{u}(t)\mathbb{E}x(t), & t \in \mathbb{N} \setminus \Lambda; \\ \mathbb{E}x(t_k) = F_2\mathbb{E}x(t_k - 1), & k \in \mathbb{Z}_+, \end{cases} \tag{15}$$

where $F_1 = [F_1^1 \ \cdots \ F_1^v]$, $F_1^z = \sum_{i=1}^{l} p_{1,i}L_{1,i}^z \in \Gamma_{2^n \times 2^{n+m}}$, $z = 1, \cdots, v$, and $F_2 = \sum_{j=1}^{r} p_{2,j}L_{2,j} \in \Gamma_{2^n \times 2^n}$.

We split each F_1^z into 2^m equal blocks as $F_1^z = [F_{1,1}^z \ F_{1,2}^z \ \cdots \ F_{1,2^m}^z]$, $z = 1, \cdots, v$. Then, F_1 is divided into $v2^m$ equal blocks. For any positive integer s, given $x(0) \in \Delta_{2^n}$ and $\{\bar{u}(0) = \delta_v^{z_0} \ltimes \delta_{2^m}^{i_0}, \cdots, \bar{u}(t_1 - 2) = \delta_v^{z_{t_1-2}} \ltimes \delta_{2^m}^{i_{t_1-2}}, \bar{u}(t_1) = \delta_v^{z_{t_1-1}} \ltimes \delta_{2^m}^{i_{t_1-1}}, \cdots\} \subseteq \Delta_{v2^m}$, through a similar procedure to (4), we have

$$\mathbb{E}x(s) = \bar{F}_s x(0),$$

where

$$\bar{F}_s = \begin{cases} \prod_{q=t_{k-1}-k+j}^{t_{k-1}-k+1} F_{1,i_q}^{z_q} F_2 \prod_{q=t_{k-1}-k}^{t_{k-2}-k+2} F_{1,i_q}^{z_q} \cdots \\ F_2 \prod_{q=t_2-3}^{t_1-1} F_{1,i_q}^{z_q} F_2 \prod_{q=t_1-2}^{0} F_{1,i_q}^{z_q}, \\ \text{when } s = t_{k-1} + j, \\ 1 \le j \le t_k - t_{k-1} - 1; \\ F_2 \prod_{q=t_k-k-1}^{t_{k-1}-k+1} F_{1,i_q}^{z_q} \cdots F_2 \prod_{q=t_2-3}^{t_1-1} F_{1,i_q}^{z_q} \\ F_2 \prod_{q=t_1-2}^{0} F_{1,i_q}^{z_q}, \ \text{when } s = t_k, \end{cases} \tag{16}$$

and $k \in \mathbb{Z}_+$.

For any $k \in \mathbb{Z}_+$, set

$$\hat{Q}_s = \begin{cases} \bigvee_{i_{s-k}, \cdots, i_0 = 1}^{2^m} \bar{F}_s, & \text{when } s = t_{k-1} + j, \\ 1 \le j \le t_k - t_{k-1} - 1; \\ \bigvee_{i_{s-k-1}, \cdots, i_0 = 1}^{2^m} \bar{F}_s, & \text{when } s = t_k. \end{cases}$$

Based on Definition 3, Theorem 1 and the construction of $\hat{Q}_s$, we have the following result.

Corollary 1: System (14) is finite-time controllable with probability one, if and only if there exists a positive integer s such that

$$\bigvee_{\tau=1}^{s} \lfloor \hat{Q}_\tau \rfloor = \mathbf{1}_{2^n \times 2^n}. \tag{17}$$

Similarly, given a switching control sequence $\{\bar{u}^z(t) : t \in \{0, \cdots, s-1\} \setminus \Lambda\}$, based on (16), we define the switching set controllability probability distribution vector as follows:

$$\hat{H}_s^z = \sum_{j \in \Omega_d} Row_j(\bar{F}_s^z). \tag{18}$$

Based on which, the s-th step switching set controllability index is defined as follows:

$$\hat{h}_s = \begin{cases} \bigvee_{z=1}^{2^{m(s-k+1)}} \hat{h}_s^z, & \text{when } s = t_{k-1} + j, \\ 1 \le j \le t_k - t_{k-1} - 1; \\ \bigvee_{z=1}^{2^{m(s-k)}} \hat{h}_s^z, & \text{when } s = t_k, \end{cases}$$

where $\hat{h}_s^z = \prod_{i \in \Omega_0} (\lfloor \hat{H}_s^z \rfloor)_i$ and $k \in \mathbb{Z}_+$.

