Basic agreements for the computation of station potential values as IHRS coordinates, geoid undulations and height anomalies within the Colorado 1 cm geoid experiment

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Preamble

During the business meeting of the JWG 0.1.2[1] held at IAG-IASPEI 2017 (Kobe, Japan), J. Ågren[2] and J. Huang[3] proposed to establish a strong interaction with the JWG 2.2.2 (the 1 cm geoid experiment). Aim of JWG 2.2.2 is the computation and comparison of geoid undulations using the same input data and the own methodologies/software of colleagues involved in the geoid computation. The comparison of the results should highlight the differences caused by disparities in the computation methodologies. In this frame, it was decided to extend the "geoid experiment" to the computation of station potential values as IHRS coordinates. With this proposal, NGS/NOAA agreed to provide terrestrial gravity data, airborne gravity, and digital terrain model for an area of about 500 km x 800 km in Colorado, USA. With these data, the different groups working on the determination of IHRF coordinates should compute potential values for some *virtual* geodetic stations located in that region. Afterwards, the results obtained individually should be compared with the Geoid Slope Validation Survey 2017 (GSVS17). In the same meeting, it was also agreed to standardise as much as possible the data processing to get as similar and compatible results as possible with the different methods. However, the definition of a "standard or unified" processing procedure/strategy is not suitable, because regions with different characteristics apply particular approaches. Therefore, at this first stage, we agreed to outline a set of basic (minimum) requirements to initiate the experiments for the computation of the potential values. The choice of the processing method is up to the gravity field modeller. This document presents a first attempt to identify that set of basic requirements, which should be followed in the 1 cm geoid experiment.

Objective

The goal of this experiment is to assess the repeatability of potential values as IHRS coordinates using different computation approaches. Based on the comparison of the results, a set of standards should be identified to get as similar and compatible results as possible. It is also the aim to compare and evaluate the corresponding quasi-geoid and geoid models.

Basics

- The determination of station potential values W(P) as IHRS coordinates is straightforward if the

[1] GGOS JWG: Strategy for the Realization of the IHRS, chair L. Sánchez.
[2] Chair of IAG SC 2.2: Methodology for geoid and physical height systems.
[3] Chair ICCT JSG 0.15: Regional geoid/quasi-geoid modelling - Theoretical framework for the sub-centimetre accuracy.

disturbing potential T(P) is known: W(P)=U(P)+T(P).

- Since the disturbing potential should be estimated with high-precision, it is proposed to compute (a) the long wavelength component (about d/o $\leq$ 200 … 250) using a satellite-only global gravity model (GGM) and (b) the short wavelength component (d/o > 200 … 250) by the combination of terrestrial (airborne, marine and land) gravity data and detailed terrain models.
- The GGM should be based at least on the combination of SLR (satellite laser ranging), GRACE and GOCE data, due to the improvement offered by these data to the long wavelengths of the Earth's gravity field modelling.
- The potential values realising the IHRS coordinates must be determined at the reference stations; i.e., at the Earth's surface and not at the geoid.
- According to the IHRS definition, the station coordinates have to be given in the mean tide system. In our meeting in Kobe, we agreed to perform the computations in zero-tide system and afterwards, to transfer the coordinates to mean-tide system at the very end, using simplified formulas. This keeps the computations consistent with the gravity/geoid work in zero-tide without introducing an awful amount of new transformations and corrections.
- However, as the gravity data and geometric coordinates provided by NGS/NOAA are in tide-free system, we should use the tide-free system for these first computations. If everything is consistent, this should not influence the comparison of results.
- For these first experiments, we assume the Earth's gravity field to be stationary; i.e., time changes are disregarded so far.

Standards

General constants (numerical values needed for the solution of several equations)

- Constant of gravitation (G) 6.674 28 × 10−11 $m^3kg^{-1}s^{-2}$
- Geocentric gravitational constant (GM) 3.986 004 415 × 10^{14} m^3s^{-2} (including the Mass of the Earth's Atmosphere)
- Nominal mean angular velocity of the Earth (ω) 7.292 115 × 10^{-5} rad s^{-1}
- Conventional reference potential value (W_0) 62 636 853.4 m^2s^{-2}
- Average density of topographic masses (ρ) 2670 kg m^{-3}. This topographic density shall be assumed when computing the geoid height.

