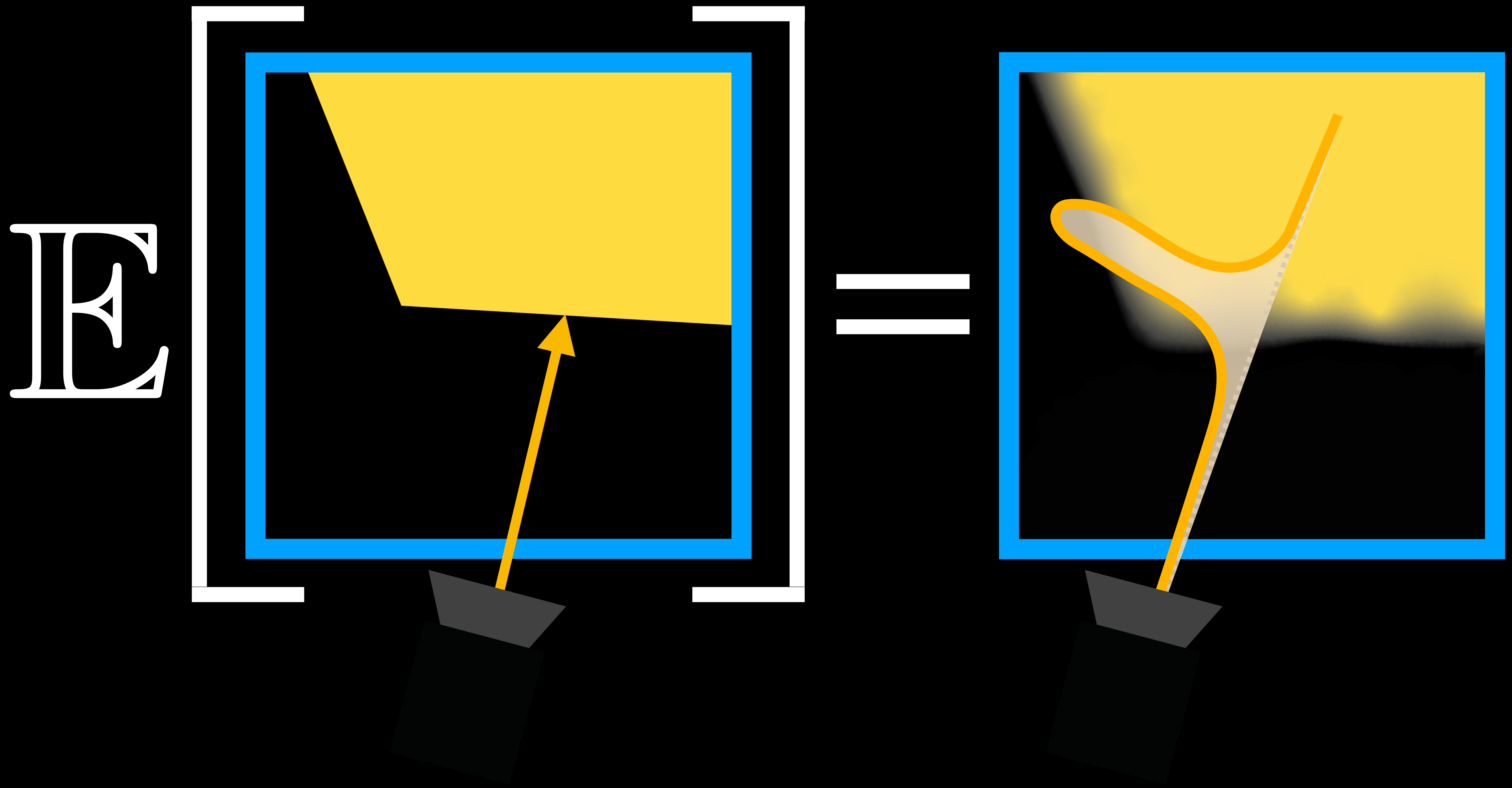
Objects as volumes

A stochastic geometry view on opaque solids



Bailey Miller, Hanyu Chen, Alice Lai, Ioannis Gkioulekas

Bailey Miller
Carnegie Mellon University

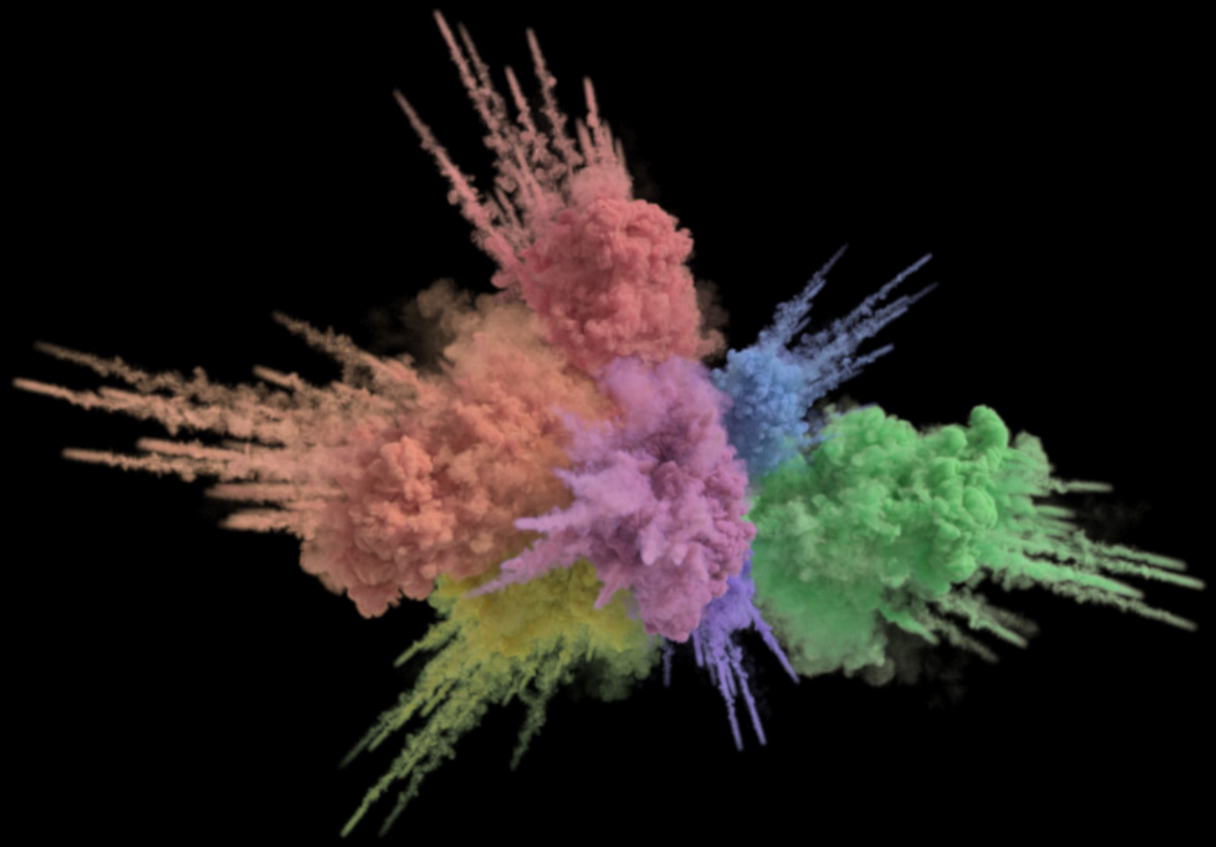
Hanyu Chen
Carnegie Mellon University
(incoming PhD at Cornell!)

Alice Lai
Carnegie Mellon University

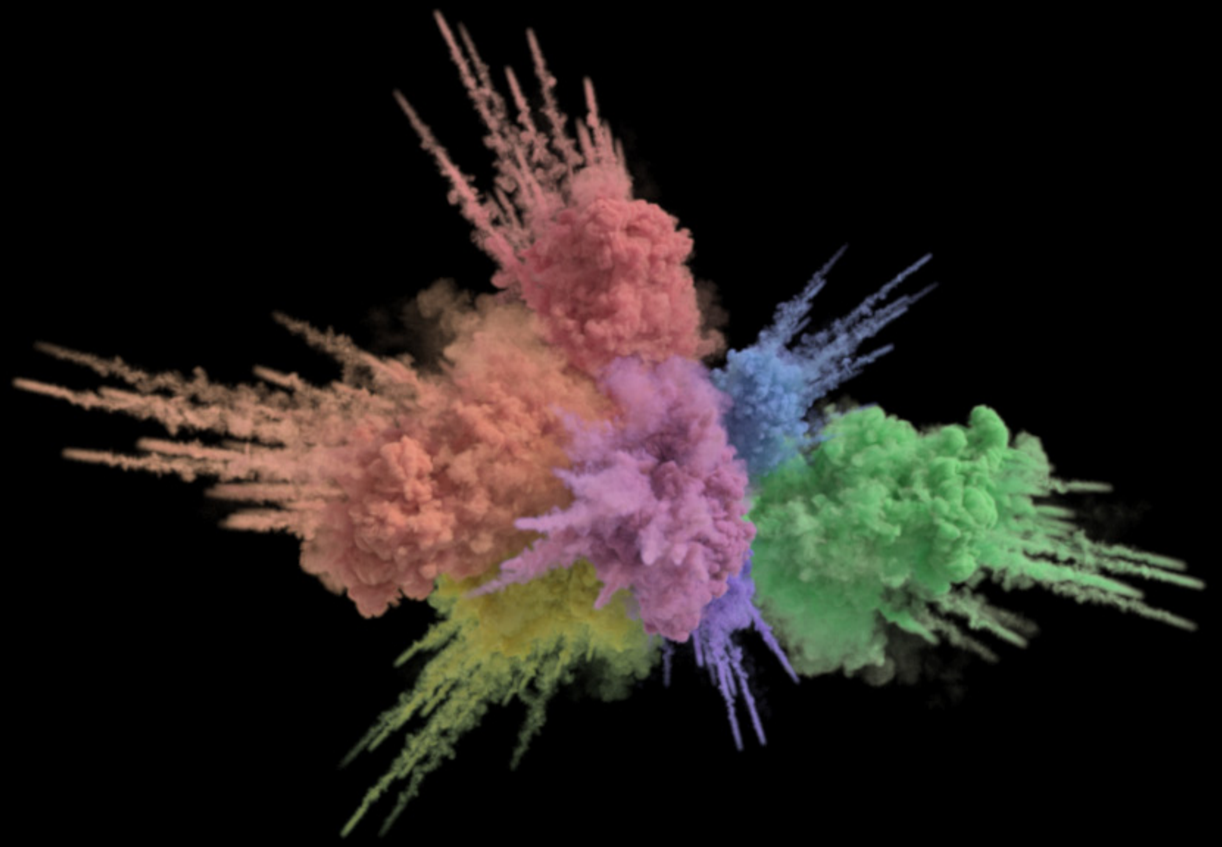
Ioannis Gkioulekas
Carnegie Mellon University

classic volume rendering (1950s-present)

classic volume rendering (1950s-present)

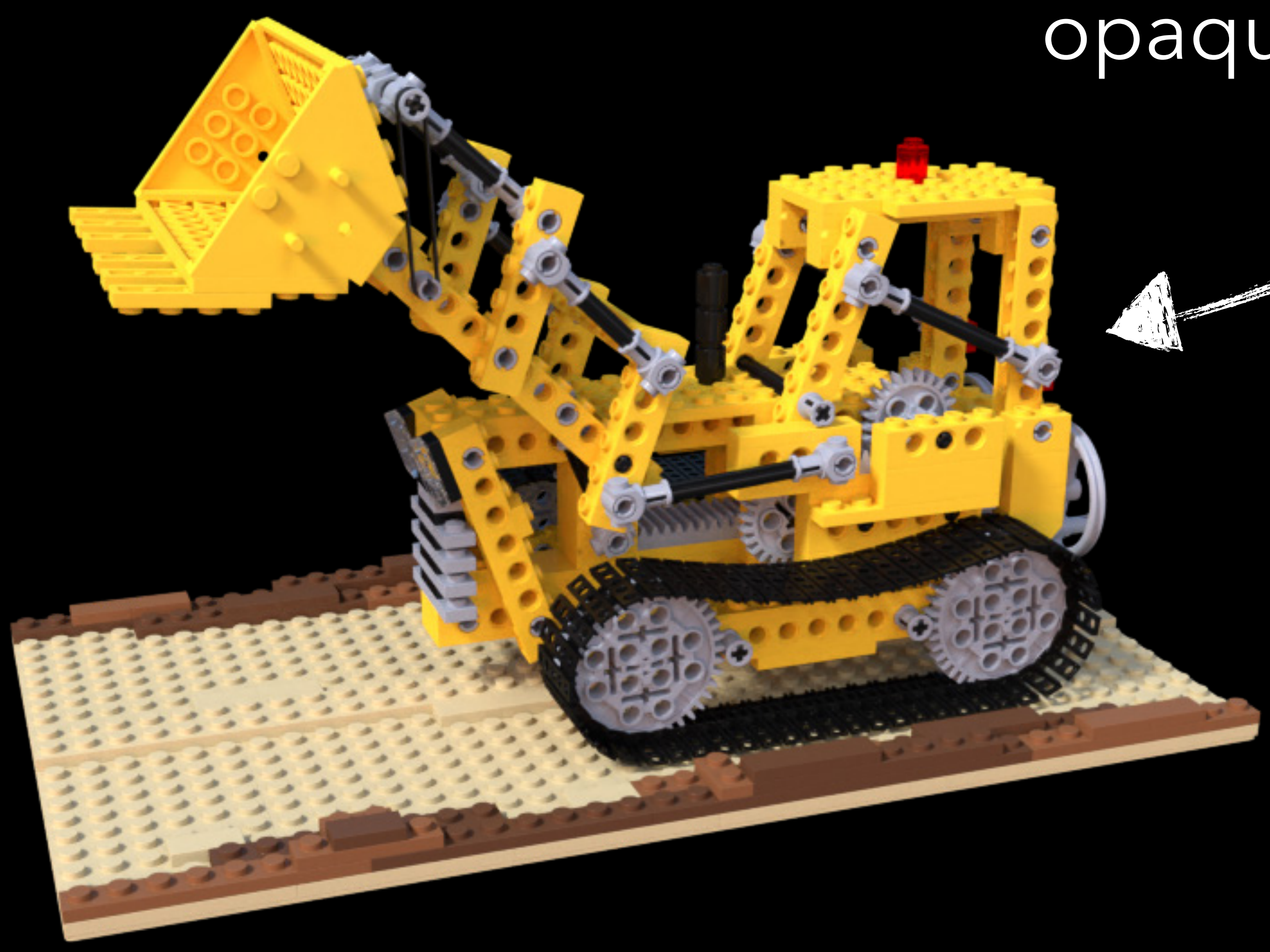


classic volume rendering (1950s-present)



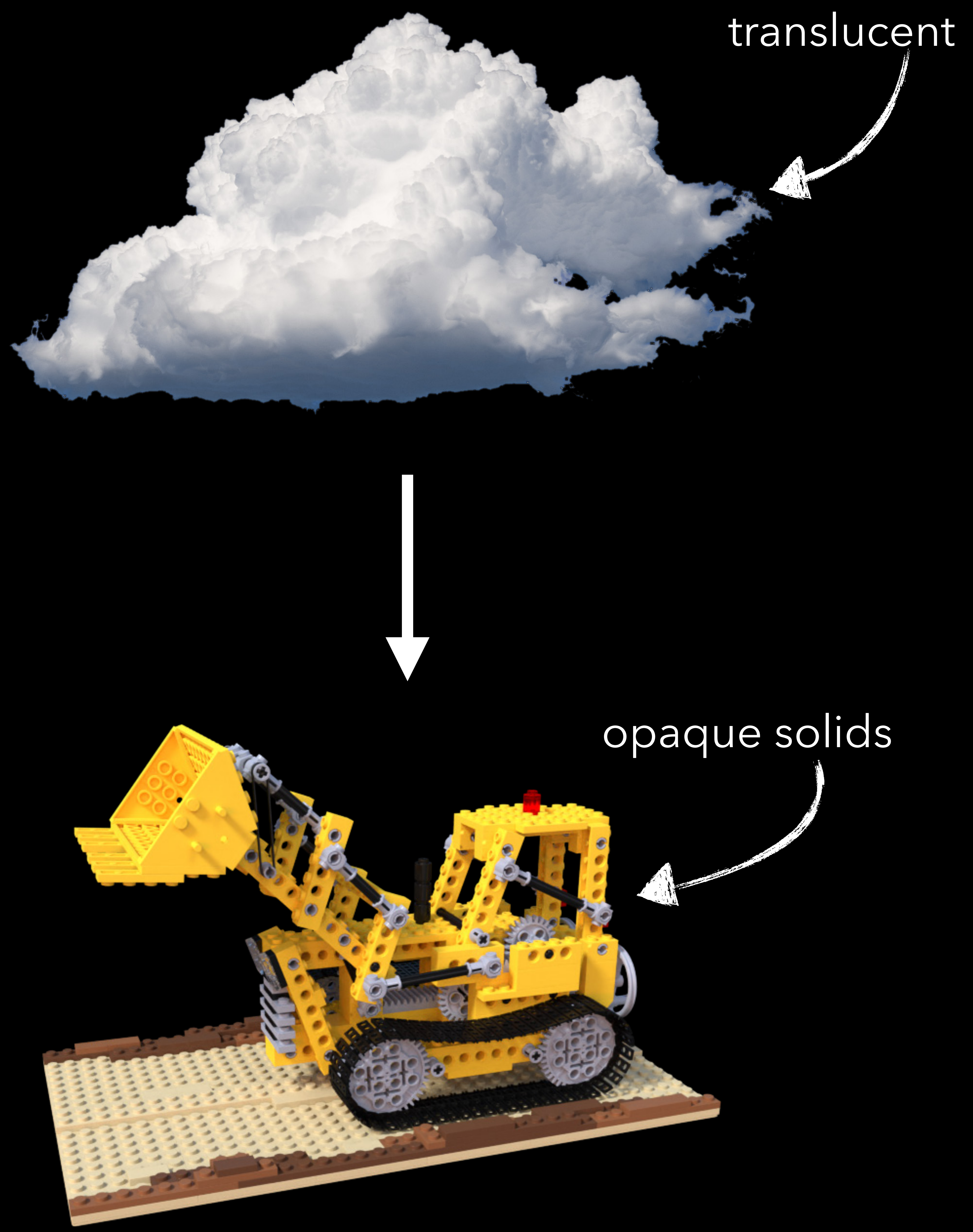
translucent

new volume rendering (2020-present)



opaque solids

How do we make volume rendering solids principled?



Why do we make volume rendering solids principled?

explains and improves prior methods

reference

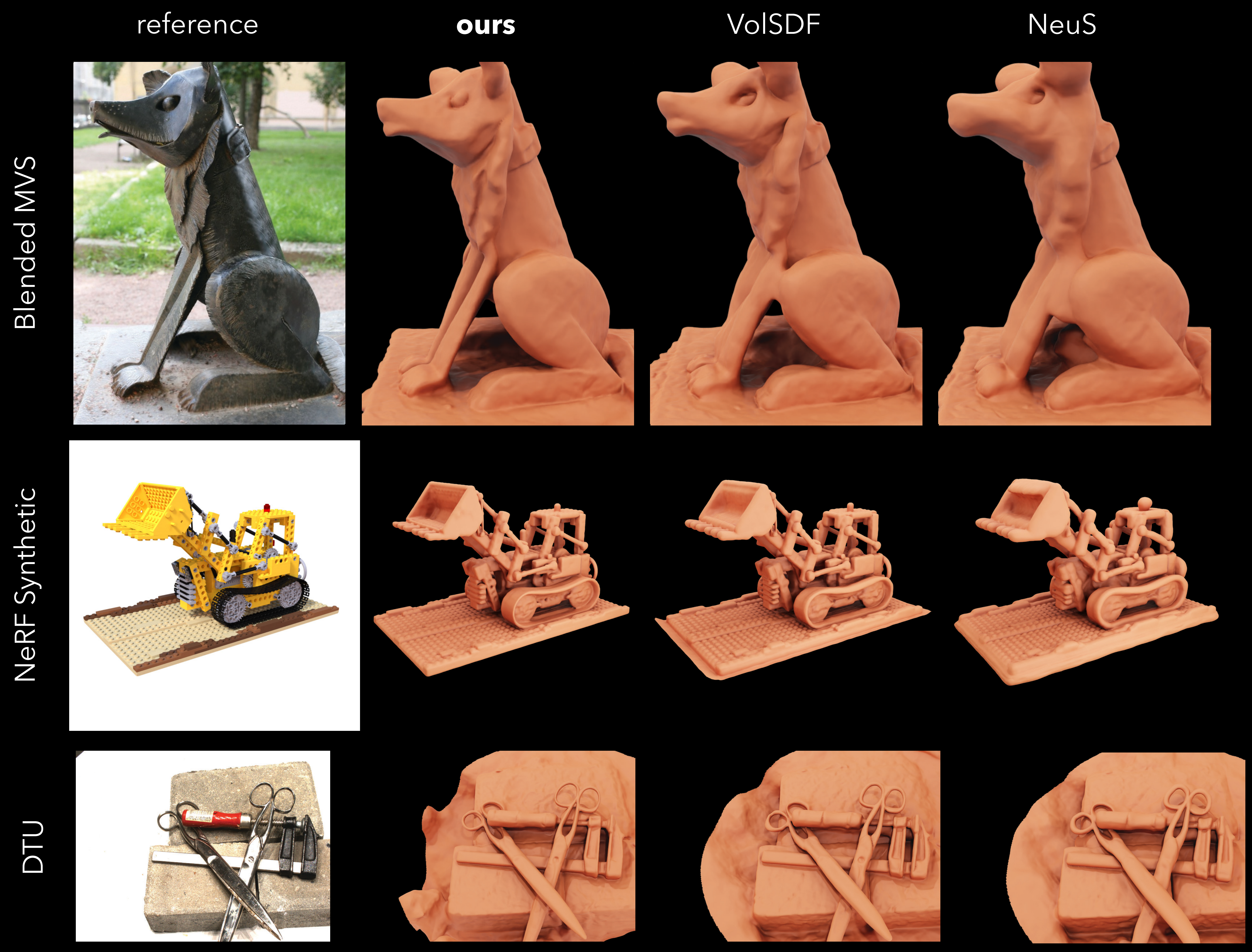
ours

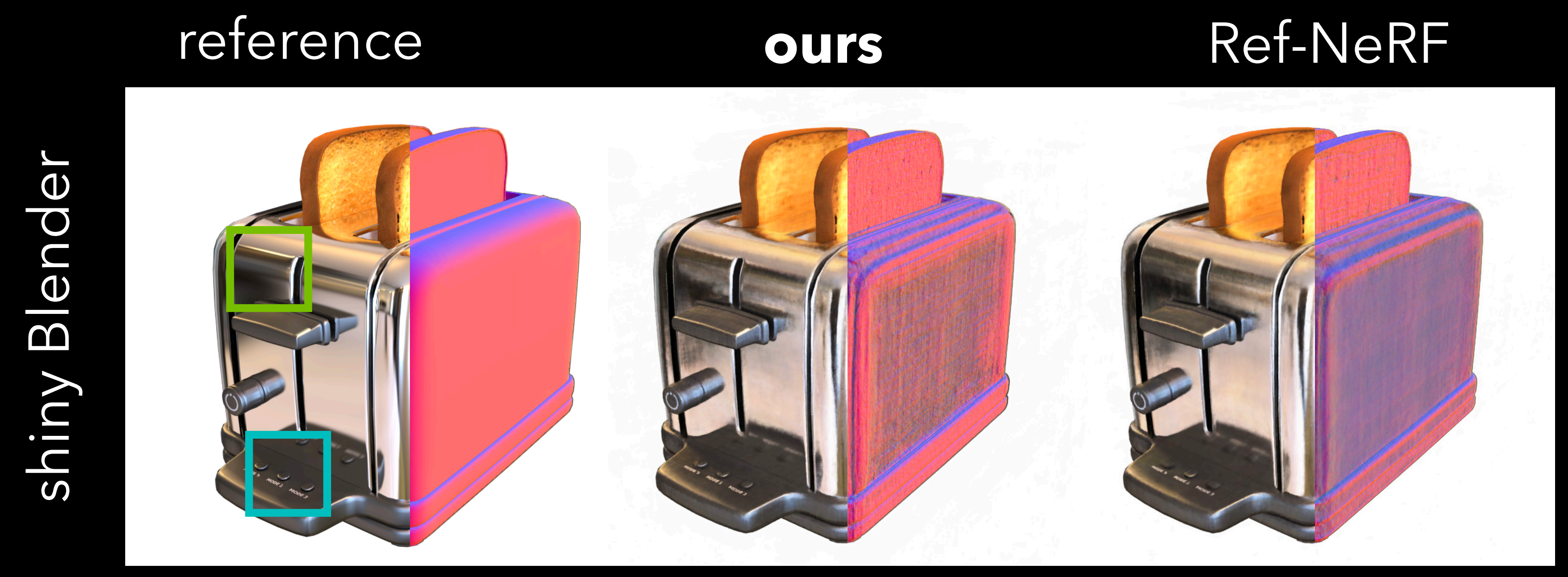
VolSDF

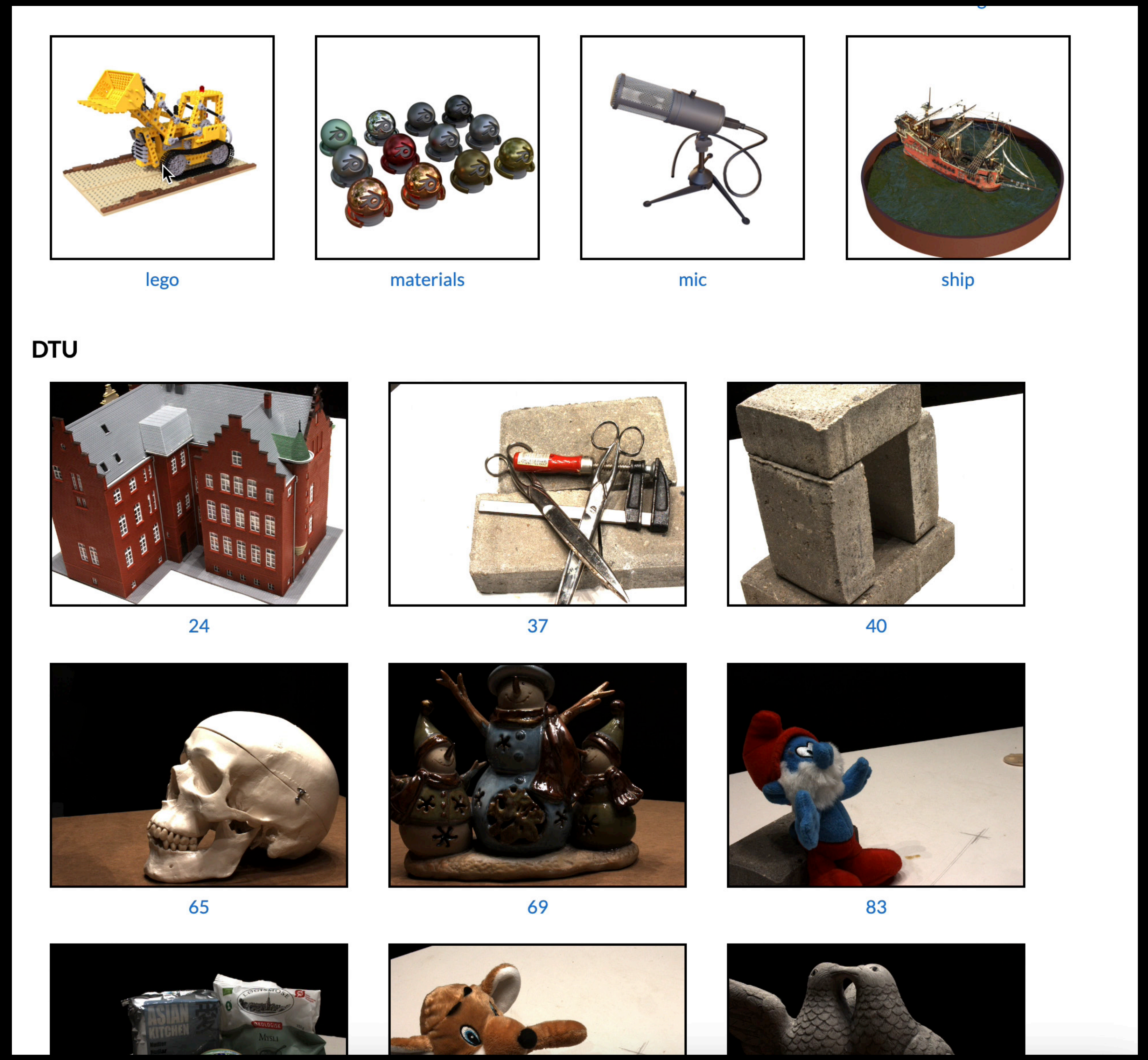
NeuS

Outperforms several recent works on a variety of datasets

project page

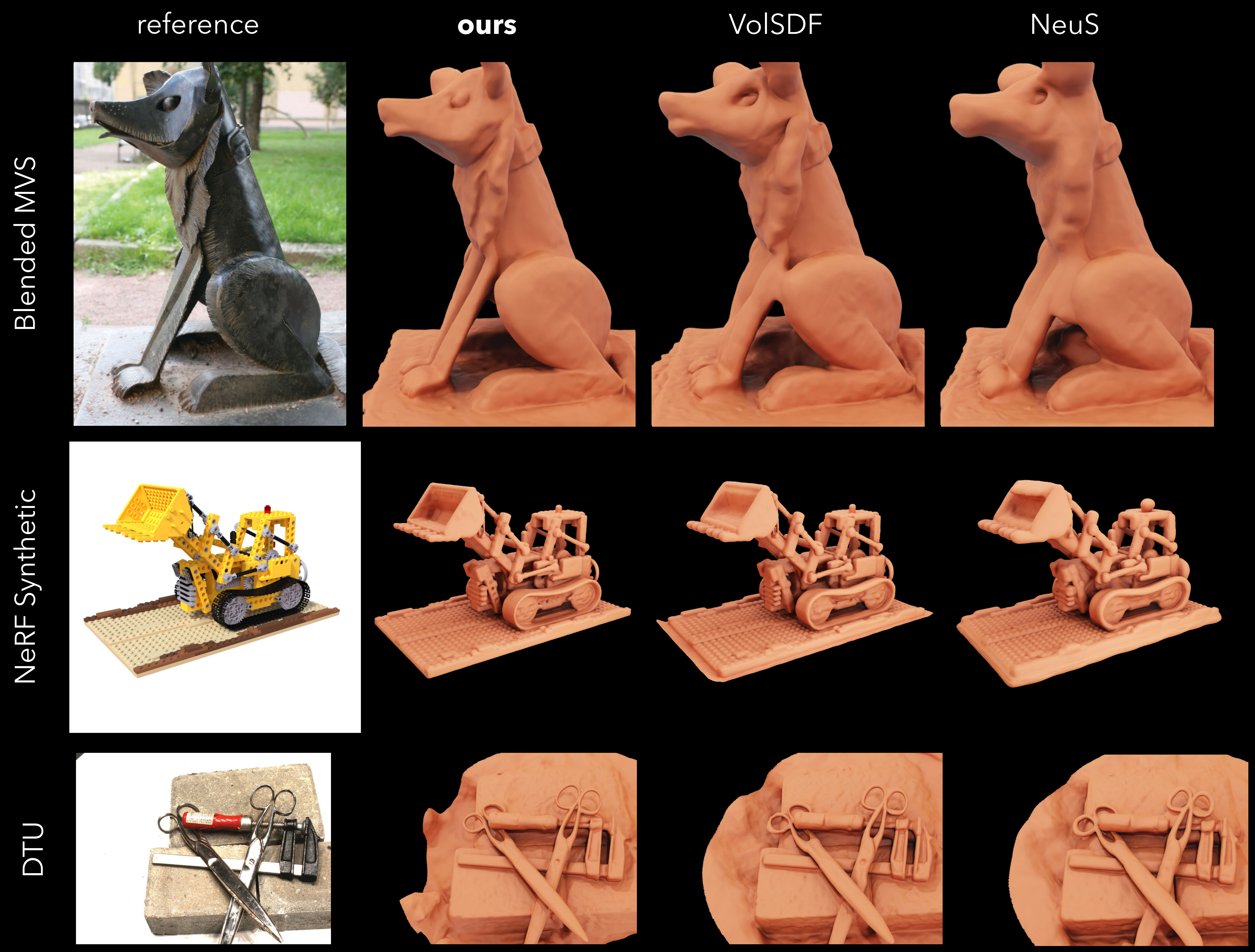


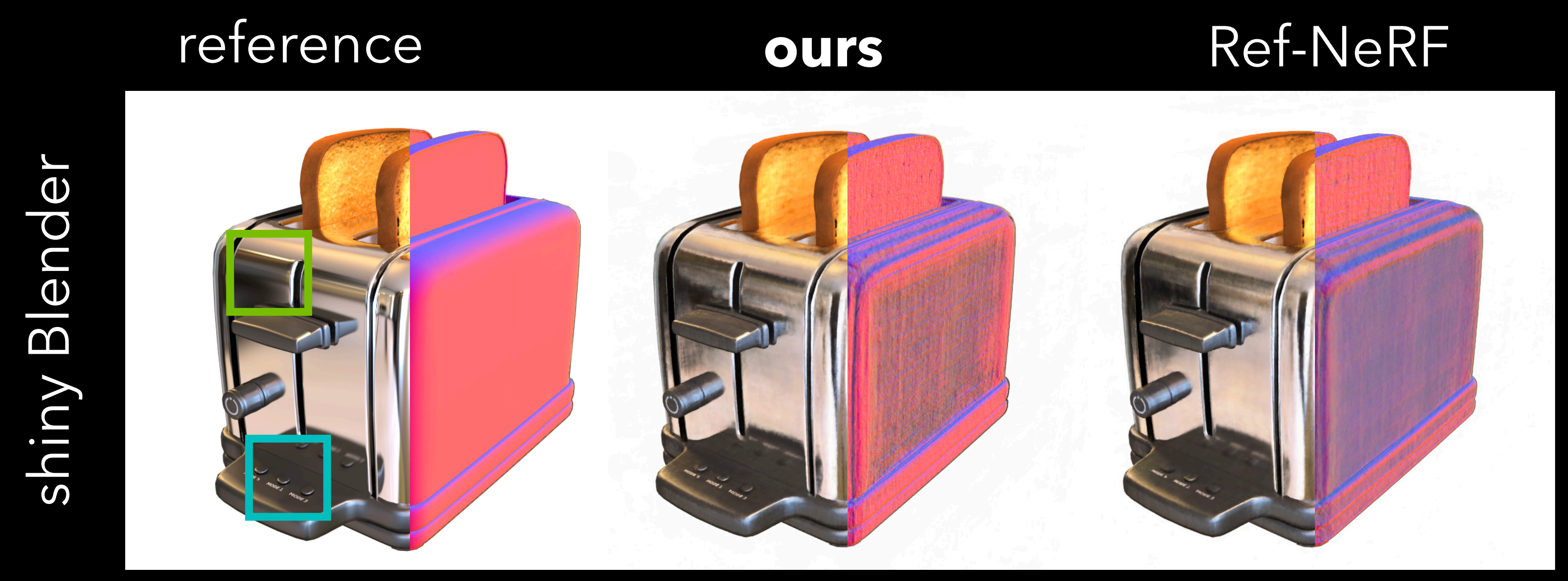


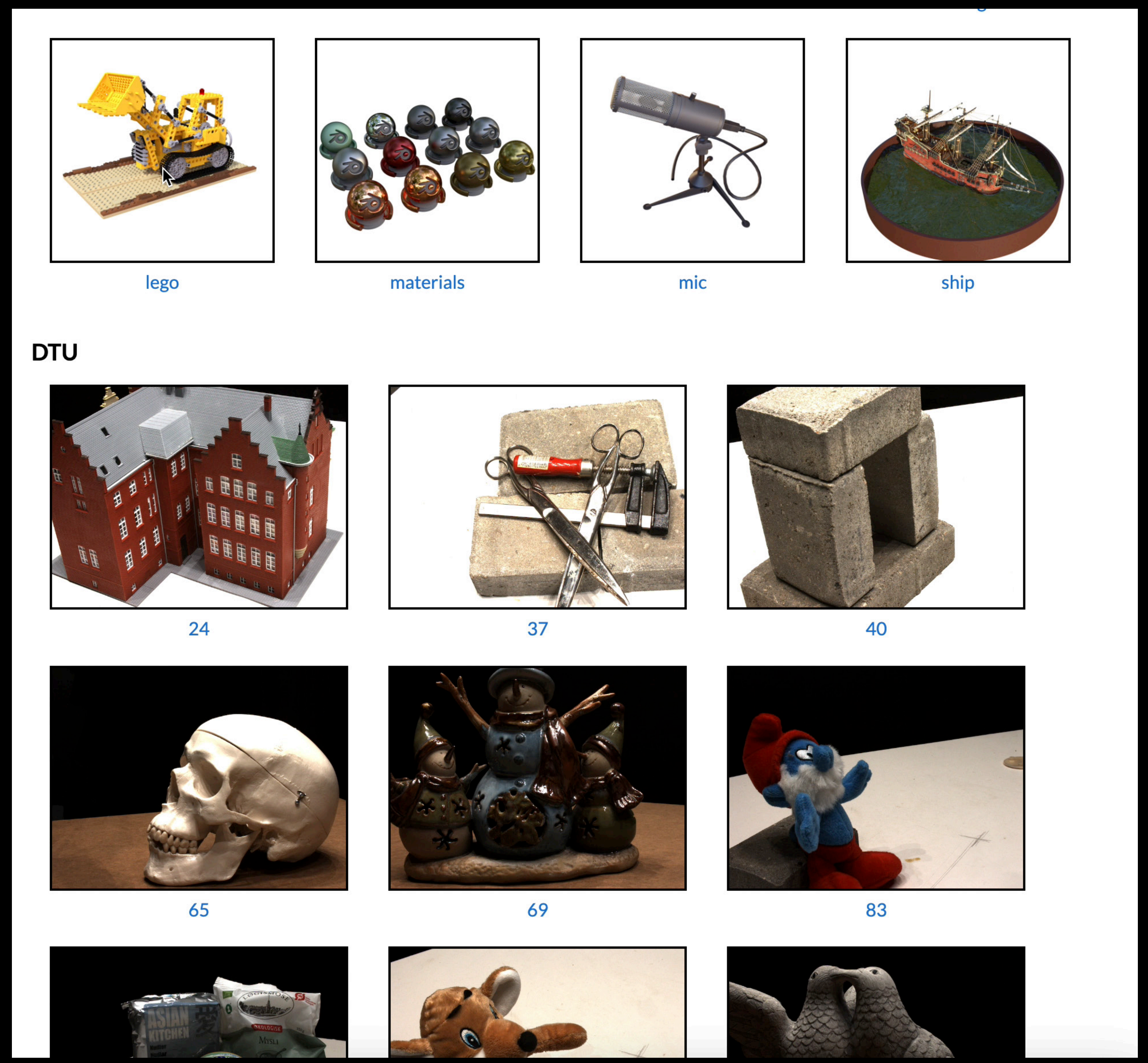


Outperforms several recent works on a variety of datasets

project page







microparticle model

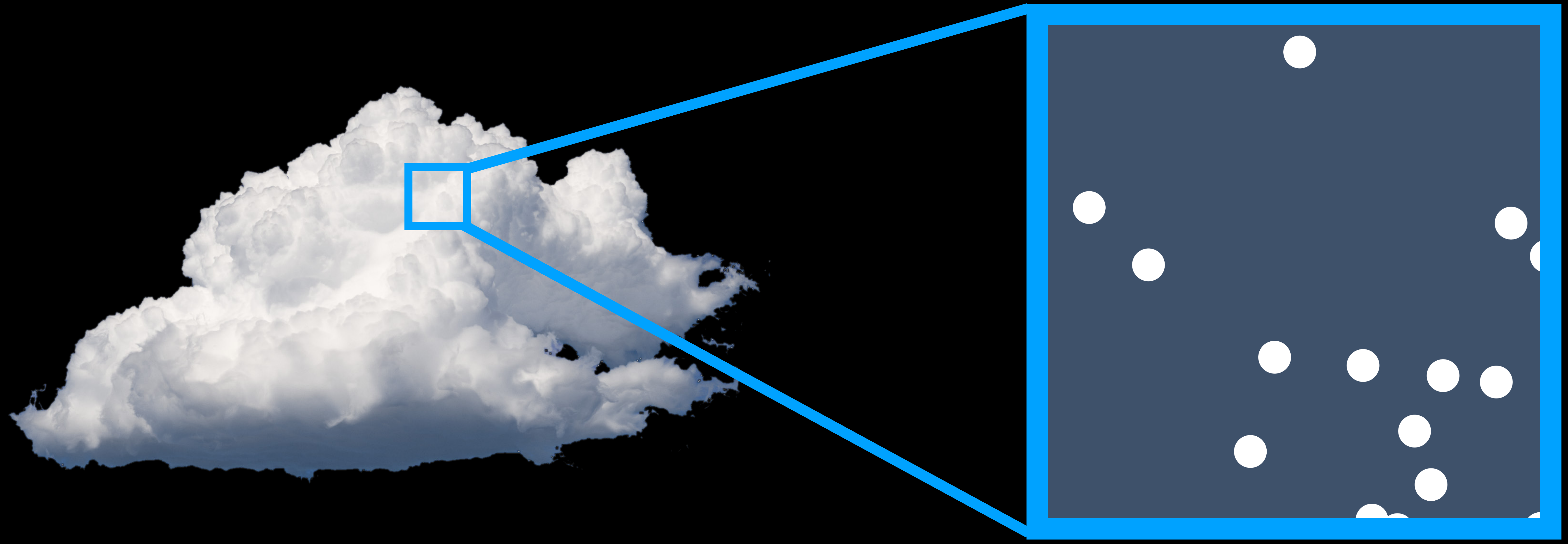
<u>exact geometry:</u>

location of particles

microparticle model

exact geometry:

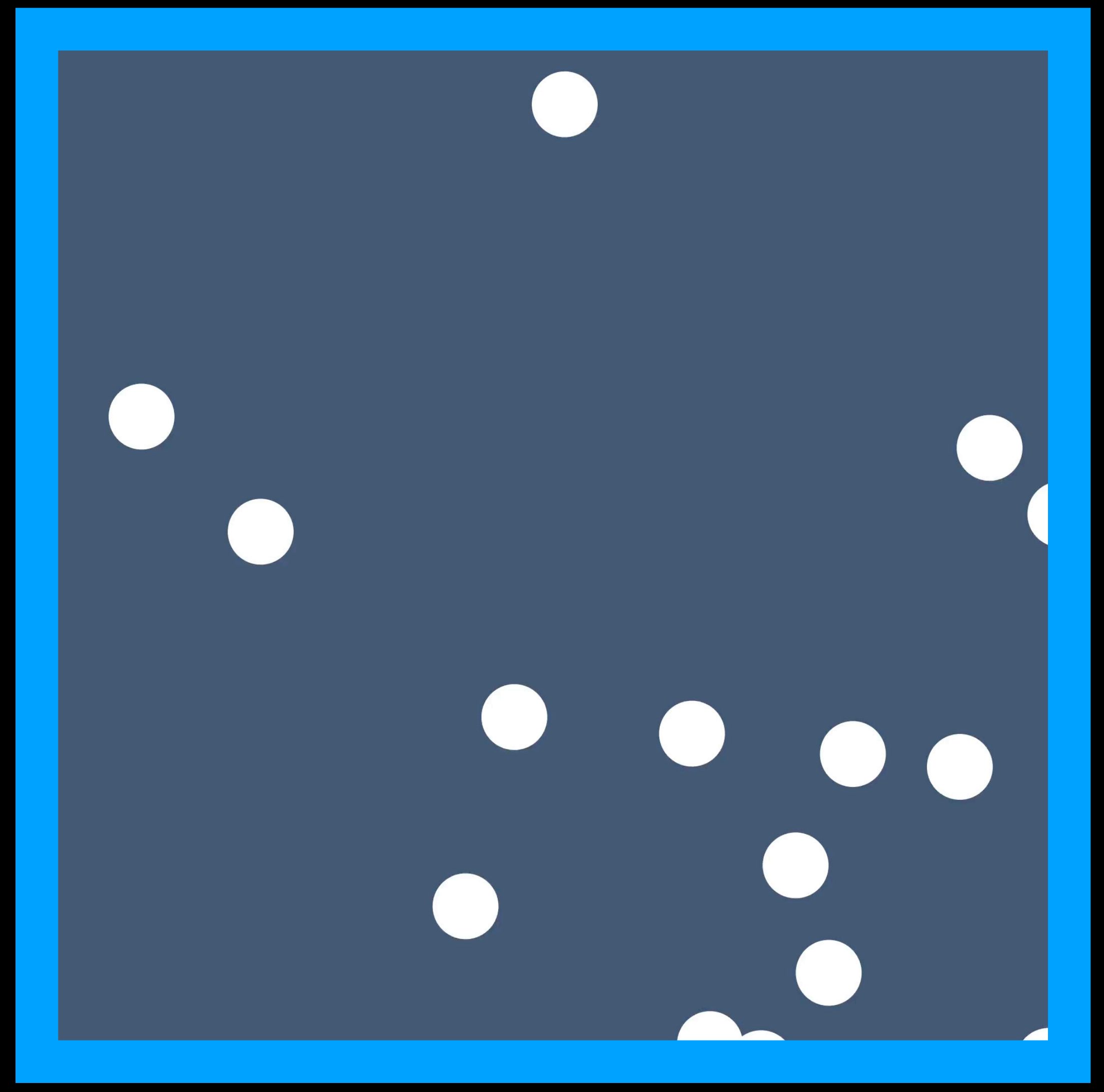
location of particles



microparticle model

<u>exact geometry:</u>

location of particles



microparticle model

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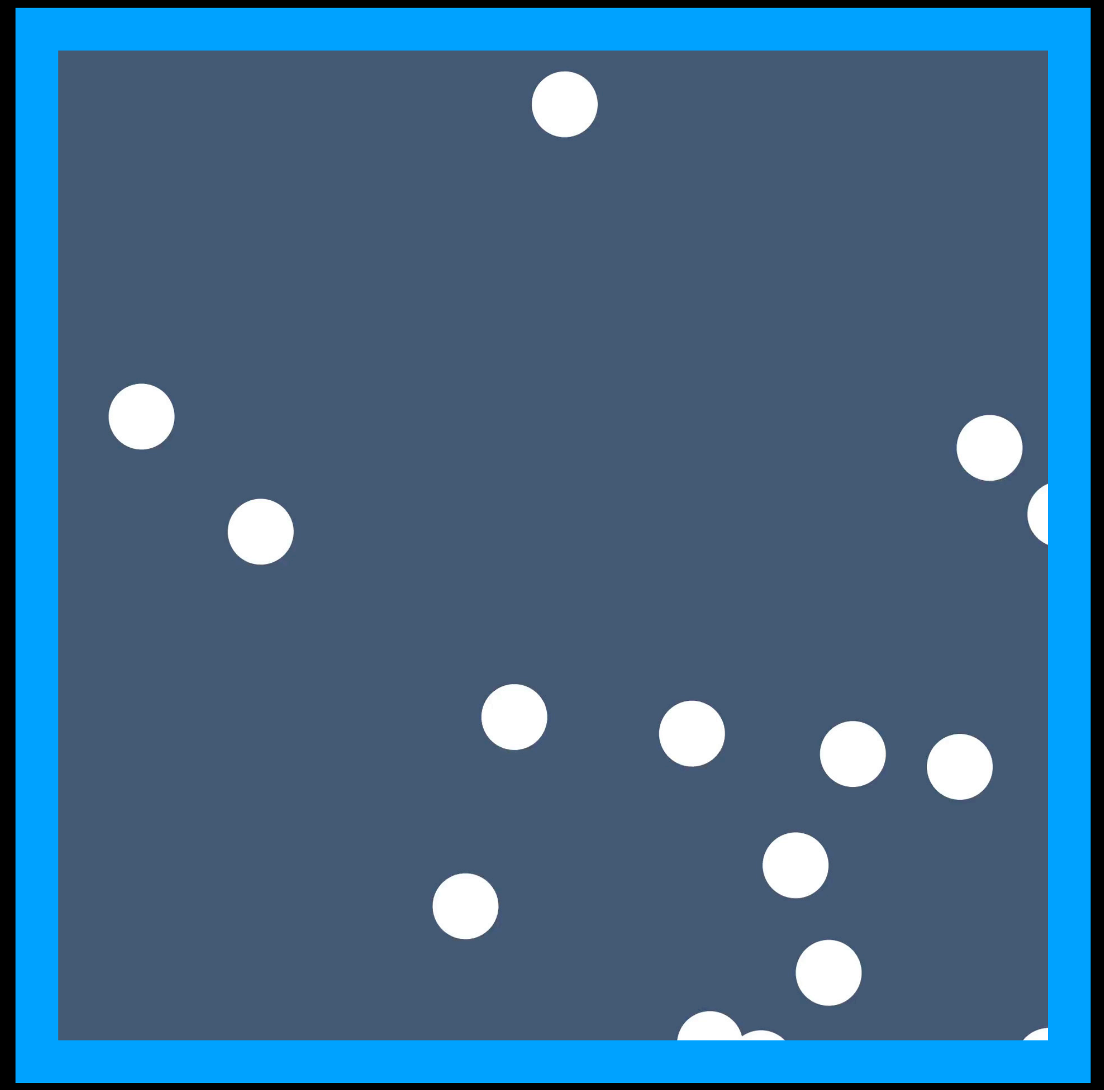
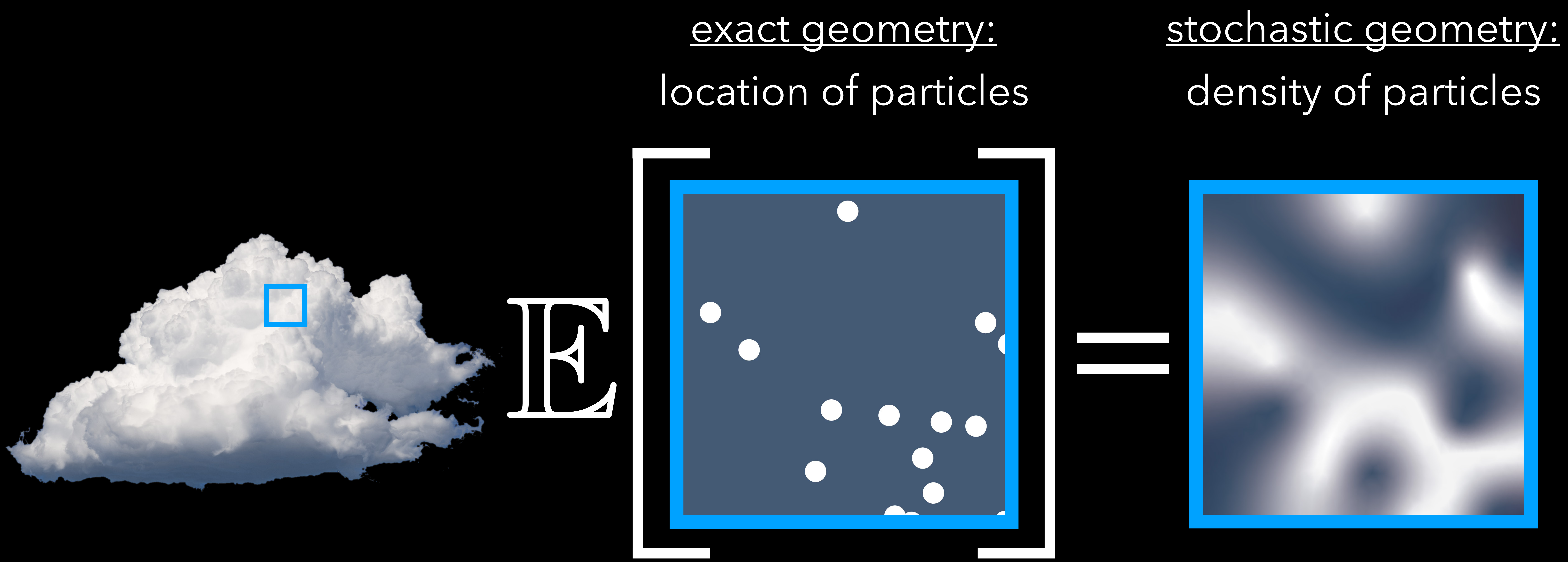


microparticle model



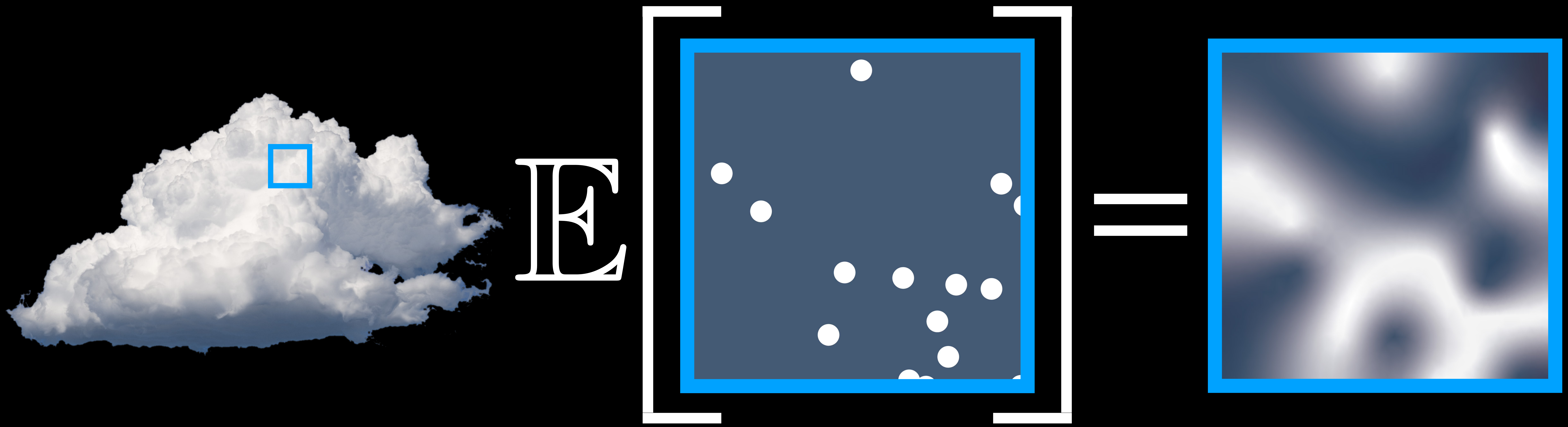
microparticle model

<u>exact geometry:</u>

location of particles

<u>stochastic geometry:</u>

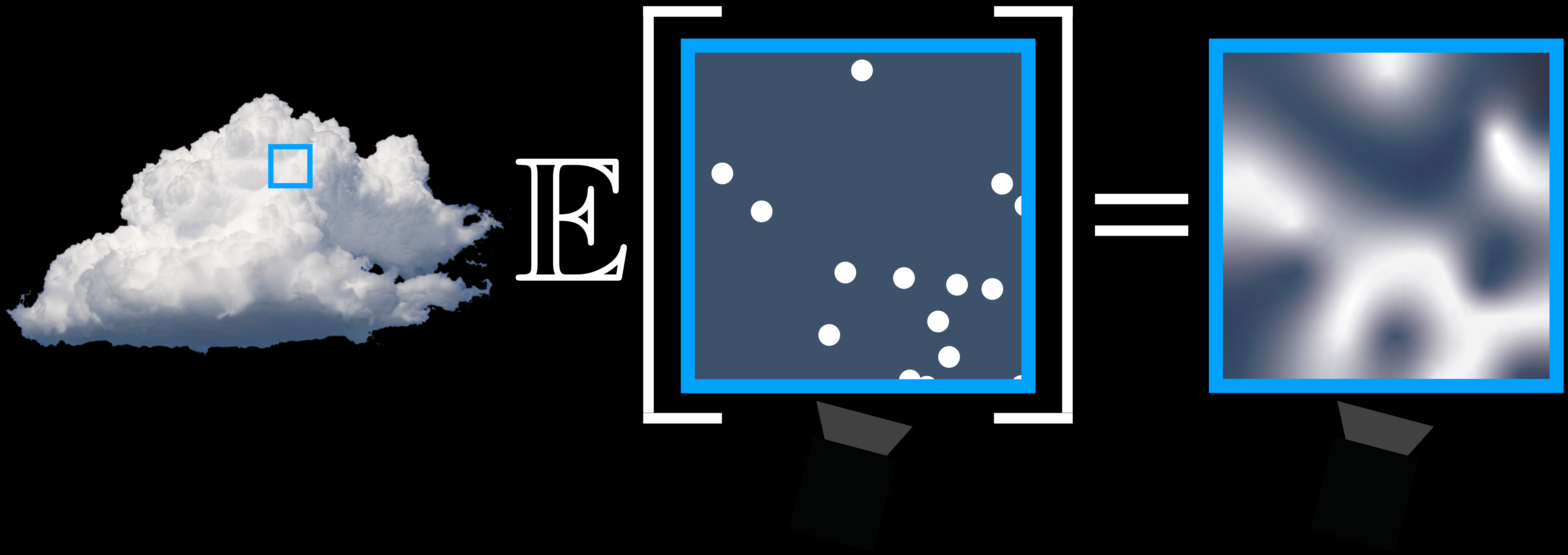
density of particles



microparticle model

exact geometry:
location of particles

stochastic geometry:
density of particles



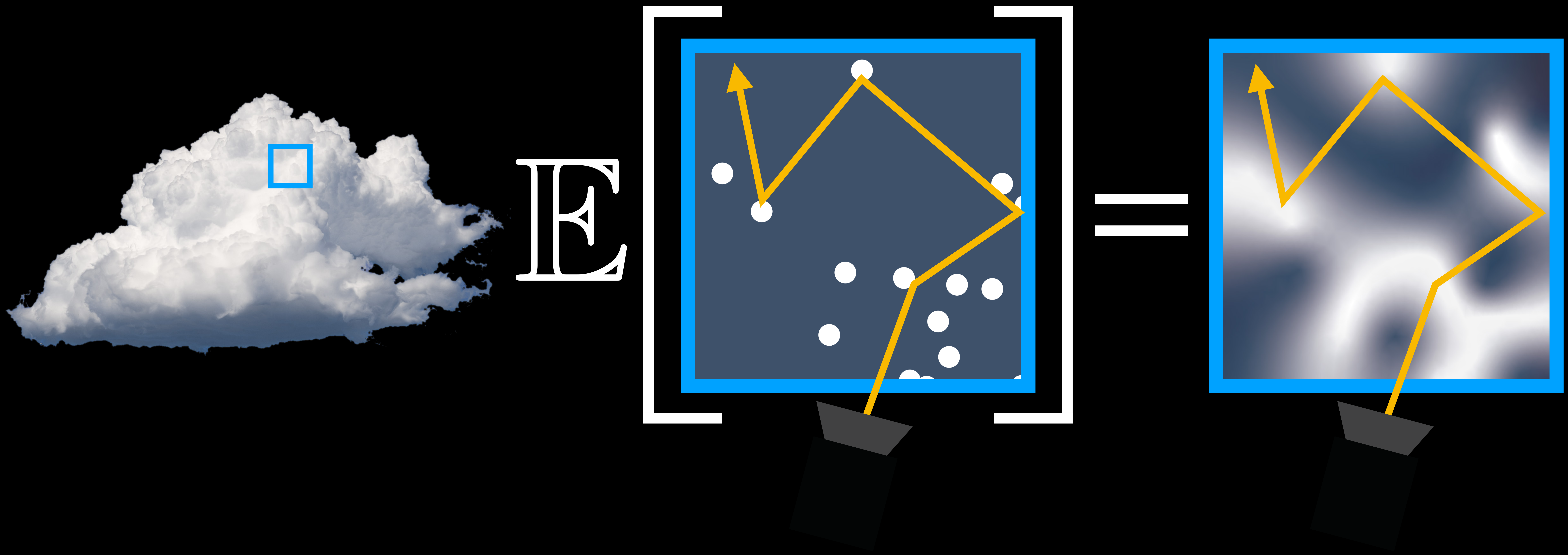
deterministic rendering

volume rendering

microparticle model

<u>exact geometry:</u>
location of particles

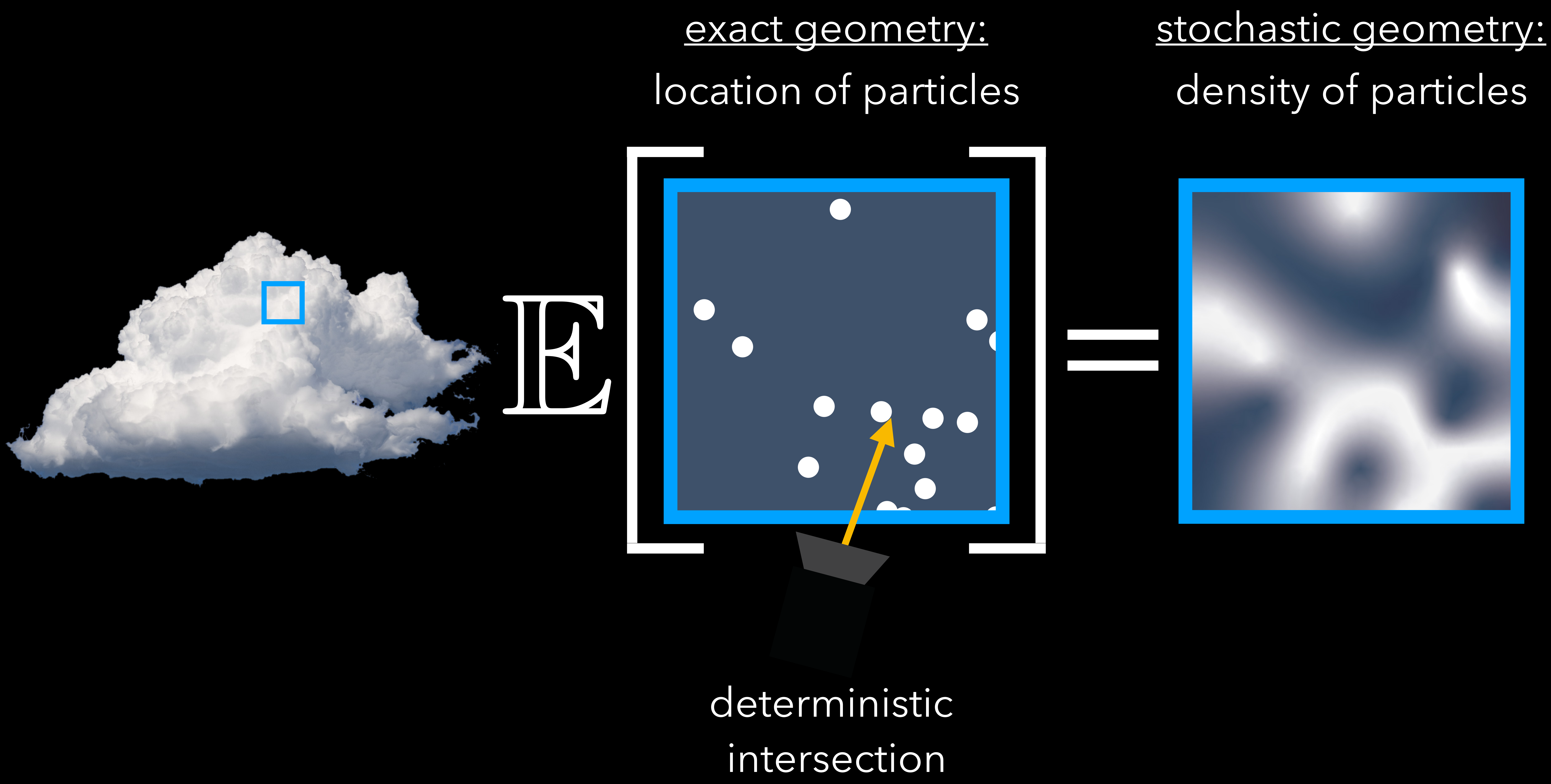
<u>stochastic geometry:</u>
density of particles



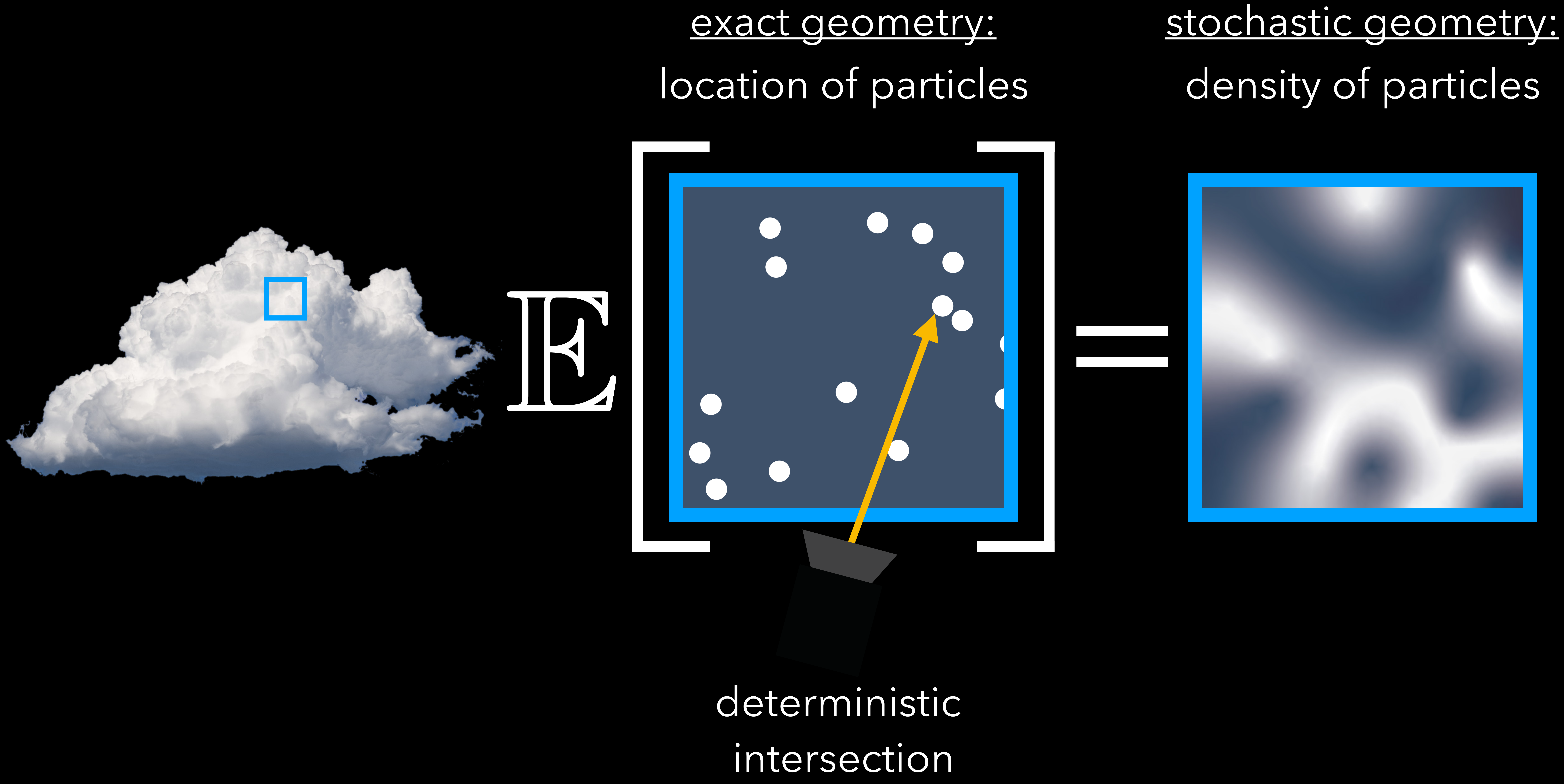
deterministic rendering

volume rendering

microparticle model



microparticle model



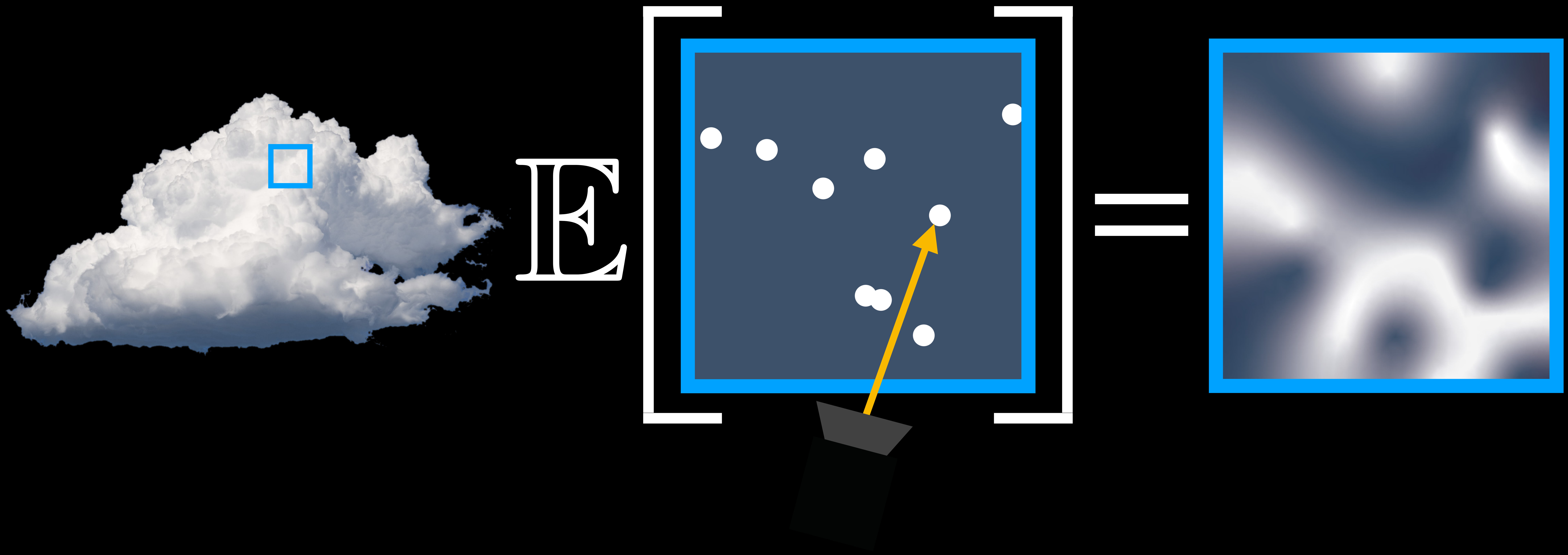
microparticle model

<u>exact geometry:</u>

location of particles

<u>stochastic geometry:</u>

density of particles

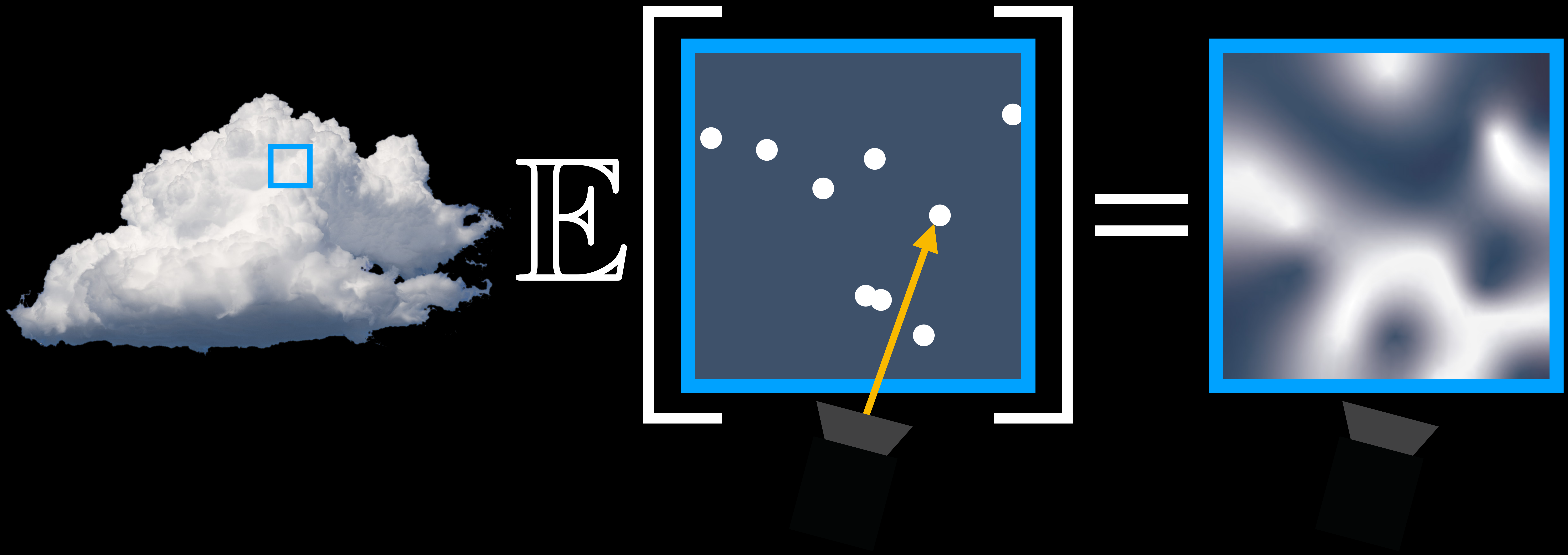


deterministic intersection

microparticle model

exact geometry: location of particles

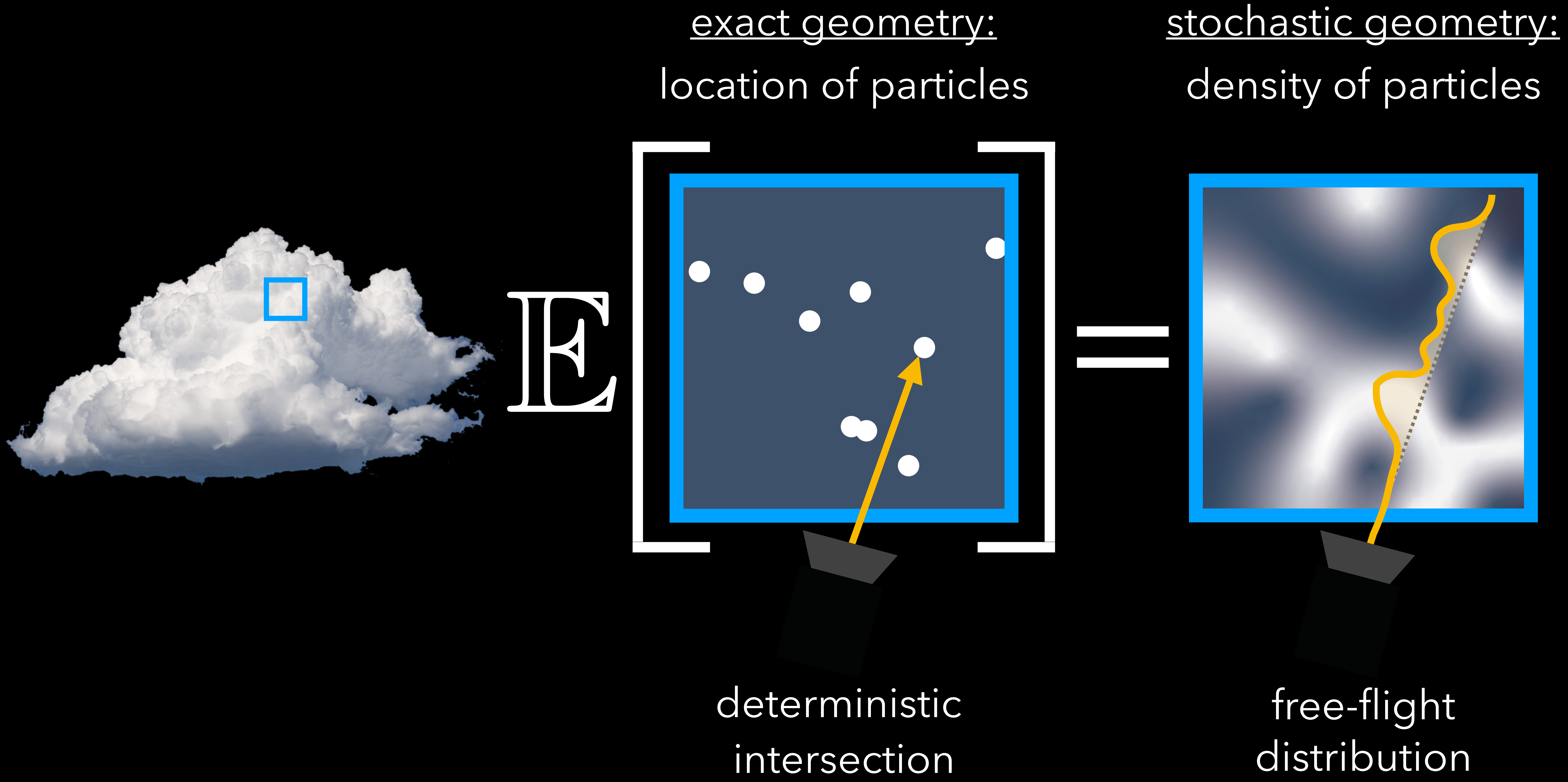
stochastic geometry: density of particles



deterministic intersection

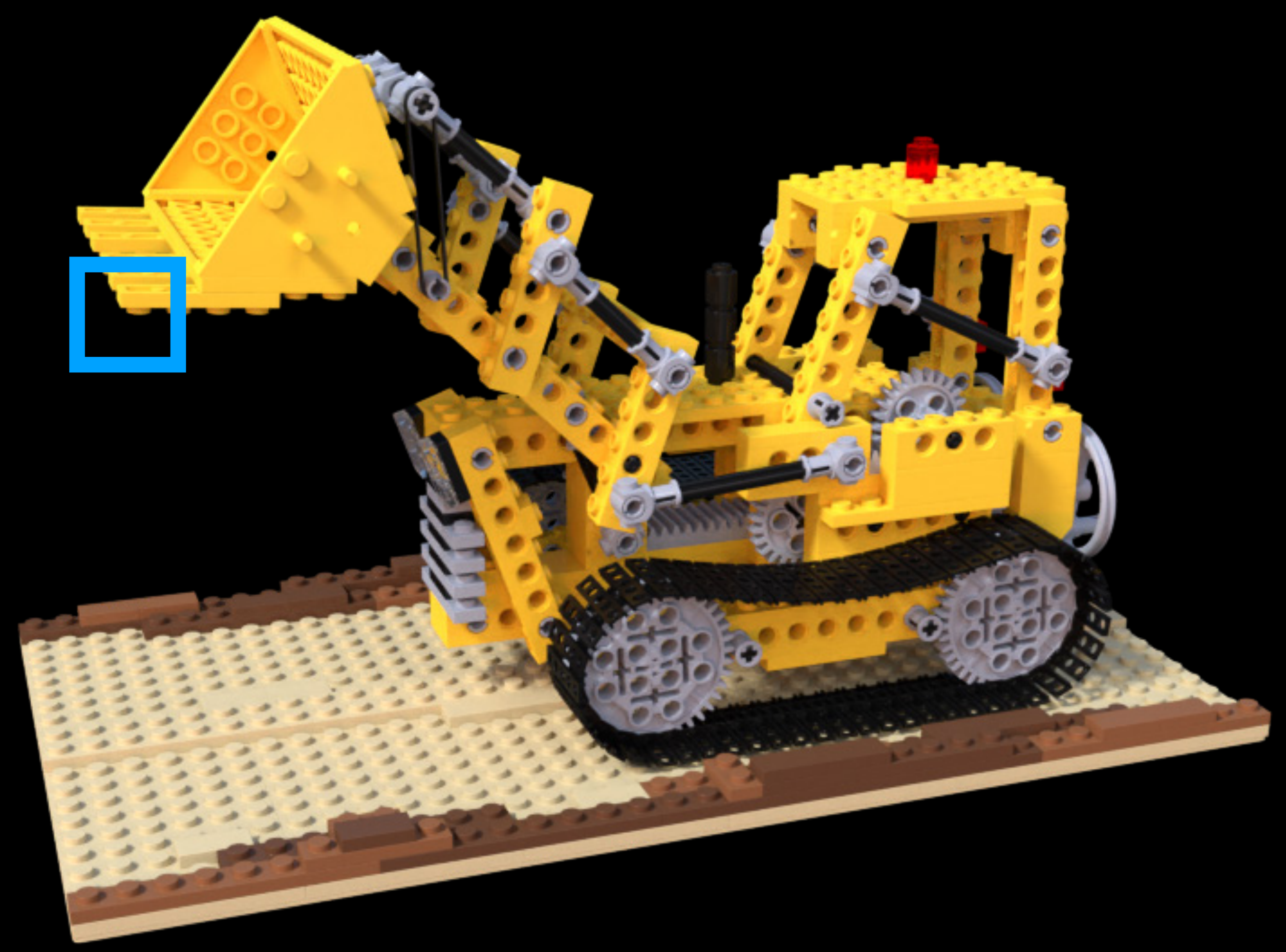
free-flight distribution

microparticle model



opaque solid model

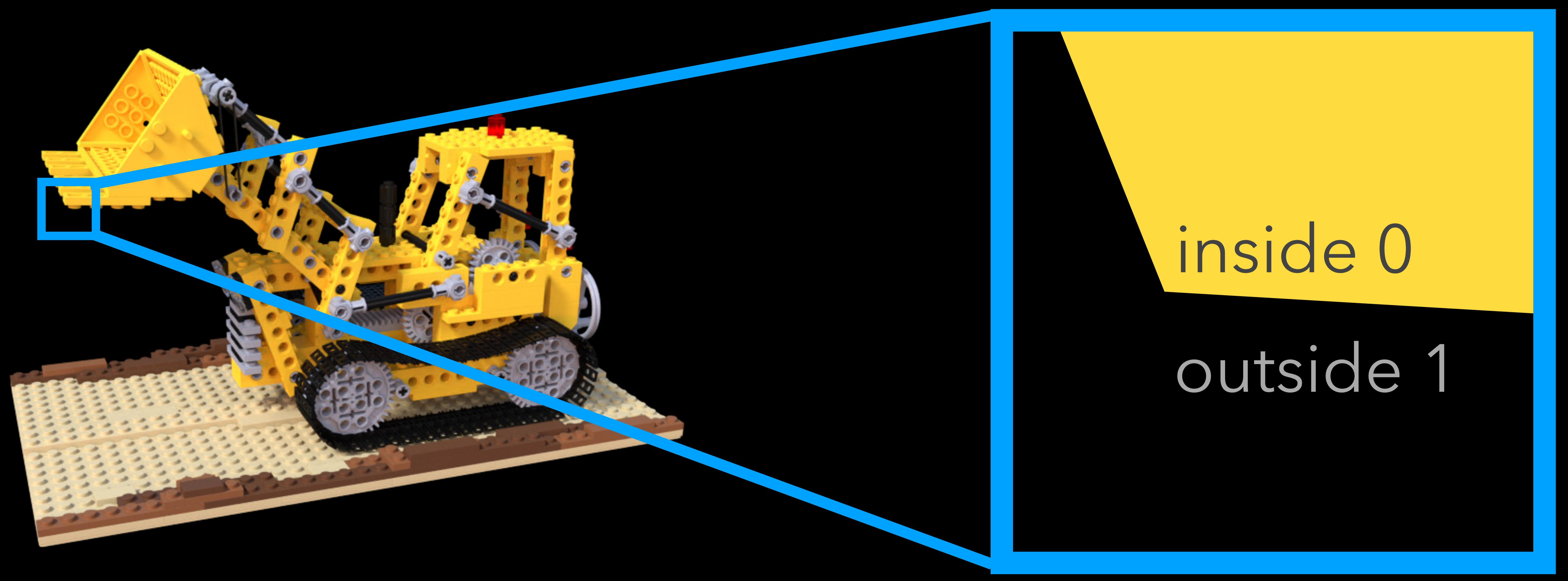
exact geometry:



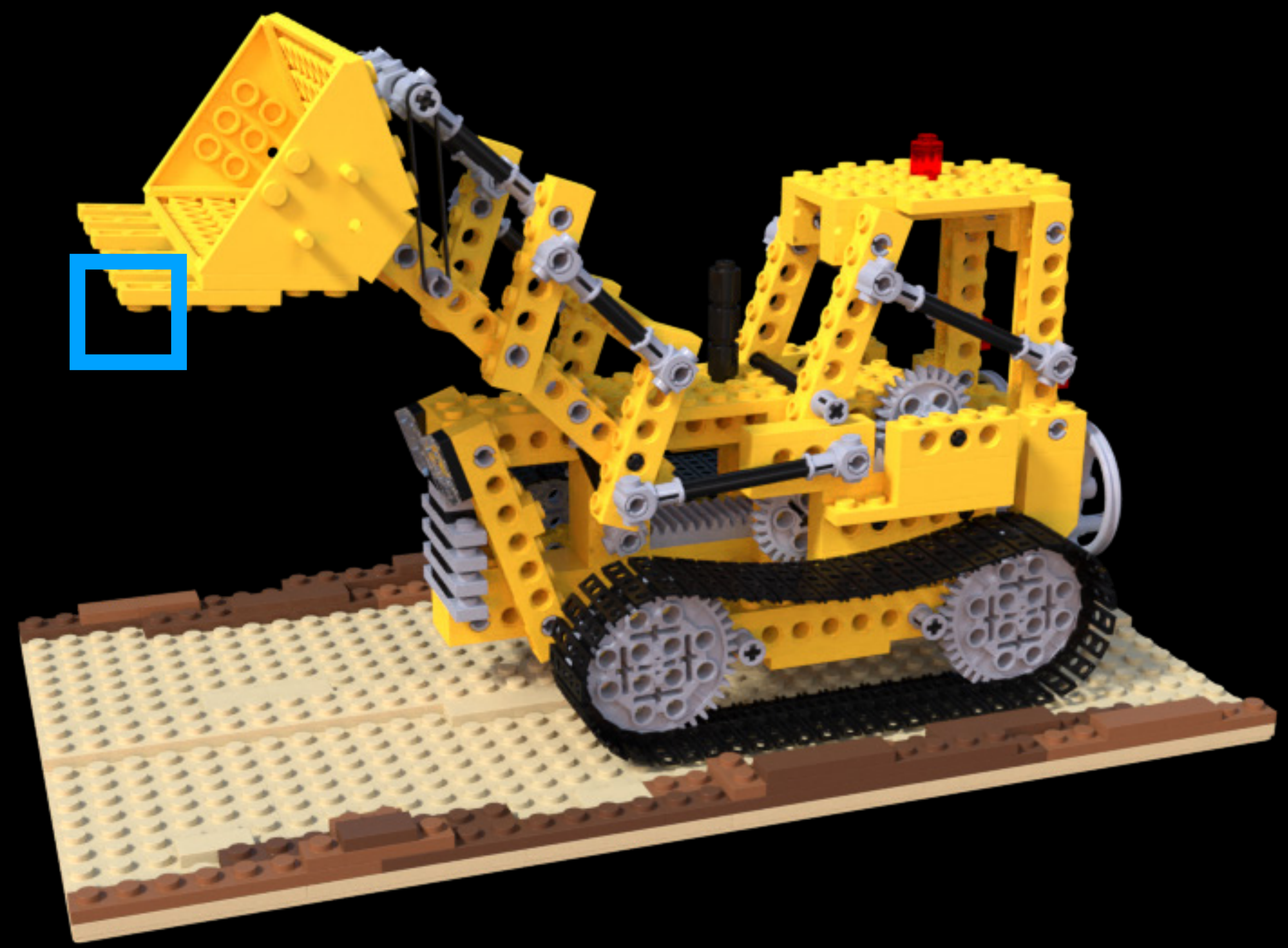
opaque solid model

<u>exact geometry:</u>

binary vacancy $\{0, 1\}$

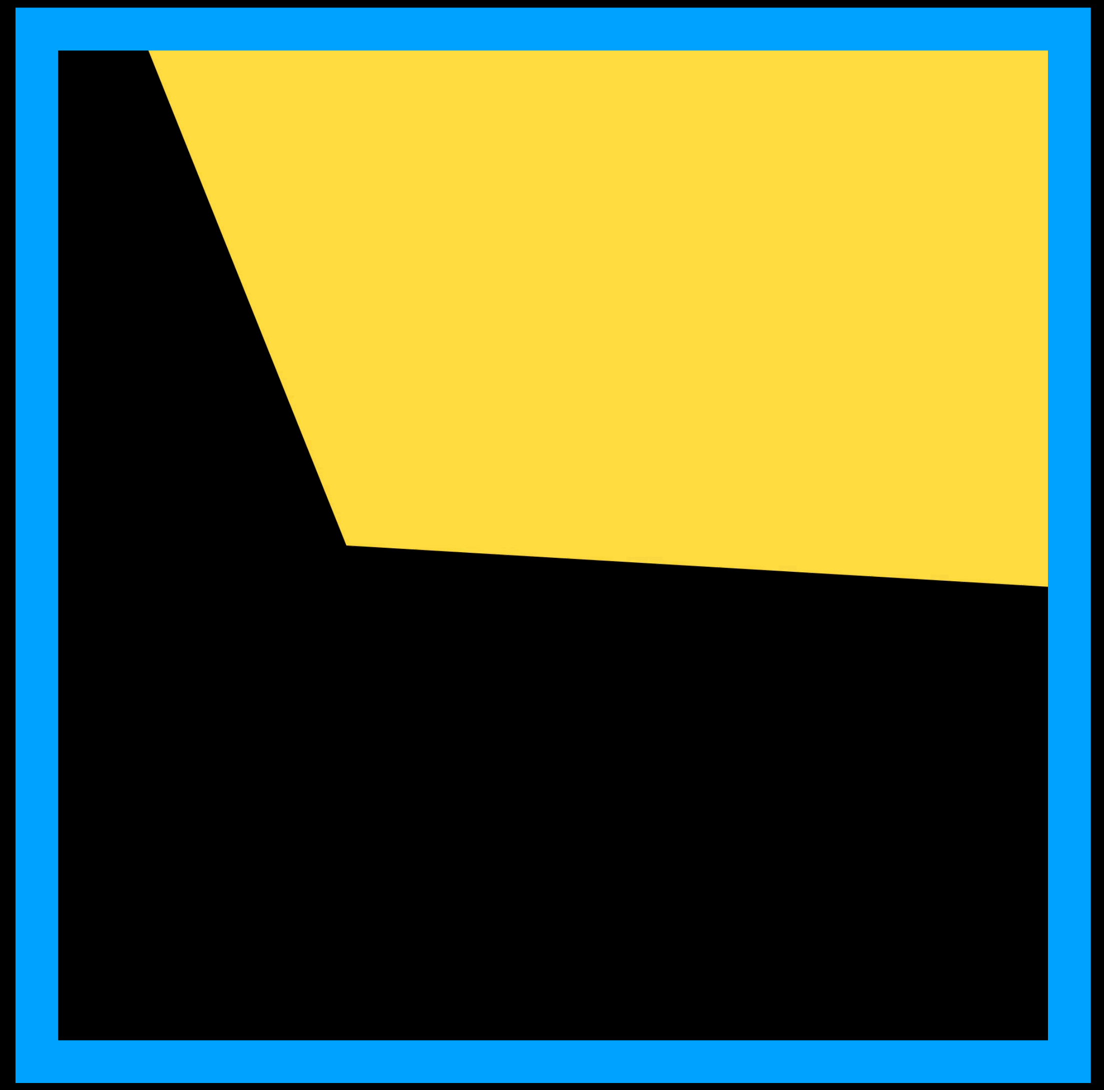


opaque solid model

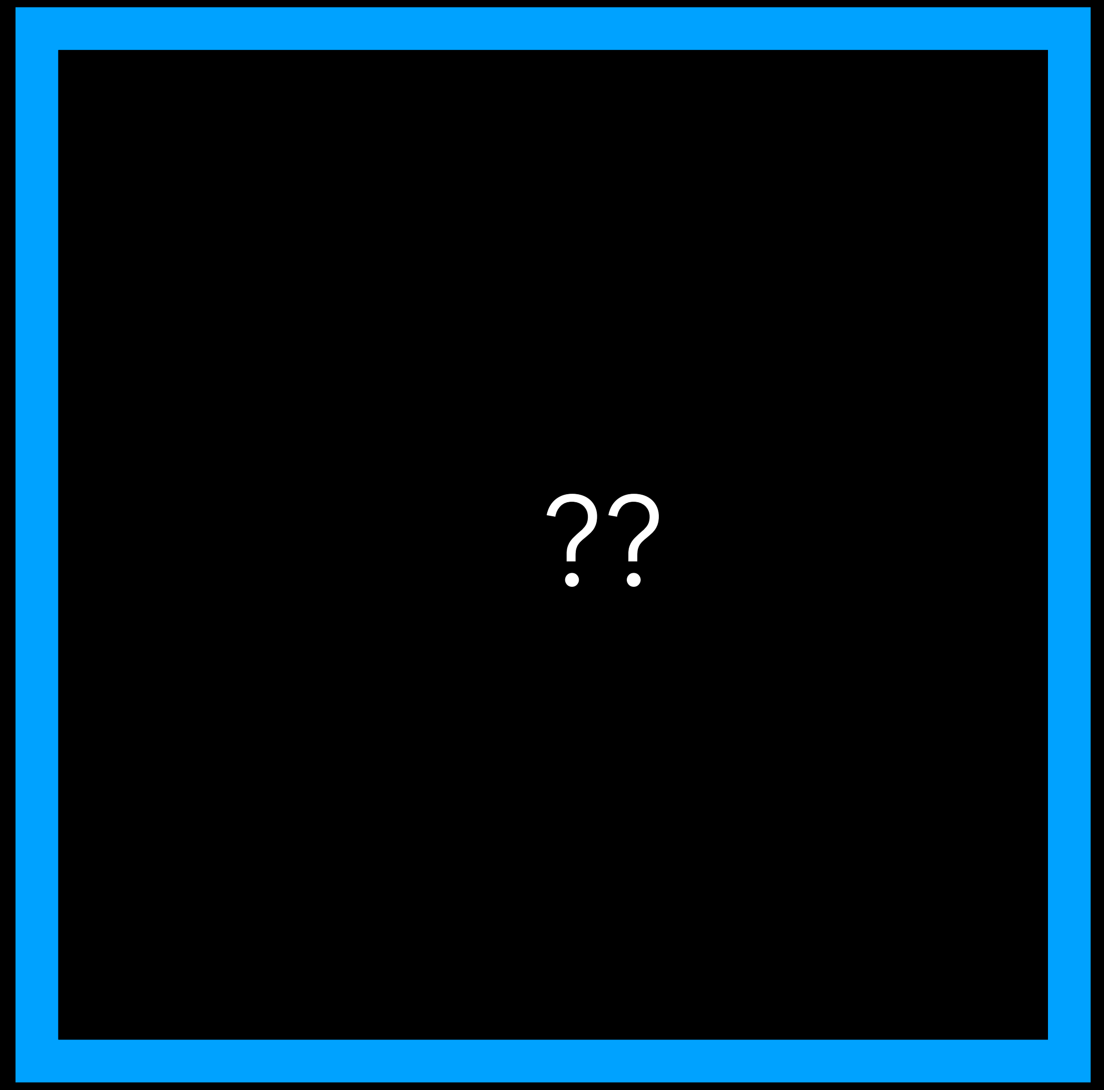


<u>exact geometry:</u>

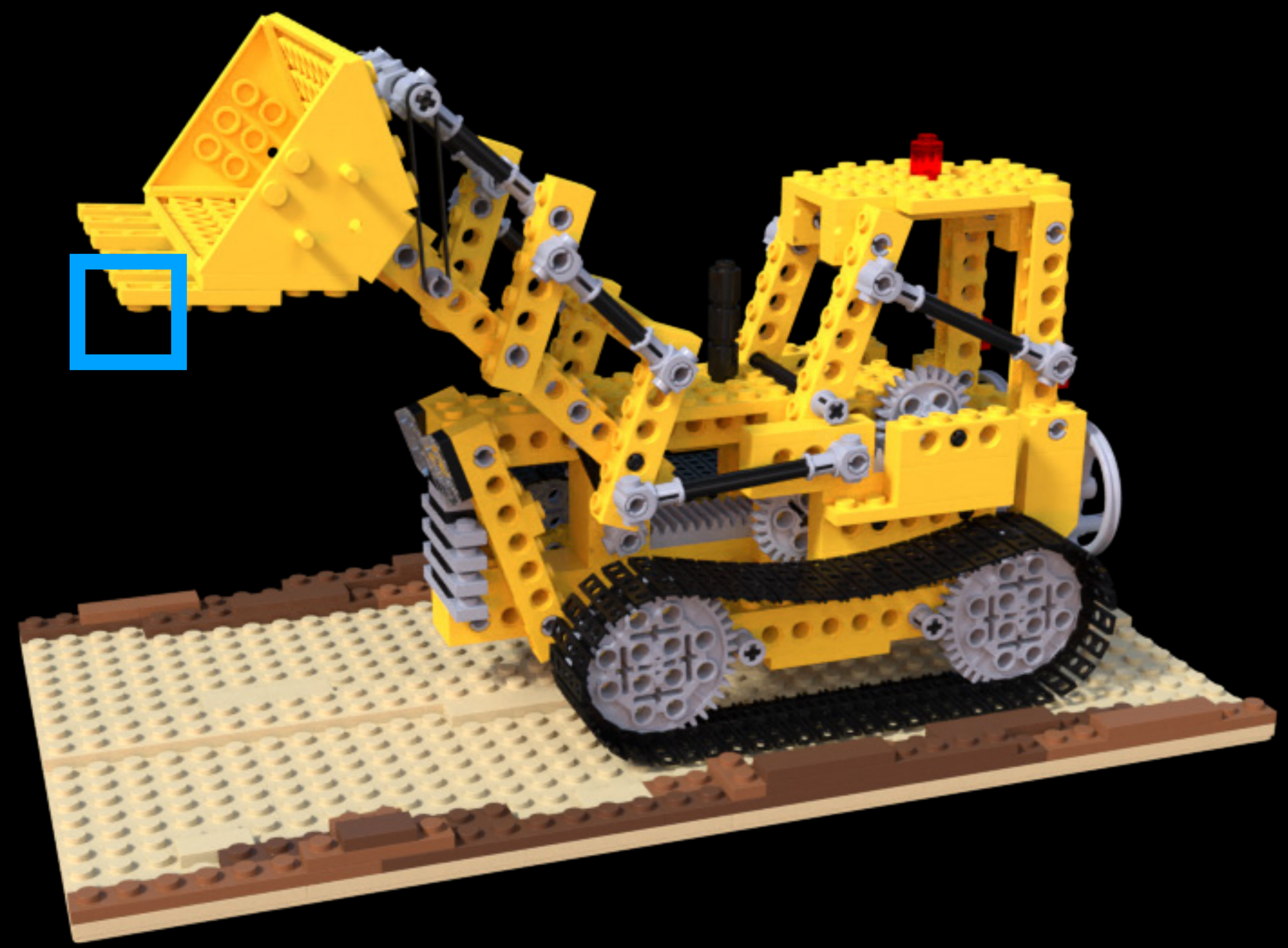
binary vacancy $\{0, 1\}$

$\mathbb{E}\Big[$  $\Big] =$

<u>stochastic geometry:</u>



opaque solid model

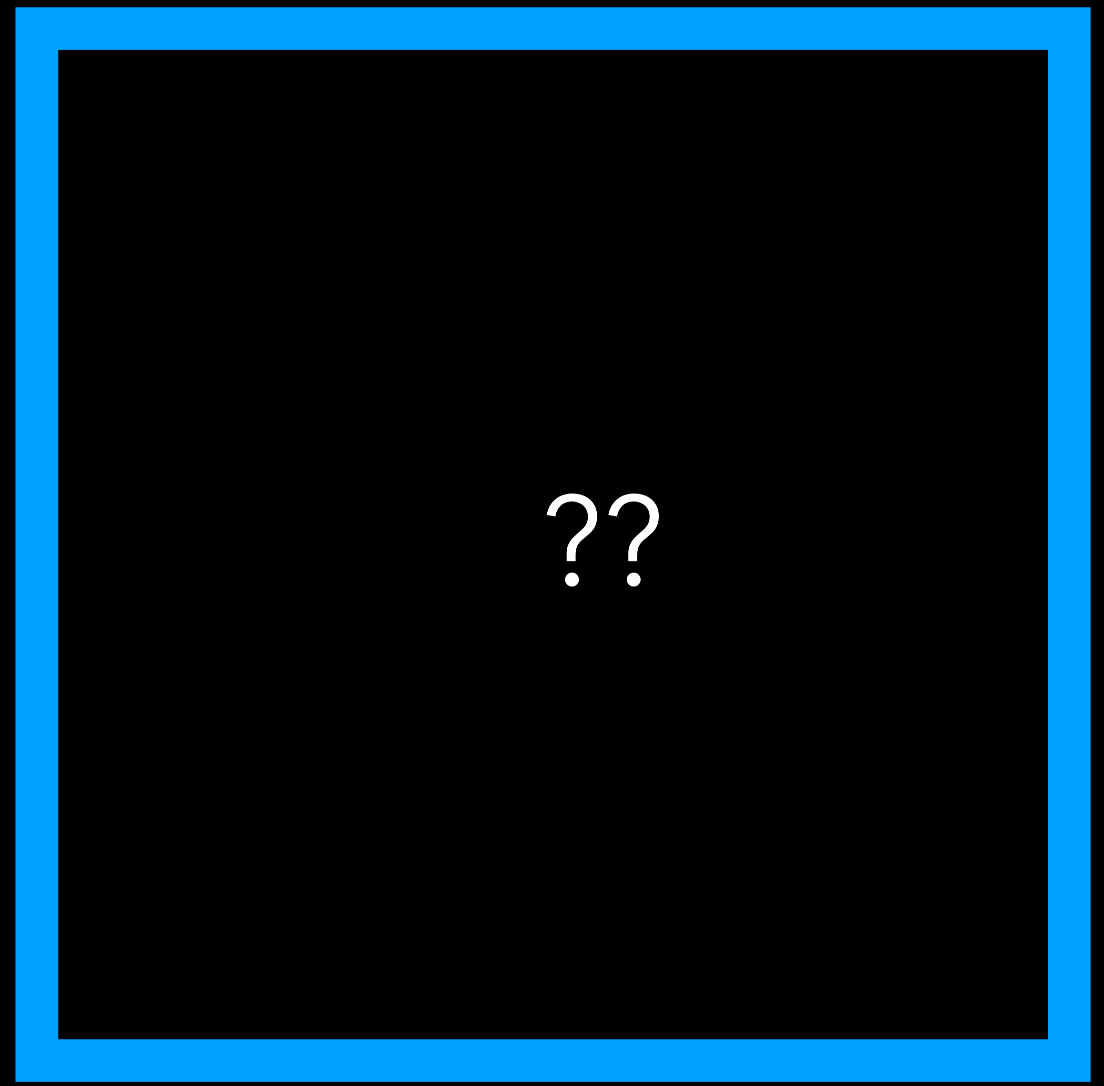


exact geometry:

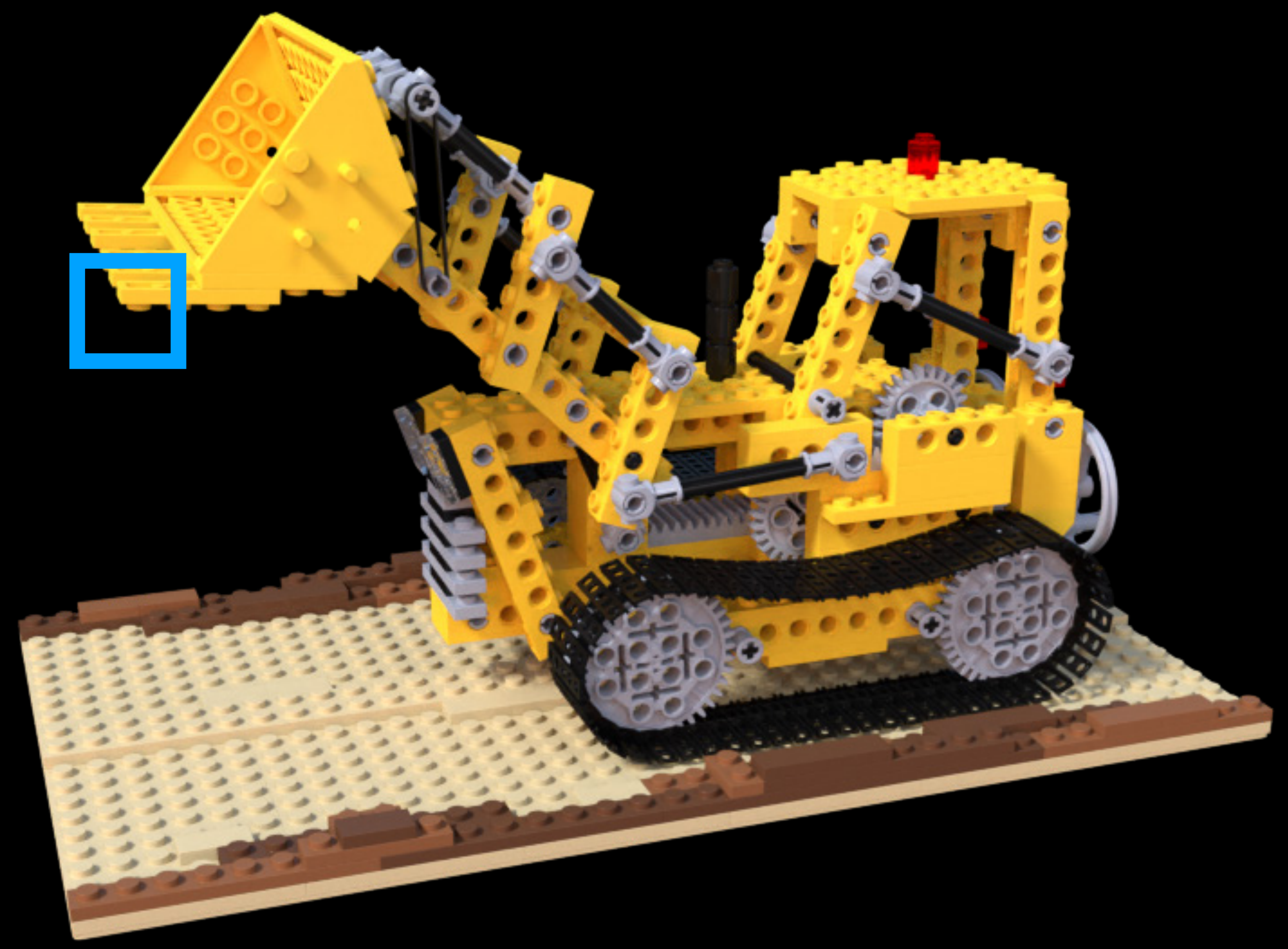
binary vacancy $\{0, 1\}$

$\mathbb{E}\Big[$  $\Big] =$

stochastic geometry:

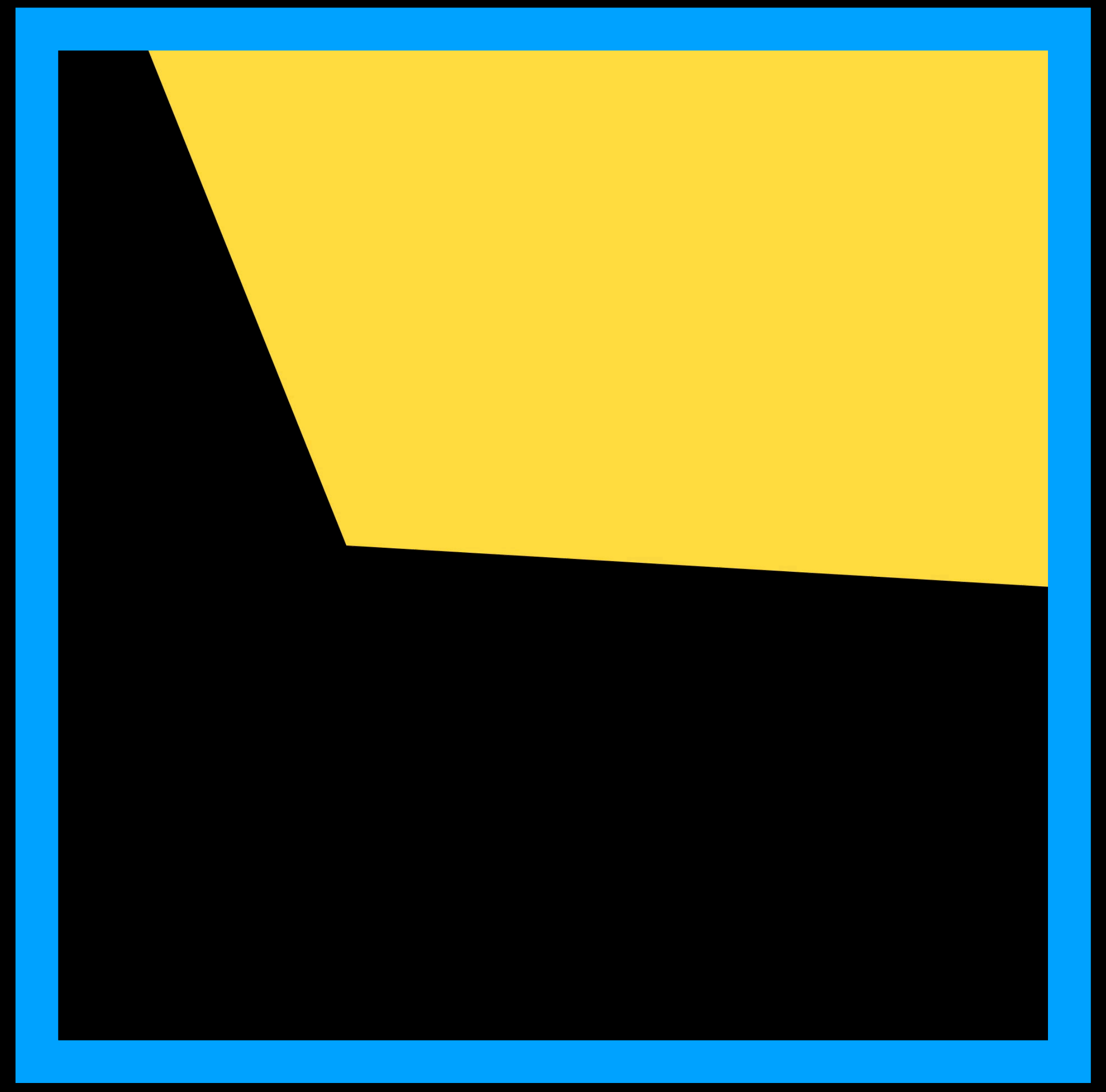


opaque solid model



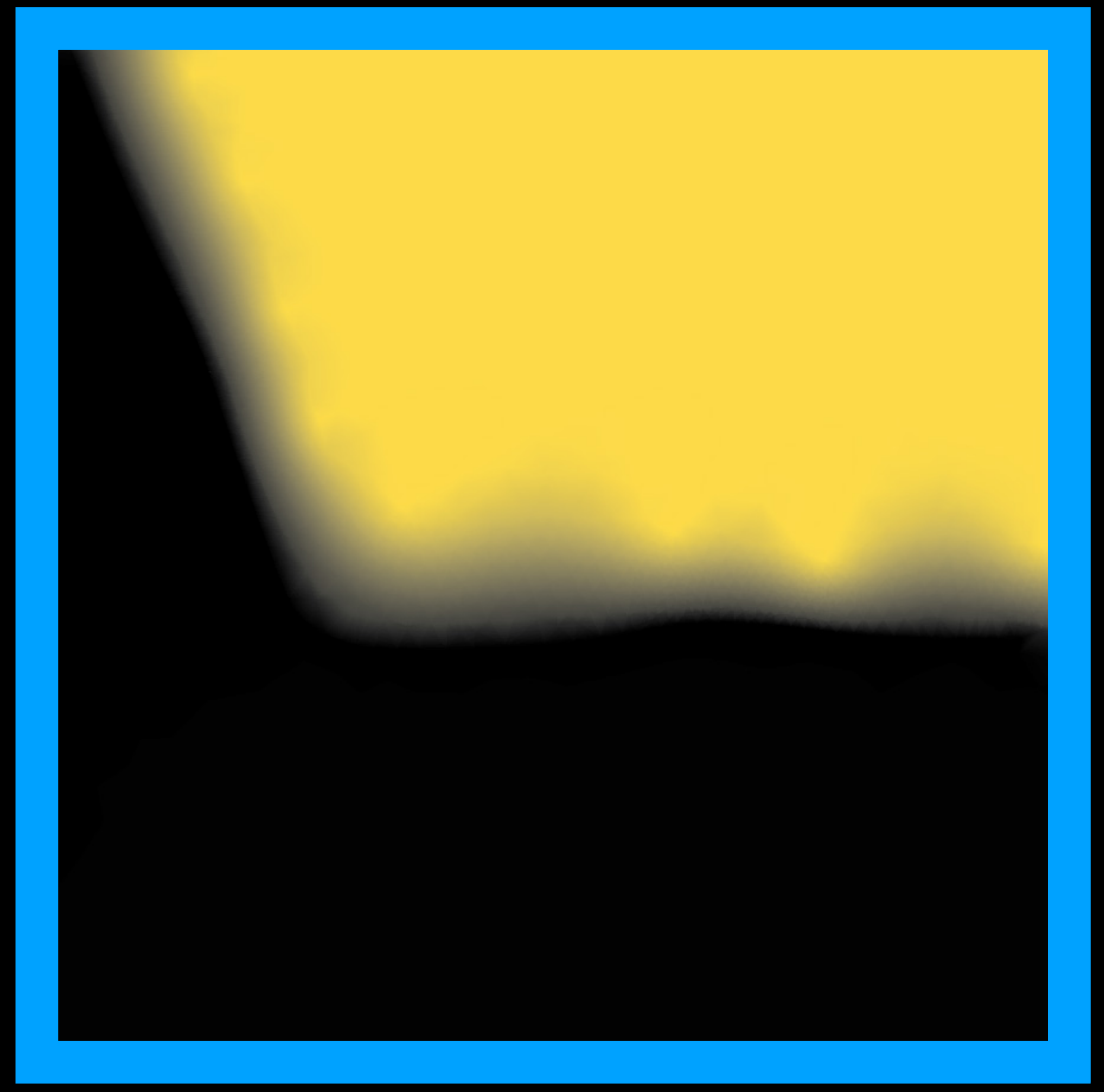
<u>exact geometry:</u>

binary vacancy $\{0, 1\}$

$\mathbb{E}\Big[$  $\Big] =$

<u>stochastic geometry:</u>

probabilistic vacancy $[0, 1]$



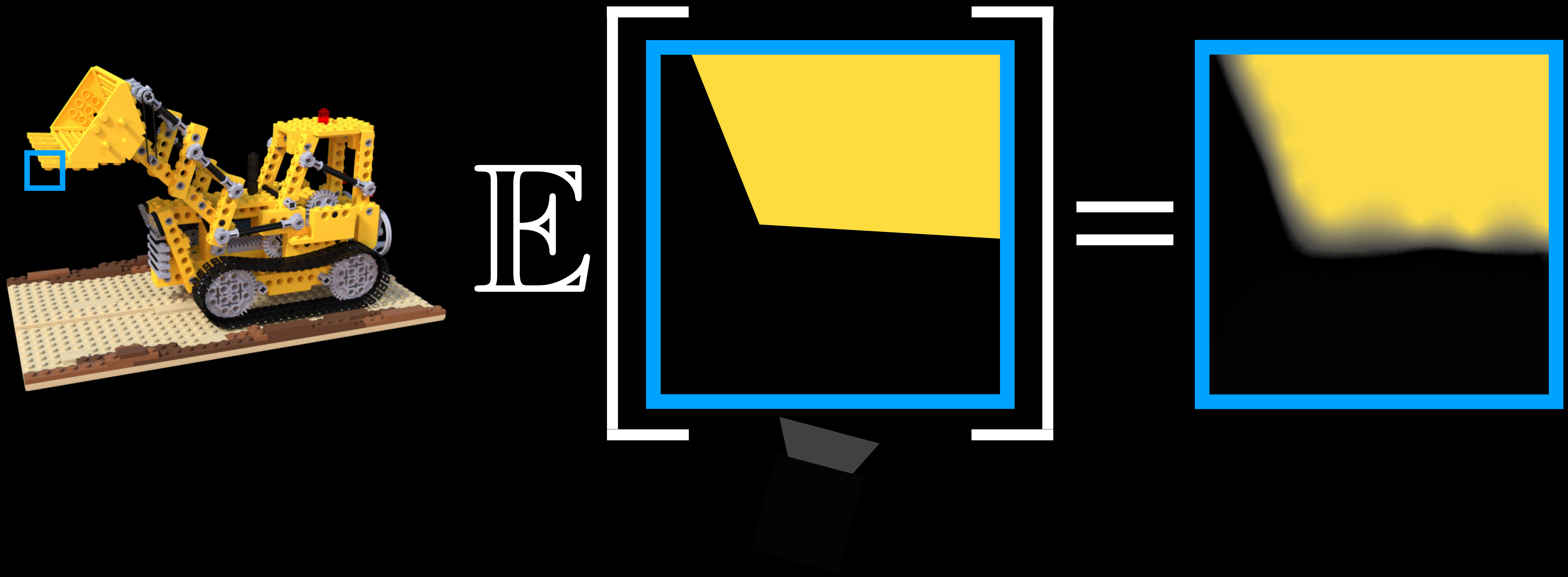
opaque solid model

<u>exact geometry:</u>

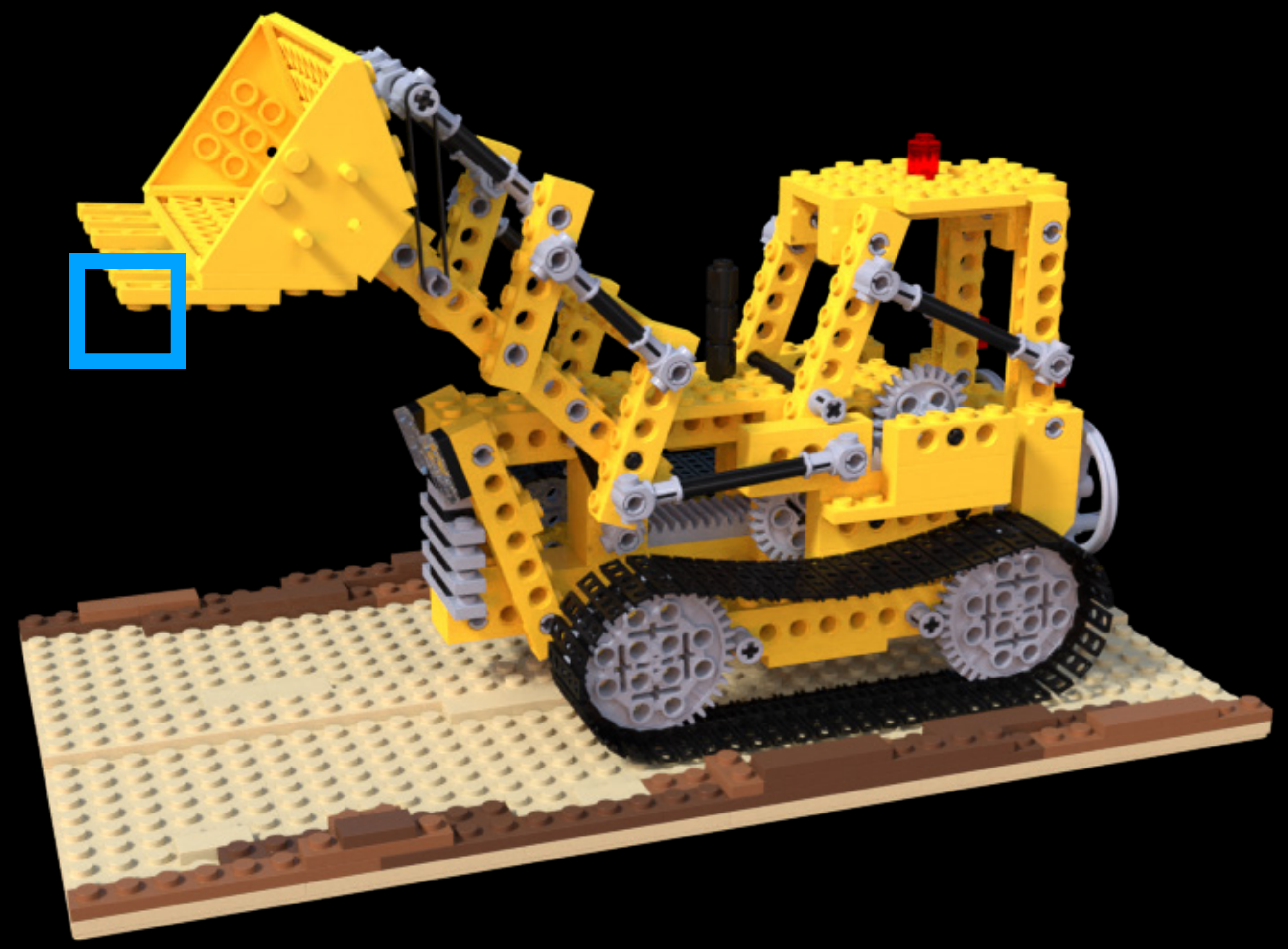
binary vacancy $\{0, 1\}$

<u>stochastic geometry:</u>

probabilistic vacancy $[0, 1]$



opaque solid model

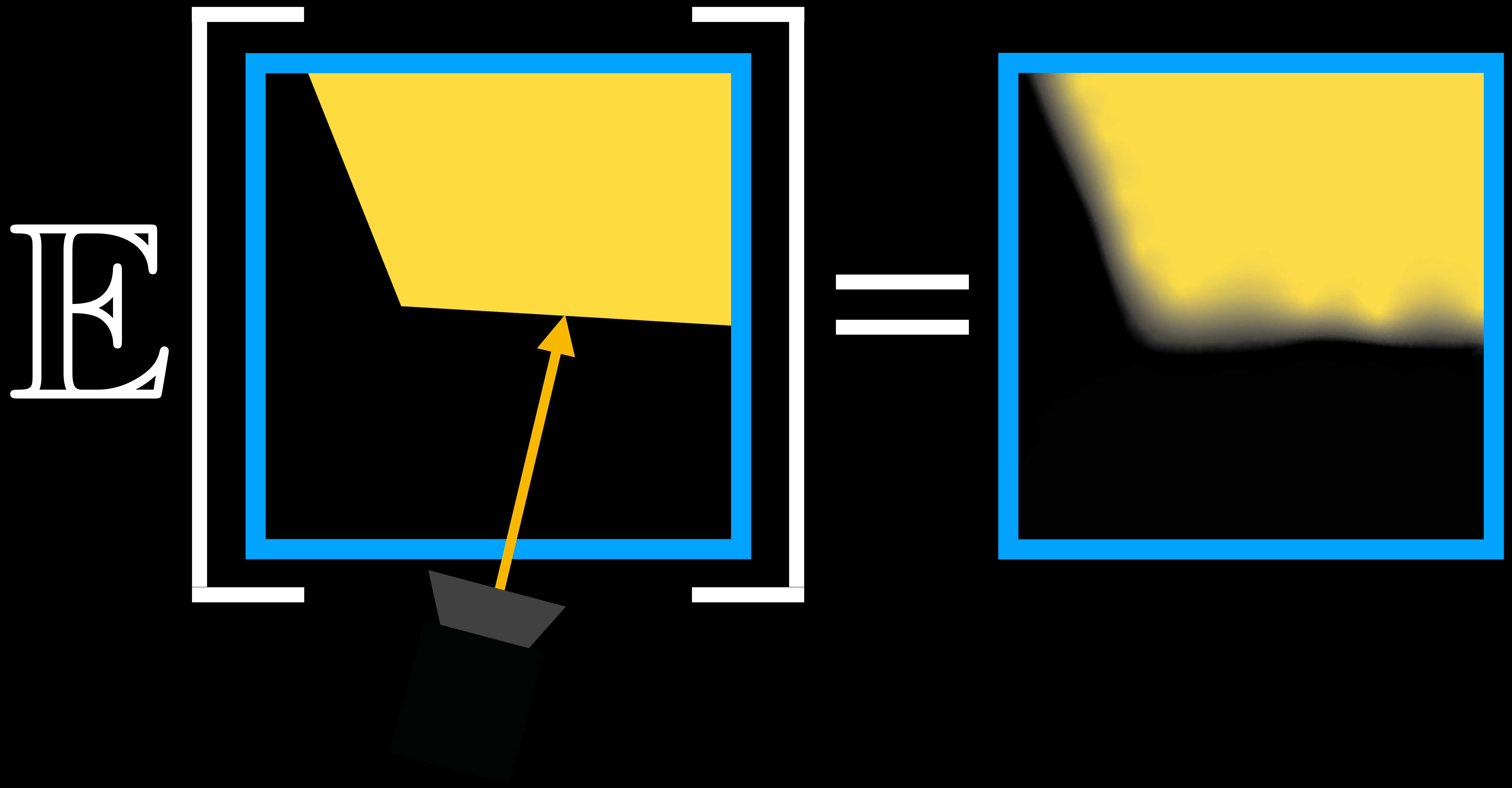


<u>exact geometry:</u>

binary vacancy $\{0, 1\}$

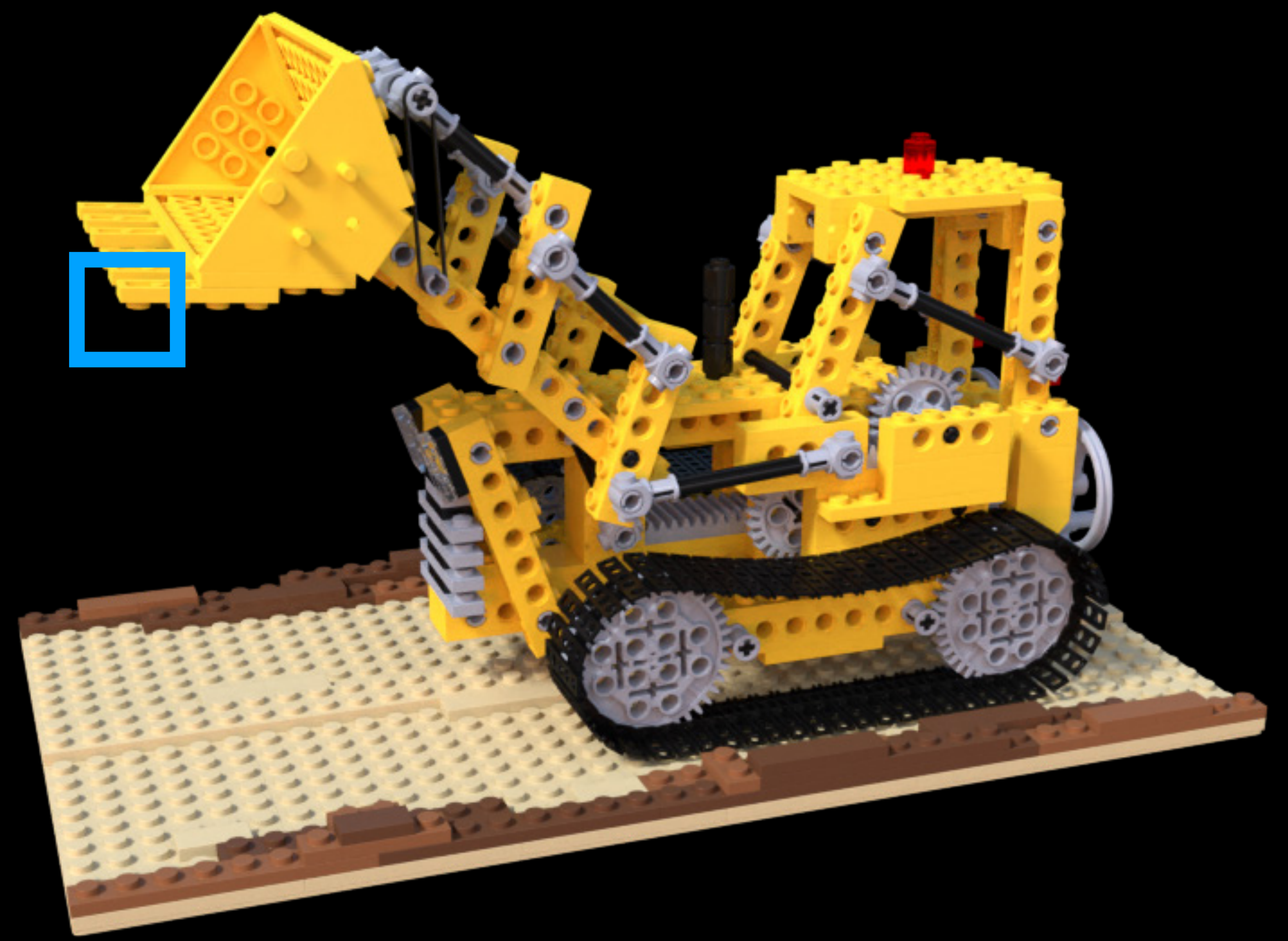
<u>stochastic geometry:</u>

probabilistic vacancy $[0, 1]$



deterministic intersection

opaque solid model

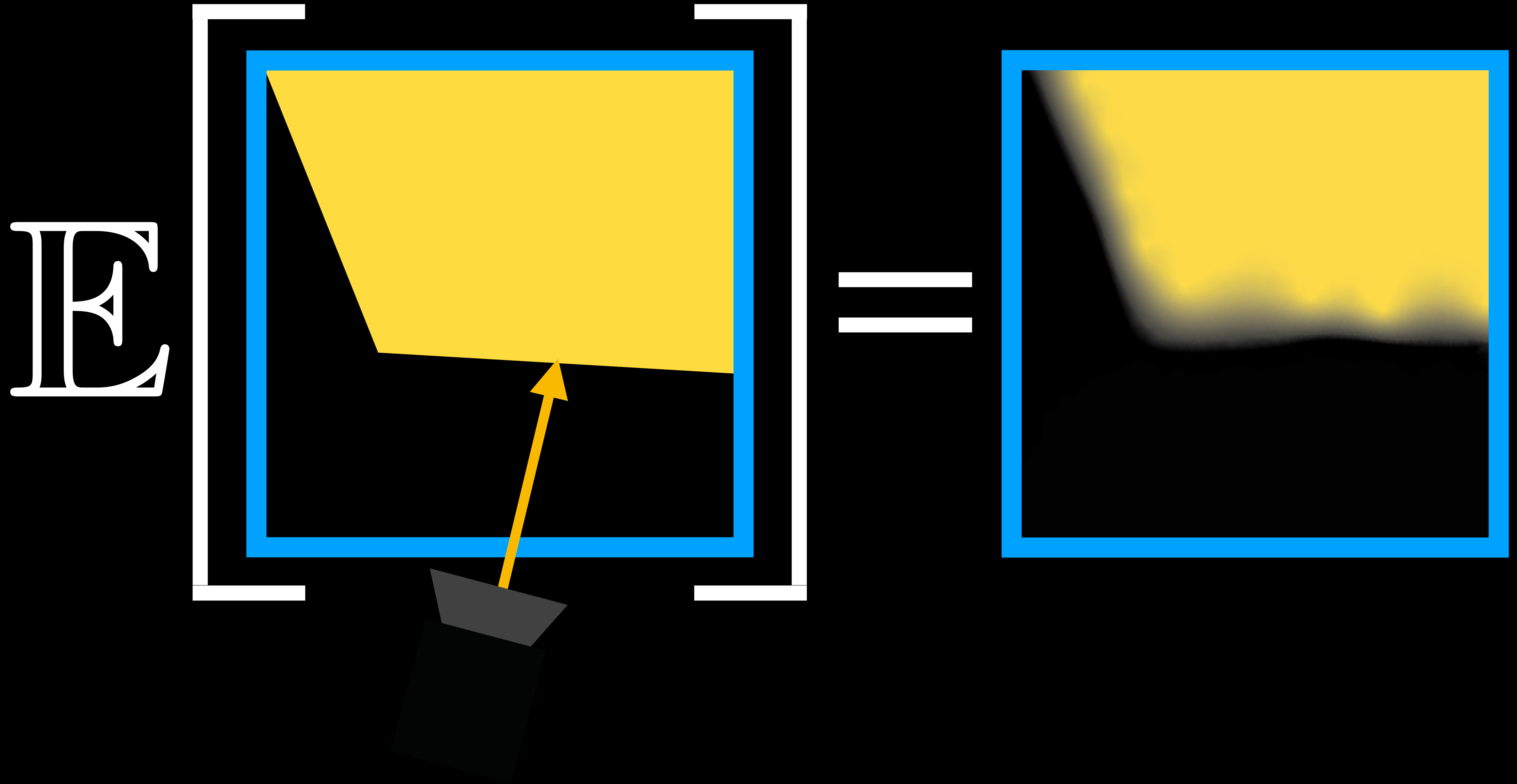


<u>exact geometry:</u>

binary vacancy $\{0, 1\}$

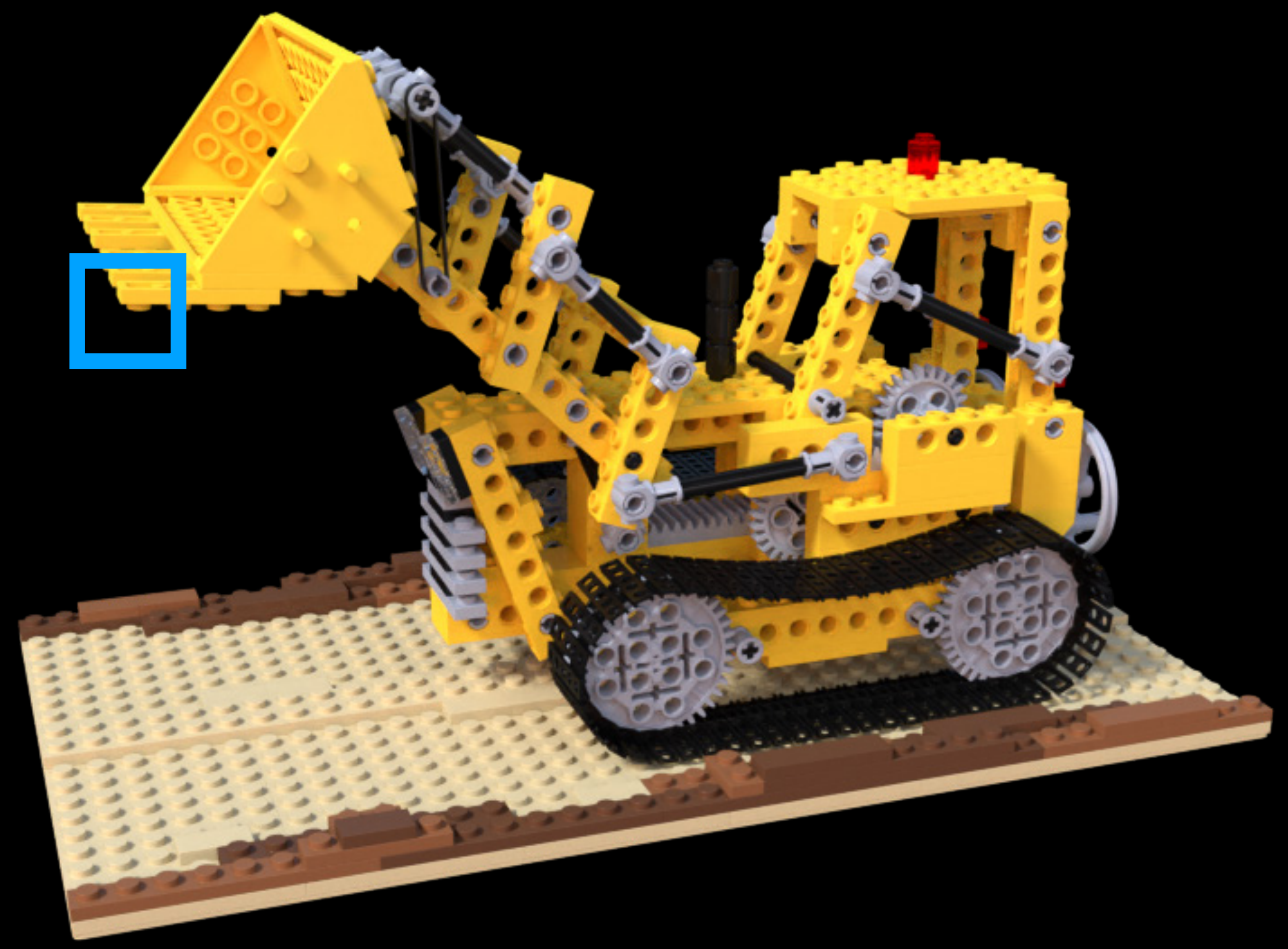
<u>stochastic geometry:</u>

probabilistic vacancy $[0, 1]$



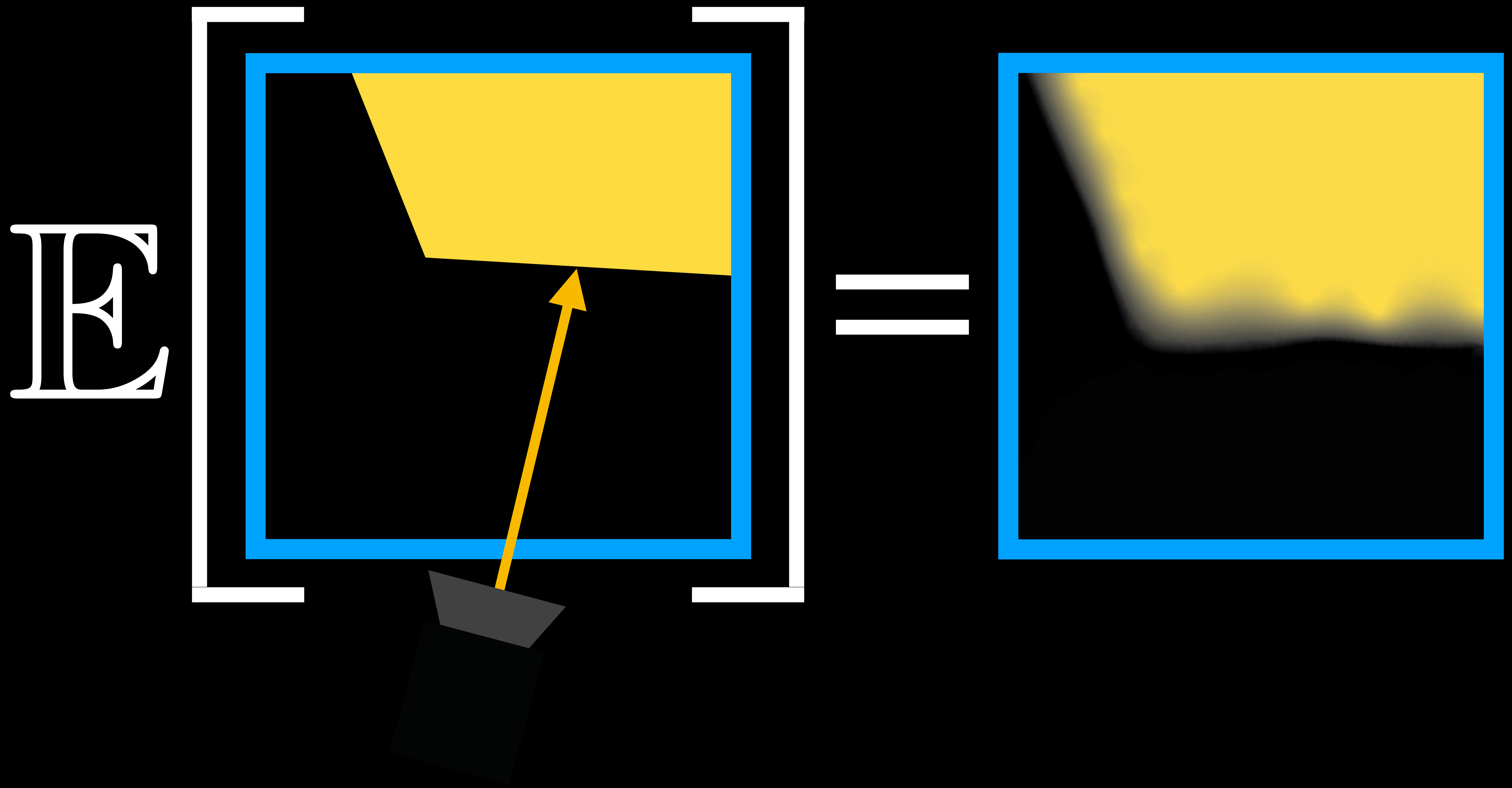
deterministic
intersection

opaque solid model



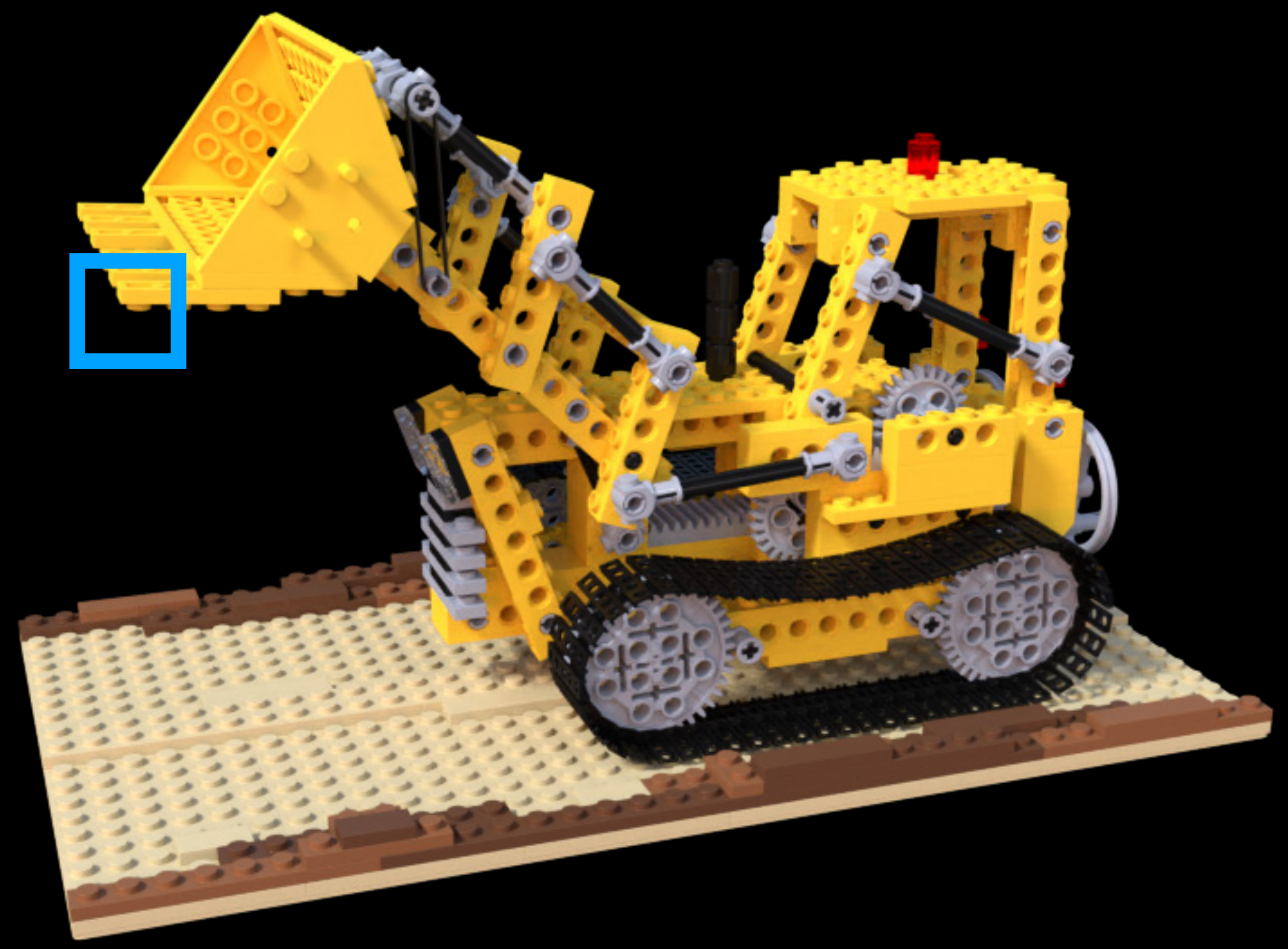
<u>exact geometry:</u> binary vacancy $\{0, 1\}$

<u>stochastic geometry:</u> probabilistic vacancy $[0, 1]$



deterministic intersection

opaque solid model

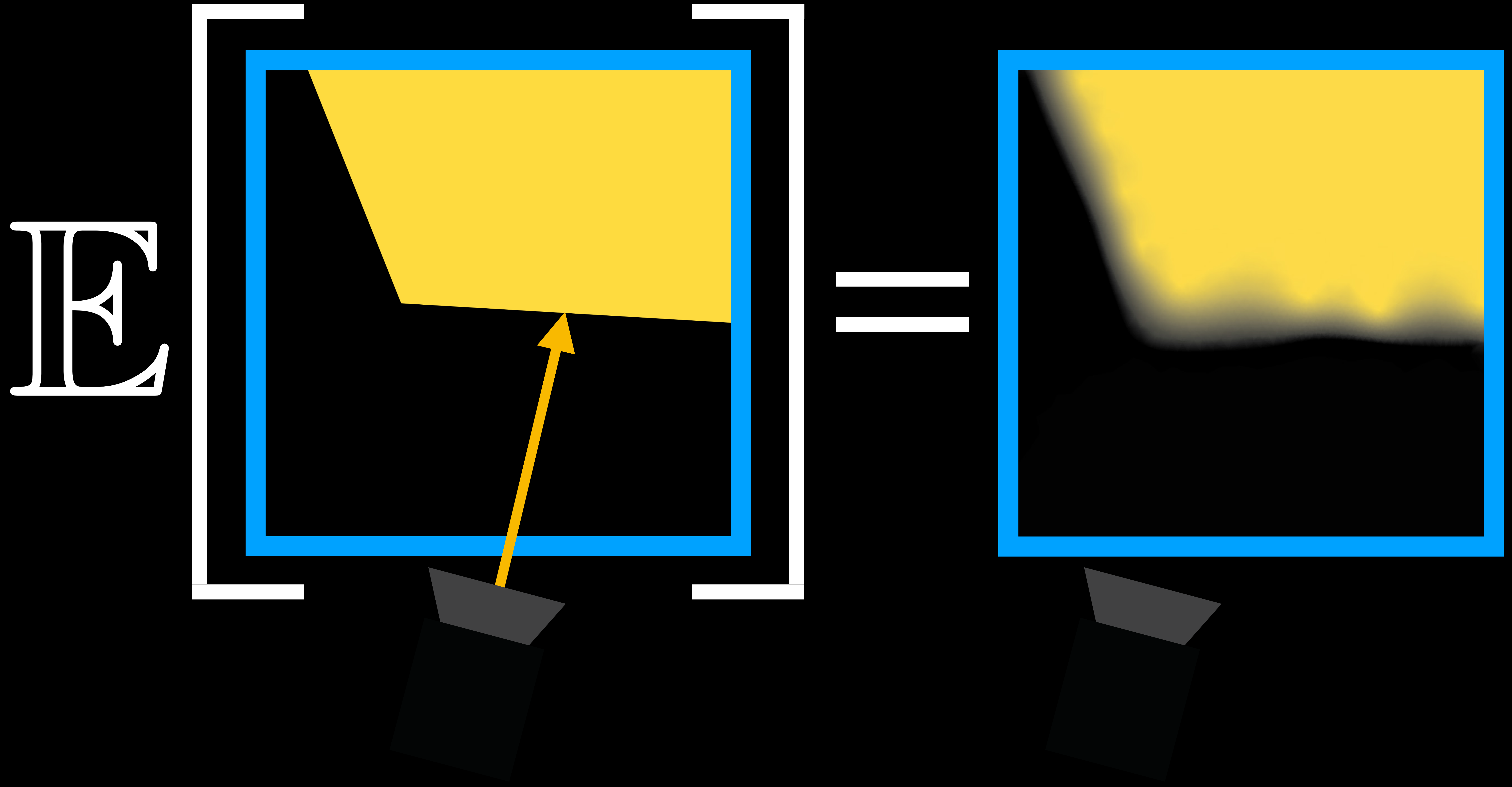


exact geometry:

binary vacancy $\{0, 1\}$

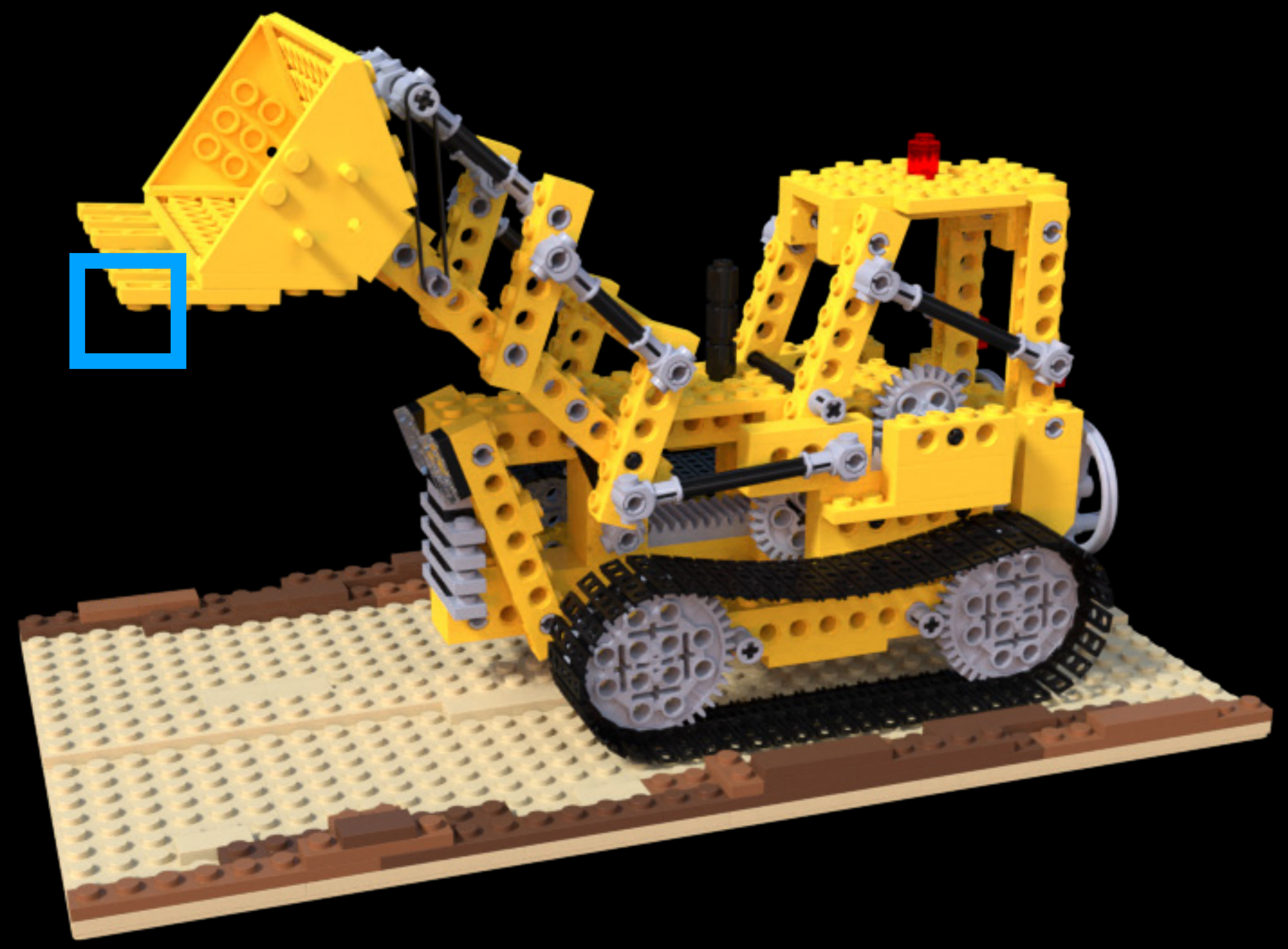
stochastic geometry:

probabilistic vacancy $[0, 1]$



deterministic intersection

opaque solid model

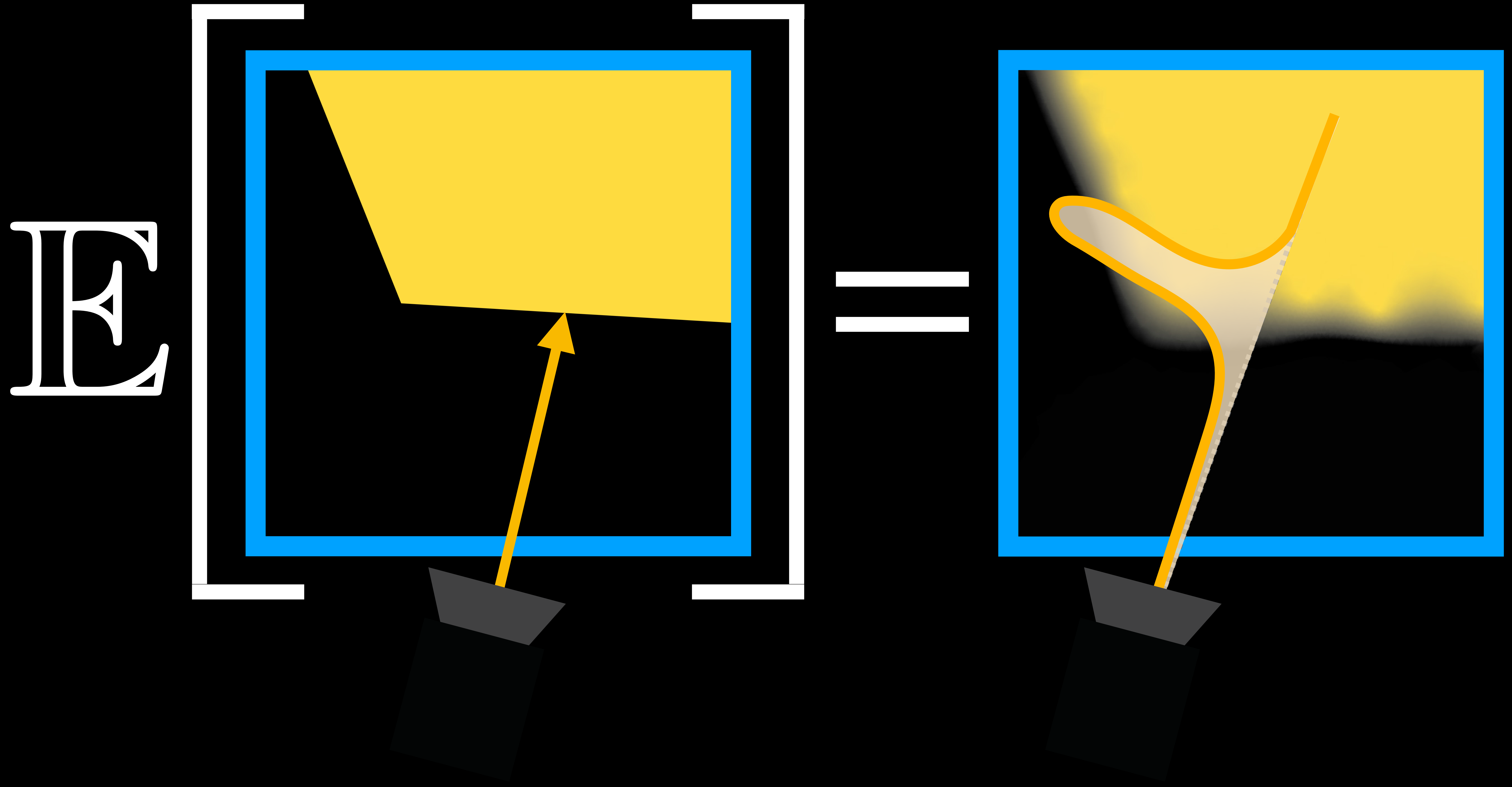


<u>exact geometry:</u>

binary vacancy $\{0, 1\}$

<u>stochastic geometry:</u>

probabilistic vacancy $[0, 1]$



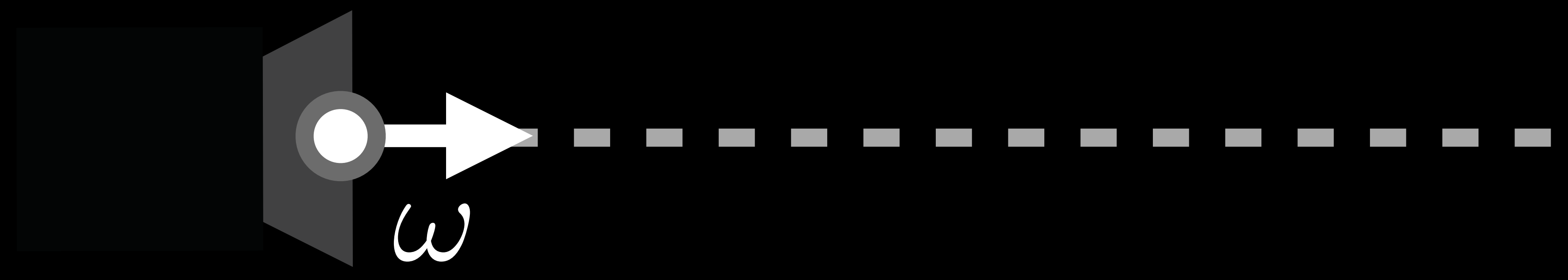
deterministic intersection

free-flight distribution

deriving free-flight for opaque solids

inside

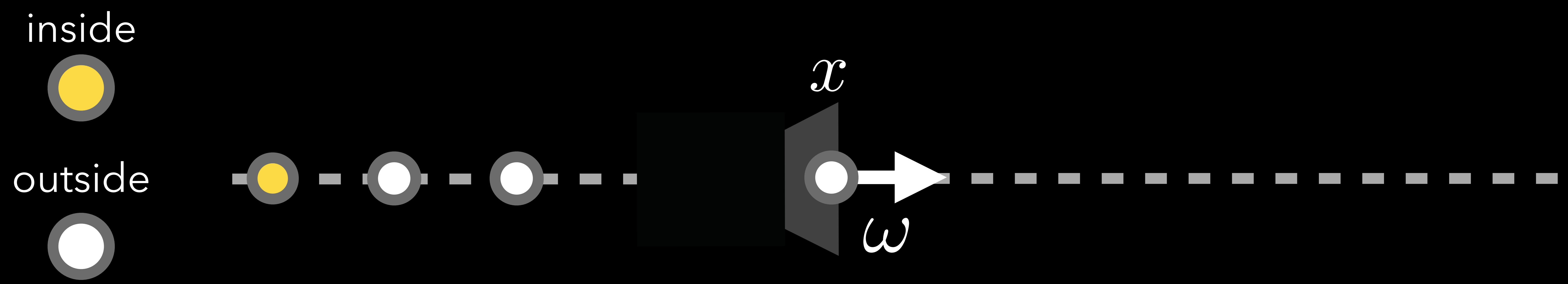
outside



$v(x)$

probabilistic vacancy $[0, 1]$

deriving free-flight for opaque solids

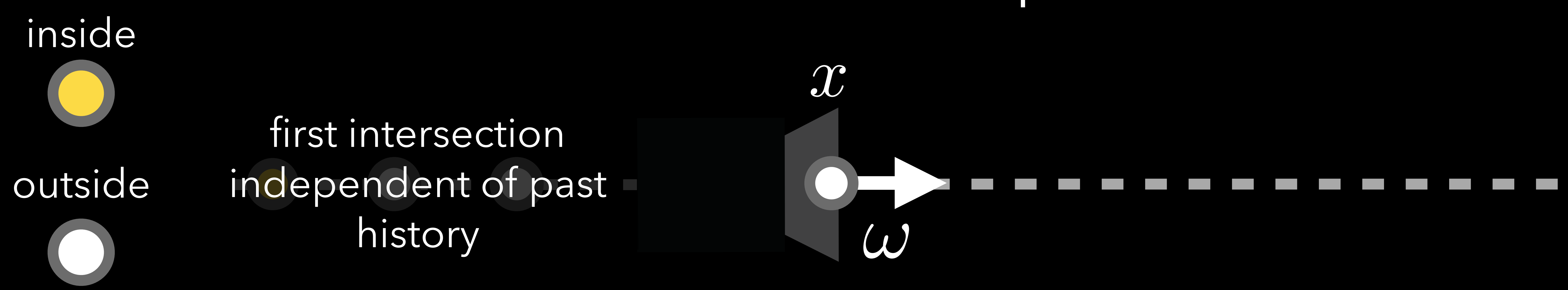


$v(x)$

probabilistic vacancy $[0, 1]$

deriving free-flight for opaque solids

Markov assumption

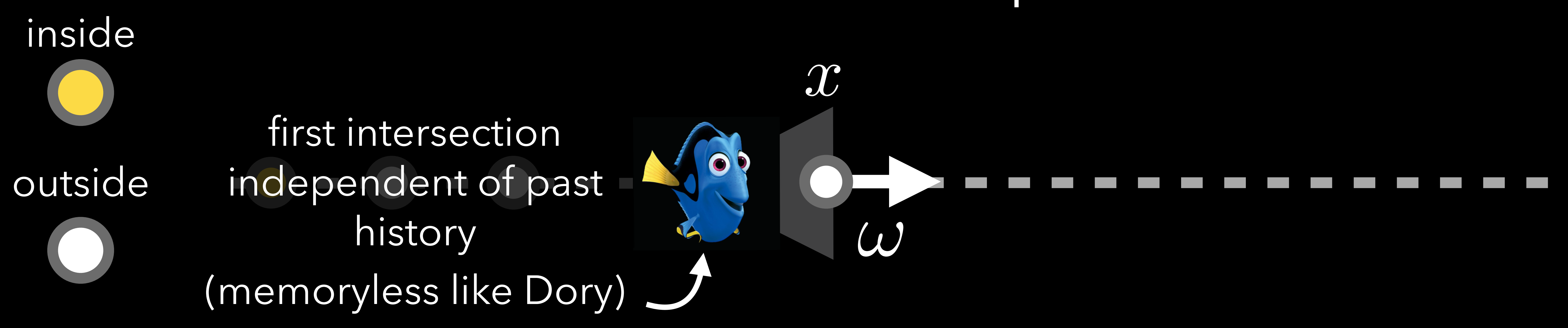


$v(x)$

probabilistic vacancy $[0, 1]$

deriving free-flight for opaque solids

Markov assumption

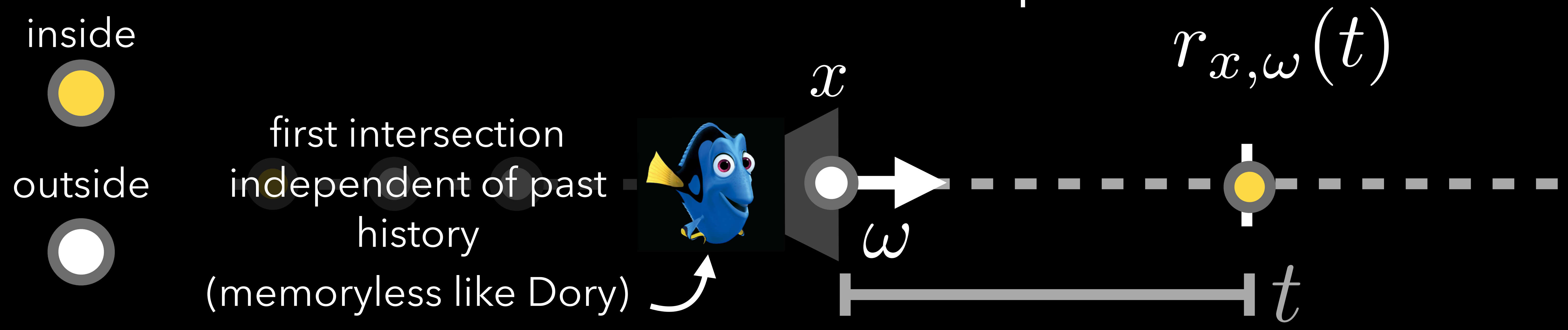


$v(x)$

probabilistic vacancy $[0, 1]$

deriving free-flight for opaque solids

Markov assumption



$v(x)$

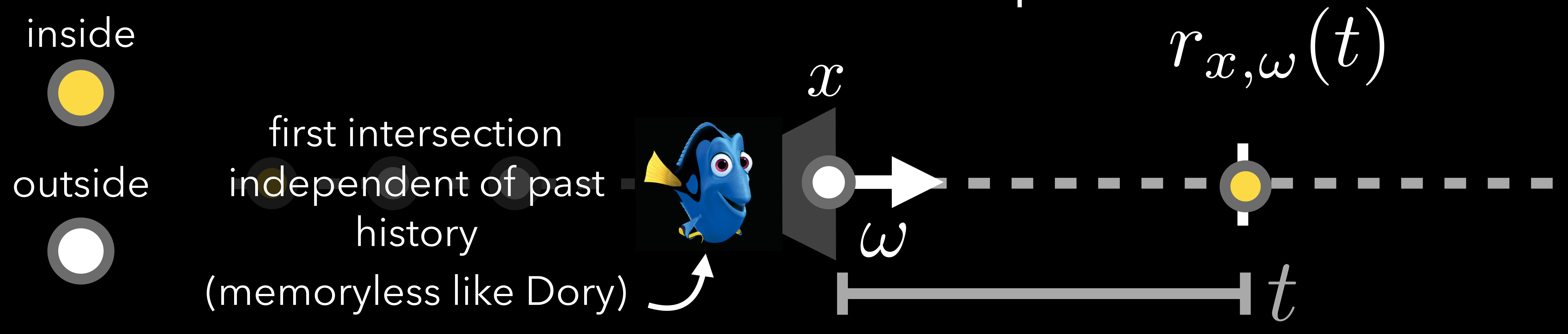
probabilistic vacancy $[0, 1]$

exponential free-flight

$$p^{\mathrm{ff}}_{x,\omega}(t) = \sigma(r_{x,\omega}(t), \omega) \exp\left(- \int_0^t \sigma(r_{x,\omega}(s), \omega) \; \mathrm{d}s \right)$$

deriving free-flight for opaque solids

Markov assumption



$v(x)$

probabilistic vacancy $[0, 1]$

exponential free-flight

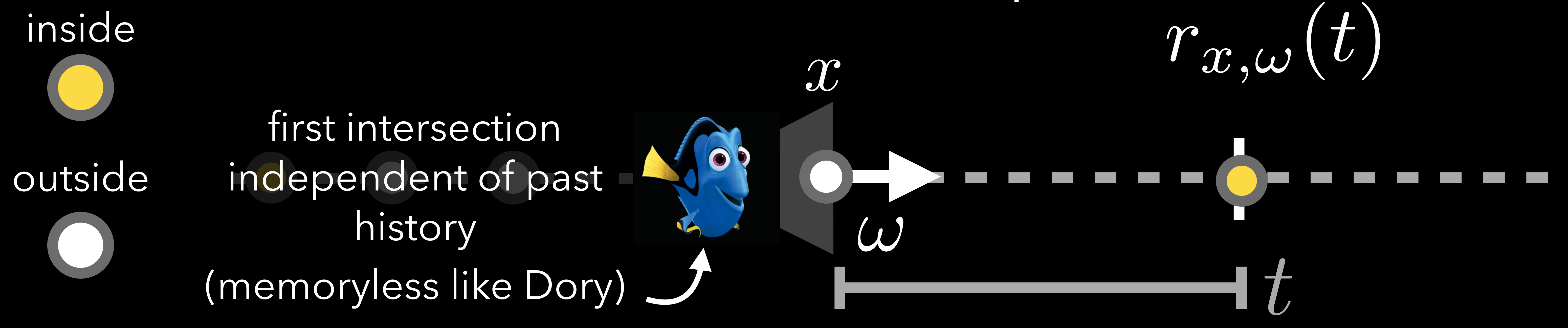
$$p^{\text{ff}}_{x,\omega}(t) = \sigma(r_{x,\omega}(t), \omega) \exp\left(-\int_0^t \sigma(r_{x,\omega}(s), \omega)\, \mathrm{d}s\right)$$

attenuation coefficient

$$\sigma(x, \omega) = \frac{|\omega \cdot \nabla v(x)|}{v(x)}$$

deriving free-flight for opaque solids

Markov assumption



$v(x)$

probabilistic vacancy $[0, 1]$

proof in paper

- Markov assumption allows us to define transition density in terms of Kolmogorov equations
- Reversibility and **reciprocity** constraints for physically valid free-flight distributions give a unique attenuation coefficient

exponential free-flight

$$p^{\mathrm{ff}}_{x,\omega}(t) = \sigma(r_{x,\omega}(t), \omega) \exp\left(-\int_0^t \sigma(r_{x,\omega}(s), \omega)\, \mathrm{d}s\right)$$

attenuation coefficient

$$\sigma(x, \omega) = \frac{|\omega \cdot \nabla v(x)|}{v(x)}$$

ensuring physically valid free-flight distribution

visibility

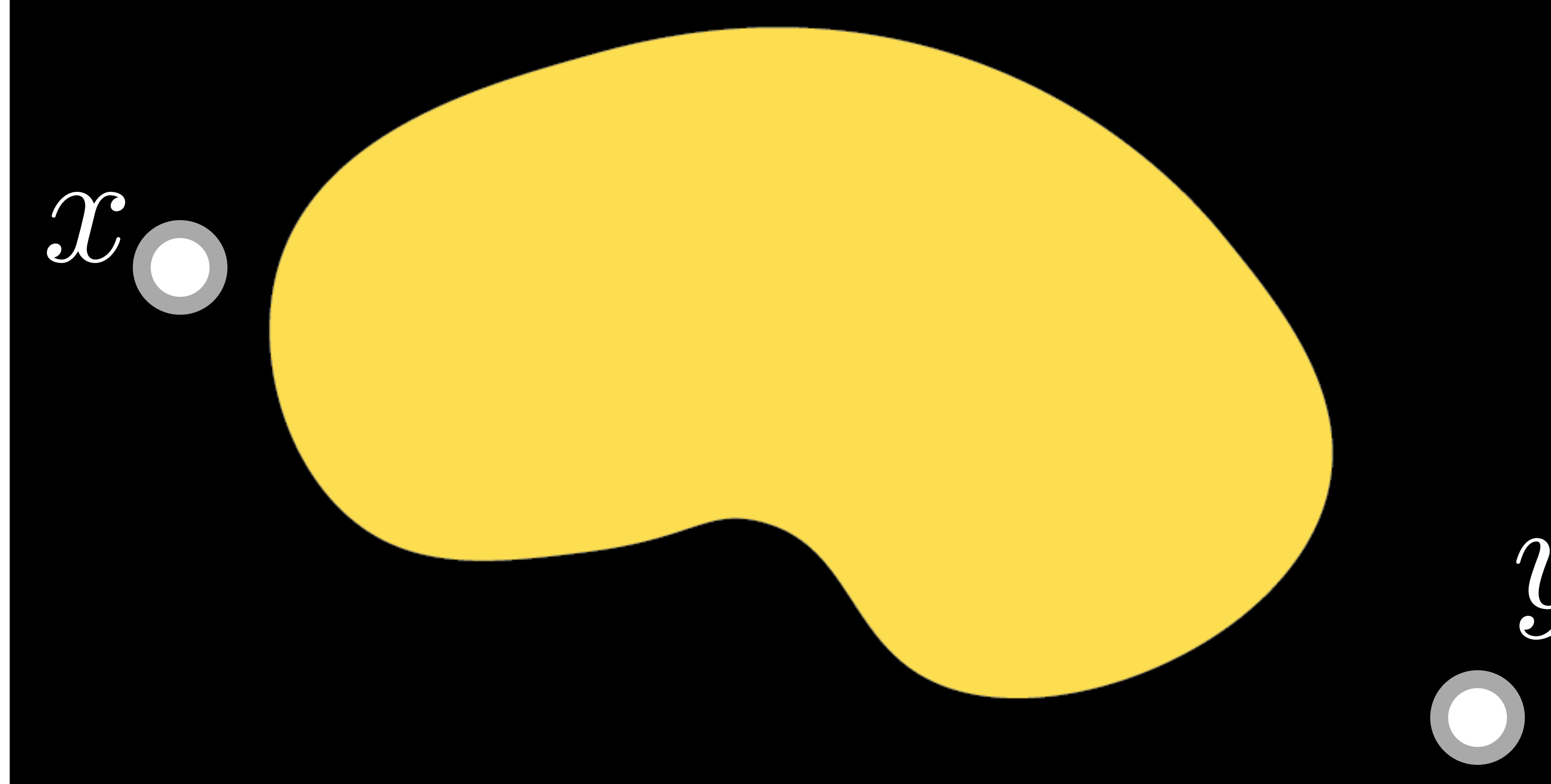
$$V(x,y) = \begin{cases} 1 & \text{if no intersections} \\ 0 & \text{otherwise} \end{cases}$$

transmittance

$$T(x,y) = 1 - \int_0^{\|x-y\|} p^{\mathrm{ff}}_{x,\omega}(t)\ \mathrm{d}t$$

$$T(y,x) = 1 - \int_0^{\|x-y\|} p^{\mathrm{ff}}_{y,-\omega}(t)\mathrm{d}t$$

$\mathbb{E}\Big[$



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ensuring physically valid free-flight distribution