According to Definition 4, Theorem 2 and the construction of $\hat{h}_s$, we have the following conclusion.

Corollary 2: Let $\mathcal{A}_0$ and $\mathcal{A}_d$ be given in (9) and (10), respectively. System (14) is finite-time set controllable from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one, if and only if there exists a positive integer s such that $\hat{h}_s = 1$.

III. ILLUSTRATIVE EXAMPLES

In this section, we give two examples to illustrate the effectiveness of the obtained results.

Example 1: Consider the following IPBCN:

$$\begin{cases} x(t+1) = L_1u(t)x(t), & t \in \mathbb{N} \setminus \Lambda; \\ x(t_k) = L_2x(t_k - 1), & k \in \mathbb{Z}_+, \end{cases} \tag{19}$$

where $t_0 := 0$, the impulsive time sequence $t_k = 2k + 1$, $k \in \mathbb{Z}_+$. $L_1 \in \{L_{1,1}, L_{1,2}\}$, $L_{1,1} = \delta_4[1 \ 2 \ 2 \ 4 \ 3 \ 1 \ 4 \ 3]$, $L_{1,2} = \delta_4[1 \ 3 \ 2 \ 4 \ 2 \ 1 \ 4 \ 3]$. $L_2 \in \{L_{2,1}, L_{2,2}\}$, $L_{2,1} = \delta_4[4 \ 1 \ 2 \ 2]$, $L_{2,2} = \delta_4[4 \ 3 \ 2 \ 1]$. $\mathbb{P}\{L_1 = L_{1,1}\} = p_{1,1} = 0.4$, $\mathbb{P}\{L_1 = L_{1,2}\} = p_{1,2} = 0.6$, $\mathbb{P}\{L_2 = L_{2,1}\} = p_{2,1} = 0.2$, and $\mathbb{P}\big\{L_2 = L_{2,2}\big\} = p_{2,2} = 0.8$.

It is easy to obtain $M_1 = \sum_{i=1}^{2} p_{1,i}L_{1,i}$, $M_2 = \sum_{j=1}^{2} p_{2,j}L_{2,j}$, that is,

$$M_1 = [\delta_4^1, \ 0.4\delta_4^2 + 0.6\delta_4^3, \ \delta_4^2, \ \delta_4^4, \ 0.6\delta_4^2 + 0.4\delta_4^3, \ \delta_4^1, \ \delta_4^4, \ \delta_4^3],$$
$$M_2 = [\delta_4^4, \ 0.2\delta_4^1 + 0.8\delta_4^3, \ \delta_4^2, \ 0.8\delta_4^1 + 0.2\delta_4^2].$$

Split M_1 into 2 equal blocks as $M_1 = [M_{1,1} \ M_{1,2}]$, where

$$M_{1,1} = [\delta_4^1, \ 0.4\delta_4^2 + 0.6\delta_4^3, \ \delta_4^2, \ \delta_4^4],$$
$$M_{1,2} = [0.6\delta_4^2 + 0.4\delta_4^3, \ \delta_4^1, \ \delta_4^4, \ \delta_4^3].$$

Let $x(0) = \delta_4^4$, after a straightforward calculation, one can obtain

$$\lfloor \hat{Q}_1 \rfloor = \lfloor M_{1,1} \vee M_{1,2} \rfloor = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

$$\lfloor \hat{Q}_2 \rfloor = \lfloor \bigvee_{i_1, i_0 = 1}^{2} M_{1,i_1} M_{1,i_0} \rfloor = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Notice that $\big(\lfloor \hat{Q}_2 \rfloor\big)_{2,4} = \big(\lfloor \hat{Q}_2 \rfloor\big)_{3,4} = \big(\lfloor \hat{Q}_2 \rfloor\big)_{4,4} = 1$. From Theorem 1 (i), $x(s) \in \big\{\delta_4^2, \delta_4^3, \delta_4^4\big\}$ is finite-time controllable with probability one from $x(0) = \delta_4^4$ at time $s = 2$.