Reference ellipsoid (to be used for the computation of gravity anomalies, disturbing potential, ellipsoidal coordinates, geoid heights, height anomalies, etc.):

- GRS80 – please use the parameters published in *Moritz H.: Geodetic Reference System 1980, J Geod 74: 128-133, doi: 10.1007/s001900050278, 2000*. Previous publications contain some typos in the normal gravity formulae. A free available copy of the paper can be found at https://link.springer.com/content/pdf/10.1007%2Fs001900050278.pdf
- Please note that no atmospheric reduction has been applied on the given terrestrial gravity nor on the GRAV-D airborne data. Therefore, it should be taken care of in the computation.
- For the EGM Harmonic Synthesis Software (NGA, http://earth-info.nga.mil/GandG/wgs84/gravitymod/new_egm/new_egm.html), please use the following standard GRS80 parameters:

```
parameter(ae    = 6378137.0d0,
&  gm     = 0.3986005d20,
&  omega  = 7292115.d-11,
&  c20    = 0.0d0,
```

& rf = 298.257222101d0).

and then consider the zero-degree term (see below).

Global Gravity Model (GGM): Although we prefer the use of a satellite-only GGM, we also open the possibility of using a combined GGM (combined means including terrestrial gravity data)

- Satellite-only GGM: GOCO05s, d/o=280 (Mayer-Gürr et al., 2015), available at http://icgem.gfz-potsdam.de/tom_longtime.
- Combined GGM: XGM2016, d/o=719 (Pail et al. 2017), available at http://icgem.gfz-potsdam.de/tom_longtime.
- Experimental global gravity model xGEOID17B (d/o=2190, model provided by NGS/NOAA within this experiment). xGEOID17B uses the same scaling parameters as EGM2008 (a = 6378136.3, GM = 3.986 004 415 × 10^{14} m^3s^{-2}) and it is given in zero-tide system. For the conversion from the zero-tide system to the tide-free system, please use:

$$\bar{C}_{20}^{TF} = \bar{C}_{20}^{ZT} + 3.11080 \cdot 10^{-8} \times 0.3 / \sqrt{5}$$

First-degree terms: The first-degree coefficients (C_1=C_{11}=S_{11}=0) are assumed to be zero to align the Earth's centre of masses with the origin of the geometric coordinate system (ITRS/ITRF). In this way, the disturbing potential T is given by (cf. Eq. 2-170, Heiskanen and Moritz, 1967):

$$T(\theta,\lambda) = T_0 + T_1(\theta,\lambda) + \sum_{n=2}^{\infty} T_n(\theta,\lambda), \text{ with} \qquad [1]$$

$$T_1(\theta,\lambda) = 0 \qquad [2]$$

Zero-degree term: The zero-degree term should be dealt with as follows:

- For the disturbing potential (T): The zero-degree term T_0 has to include the difference between Earth's and reference ellipsoid's GM constant (cf. Eq. 2-172, Heiskanen and Moritz, 1967).

$$\begin{aligned} T_0 &= (GM_{GGM} - GM_{GRS80})/r_P \\ &= (3.986\,004\,415 \times 10^{14} m^3s^{-2} - 3.986\,00 \times 10^{14} m^3s^{-2})/r_P. \end{aligned} \qquad [3]$$

r_p is the geocentric radial distance of the computation point P.

- For the quasi-geoid (ζ) or the geoid (N): In addition to the difference between the two GM values, we also have to consider the difference between the reference potential W_0 value adopted by the IHRS and the potential U_0 on the reference ellipsoid: (cf. the generalized Brun's formula in Eq. 2-178, and also Eq. 2-182, Heiskanen and Moritz, 1967):

$$\zeta_0 = \frac{(GM_{GGM} - GM_{GRS80})}{r_P \cdot \gamma_Q} - \frac{\Delta W_0}{\gamma_Q} \qquad [4a]$$

$$N_0 = \frac{(GM_{GGM} - GM_{GRS80})}{r_{P_0} \cdot \gamma_{Q_0}} - \frac{\Delta W_0}{\gamma_{Q_0}} \qquad [4b]$$

with

$$\Delta W_0 = W_0 - U_0 = 62\,636\,853.4\ m^2s^{-2} - 62\,636\,860.850\ m^2s^{-2} = -7.45\ m^2s^{-2} \qquad [5]$$

Please see figure 1 for the positions of P, Q, P_0 and Q_0.

As it is specified above that the geoid/quasi-geoid should be consistent with the IHRS reference level W_0 and that GRS80 is to be used as normal gravity field/ellipsoid, we conclude:

1) To compute the quasi-geoid, you may first compute ζ starting with n=2 and then add Eq. [4a].
2) To compute the geoid, you may first compute N starting with n=2 and then add Eq. [4b].