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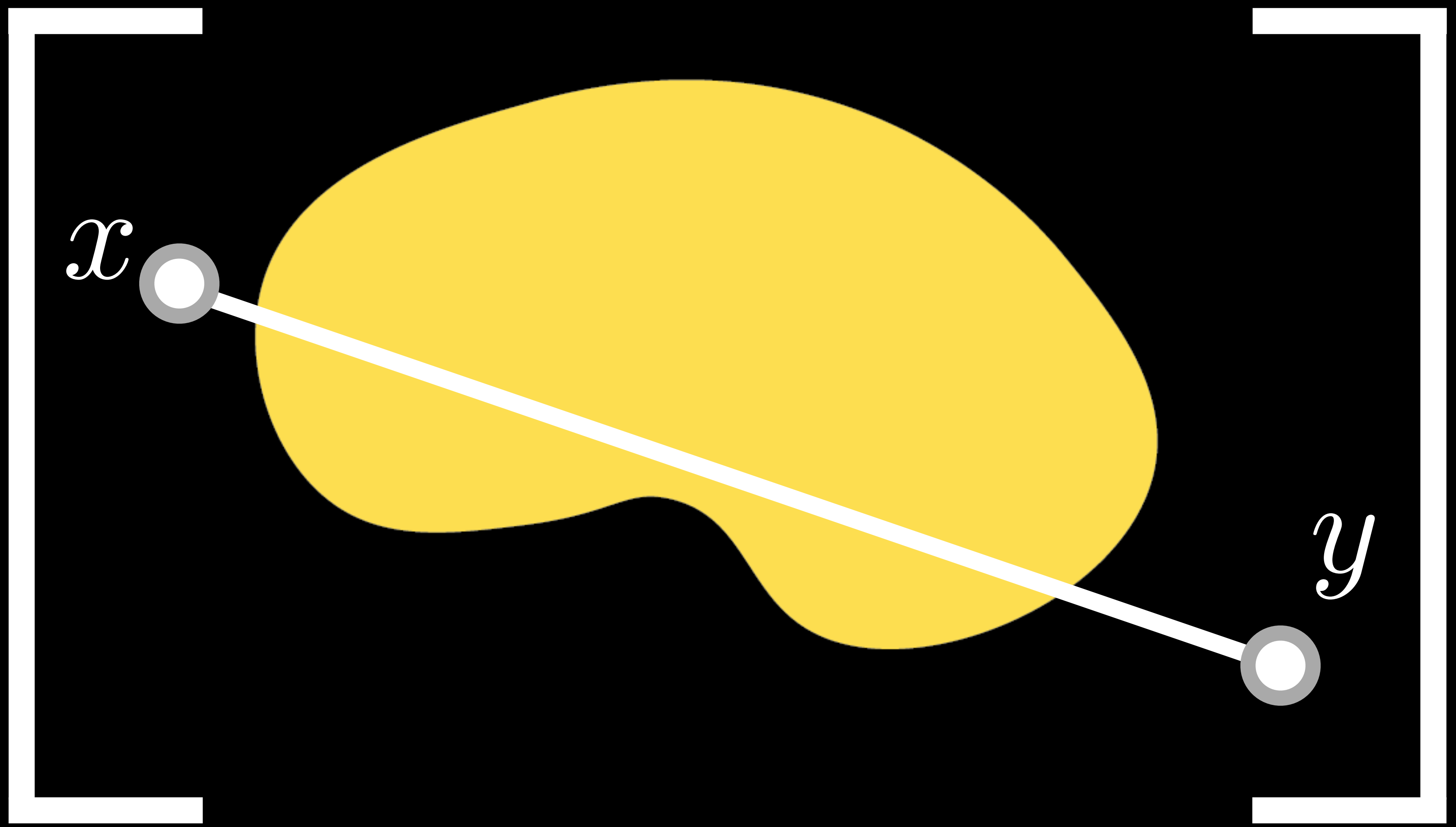
$\mathbb{E}$ [] = 

$V(x,y) = V(y,x)$

visibility is reciprocal

ensuring physically valid free-flight distribution

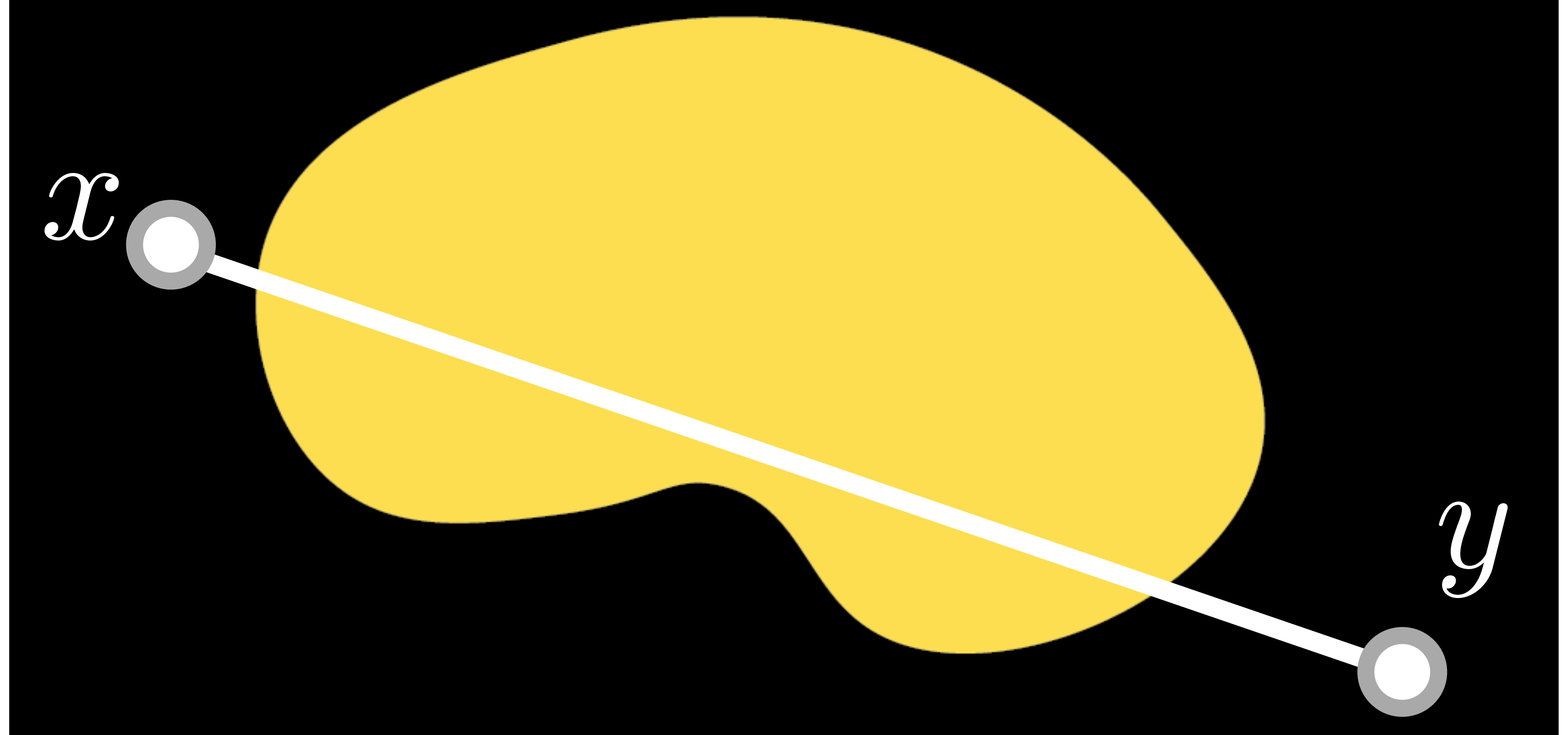
visibility

$$V(x,y) = \begin{cases} 1 & \text{if no intersections} \\ 0 & \text{otherwise} \end{cases}$$

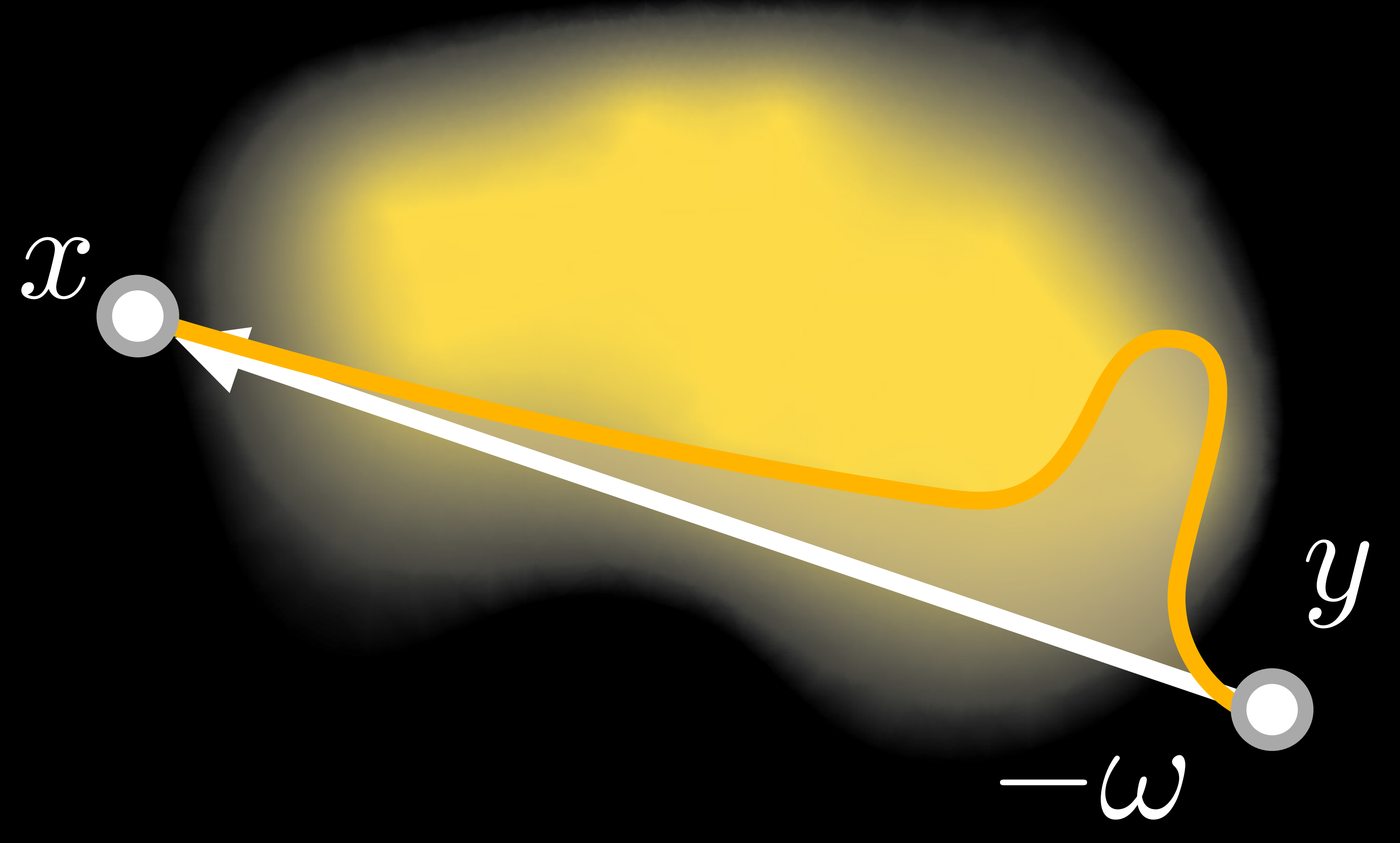
transmittance

$$T(x,y) = 1 - \int_0^{\|x-y\|} p^{\text{ff}}_{x,\omega}(t)\ \mathrm{d}t$$

$$T(y,x) = 1 - \int_0^{\|x-y\|} p^{\text{ff}}_{y,-\omega}(t)\mathrm{d}t$$

$\mathbb{E}\Big[$  $\Big]$

$=$



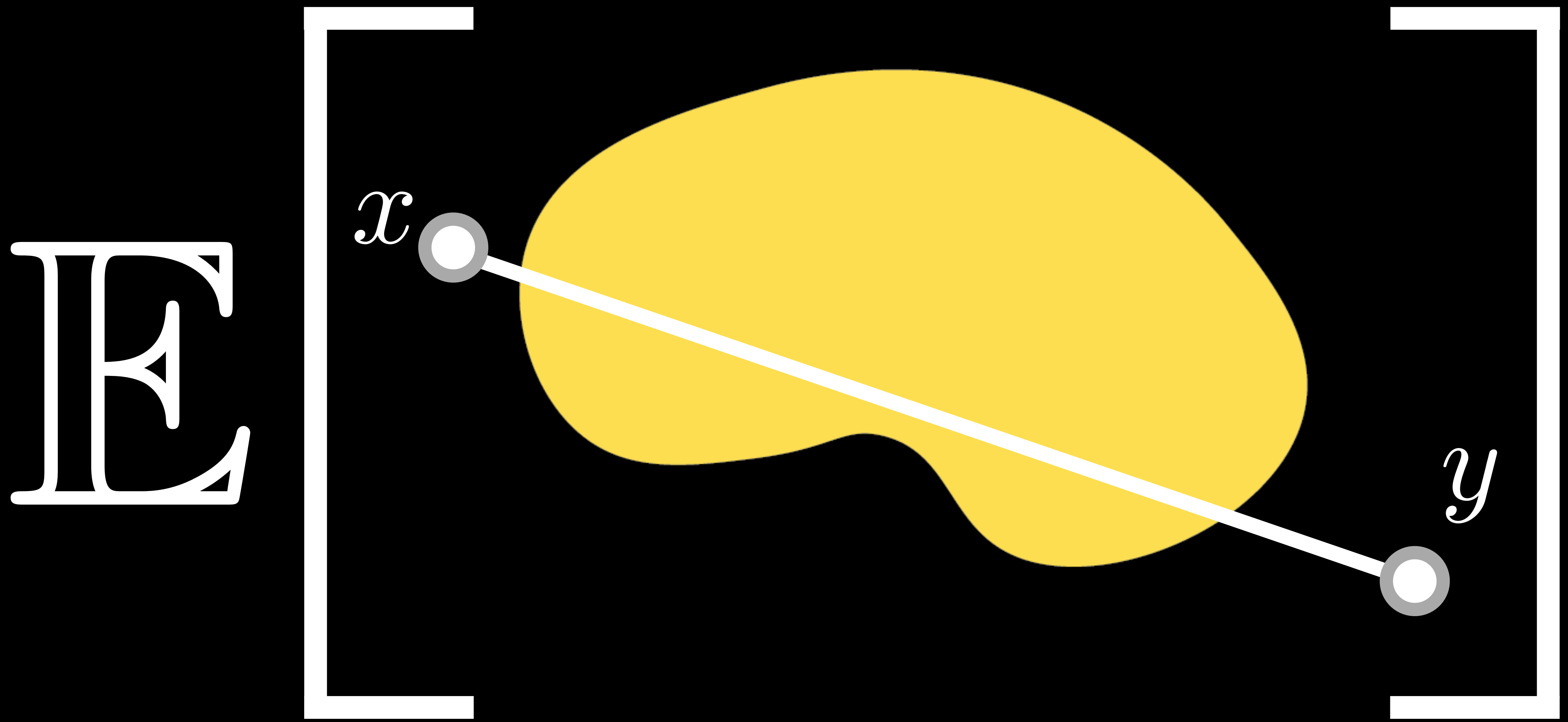
$$V(x,y) = V(y,x)$$

visibility is reciprocal

ensuring physically valid free-flight distribution

visibility

$$V(x,y) = \begin{cases} 1 & \text{if no intersections} \\ 0 & \text{otherwise} \end{cases}$$



$V(x,y) = V(y,x)$

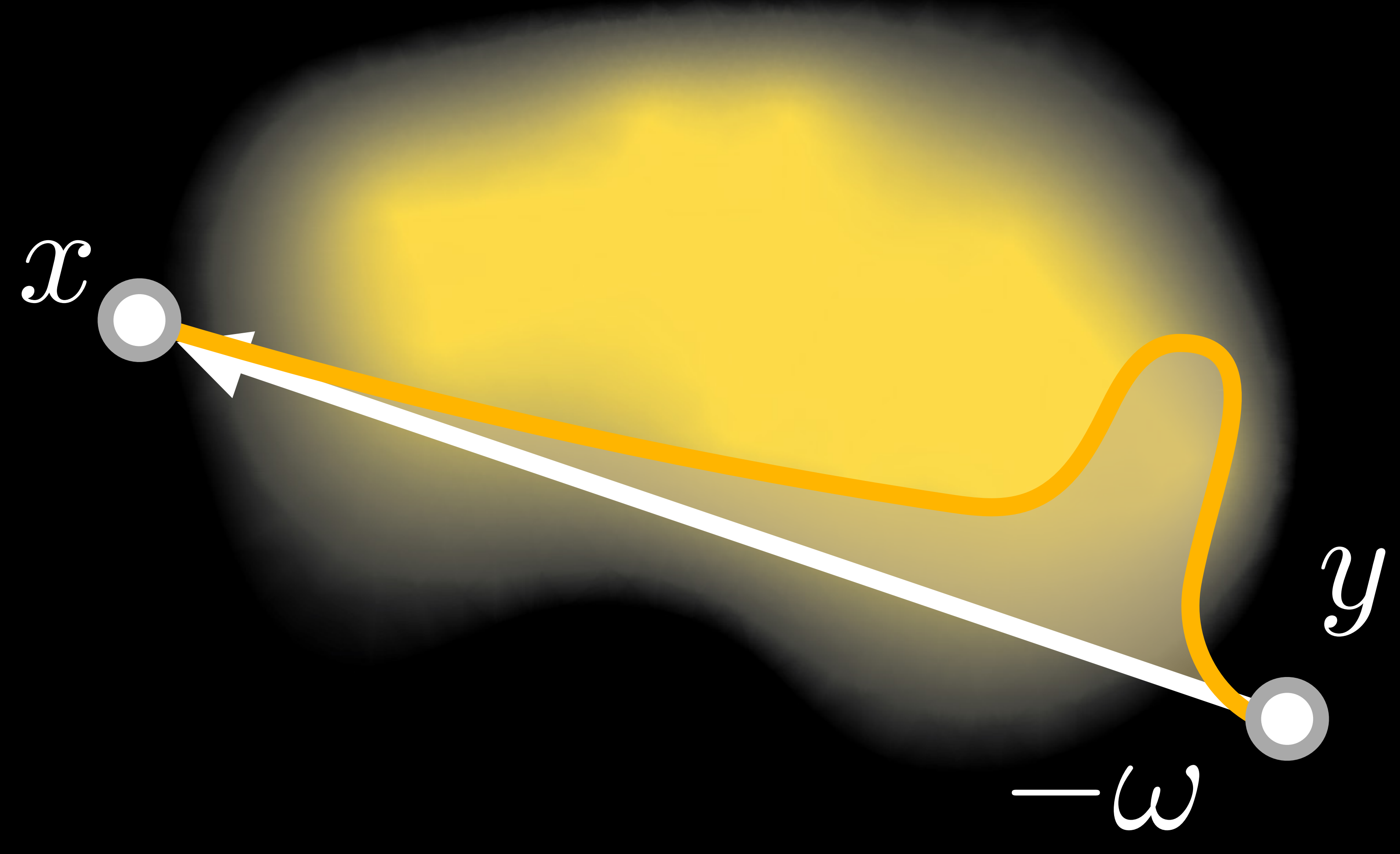
visibility is reciprocal

transmittance

$$E[V(x,y)] = 1 - \int_0^{\|x-y\|} p^{\text{ff}}_{x,\omega}(t)\ \text{d}t$$

$$E[V(y,x)] = 1 - \int_0^{\|x-y\|} p^{\text{ff}}_{y,-\omega}(t)\text{d}t$$

=



$E[V(x,y)] = E[V(y,x)]$

transmittance should be reciprocal

ensuring physically valid free-flight distribution

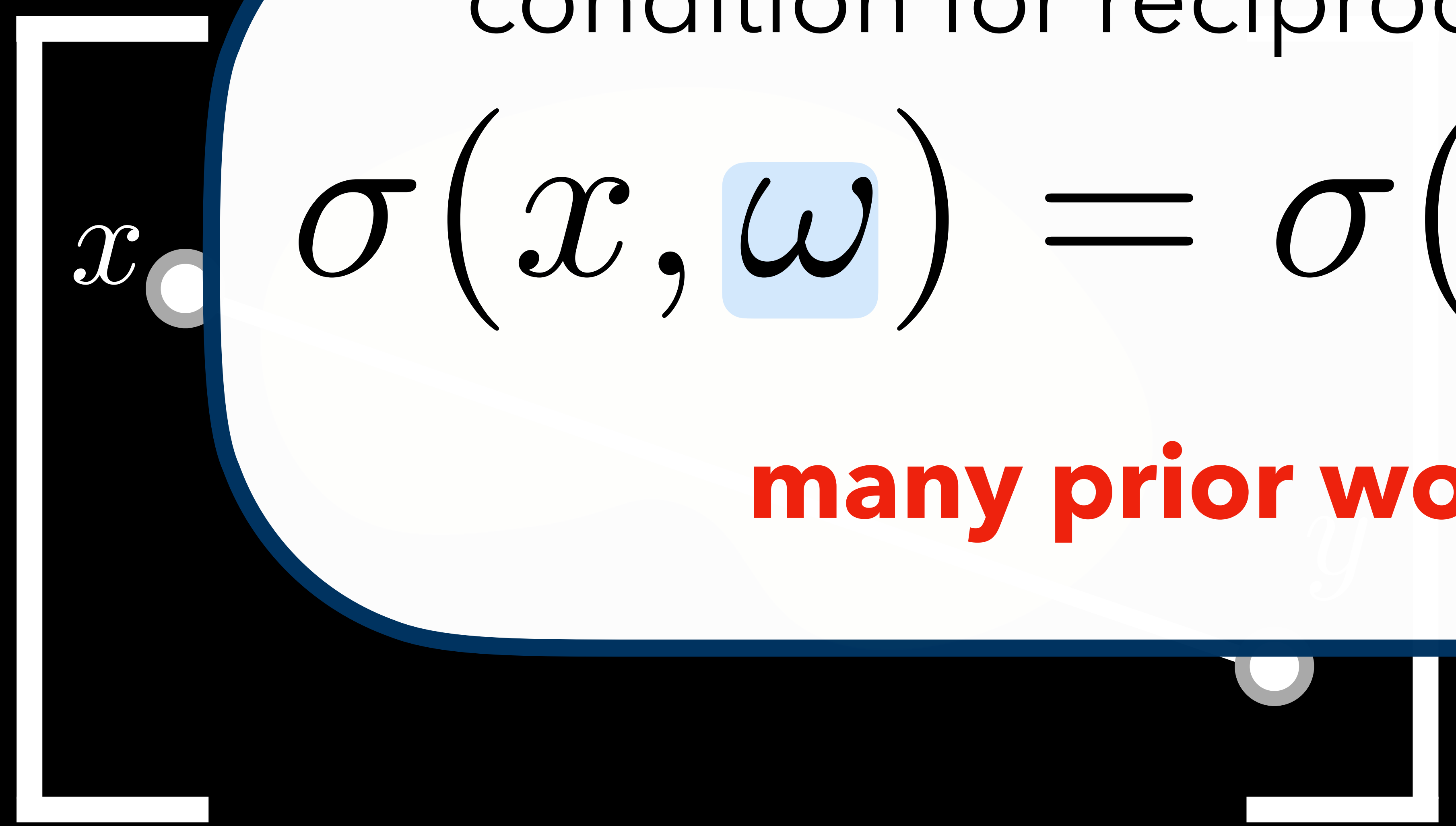
visibility

$$V(x,y) = \begin{cases} 1 & \text{if no intersections} \\ 0 & \text{otherwise} \end{cases}$$

transmittance

$$E[V(x,y)] = 1 - \int_0^{\|x-y\|} p^{\text{ff}}_{x,\omega}(t)\ \mathrm{d}t$$

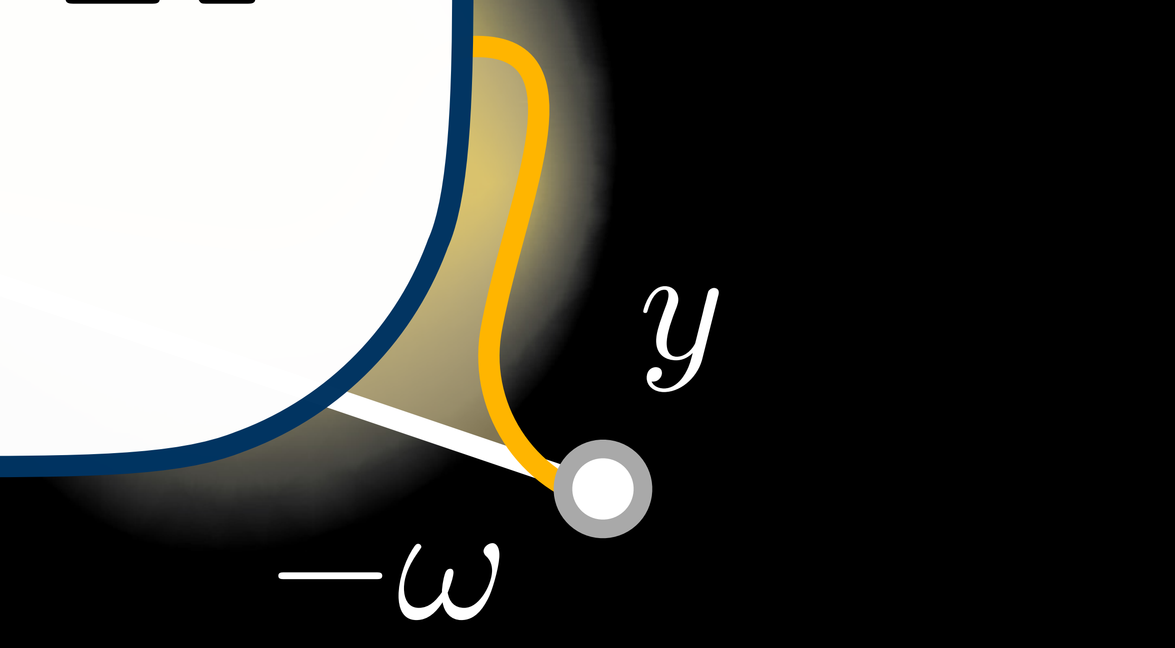
$$E[V(y,x)] = 1 - \int_0^{\|x-y\|} p^{\text{ff}}_{y,-\omega}(t)\mathrm{d}t$$

$\mathbb{E}$ 

condition for reciprocal exponential transmittance:

$$\sigma(x,\omega) = \sigma(x,-\omega) \quad \forall x \in \mathbb{R}^3$$

many prior works violate reciprocity



$$V(x,y) = V(y,x)$$

visibility is reciprocal

$$E[V(x,y)] = E[V(y,x)]$$

transmittance should be reciprocal

understanding attenuation for opaque solids

$$\sigma(x, \omega) = \frac{|\omega \cdot \nabla v(x)|}{v(x)}$$

understanding attenuation for opaque solids

density　　projected area　　　　normals

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

$$n(x) = \frac{\nabla v(x)}{\|v(x)\|}$$

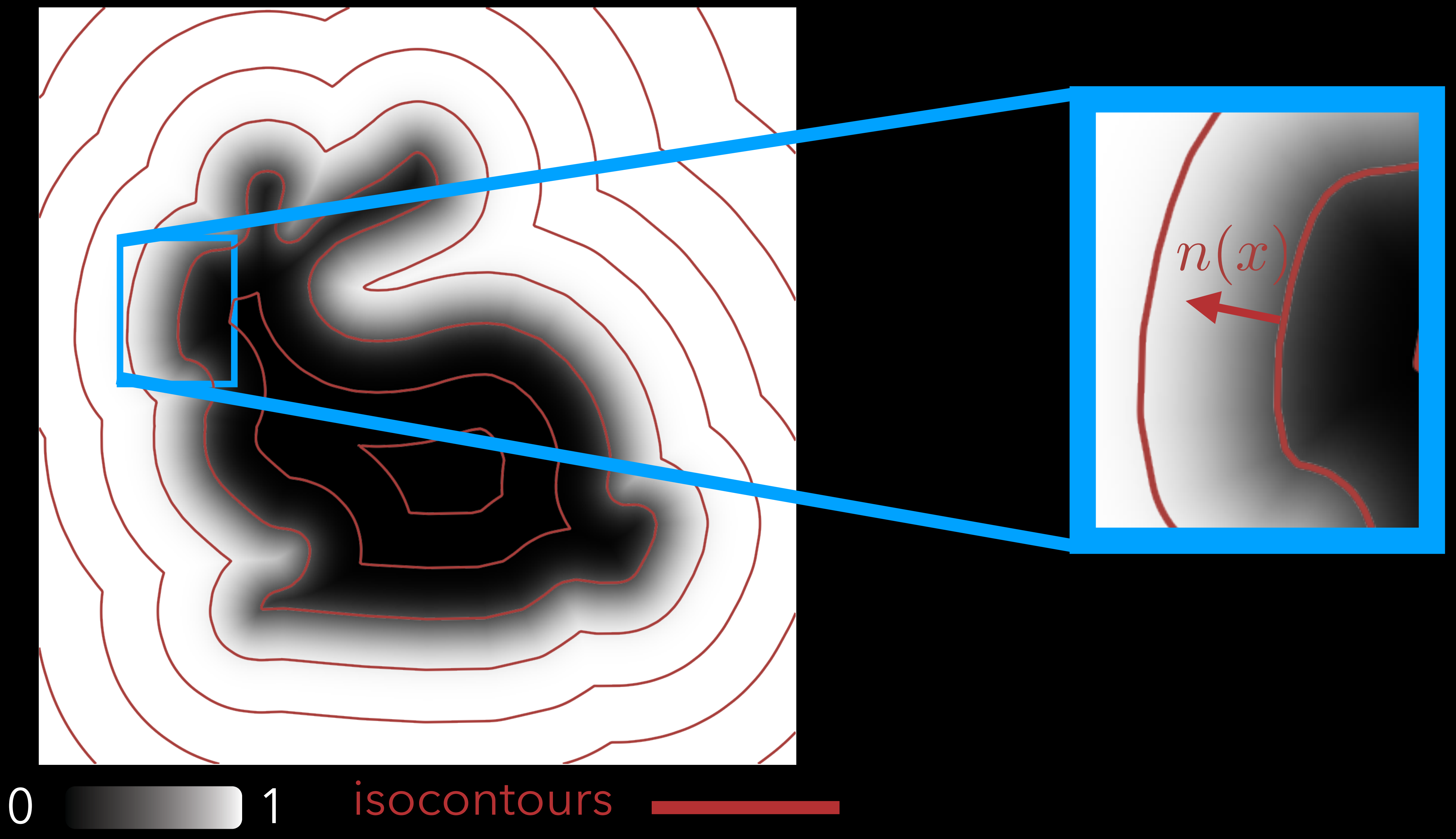
understanding attenuation for opaque solids

density · projected area · normals

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

$$n(x) = \frac{\nabla v(x)}{\|v(x)\|}$$

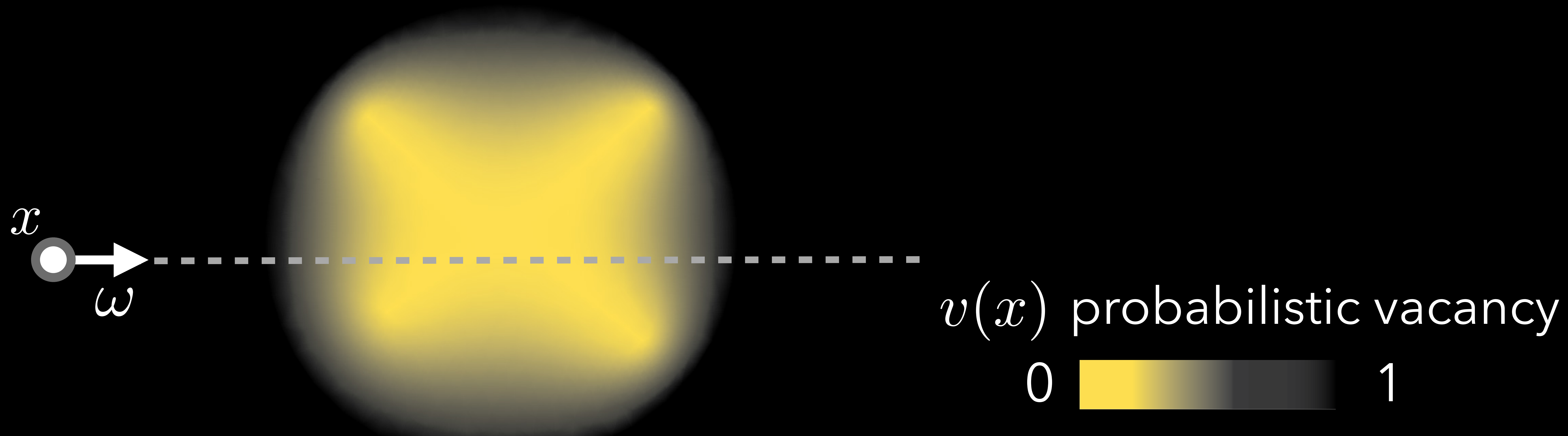
probabilistic vacancy



understanding density for opaque solids

density — projected area

$$\sigma(x,\omega) = \frac{||\nabla v(x)||}{v(x)} \cdot |\omega \cdot n(x)|$$

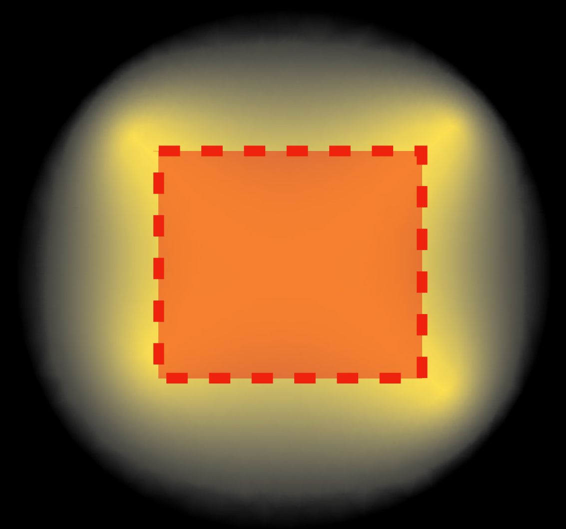


understanding density for opaque solids

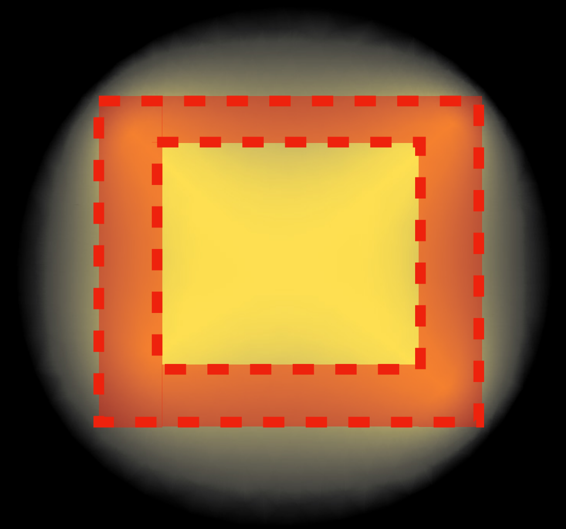
density | projected area

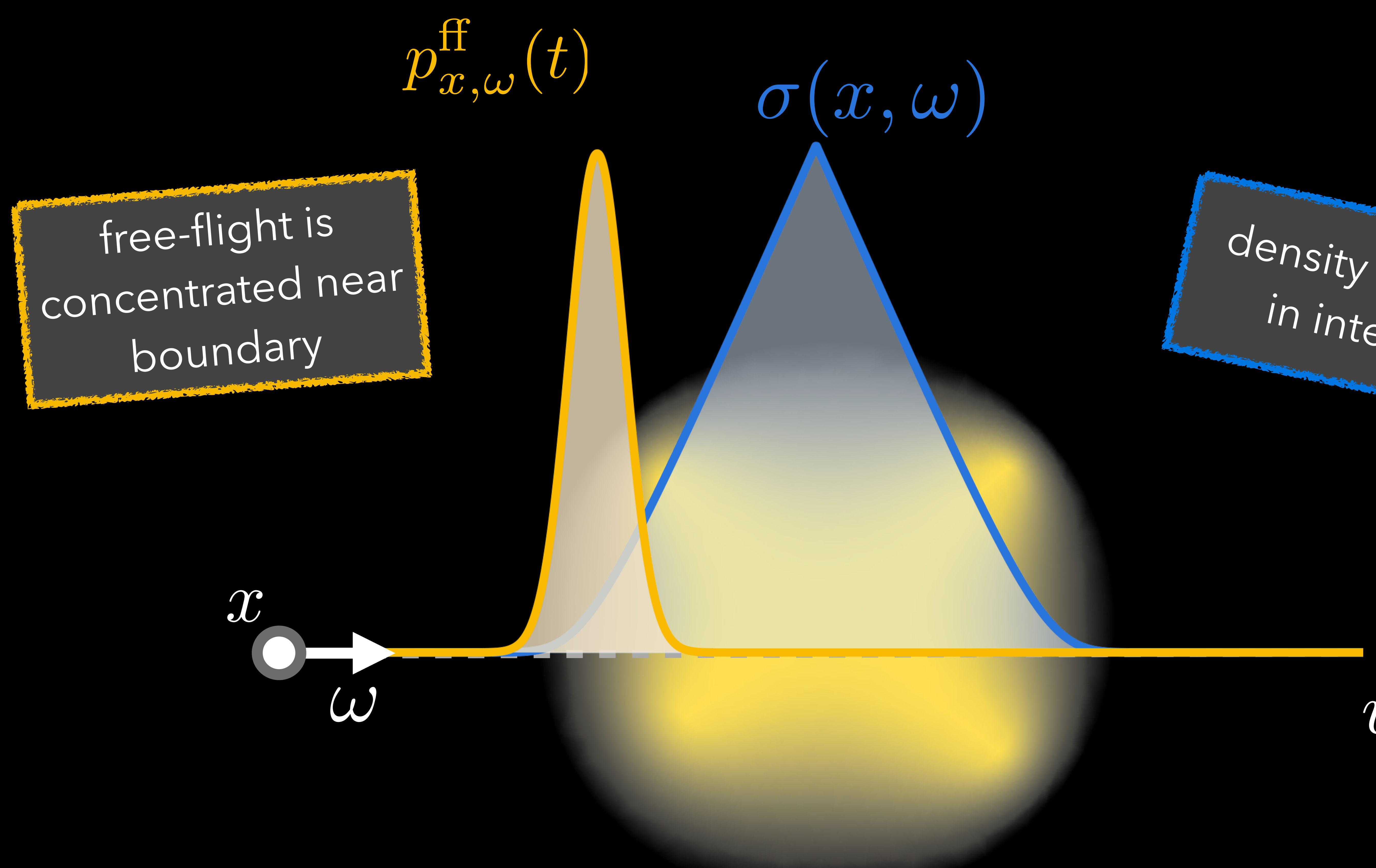
$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

interior: $v(x) < 0.5$



boundary: $v(x) \approx 0.5$

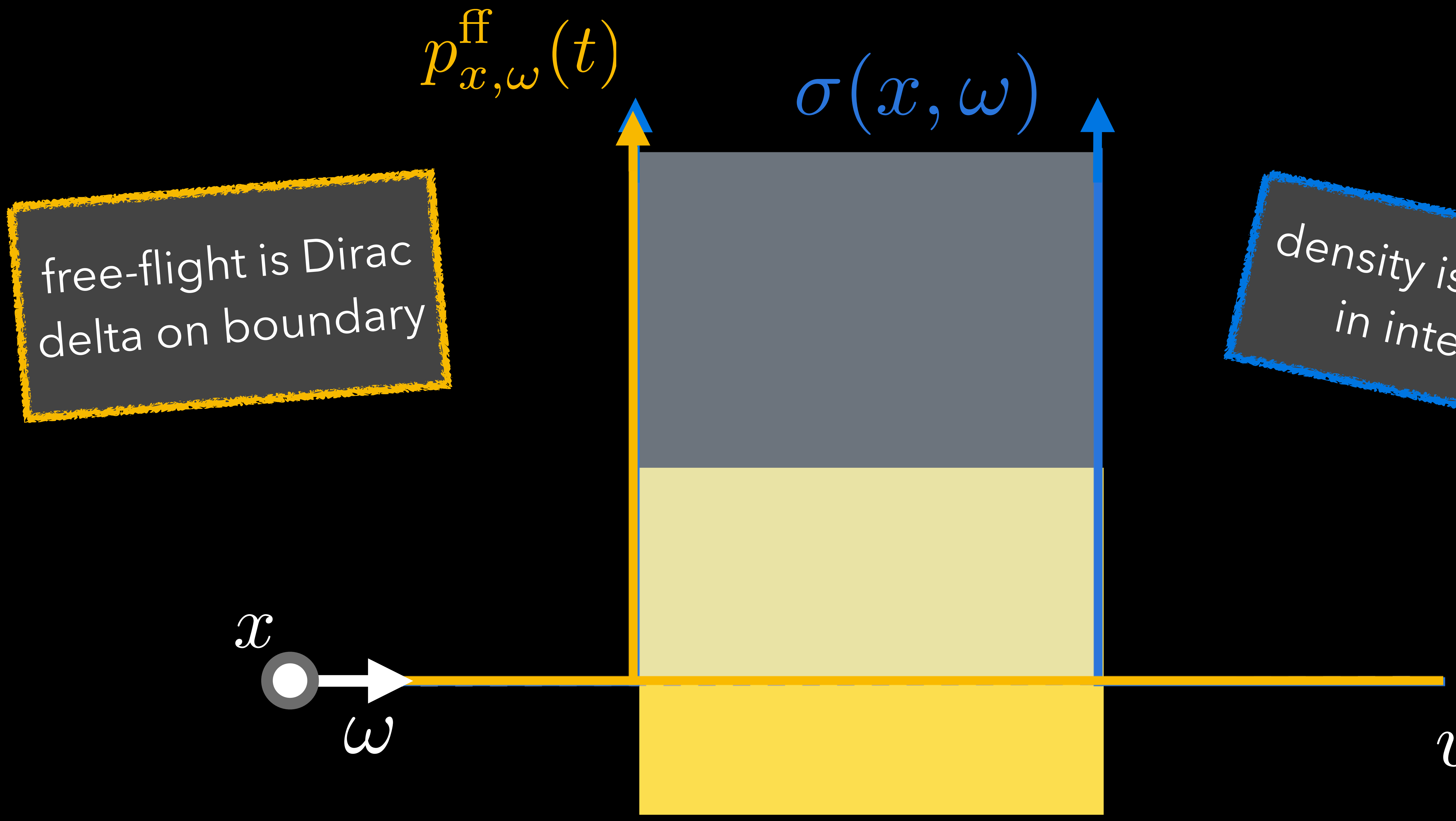




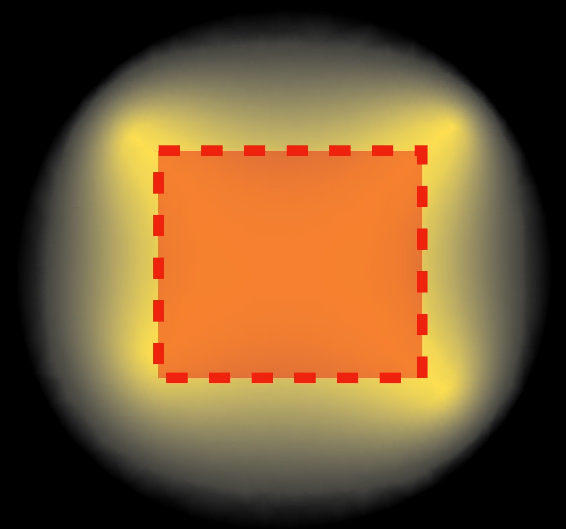
understanding density for opaque solids

density — projected area

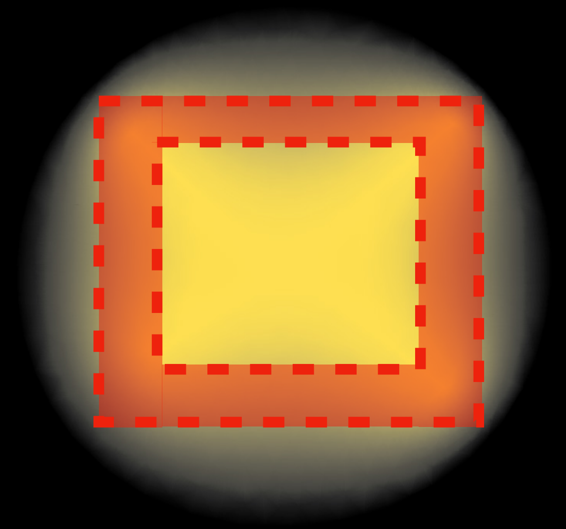
$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$