Now, we consider the finite-time controllability with probability one of system (19).

Continuing the above calculation procedure, we have

$$\lfloor \bar{Q}_3 \rfloor = \Big\lfloor \bigvee_{i_1,i_0=1}^{2} M_2 M_{1,i_1} M_{1,i_0} \Big\rfloor = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix},$$

$$\lfloor \bar{Q}_4 \rfloor = \Big\lfloor \bigvee_{i_2,i_1,i_0=1}^{2} M_{1,i_2} M_2 M_{1,i_1} M_{1,i_0} \Big\rfloor = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix},$$

$$\lfloor \bar{Q}_5 \rfloor = \Big\lfloor \bigvee_{i_3,i_2,i_1,i_0=1}^{2} M_{1,i_3} M_{1,i_2} M_2 M_{1,i_1} M_{1,i_0} \Big\rfloor = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

Hence, $\bigvee_{\tau=1}^{s} \lfloor \bar{Q}_\tau \rfloor = \mathbf{1}_{4\times 4}$ holds for $s = 5$. From Theorem 1, system (19) is finite-time controllable with probability one.

Finally, given the initial set $\mathcal{A}_0 = \{\delta_4^3, \delta_4^4\}$ and the target set $\mathcal{A}_d = \{\delta_4^1, \delta_4^2\}$, we discuss the finite-time set controllability with probability one.

For the control sequence $\{u^z(0) = \delta_2^2, u^z(1) = \delta_2^2\}$, after a simple calculation, we have

$$\bar{M}_3^z = M_2 (M_{1,2})^2 = \begin{bmatrix} 0.32 & 0.12 & 0 & 0.8 \\ 0.08 & 0.4 & 1 & 0.2 \\ 0 & 0.48 & 0 & 0 \\ 0.6 & 0 & 0 & 0 \end{bmatrix},$$

which implies that $\sum_{j\in\{1,2\}} Row_j(\bar{M}_3^z) = [0.4\ 0.52\ 1\ 1]$. From the construction of h_s, it holds that $h_3 = 1$. Hence, from Theorem 2, system (19) is finite-time set controllable with probability one from $\mathcal{A}_0$ to $\mathcal{A}_d$.

Remark 4: In order to make some comparisons, we consider the finite-time controllability of system (19) without impulsive effects. In this case, $\lfloor \bar{Q}_1 \rfloor$ and $\lfloor \bar{Q}_2 \rfloor$ remain unchanged, but $\lfloor \bar{Q}_i \rfloor$, $i = 3, 4, 5$ are changed to

$$\lfloor \bar{Q}_3 \rfloor = \lfloor \bar{Q}_4 \rfloor = \lfloor \bar{Q}_5 \rfloor = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Therefore, $\bigvee_{\tau=1}^{5} \lfloor \bar{Q}_\tau \rfloor \neq \mathbf{1}_{4\times 4}$. Hence, the finite-time controllability of system (19) is affected by impulsive effects, which shows that impulsive effects play an important role in the finite-time controllability analysis of PBCNs.

Example 2: Consider the following IPBCN with switching topology:

$$\begin{cases} x(t+1) = L_1^{\sigma(t)} u(t) x(t), & t \in \mathbb{N} \setminus \Lambda; \\ x(t_k) = L_2 x(t_k - 1), & k \in \mathbb{Z}_+, \end{cases} \tag{20}$$