<u>Potential values W(P) as IHRS/IHRF coordinates</u>

To determine the potential value W(P) at the stations located on the Earth's surface, consistency with the approach used for the estimation of the disturbing potential should be ensured. If you start from the quasi-geoid computation, the disturbing potential is determined at the point P on the Earth's surface (see figure 1) and the estimation of W(P) is straightforward:

$$W(P) = U(P) + T(P) = U(P) + \left(T_0 + \sum_{n=2}^{\infty} T_n(P) \right), \text{ or} \qquad [6]$$

$$W(P) = U(P) + \gamma \cdot \zeta(P) + \Delta W_0 \,. \qquad [7]$$

When you compute the geoid, the disturbing potential is determined at the point P_0 on the geoid (inside the Earth's topographic masses, see figure 1) and an upward continuation would be necessary to estimate W(P) on the Earth's surface. This upward continuation must be consistent with the hypotheses applied to reduce the gravity values from the Earth's surface to the geoid. We recommend therefore, to start from the quasi-geoid or disturbing potential at surface and then to infer the potential values W(P) using [6] or [7]. If you prefer to compute the geoid first, we recommend to transform N to ζ and then to infer the potential values W(P) with [6] or [7]. The transformation from N to ζ must also be consistent with the hypotheses applied for the geoid computation.

<u>Expected results</u>

Each group should provide three excel or text files containing:

1) Quasi-geoid model grid at a resolution of 1'x1' with latitude, longitude, and height anomaly (one grid-node per row).
2) Geoid model grid at a resolution of 1'x1' with latitude, longitude, and N (one grid-node per row). Remember that average density of the topographic masses (ρ = 2670 kg m^{-3}) shall be assumed.
3) Values at the GSVS17 test points with ID, geoid height (m), height anomaly (m), potential value W(P) (m^2/s^2).
4) Optionally at the GSVS17 test points: gravity values and deflections of the vertical (at the Earth's surface).

Please provide the geoid and quasi-geoid models for the target area with latitude 36°N to 39°N and longitude 251° to 257°.

Please provide a brief description of the computation method and standards/constants you used (around 2-3 pages, including a few references).

Please let us know if you agree to contribute with a publication about this work to a special issue in a geodetic journal.

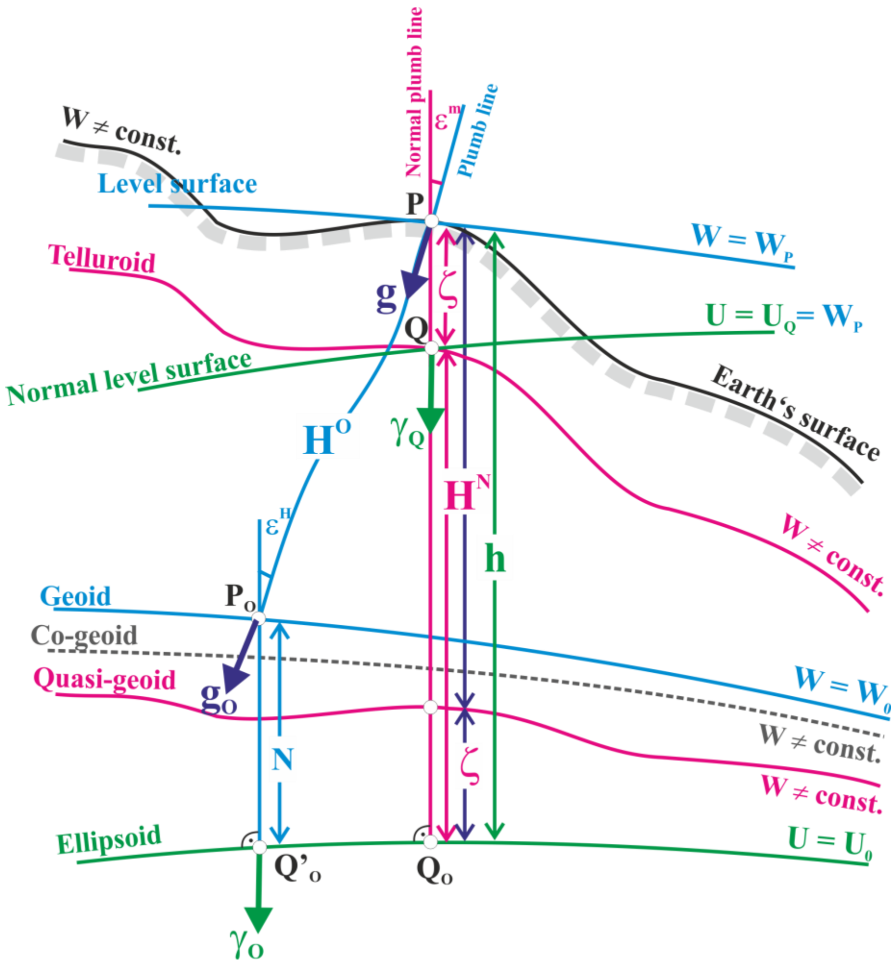

Fig. 1: Heights and reference surfaces.