interior: $v(x) < 0.5$



boundary: $v(x) \approx 0.5$



$v(x)$ probabilistic vacancy

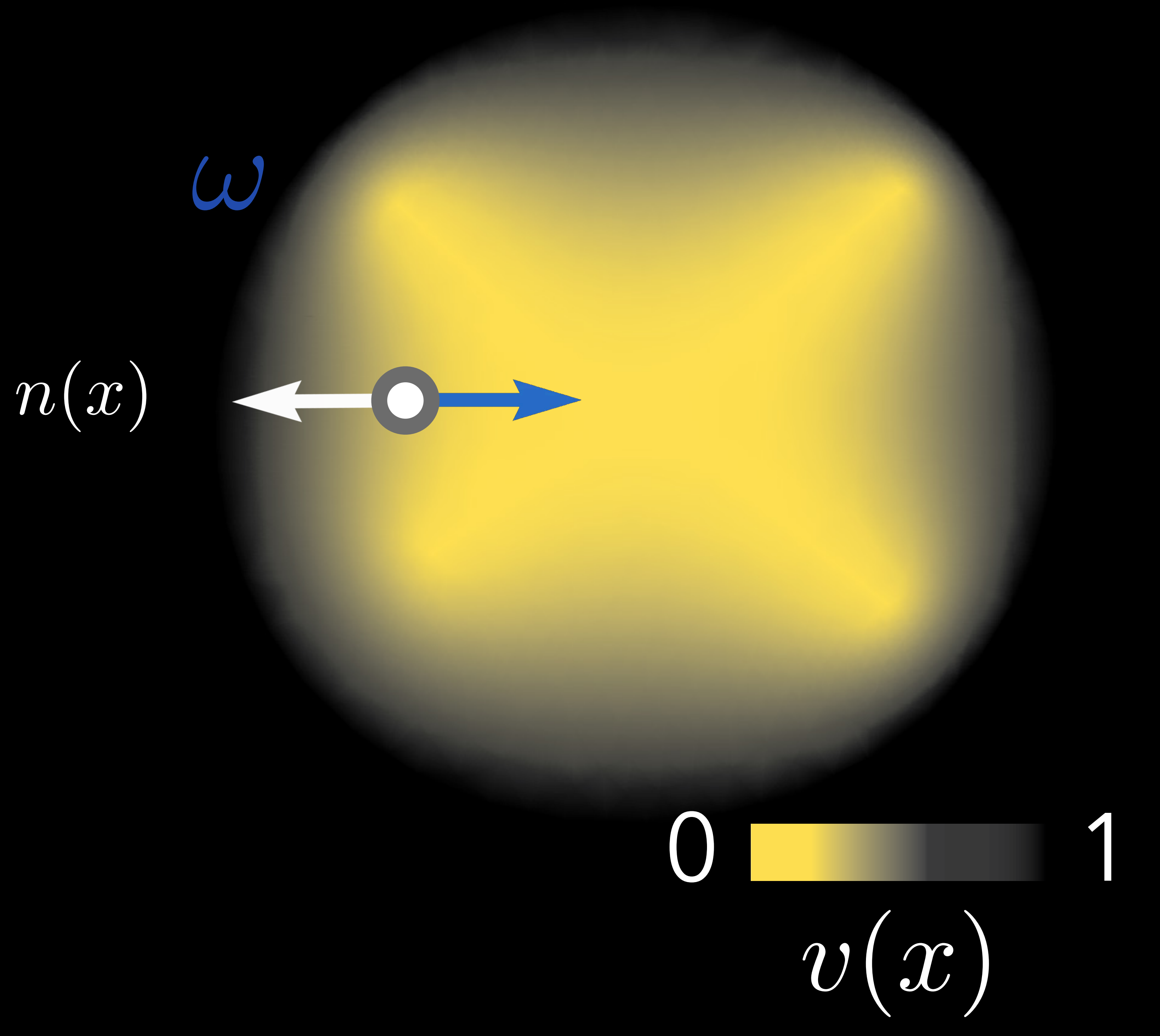
0 — 1

understanding projected area for opaque solids

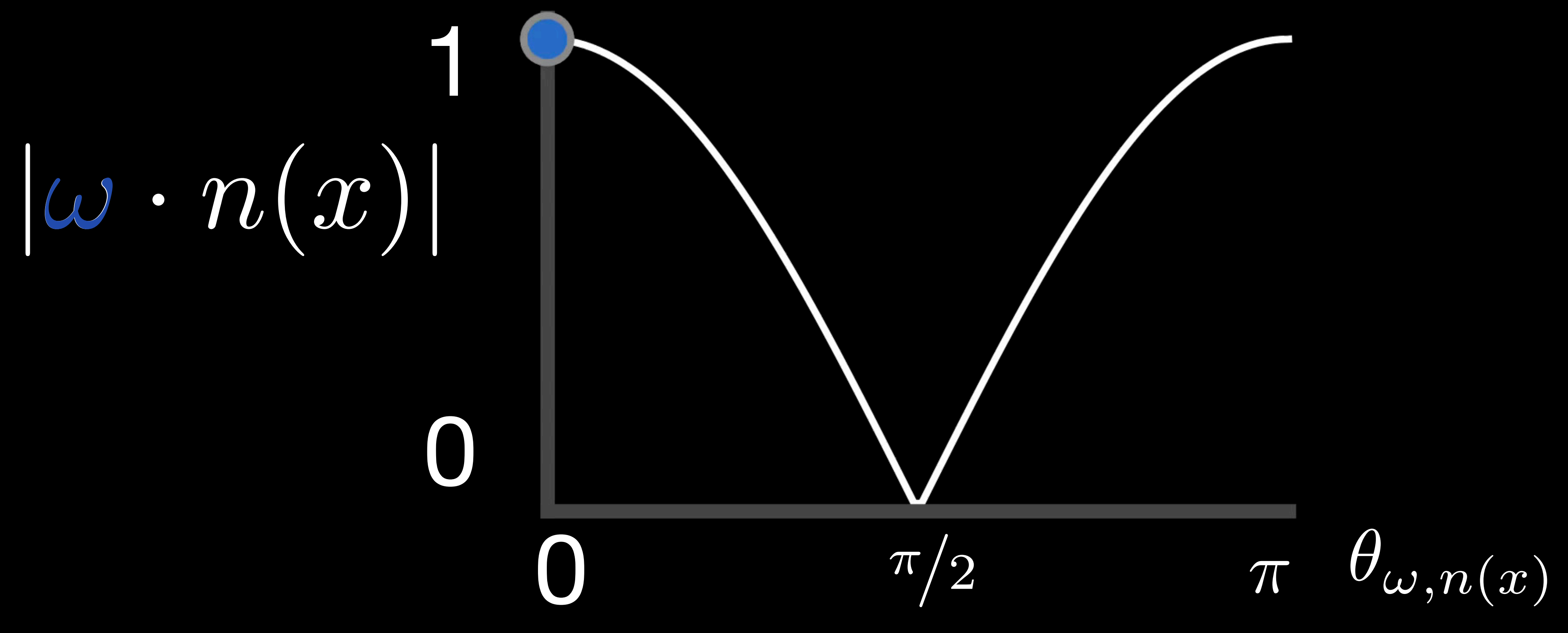
density projected area

$$\sigma(x, \omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

direction relative to vacancy



projected area

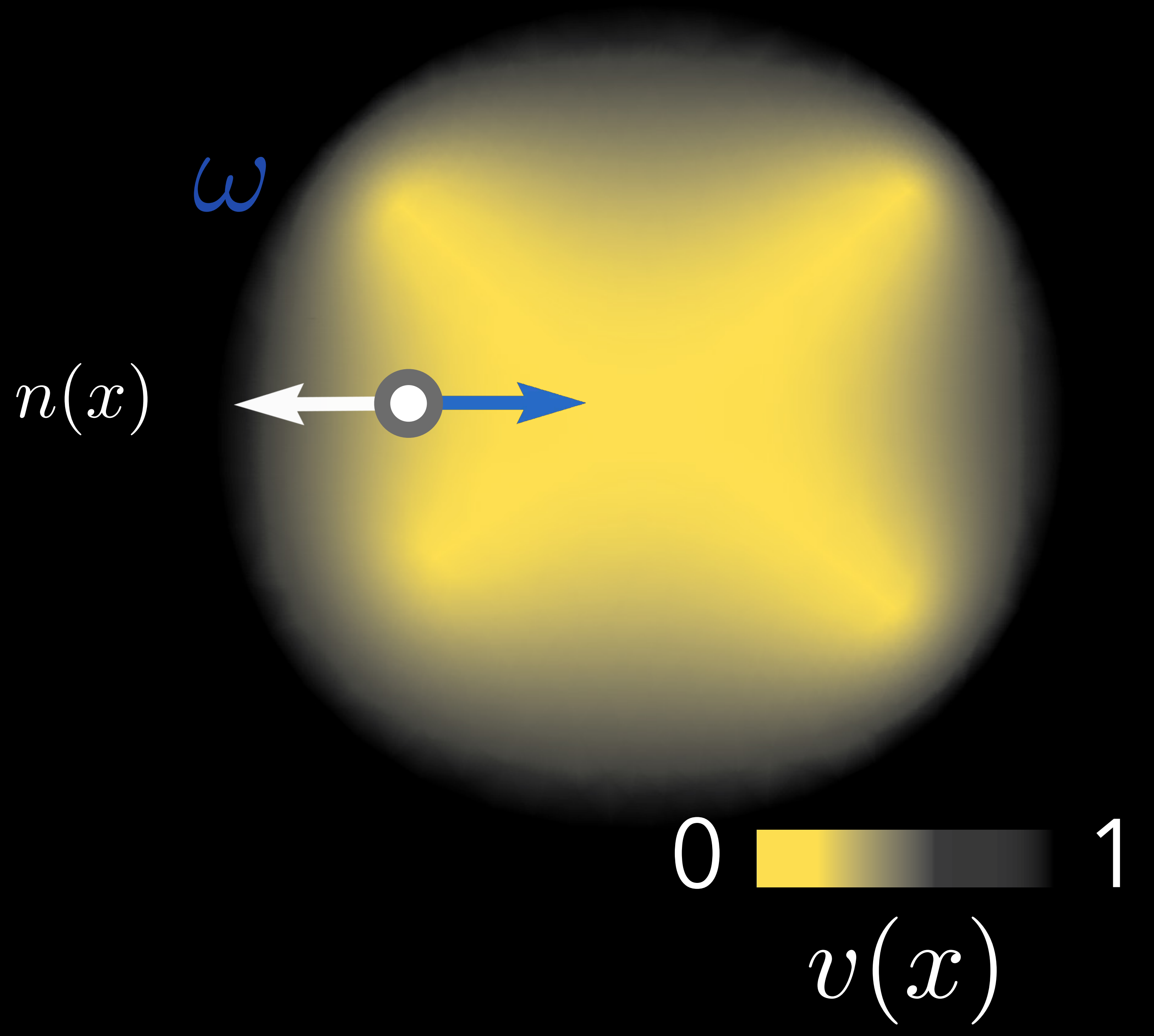


understanding projected area for opaque solids

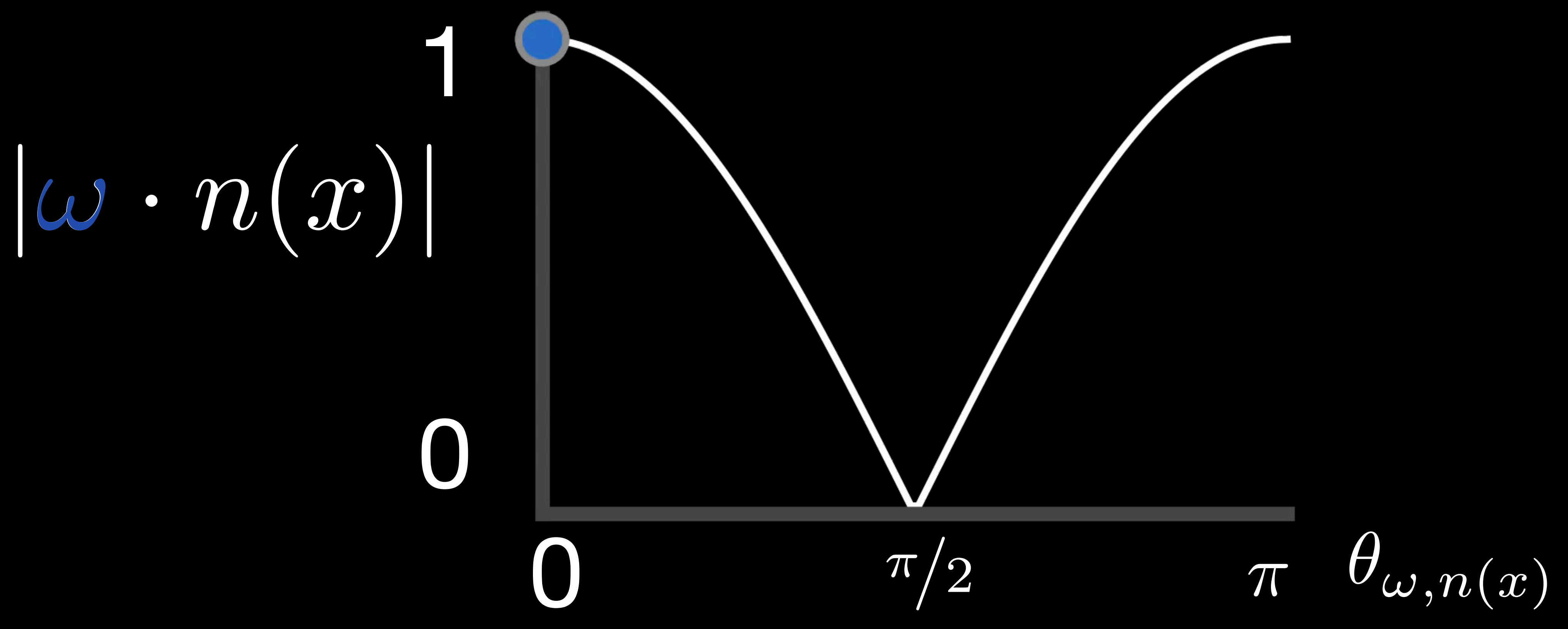
density — projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

direction relative to vacancy



projected area

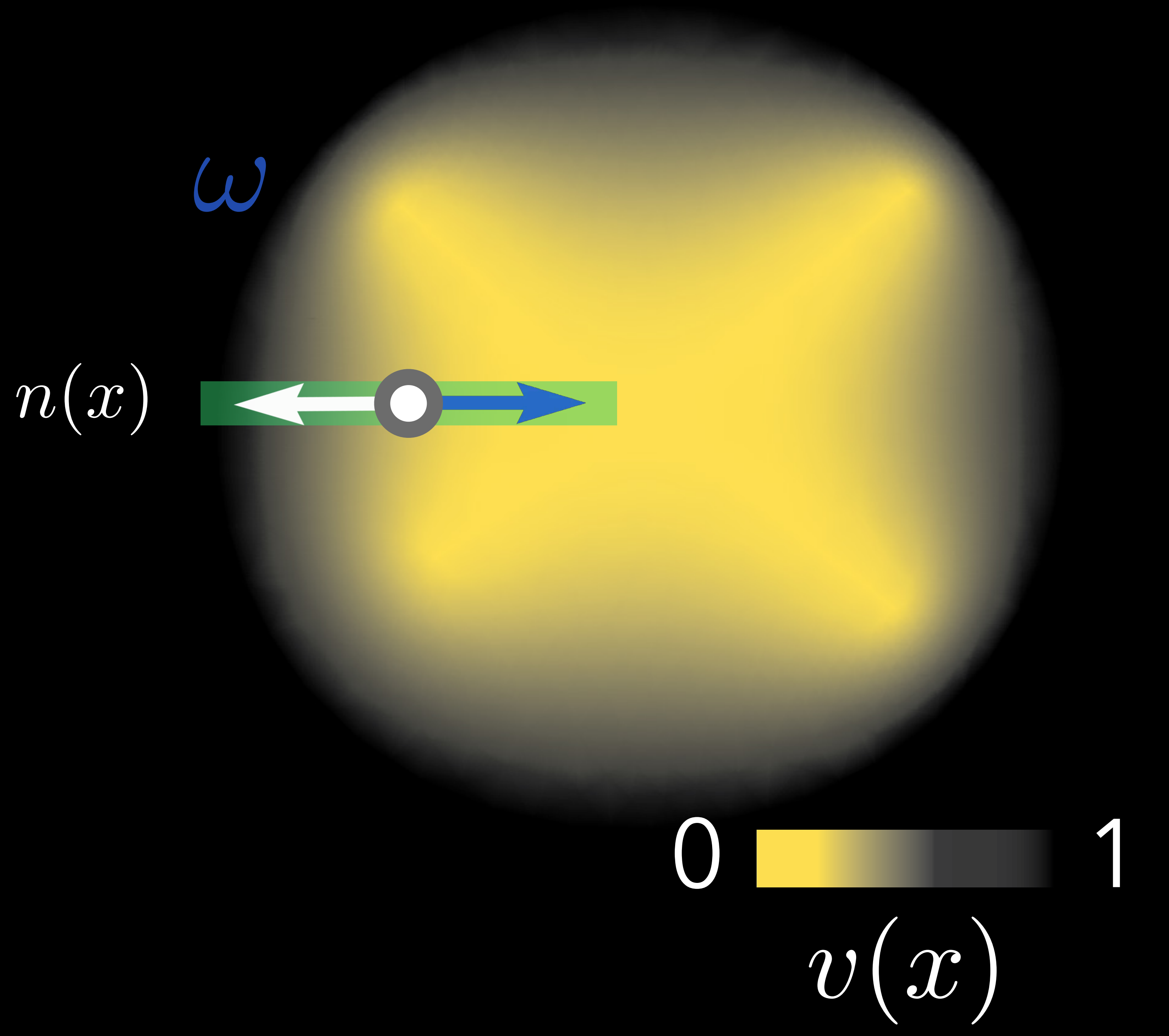


understanding projected area for opaque solids

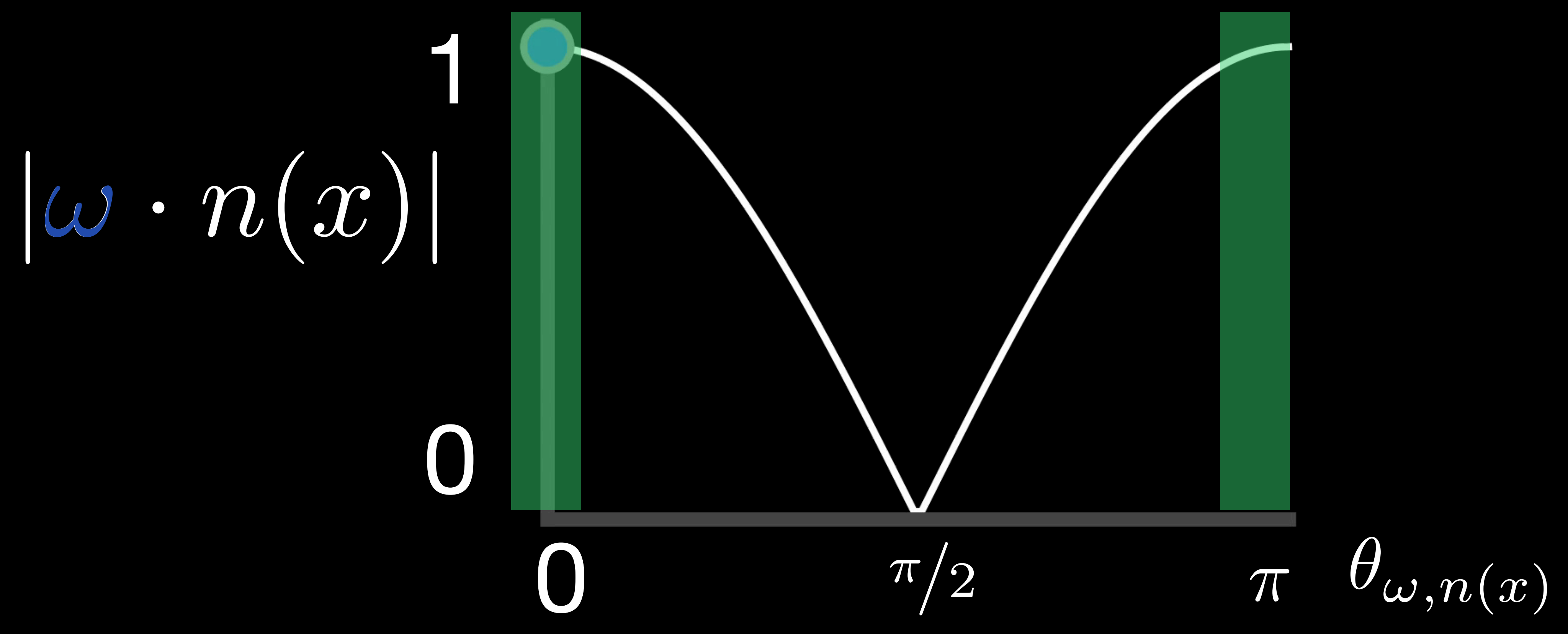
density   projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

direction relative to vacancy



projected area



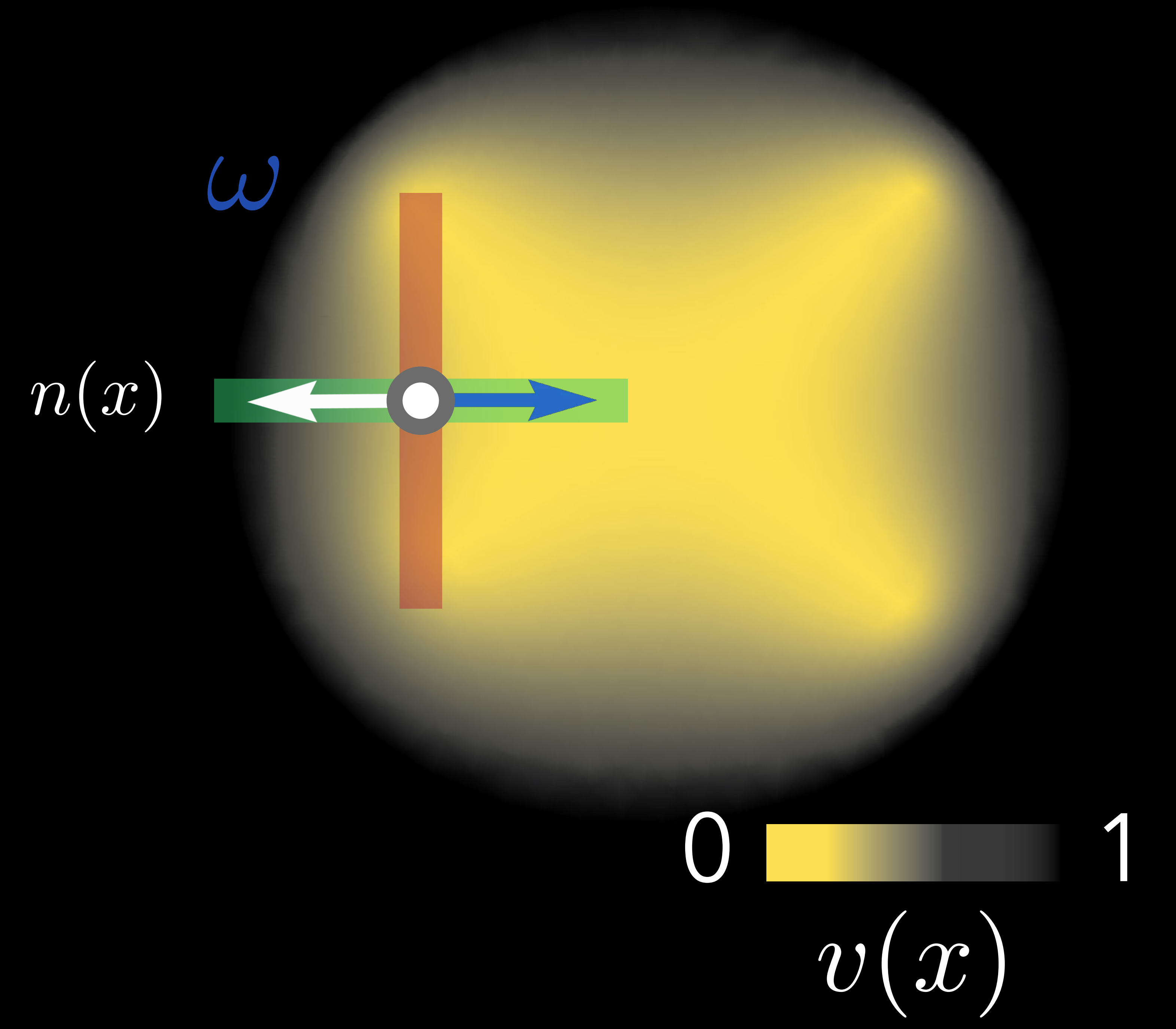
projected area is large when perpendicular

understanding projected area for opaque solids

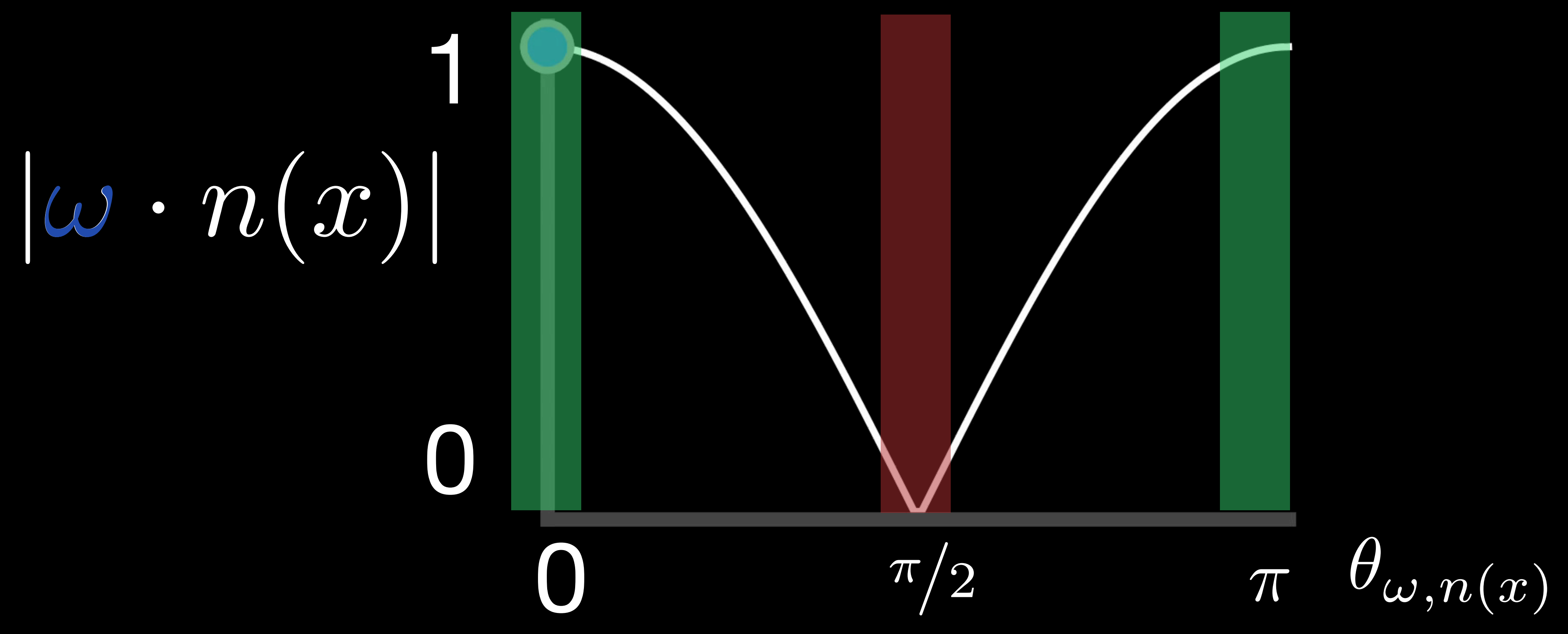
density · projected area

$$\sigma(x, \omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

direction relative to vacancy



projected area



projected area is large when perpendicular

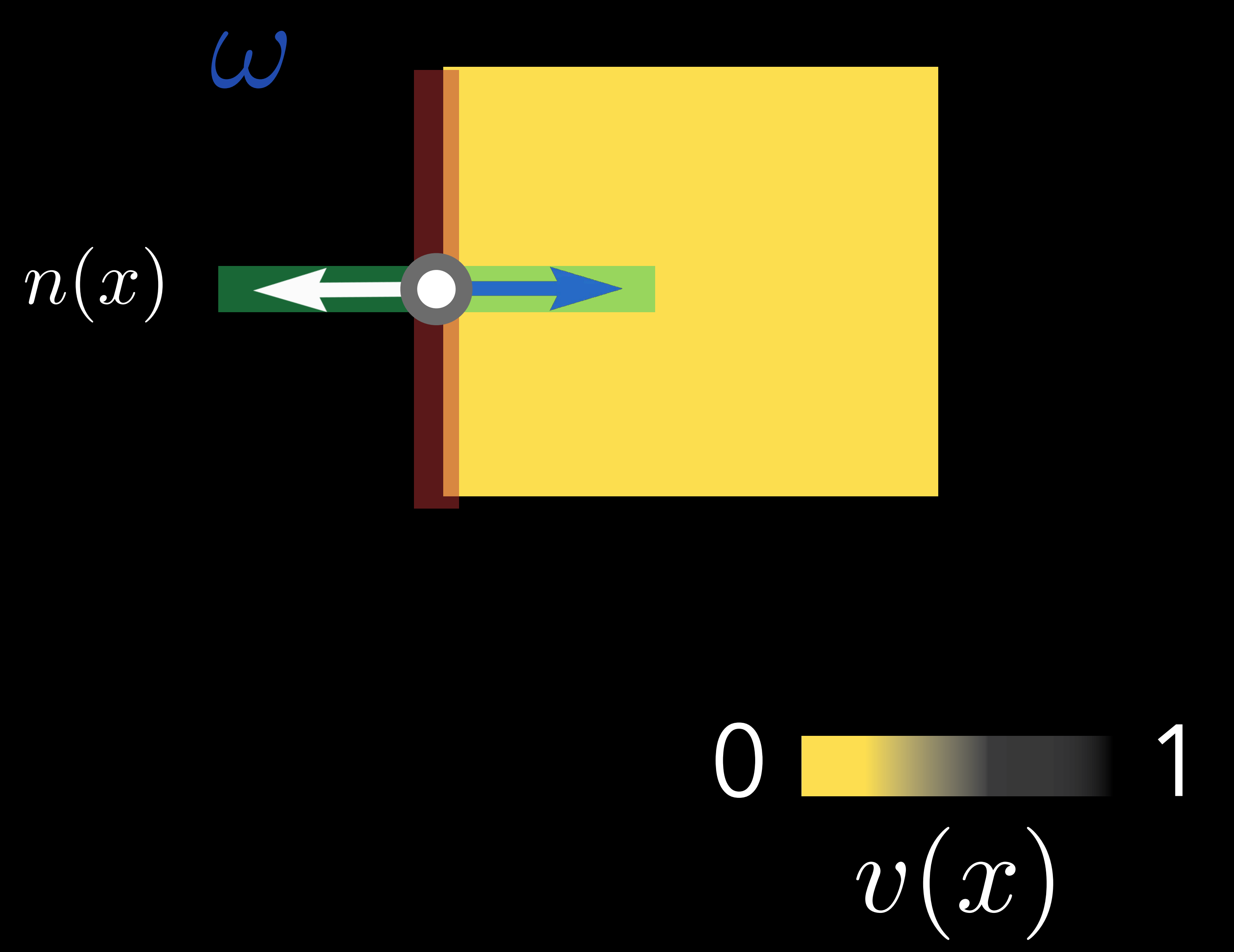
projected area is small when grazing

understanding projected area for opaque solids

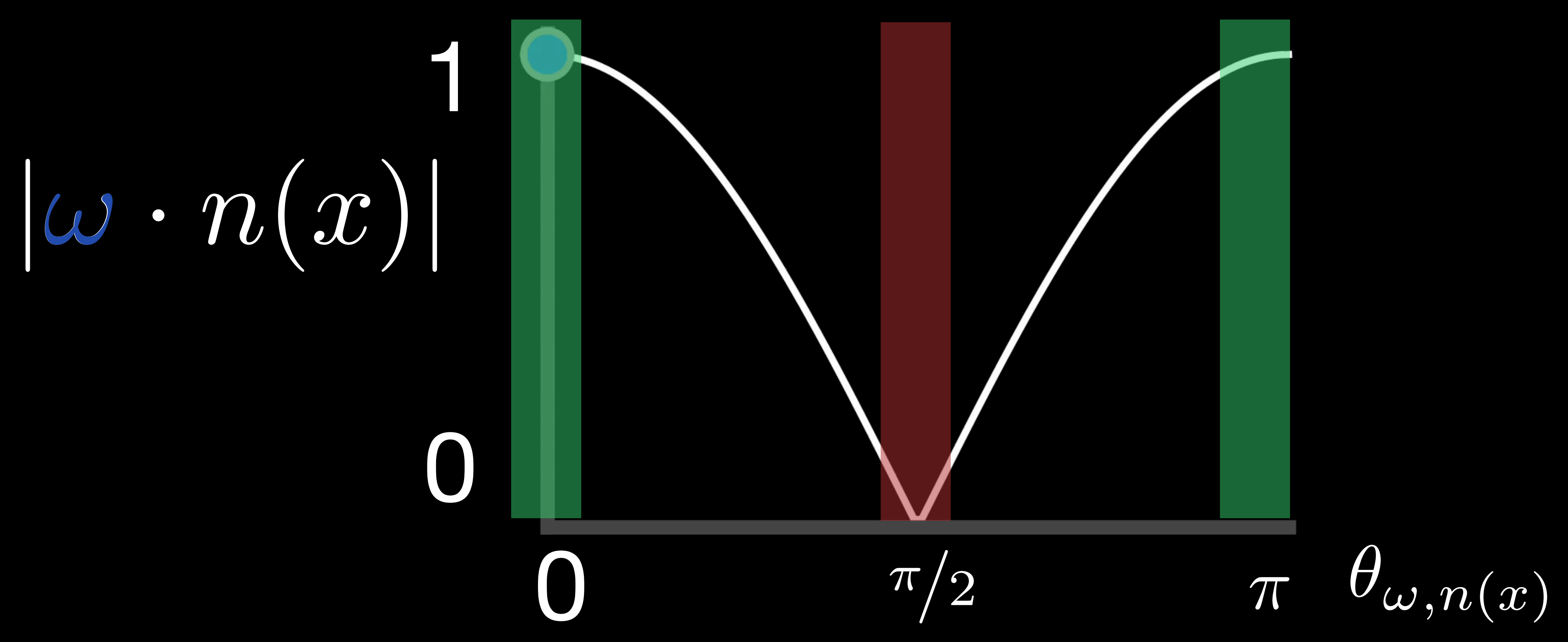
density   projected area

$$\sigma(x,\omega) = \frac{||\nabla v(x)||}{v(x)} \cdot |\omega \cdot n(x)|$$

direction relative to vacancy



projected area



projected area is large when perpendicular

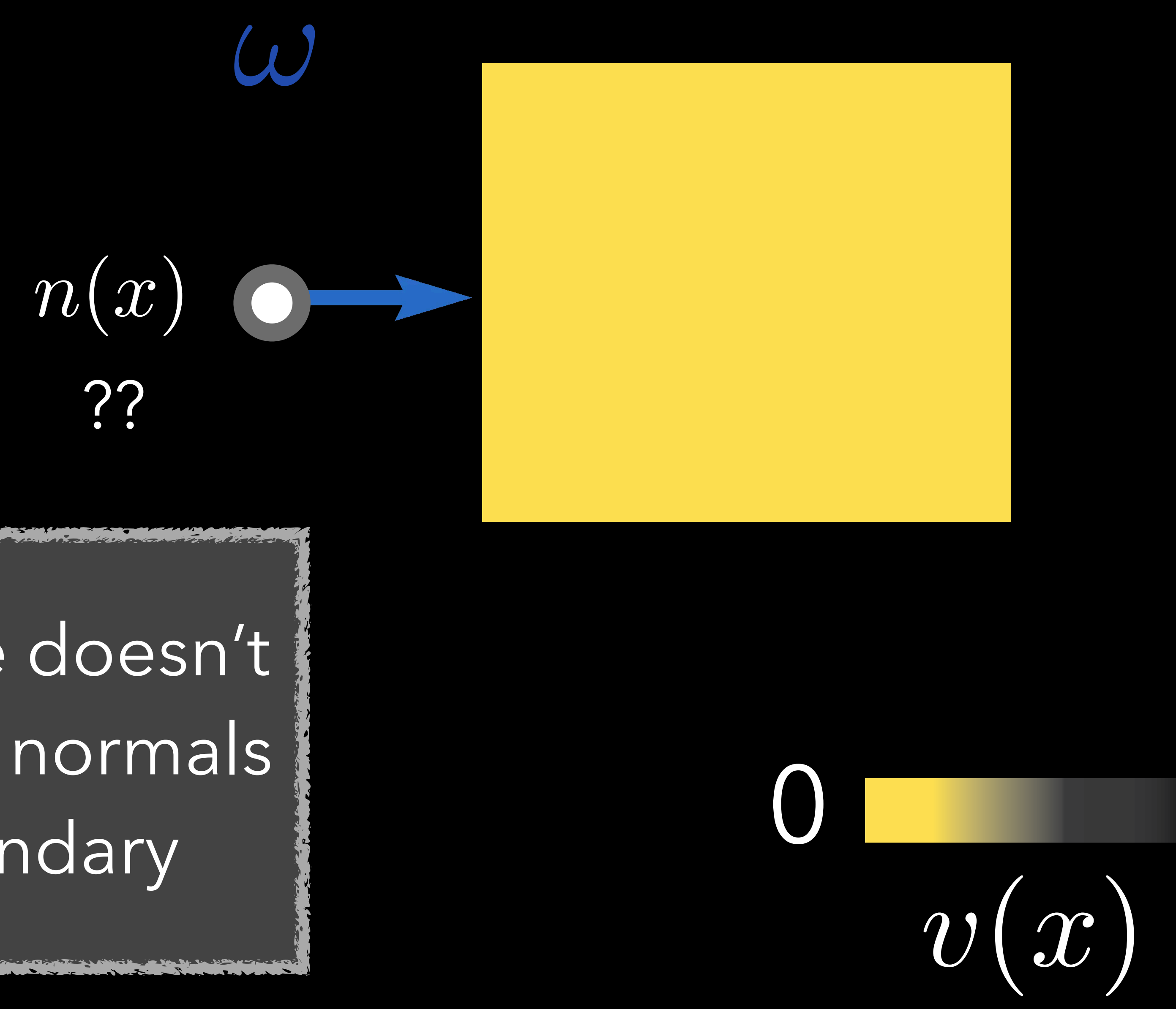
projected area is small when grazing

understanding projected area for opaque solids

density / projected area

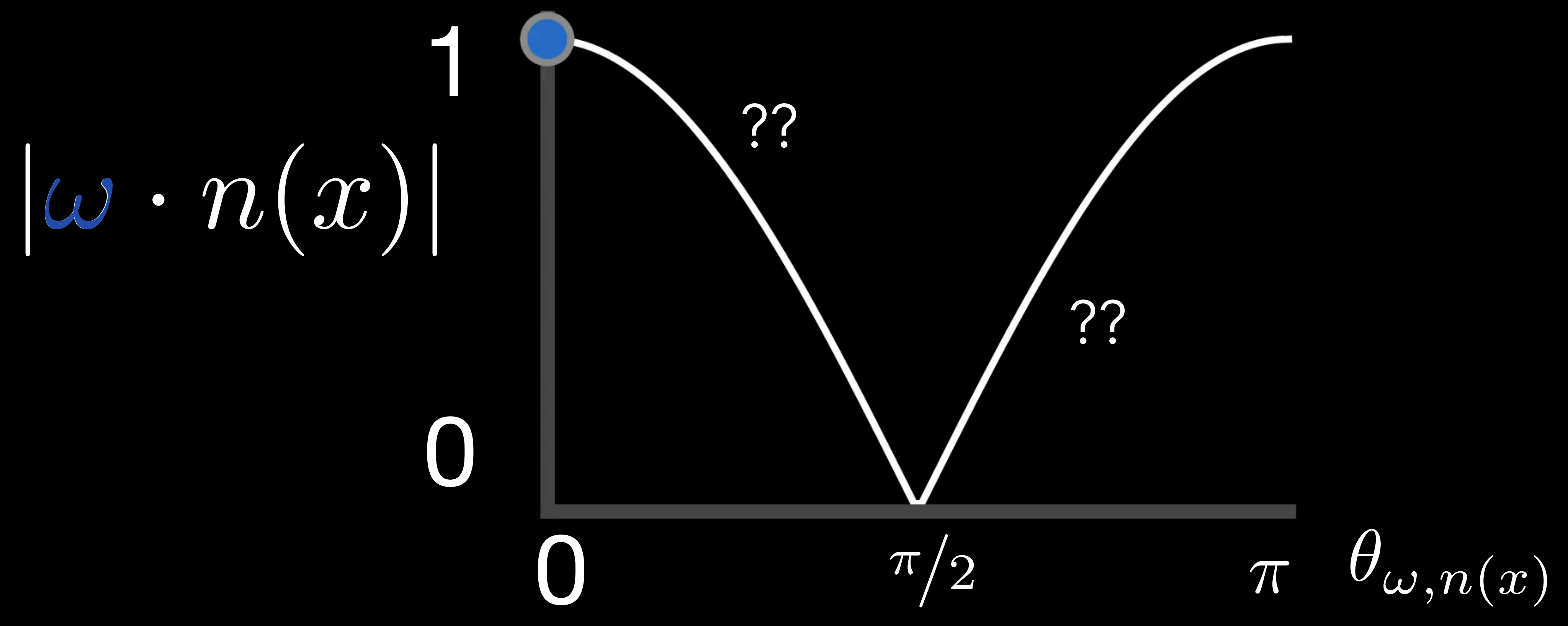
$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot |\omega \cdot n(x)|$$

direction relative to vacancy



deterministic case doesn't have well defined normals away from boundary

projected area



projected area is large when perpendicular

projected area is small when grazing

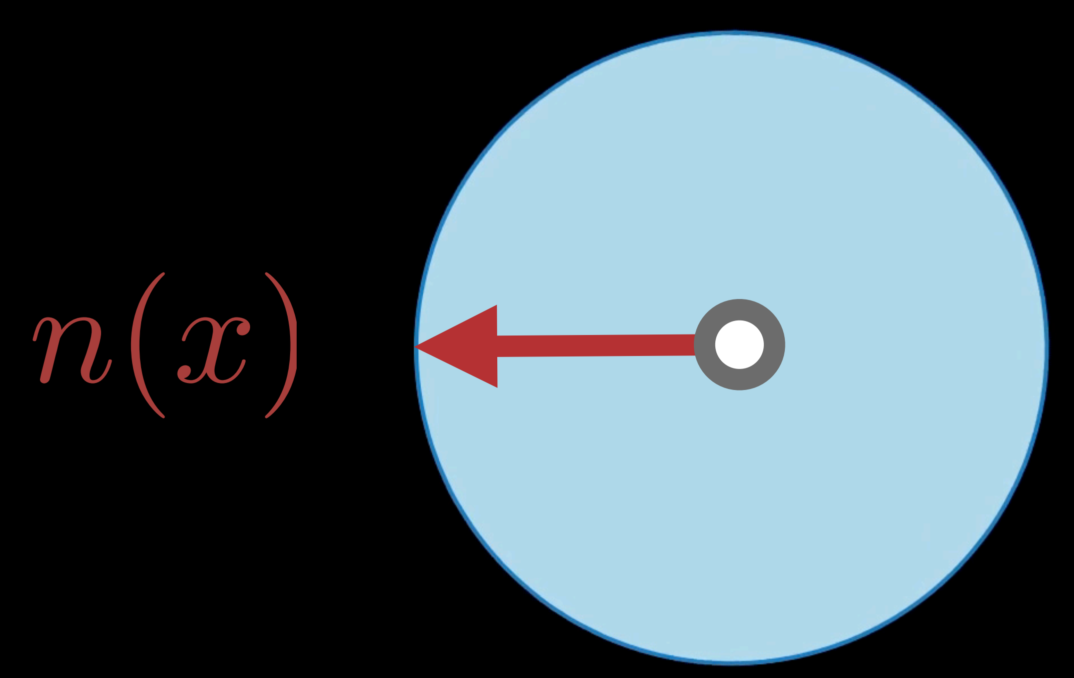
understanding projected area for opaque solids

density — projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

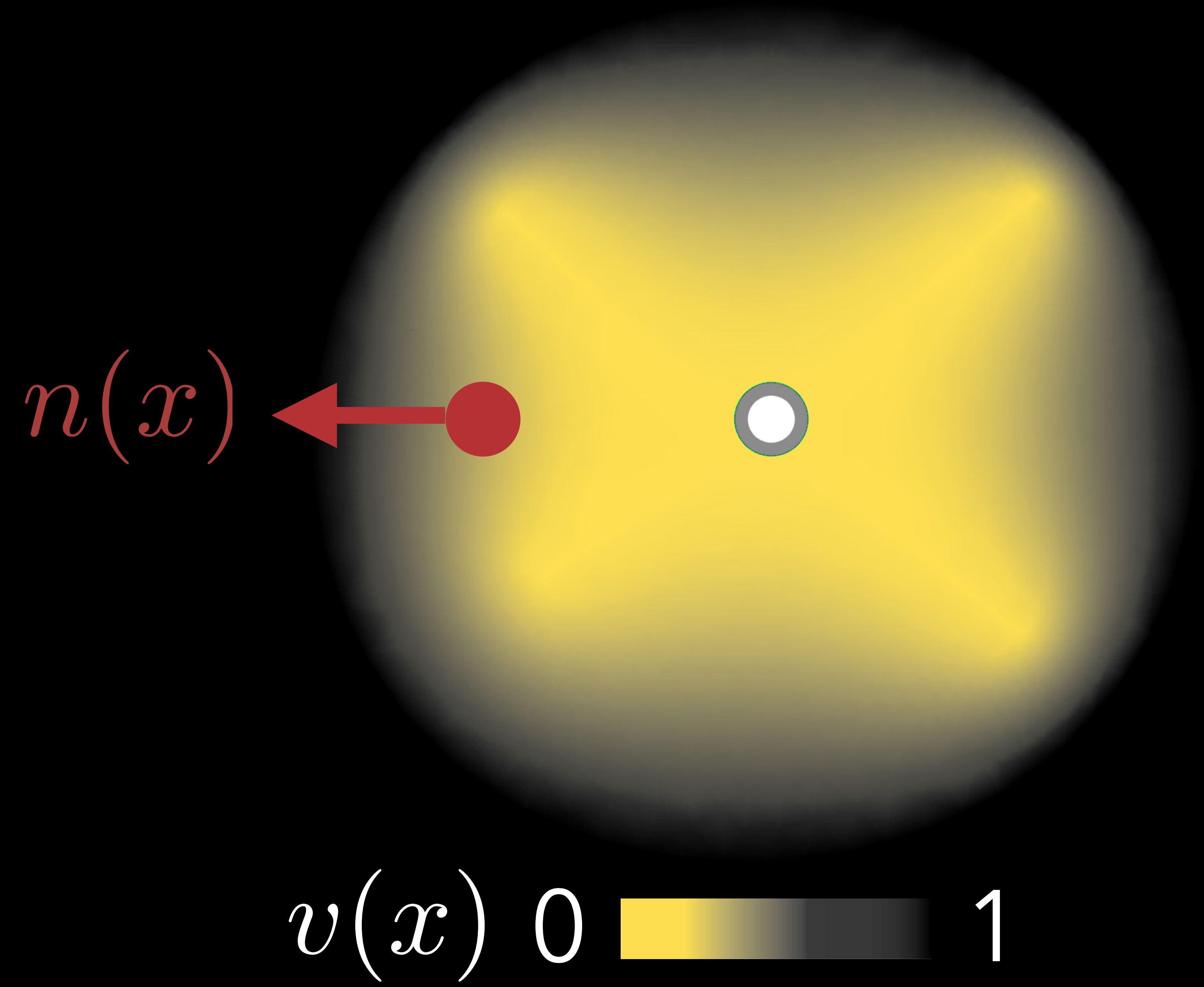
distribution of normals

$D_x(m)$

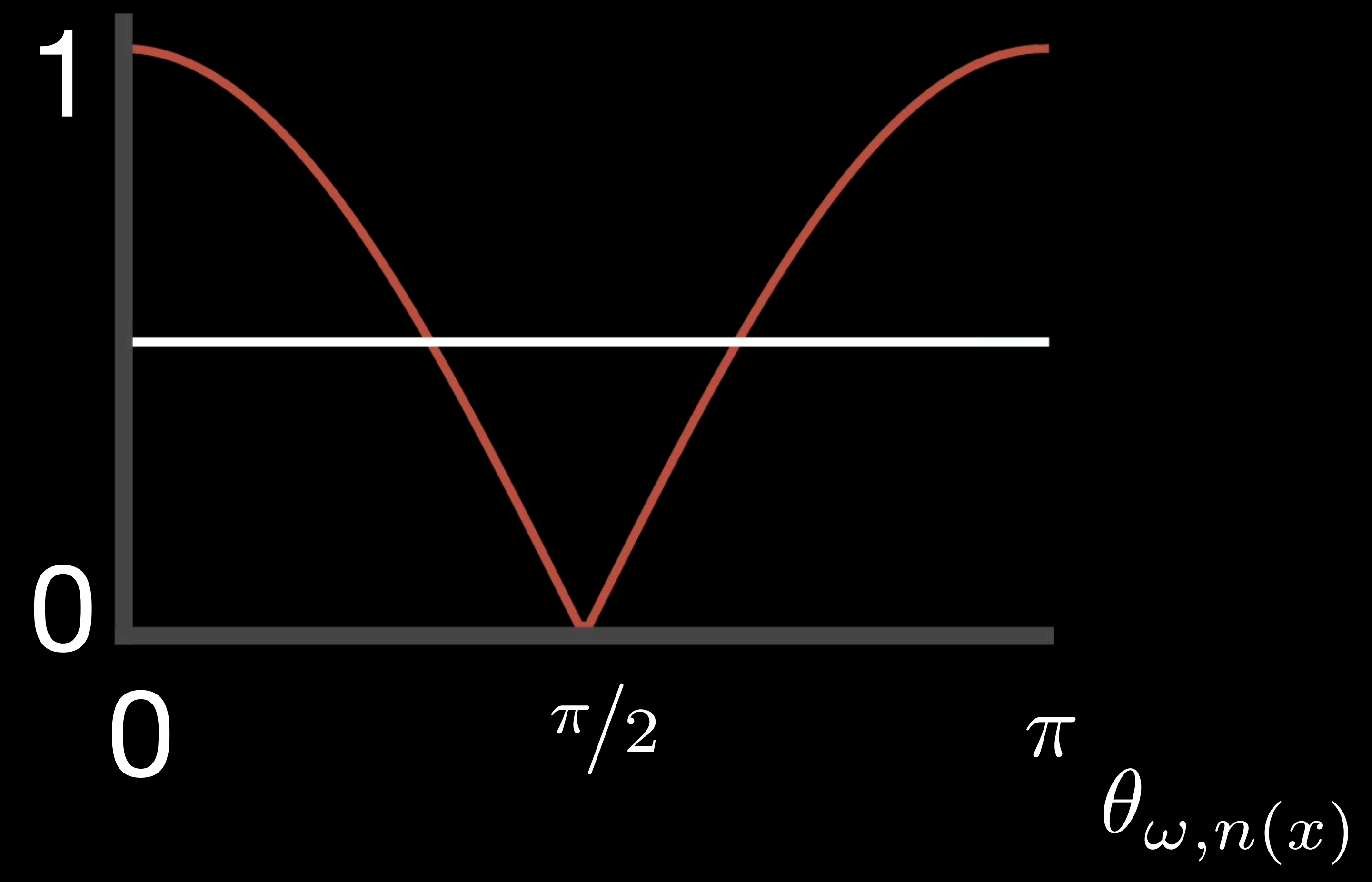


boundary vs interior point

x



projected area



understanding projected area for opaque solids

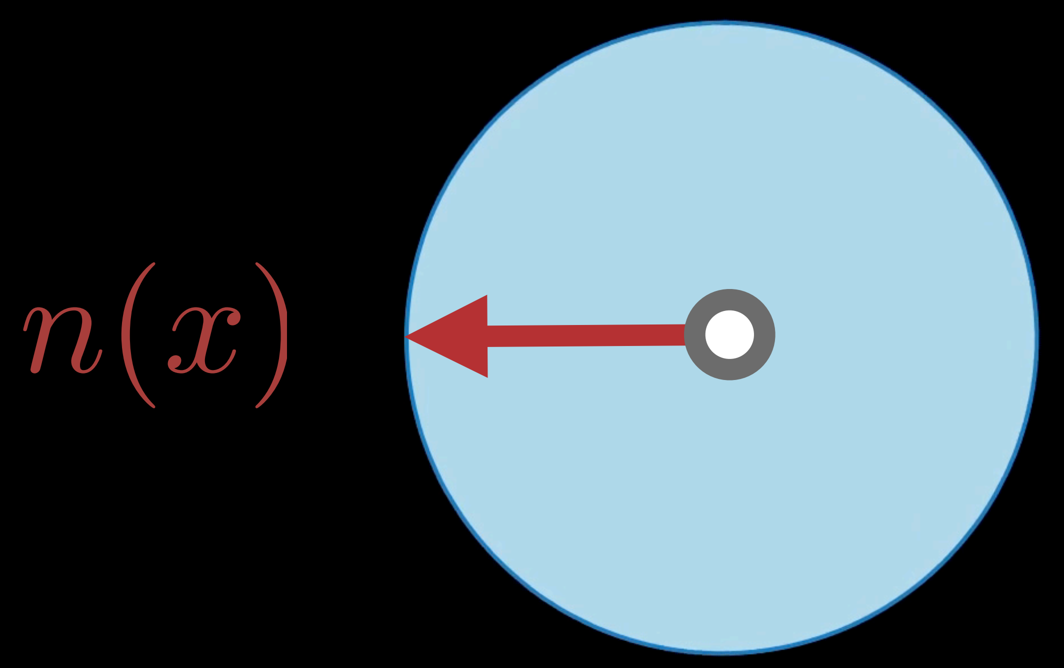
density

projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m)\,\mathrm{d}m$$

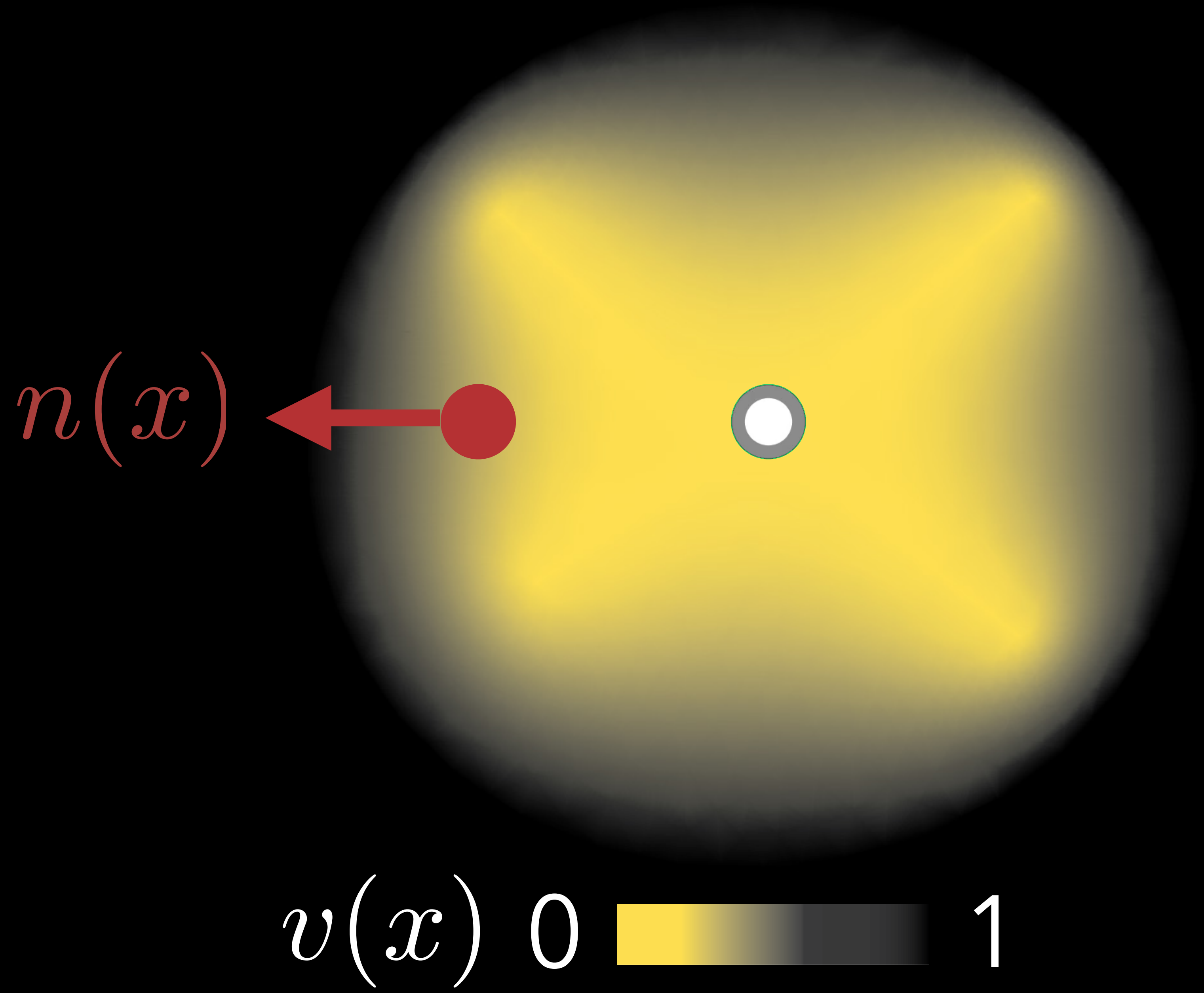
distribution of normals

$D_x(m)$

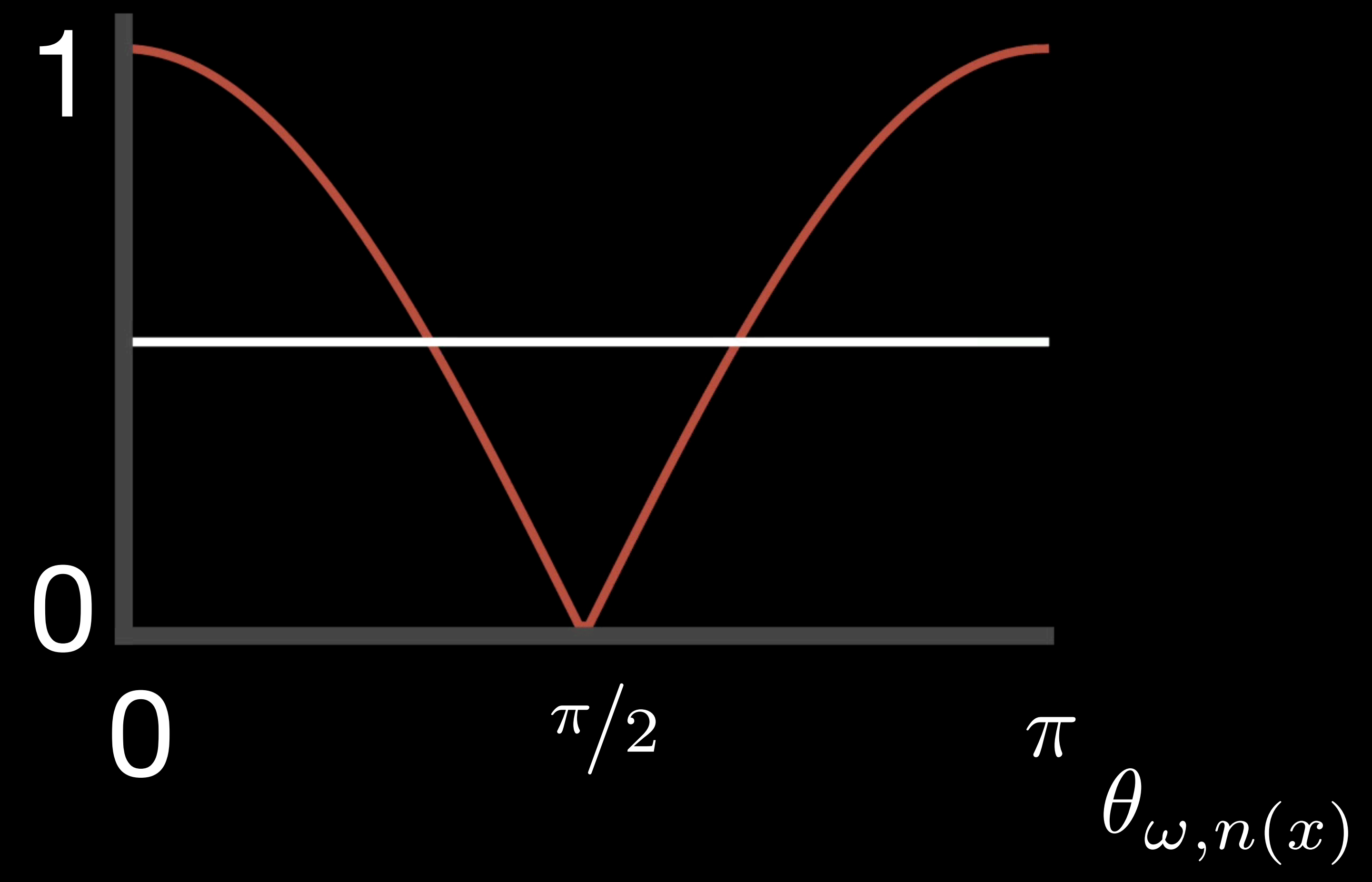


boundary vs interior point

x



projected area

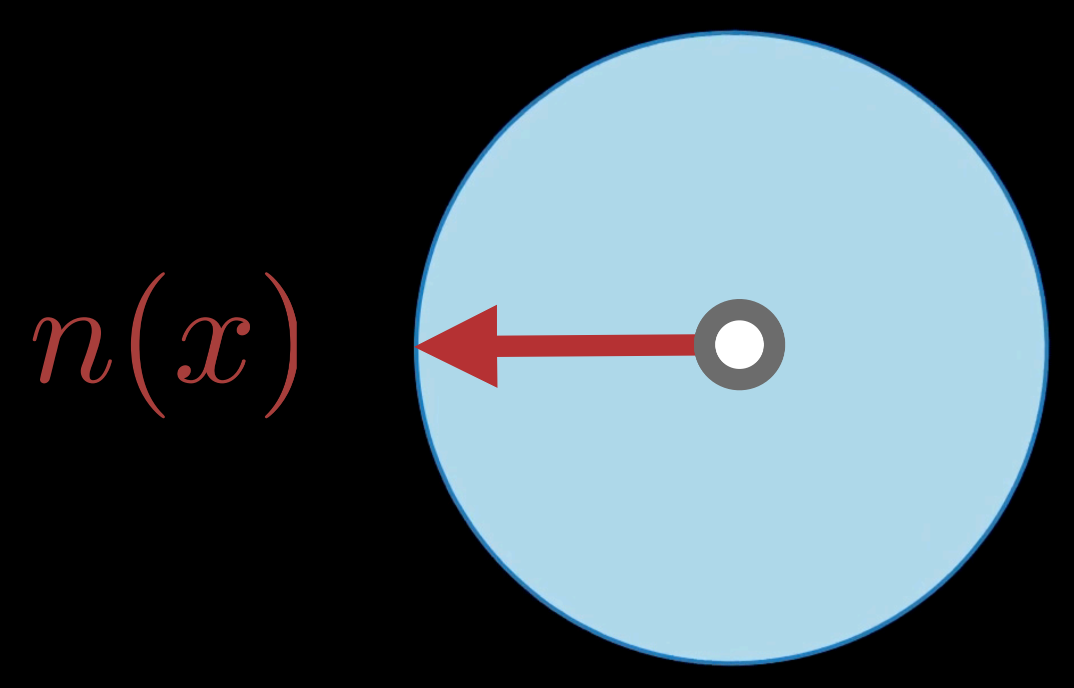


understanding projected area for opaque solids

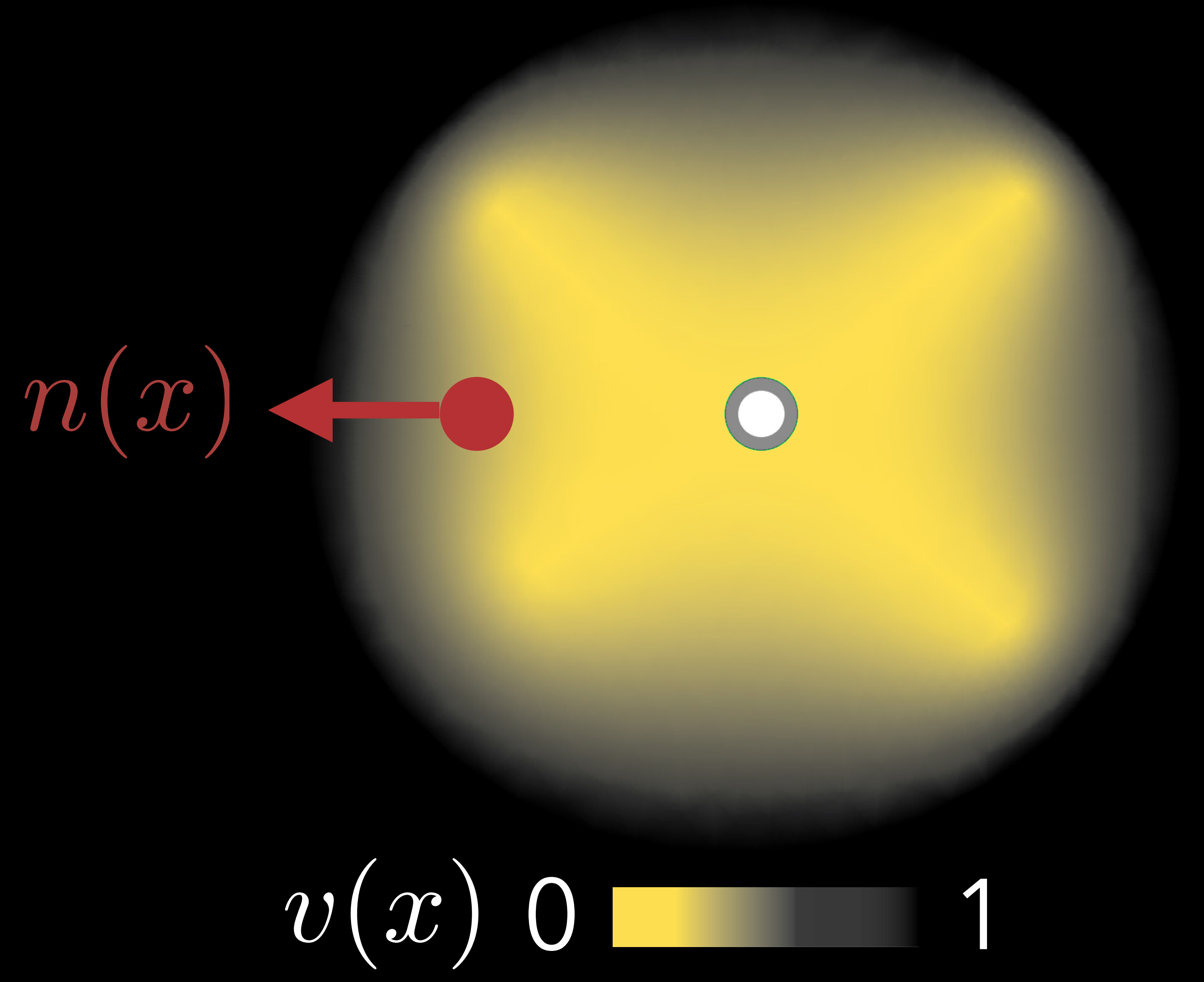
density · projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

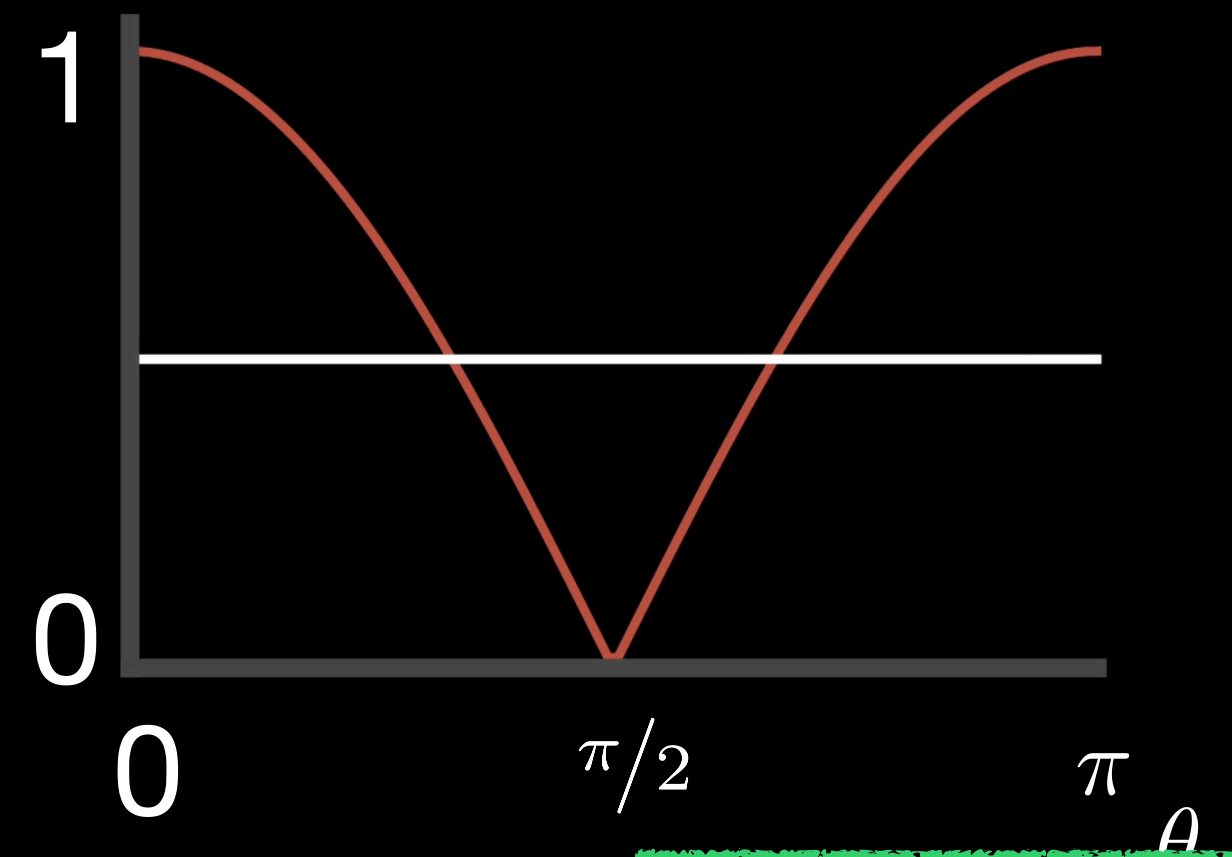
distribution of normals $D_x(m)$



boundary vs interior point x



projected area



projected area is a constant function in interior

understanding projected area for opaque solids

Prior works choose use
either boundary or interior
model without adapting

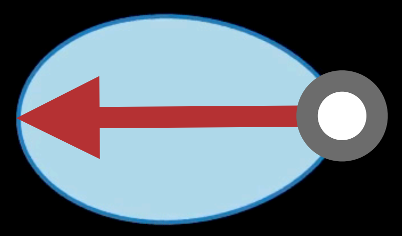
density — projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m)\mathrm{d}m$$

distribution of normals

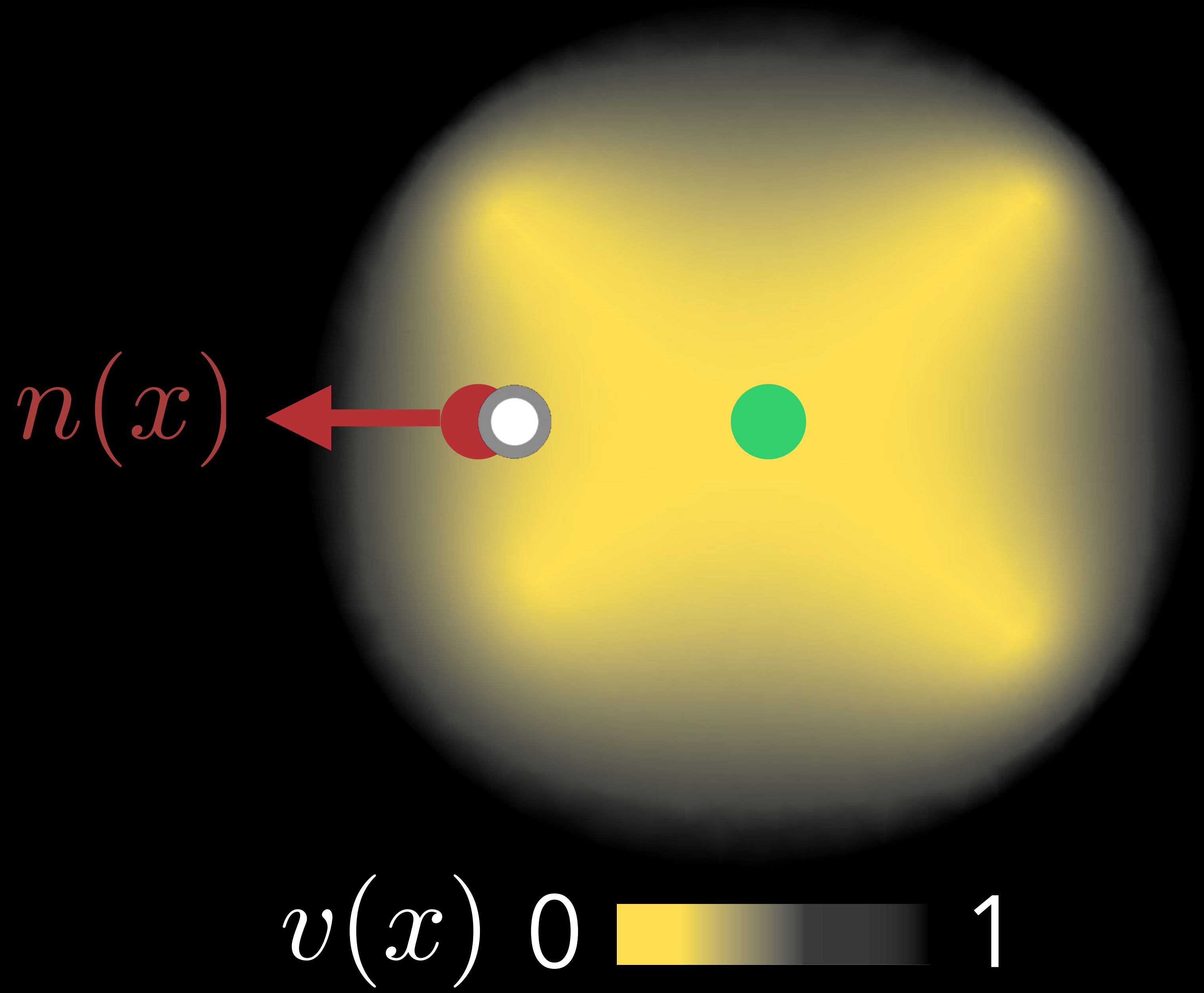
$D_x(m)$

$n(x)$

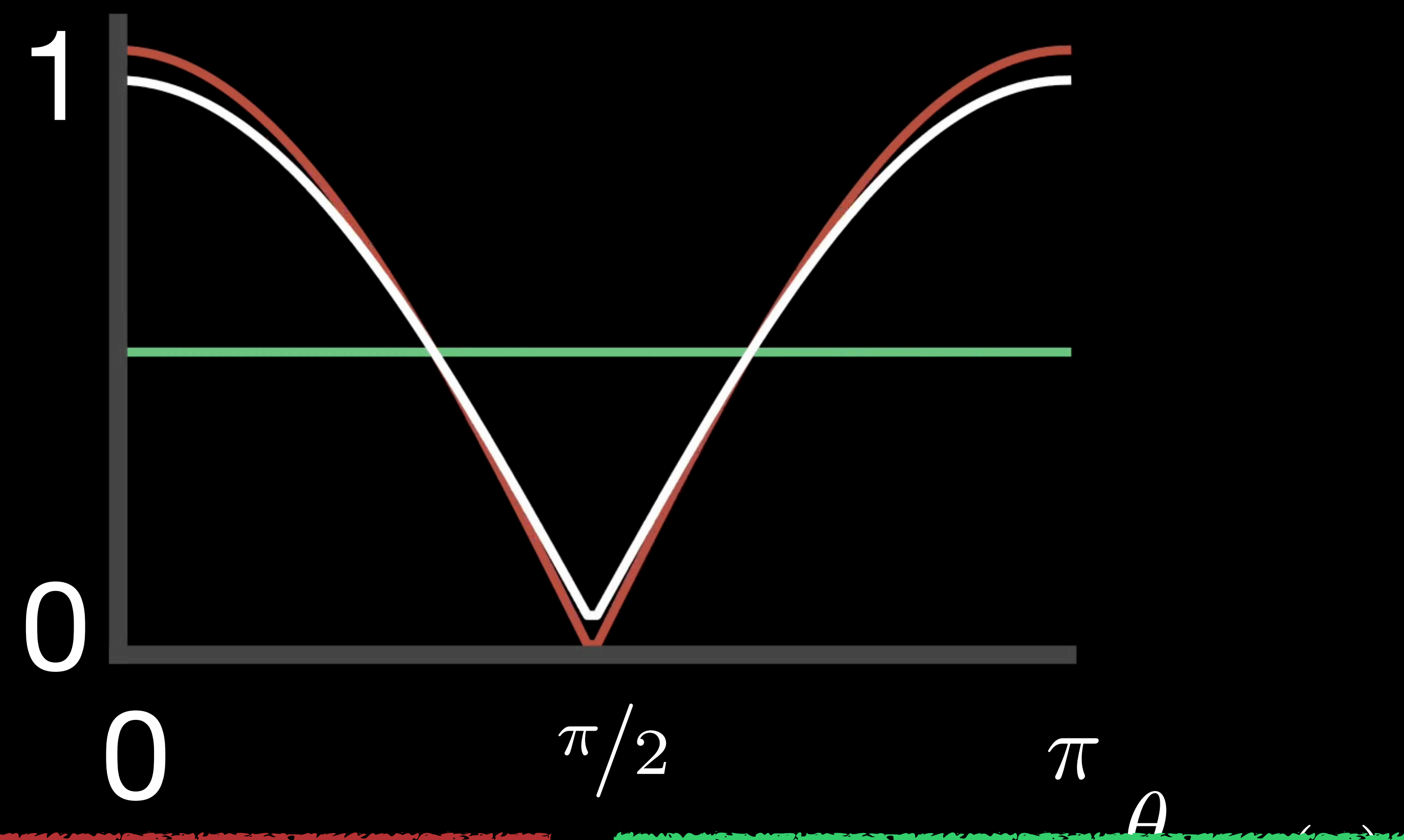


boundary vs interior point

x



projected area



projected area includes grazing at boundary

projected area is a constant function in interior

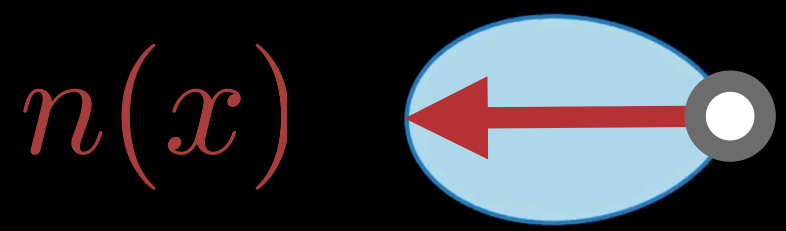
understanding projected area for opaque solids

Prior works choose use <u>either</u> boundary <u>or</u> interior model without adapting

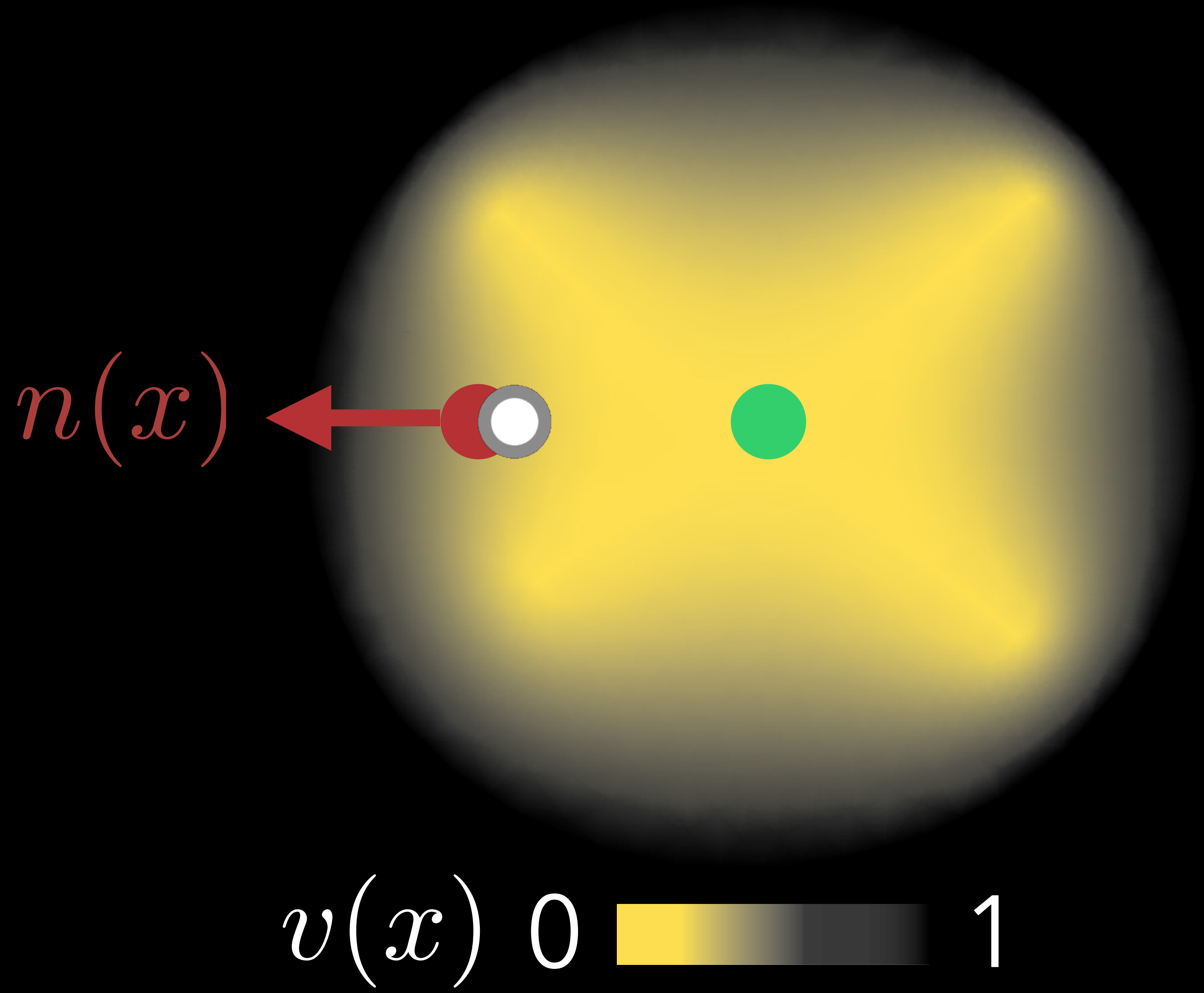
density   projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

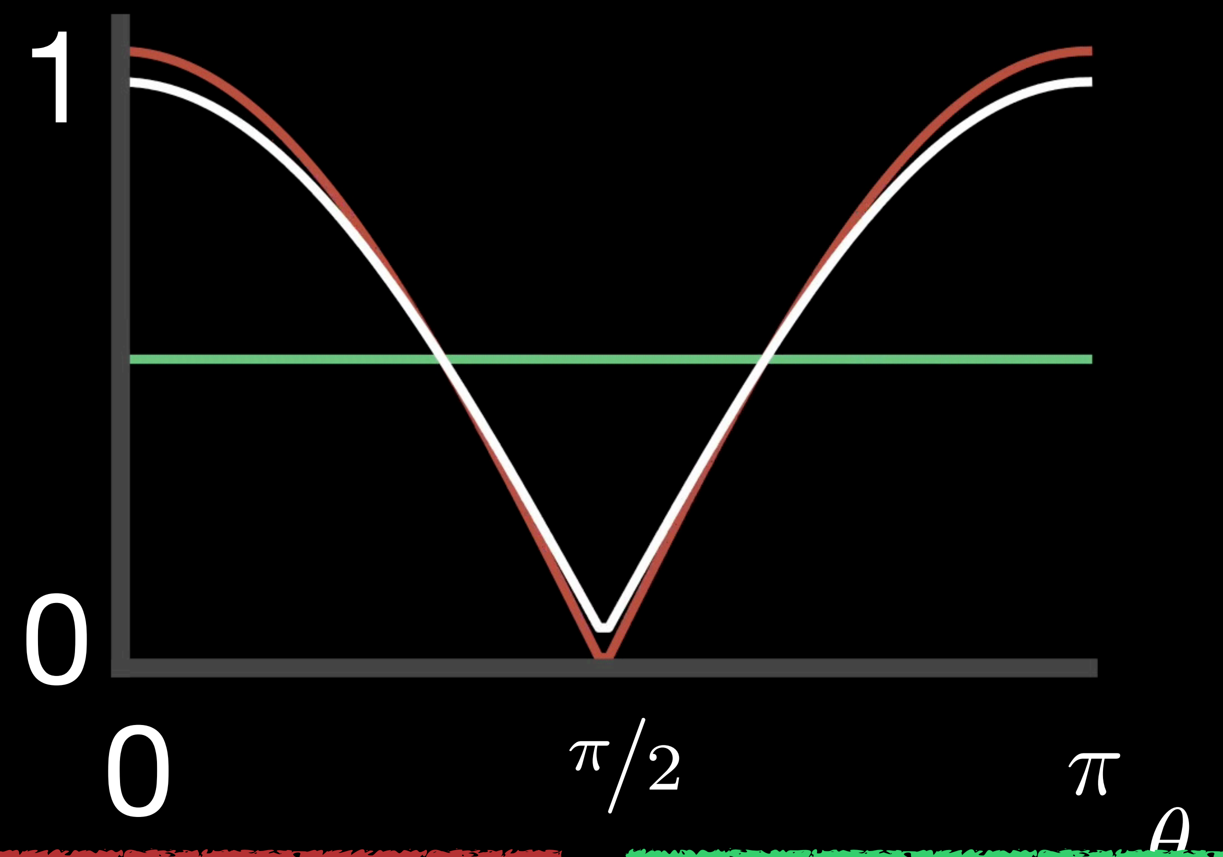
distribution of normals $D_x(m)$



boundary vs interior point x



projected area



projected area includes grazing at boundary

projected area is a constant function in interior

understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

distribution of normals	microflake model [Jakob et al. 2010, Heitz et al. 2015]	opaque solids (ours)
$D_x(m)$	particle density · projected area $\rho(x) \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$	solid density · projected area $\frac{\lVert \nabla v(x) \rVert}{v(x)} \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$

understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

distribution of normals $D_x(m)$	microflake model [Jakob et al. 2010, Heitz et al. 2015]	opaque solids (ours)
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 uniform		

understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

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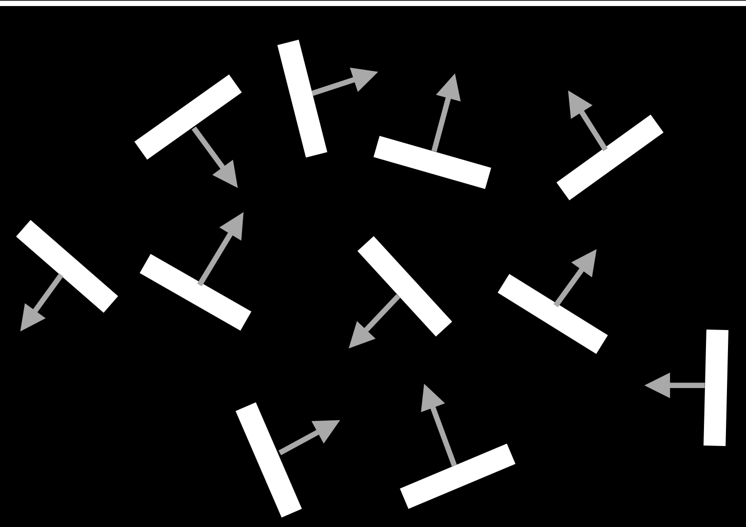
 uniform		

understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

microflake model

[Jakob et al. 2010, Heitz et al. 2015]

particle density · projected area

$$\rho(x) \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

opaque solids

(ours)