where $t_0 := 0$, the impulsive time sequence $t_k = k^2 + 1$, $k \in \mathbb{Z}_+$. $L_1^1 \in \{L_{1,1}^1, L_{1,2}^1\}$, $L_{1,1}^1 = \delta_8[1\ 3\ 5\ 3\ 4\ 4\ 4\ 8\ 2\ 5\ 5\ 3\ 5\ 6\ 6\ 7]$, $L_{1,2}^1 = \delta_8[1\ 7\ 5\ 3\ 3\ 4\ 4\ 8\ 2\ 4\ 5\ 3\ 3\ 3\ 6\ 8]$; $L_1^2 \in \{L_{1,1}^2, L_{1,2}^2\}$, $L_{1,1}^2 = \delta_8[3\ 3\ 6\ 3\ 2\ 1\ 3\ 5\ 1\ 4\ 6\ 7\ 7\ 5\ 3\ 1]$, $L_{1,2}^2 = \delta_8[3\ 1\ 6\ 8\ 2\ 1\ 4\ 5\ 1\ 1\ 6\ 1\ 7\ 8\ 3\ 1]$; $L_2 \in \{L_{2,1}, L_{2,2}, L_{2,3}\}$, $L_{2,1} = \delta_8[4\ 3\ 3\ 4\ 2\ 5\ 8\ 1]$, $L_{2,2} = \delta_8[4\ 3\ 8\ 1\ 3\ 2\ 5\ 1]$, $L_{2,3} = \delta_8[4\ 3\ 6\ 6\ 3\ 7\ 5\ 8]$. $\mathbb{P}\{L_1^i = L_{1,1}^i\} = p_{1,1} = 0.4$, $\mathbb{P}\{L_1^z = L_{1,2}^z\} = p_{1,2} = 0.6$, $i = 1, 2$. $\mathbb{P}\{L_2 = L_{2,1}\} = p_{2,1} = 0.1$, $\mathbb{P}\{L_2 = L_{2,2}\} = p_{2,2} = 0.2$, and $\mathbb{P}\{L_2 = L_{2,3}\} = p_{2,3} = 0.7$.

Now we investigate the finite-time set controllability from $\mathcal{A}_0$ to $\mathcal{A}_d$ with probability one, where $\mathcal{A}_0 = \{\delta_8^1, \delta_8^2, \delta_8^5\}$ and $\mathcal{A}_d = \{\delta_8^3, \delta_8^4, \delta_8^6, \delta_8^8\}$.

Consider the control $\bar{u}^z(0) = \delta_4^3$. According to $F_1^2 = \sum_{i=1}^{2} p_{1,i} L_{1,i}^2$, $F_2 = \sum_{j=1}^{3} p_{2,j} L_{2,j}$ and $F_1^2 = [F_{1,1}^2\ F_{1,2}^2]$, one can obtain that

$$F_{1,1}^2 = \begin{bmatrix} 0 & 0.6 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0.4 & 0 & 0.4 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 & & 0 & 0 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 0.1 & 0.2 & 0 & 0 \\ 0 & 1 & 0.1 & 0 & 0.9 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0.9 & 0 \\ 0 & 0 & 0.7 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 & 0.1 & 0.7 \end{bmatrix}.$$

Based on (16), we have $\hat{H}_2^z = \sum_{j\in\{3,4,6,8\}} Row_j(\bar{F}_2^z) = [1\ 1\ 0\ 0.82\ 1\ 1\ 0.88\ 0.9]$. Obviously, $\hat{h}_s = \prod_{i\in\{1,2,5\}} (\lfloor \hat{H}_2^z \rfloor)_i = 1$. Hence, by Corollary 2, system (20) is finite-time set controllable with probability one from $\mathcal{A}_0$ to $\mathcal{A}_d$.

IV. CONCLUSION

In this paper, we have investigated the finite-time controllability and set controllability with probability one of IPBCNs. We have constructed the finite step reachability matrix with probability one, and proposed several criteria for the finite-time controllability with probability one of IPBCNs. By constructing the set controllability probability distribution vector, we have obtained a criterion for finite-time set controllability with probability one of IPBCNs. Moreover, as a generalization, we have studied the finite-time controllability and set controllability of IPBCNs with switching topology. The study of two examples has shown that impulsive effects may prohibit the finite-time controllability with probability one of IPBCNs.

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REFERENCES

[1] A. E. Bashirov and N. I. Mahmudov, "On concepts of controllability for deterministic and stochastic systems," *SIAM J. Control Optim.*, vol. 37, no. 6, pp. 1808–1821, Jan. 1999.

[2] H. Chen, X. Li, and J. Sun, "Stabilization, controllability and optimal control of Boolean networks with impulsive effects and state constraints," *IEEE Trans. Autom. Control*, vol. 60, no. 3, pp. 806–811, Mar. 2015.

[3] H. Chen and J. Liang, "Local synchronization of interconnected Boolean networks with stochastic disturbances," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 2, pp. 452–463, Feb. 2020.

[4] H. Chen, Z. Wang, J. Liang, and M. Li, "State estimation for stochastic time-varying Boolean networks," *IEEE Trans. Autom. Control*, early access, Feb. 2020, doi: 10.1109/TAC.2020.2973817.

[5] H. Chen, B. Wu, and J. Lu, "A minimum-time control for Boolean control networks with impulsive disturbances," *Appl. Math. Comput.*, vol. 273, pp. 477–483, Jan. 2016.

[6] D. Cheng and X. Hu, "Set stability and controllability for switched non-homogeneous linear systems," in *Proc. 25th Chin. Control Conf.*, Aug. 2006, pp. 7–11.

[7] D. Cheng, C. Li, and F. He, "Observability of Boolean networks via set controllability approach," *Syst. Control Lett.*, vol. 115, pp. 22–25, May 2018.

[8] D. Cheng, H. Qi, and Z. Li, *Analysis and Control of Boolean Networks: A Semi-tensor Product Approach*, London, U.K.: Springer, 2011.

[9] X. Ding, H. Li, X. Li, and W. Sun, "Stability analysis of Boolean networks with stochastic function perturbations," *IEEE Access*, vol. 7, pp. 1323–1329, 2019.

[10] X. Ding, H. Li, and S. Wang, "Set stability and synchronization of logical networks with probabilistic time delays," *J. Franklin Inst.*, vol. 355, no. 15, pp. 7735–7748, Oct. 2018.

[11] X. Ding, H. Li, Q. Yang, Y. Zhou, A. Alsaedi, and F. E. Alsaadi, "Stochastic stability and stabilization of n -person random evolutionary Boolean games," *Appl. Math. Comput.*, vol. 306, pp. 1–12, Aug. 2017.

[12] M. Ehrhardt and W. Kliemann, "Controllability of linear stochastic systems," *Syst. Control Lett.*, vol. 2, no. 3, pp. 145–153, Oct. 1982.

[13] E. Fornasini and M. E. Valcher, "Observability, reconstructibility and state observers of Boolean control networks," *IEEE Trans. Autom. Control*, vol. 58, no. 6, pp. 1390–1401, Jun. 2013.

[14] Y. Guo, R. Zhou, Y. Wu, W. Gui, and C. Yang, "Stability and set stability in distribution of probabilistic Boolean networks," *IEEE Trans. Autom. Control*, vol. 64, no. 2, pp. 736–742, Feb. 2019.

[15] X. Hu, C. Huang, J. Lu, and J. Cao, "Stabilization of Boolean control networks with stochastic impulses," *J. Franklin Inst.*, vol. 356, no. 13, pp. 7164–7182, Sep. 2019.

[16] R. Kalman, Y. Ho, and K. Narendra, "Controllability of linear dynamical systems," *Contributions Differ. Equ.*, vol. 1, no. 2, pp. 189–213, 1963.

[17] S. Karthikeyan and K. Balachandran, "Controllability of nonlinear stochastic neutral impulsive systems," *Nonlinear Anal. Hybrid Syst.*, vol. 3, no. 3, pp. 266–276, Aug. 2009.

[18] D. Laschov and M. Margaliot, "Minimum-time control of Boolean networks," *SIAM J. Control Optim.*, vol. 51, no. 4, pp. 2869–2892, Jan. 2013.

[19] B. Li and J. Lu, "Boolean-network-based approach for the construction of filter generators," *Sci. China Inf. Sci.*, to be published, doi: 10.1007/s11432-019-2813-7.

[20] F. Li and J. Sun, "Controllability of probabilistic Boolean control networks," *Automatica*, vol. 47, no. 12, 2011, pp. 2765–2772.

[21] F. Li and J. Sun, "Observability analysis of Boolean control networks with impulsive effects," *IET Control Theory Appl.*, vol. 5, no. 14, pp. 1609–1616, Sep. 2011.

[22] F. Li and J. Sun, "Stability and stabilization of Boolean networks with impulsive effects," *Syst. Control Lett.*, vol. 61, no. 1, pp. 1–5, Jan. 2012.

[23] F. Li and Y. Tang, "Set stabilization for switched Boolean control networks," *Automatica*, vol. 78, no. 4, pp. 223–230, Apr. 2017.

[24] H. Li and X. Ding, "A control Lyapunov function approach to feedback stabilization of logical control networks," *SIAM J. Control Optim.*, vol. 57, no. 2, pp. 810–831, Jan. 2019.

[25] H. Li, X. Xu, and X. Ding, "Finite-time stability analysis of stochastic switched Boolean networks with impulsive effect," *Appl. Math. Comput.*, vol. 347, pp. 557–565, Apr. 2019.

[26] H. Li, Y. Zheng, and F. E. Alsaadi, "Algebraic formulation and topological structure of Boolean networks with state-dependent delay," *J. Comput. Appl. Math.*, vol. 350, pp. 87–97, Apr. 2019.

[27] R. Li, M. Yang, and T. Chu, "State feedback stabilization for probabilistic Boolean networks," *Automatica*, vol. 50, no. 4, pp. 1272–1278, Apr. 2014.

[28] X. Li, J. Shen, and R. Rakkiyappan, "Persistent impulsive effects on stability of functional differential equations with finite or infinite delay," *Appl. Math. Comput.*, vol. 329, pp. 14–22, Jul. 2018.

[29] Y. Li, B. Li, Y. Liu, J. Lu, Z. Wang, and F. E. Alsaadi, "Set stability and stabilization of switched Boolean networks with state-based switching," *IEEE Access*, vol. 6, pp. 35624–35630, 2018.

[30] J. Liang, H. Chen, and Y. Liu, "On algorithms for state feedback stabilization of Boolean control networks," *Automatica*, vol. 84, pp. 10–16, Oct. 2017.

[31] J. Liu, Y. Liu, Y. Guo, and W. Gui, "Sampled-data state-feedback stabilization of probabilistic Boolean control networks: A control Lyapunov function approach," *IEEE Trans. Cybern.*, early access, Aug. 2019, doi: 10.1109/TCYB.2019.2932914.

[32] Y. Liu, H. Chen, J. Lu, and B. Wu, "Controllability of probabilistic Boolean control networks based on transition probability matrices," *Automatica*, vol. 52, pp. 340–345, Feb. 2015.

[33] Y. Liu, H. Chen, and B. Wu, "Controllability of Boolean control networks with impulsive effects and forbidden states," *Math. Methods Appl. Sci.*, vol. 37, no. 1, pp. 1–9, Jan. 2014.

[34] Y. Liu, Y. Zheng, H. Li, F. E. Alsaadi, and B. Ahmad, "Control design for output tracking of delayed Boolean control networks," *J. Comput. Appl. Math.*, vol. 327, pp. 188–195, Jan. 2018.

[35] N. Mahmudov, "On controllability of linear stochastic systems," *Int. J. Control*, vol. 46, no. 5, pp. 1476–1481, 1999.

[36] M. Meng, J. Lam, J.-E. Feng, and K. Chung Cheung, "Stability and stabilization of Boolean networks with stochastic delays," *IEEE Trans. Autom. Control*, vol. 64, no. 2, pp. 790–796, Feb. 2019.

[37] O. B. Oyediran, "δ-controllability of impulsive systems and applications to some physical and biological control system," *Int. J. Differ. Equ. Appl.*, vol. 12, no. 3, 2013, pp. 171–191.

[38] R. Pal, A. Datta, and E. R. Dougherty, "Optimal infinite-horizon control for probabilistic Boolean networks," *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2375–2387, Jun. 2006.

[39] X. Qian and E. R. Dougherty, "Effect of function perturbation on the steady-state distribution of genetic regulatory networks: Optimal structural intervention," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 4966–4976, Oct. 2008.

[40] J. Qiu, K. Sun, C. Yang, X. Chen, X. Chen, and A. Zhang, "Finite-time stability of genetic regulatory networks with impulsive effects," *Neurocomputing*, vol. 219, pp. 9–14, Jan. 2017.

[41] I. Shmulevich, E. R. Dougherty, S. Kim, and W. Zhang, "Probabilistic Boolean networks: A rule-based uncertainty model for gene regulatory networks," *Bioinformatics*, vol. 18, no. 2, pp. 261–274, Feb. 2002.

[42] I. Shmulevich, I. Gluhovsky, R. F. Hashimoto, E. R. Dougherty, and W. Zhang, "Steady-state analysis of genetic regulatory networks modelled by probabilistic Boolean networks," *Comparative Funct. Genomics*, vol. 4, no. 6, pp. 601–608, 2003.

[43] J. Suo and J. Sun, "Asymptotic stability of differential systems with impulsive effects suffered by logic choice," *Automatica*, vol. 51, pp. 302–307, Jan. 2015.

[44] H. Sussmann and V. Jurdjevic, "Controllability of nonlinear systems," *J. Differ. Equ.*, vol. 12, no. 1, 1972, pp. 95–116.

[45] Y. Tang, X. Wu, P. Shi, and F. Qian, "Input-to-state stability for nonlinear systems with stochastic impulses," *Automatica*, vol. 113, Mar. 2020, Art. no. 108766.

[46] Y. Wu, X.-M. Sun, X. Zhao, and T. Shen, "Optimal control of Boolean control networks with average cost: A policy iteration approach," *Automatica*, vol. 100, pp. 378–387, Feb. 2019.

[47] Z. Wu, "Stability criteria of random nonlinear systems and their applications," *IEEE Trans. Autom. Control*, vol. 60, no. 4, pp. 1038–1049, Apr. 2015.

[48] Z. Wu, M. Cui, P. Shi, and H. R. Karimi, "Stability of stochastic nonlinear systems with state-dependent switching," *IEEE Trans. Autom. Control*, vol. 58, no. 8, pp. 1904–1918, Aug. 2013.

[49] X. Xu, H. Li, Y. Li, and F. E. Alsaadi, "Output tracking control of Boolean control networks with impulsive effects," *Math. Methods Appl. Sci.*, vol. 41, no. 4, pp. 1554–1564, Mar. 2018.

[50] X. Xu, Y. Liu, H. Li, and F. E. Alsaadi, "Robust set stabilization of Boolean control networks with impulsive effects," *Nonlinear Anal. Model. Control*, vol. 23, no. 4, pp. 553–567, Aug. 2018.

[51] J. Yang, J. Lu, J. Lou, and Y. Liu, "Synchronization of drive-response Boolean control networks with impulsive disturbances," *Appl. Math. Comput.*, vol. 364, Jan. 2020, Art. no. 124679.

[52] K. Zhang, L. Zhang, and L. Xie, "Invertibility and nonsingularity of Boolean control networks," *Automatica*, vol. 60, pp. 155–164, Oct. 2015.

[53] D. Zhao, Y. Liu, and X. Li, "Controllability for a class of semilinear fractional evolution systems via resolvent operators," *Commun. Pure Appl. Anal.*, vol. 18, no. 1, pp. 455–478, 2019.

[54] X. Zhao and F. Deng, "Operator-type stability theorem for retarded stochastic systems with application," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4203–4209, Dec. 2016.

[55] Y. Zheng, H. Li, and J.-E. Feng, "State-feedback set stabilization of logical control networks with state-dependent delay," *Sci. China Inf. Sci.*, vol. 64, no. 6, Jun. 2021, Art. no. 169203.

[56] J. Zhong, J. Lu, T. Huang, and J. Cao, "Synchronization of master–slave Boolean networks with impulsive effects: Necessary and sufficient criteria," *Neurocomputing*, vol. 143, pp. 269–274, Nov. 2014.

[57] R. Zhou, Y. Guo, and W. Gui, "Set reachability and observability of probabilistic Boolean networks," *Automatica*, vol. 106, pp. 230–241, Aug. 2019.

[58] Y. Zou and J. Zhu, "System decomposition with respect to inputs for Boolean control networks," *Automatica*, vol. 50, no. 4, pp. 1304–1309, Apr. 2014.