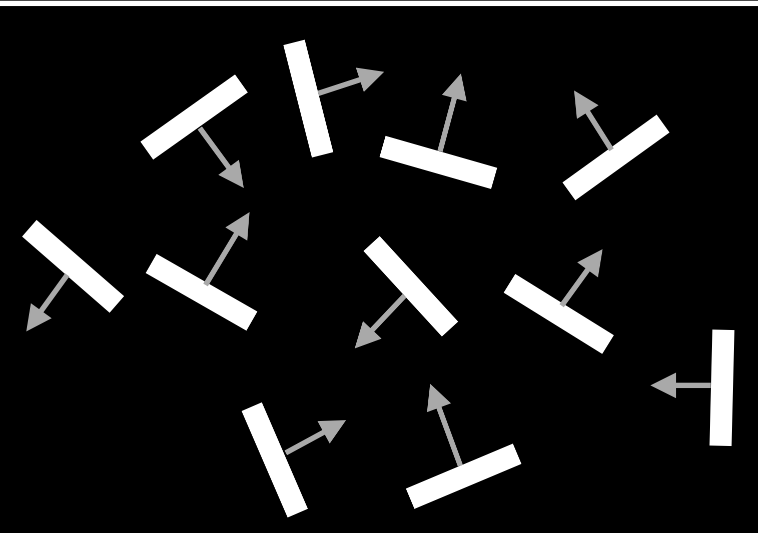
solid density · projected area

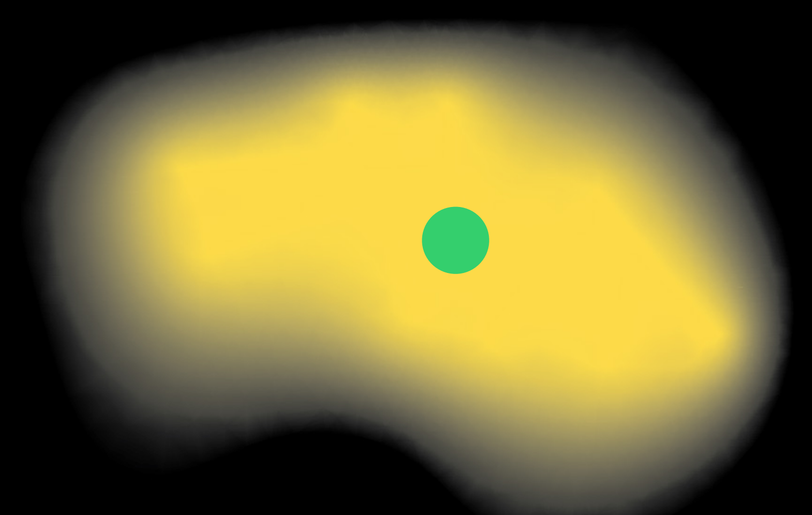
$$\frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

distribution of normals

$D_x(m)$

uniform





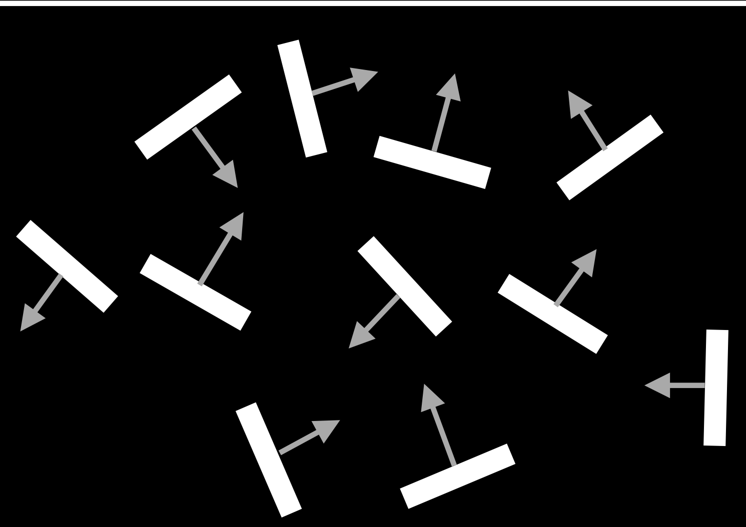
interior

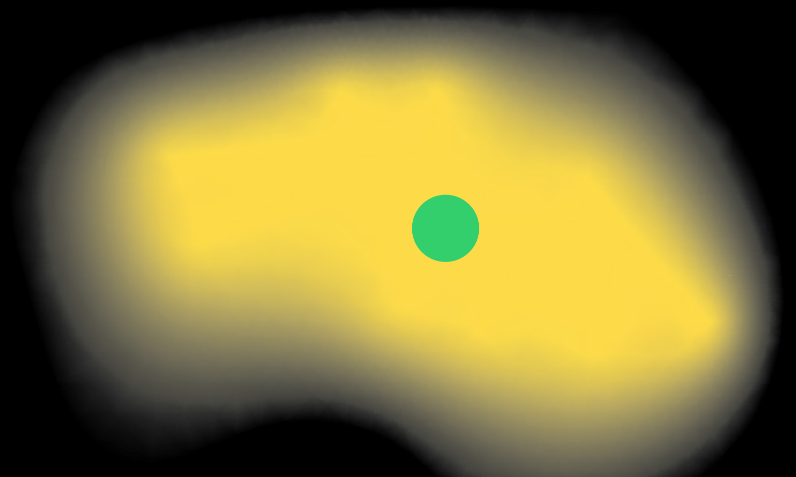
understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

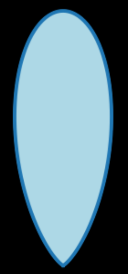
distribution of normals $D_x(m)$	microflake model [Jakob et al. 2010, Heitz et al. 2015]	opaque solids (ours)
	particle density · projected area	solid density · projected area
	$\rho(x) \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$	$\frac{\lVert\nabla v(x)\rVert}{v(x)} \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$

uniform





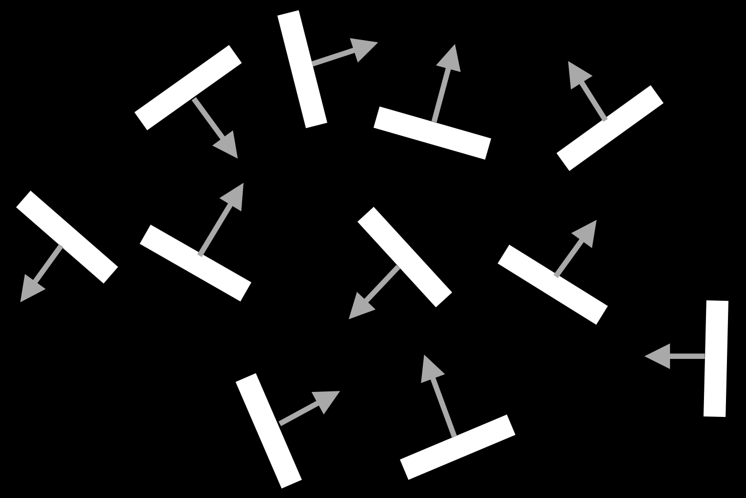
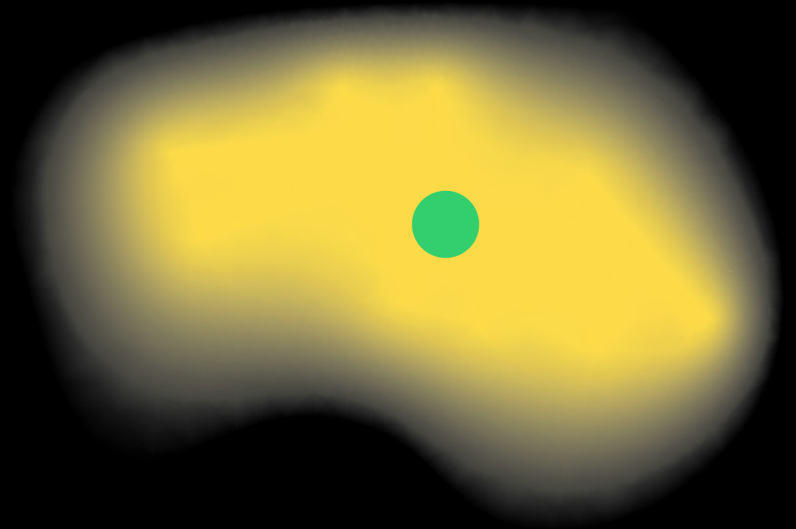
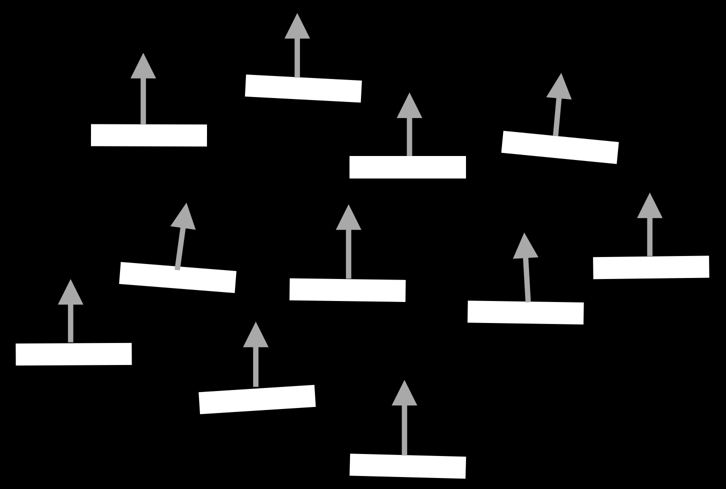
interior



concentrated

understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

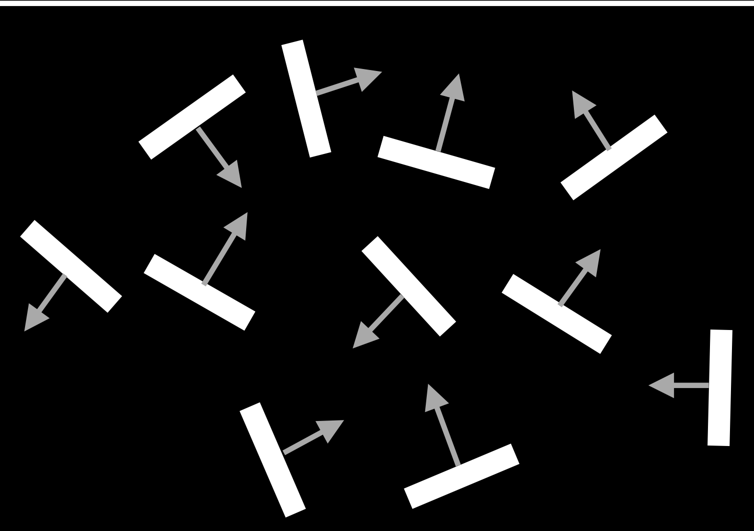
	microflake model [Jakob et al. 2010, Heitz et al. 2015]	opaque solids (ours)
	particle density · projected area	solid density · projected area
distribution of normals $D_x(m)$	$\rho(x) \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$	$\frac{\lVert \nabla v(x) \rVert}{v(x)} \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$
 uniform		 interior
 concentrated		

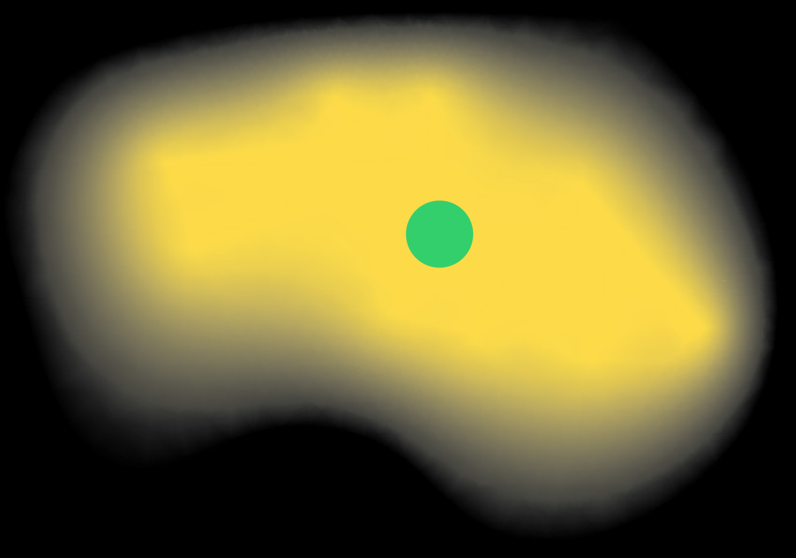
understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

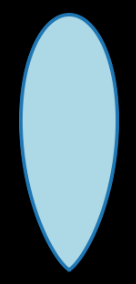
distribution of normals	microflake model [Jakob et al. 2010, Heitz et al. 2015]	opaque solids (ours)
$D_x(m)$	particle density · projected area: $\rho(x) \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$	solid density · projected area: $\frac{\lVert\nabla v(x)\rVert}{v(x)} \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$

uniform

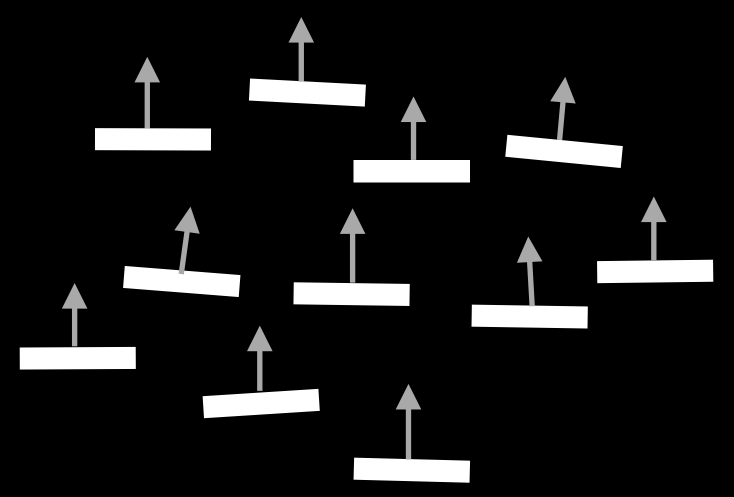


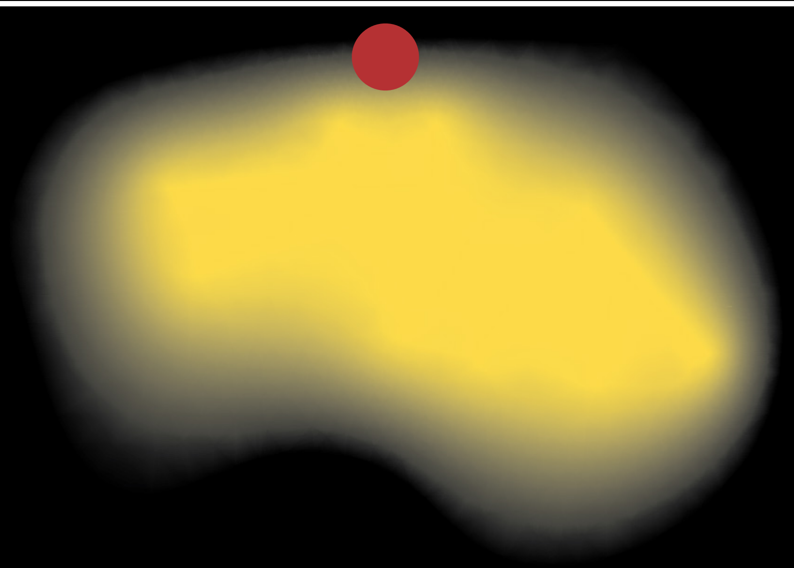


interior



concentrated





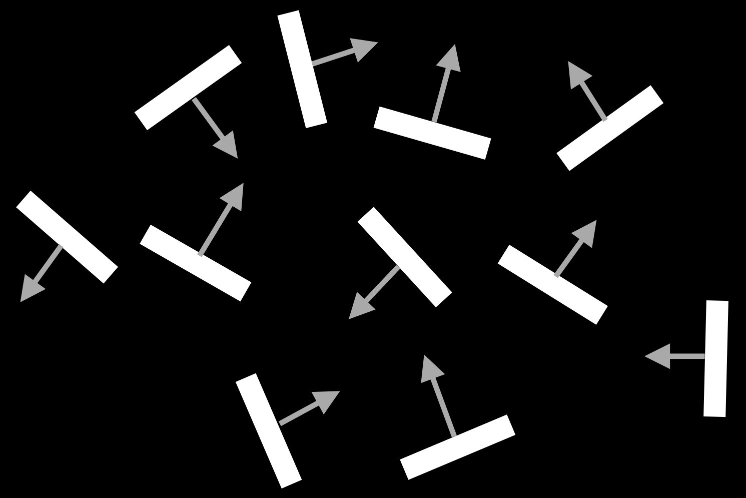
boundary

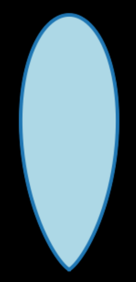
understanding projected area for opaque solids

Directly analogous to distribution of normals used in microflake models

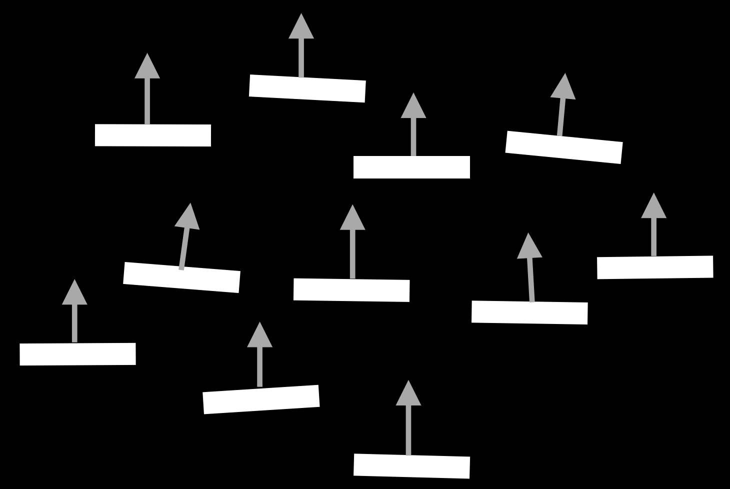
	microflake model [Jakob et al. 2010, Heitz et al. 2015]	opaque solids (ours)
distribution of normals $D_x(m)$	particle density · projected area $\rho(x) \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$	solid density · projected area $\dfrac{\lVert\nabla v(x)\rVert}{v(x)} \cdot \int_{S^2} \lvert\omega \cdot m\rvert D_x(m) \mathrm{d}m$

uniform

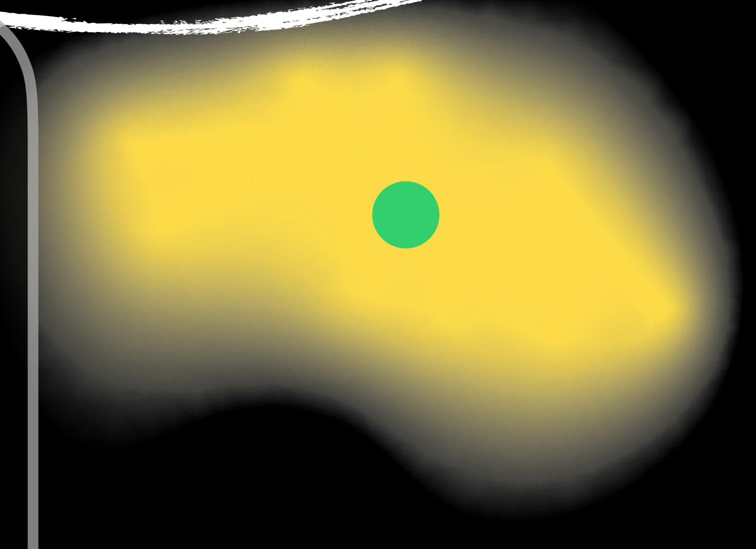




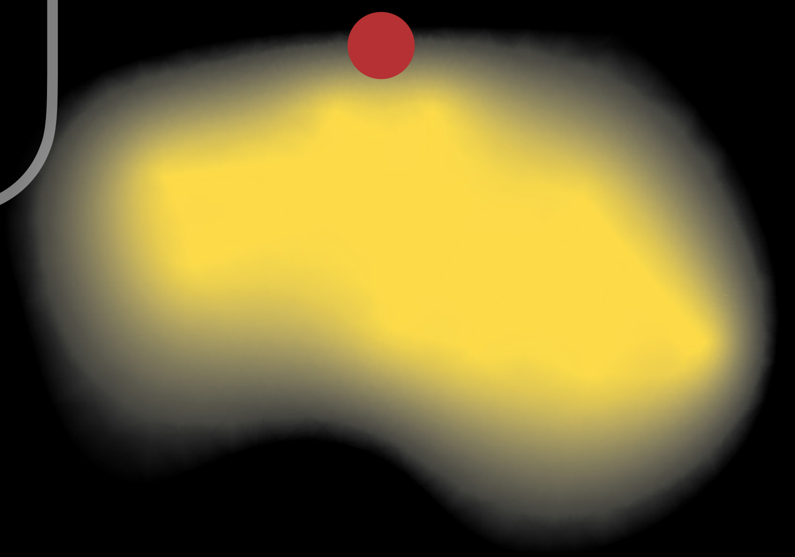
concentrated



SGGX, Phong, linear mixture, etc.



interior



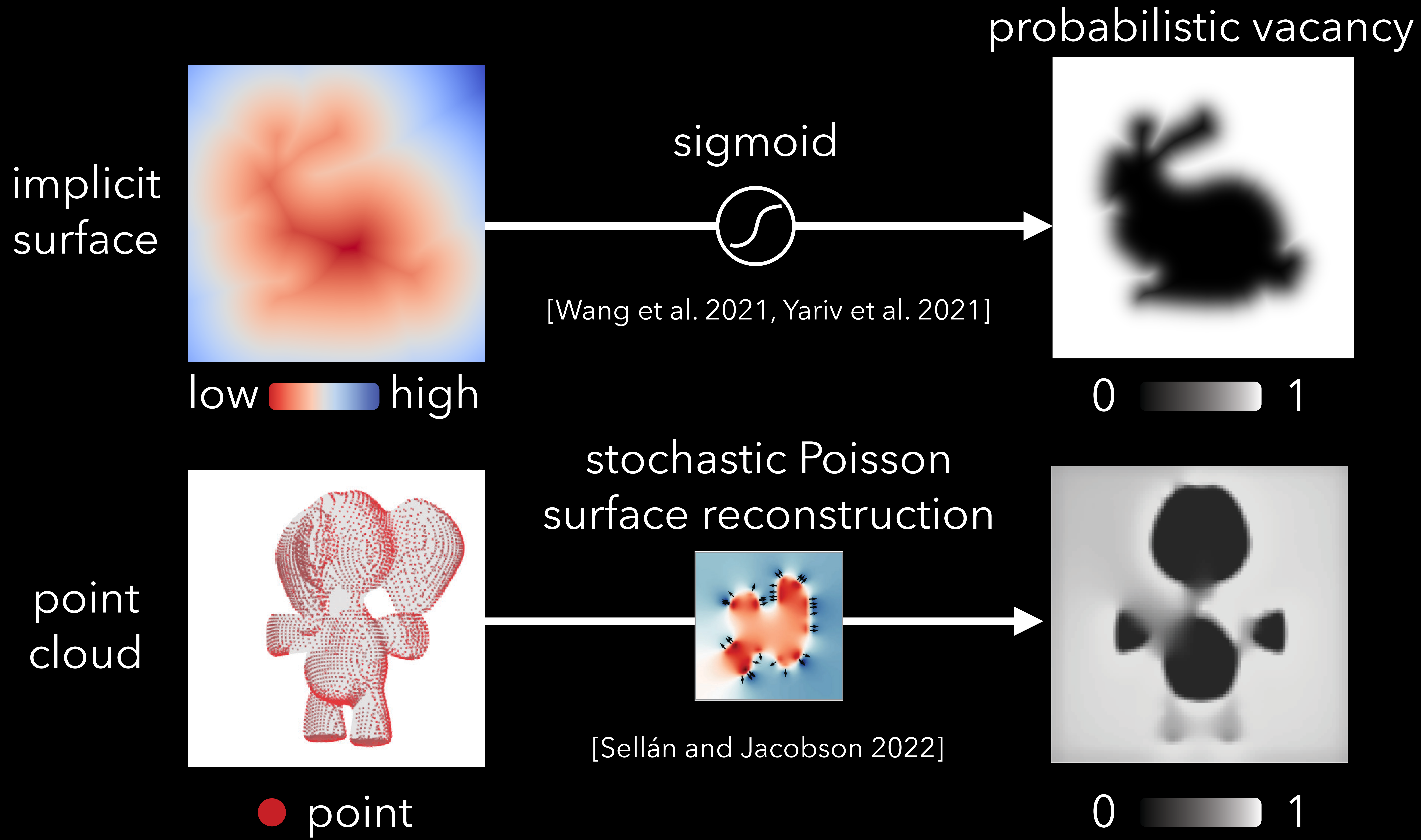
boundary

Where does vacancy $v(x)$ come from?

density   projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

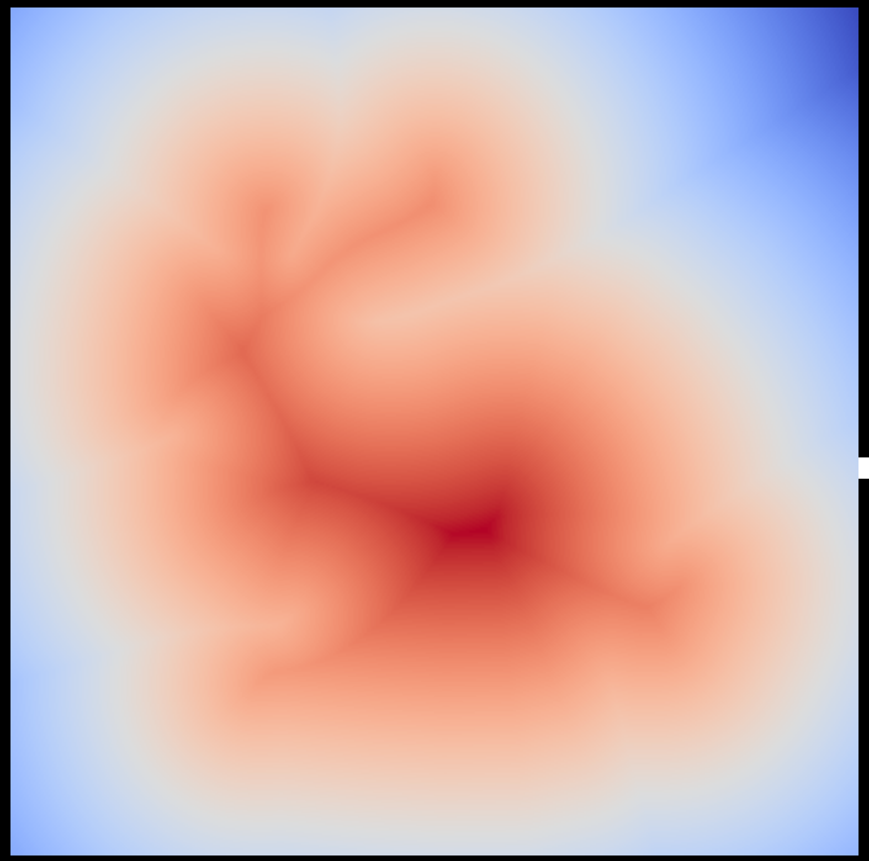
parameterizing vacancy with geometric representations



parameterizing vacancy with geometric representations

probabilistic vacancy

implicit surface



low — high

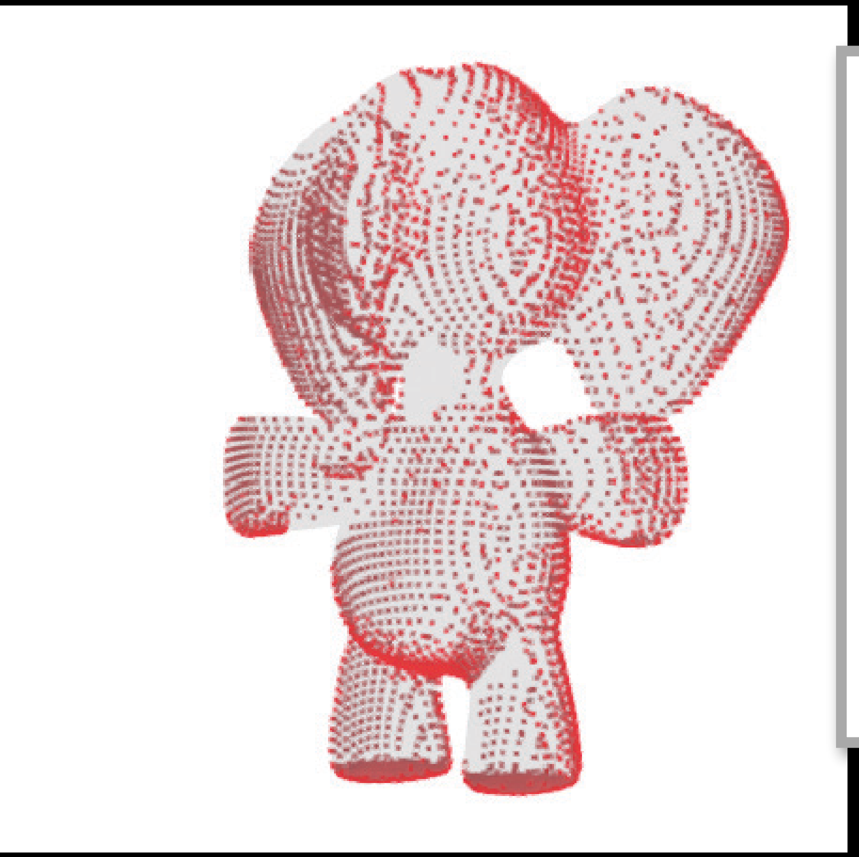
sigmoid

[Wang et al. 2021, Yariv et al. 2021]

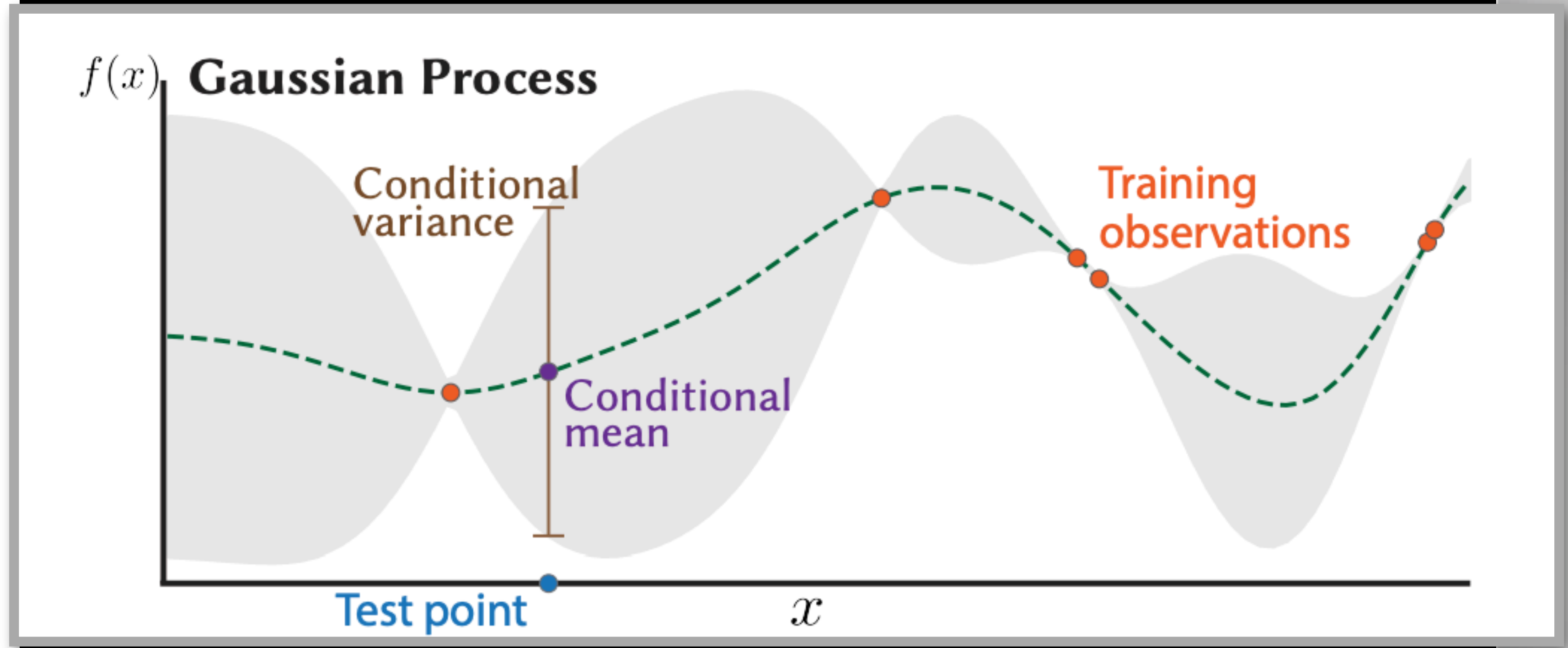
0 — 1

stochastic Poisson

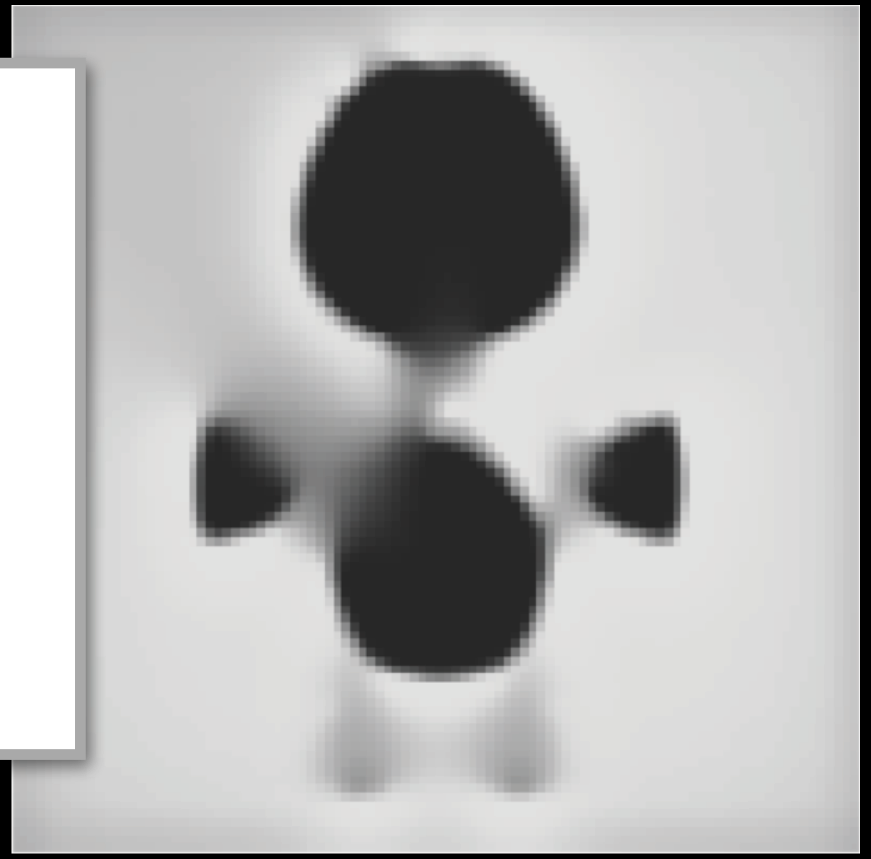
point cloud



● point



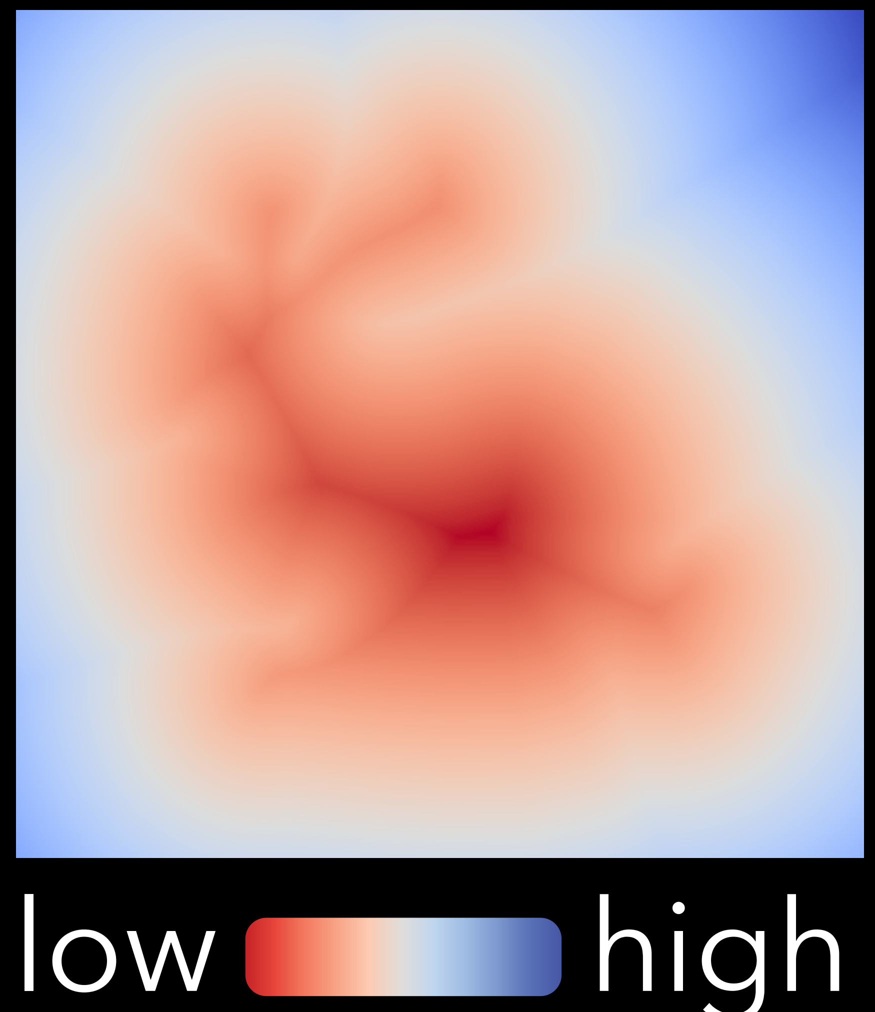
[Sellán and Jacobson 2022]



0 — 1

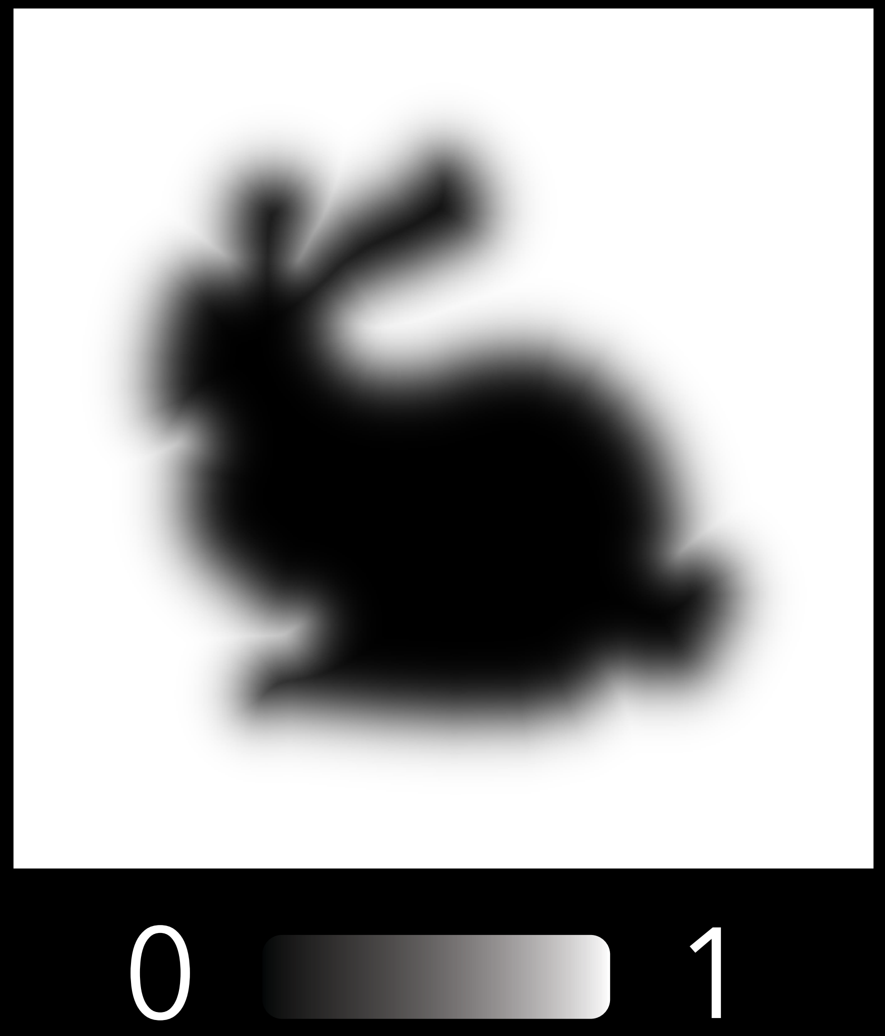
stochastic implicit surfaces

implicit surface



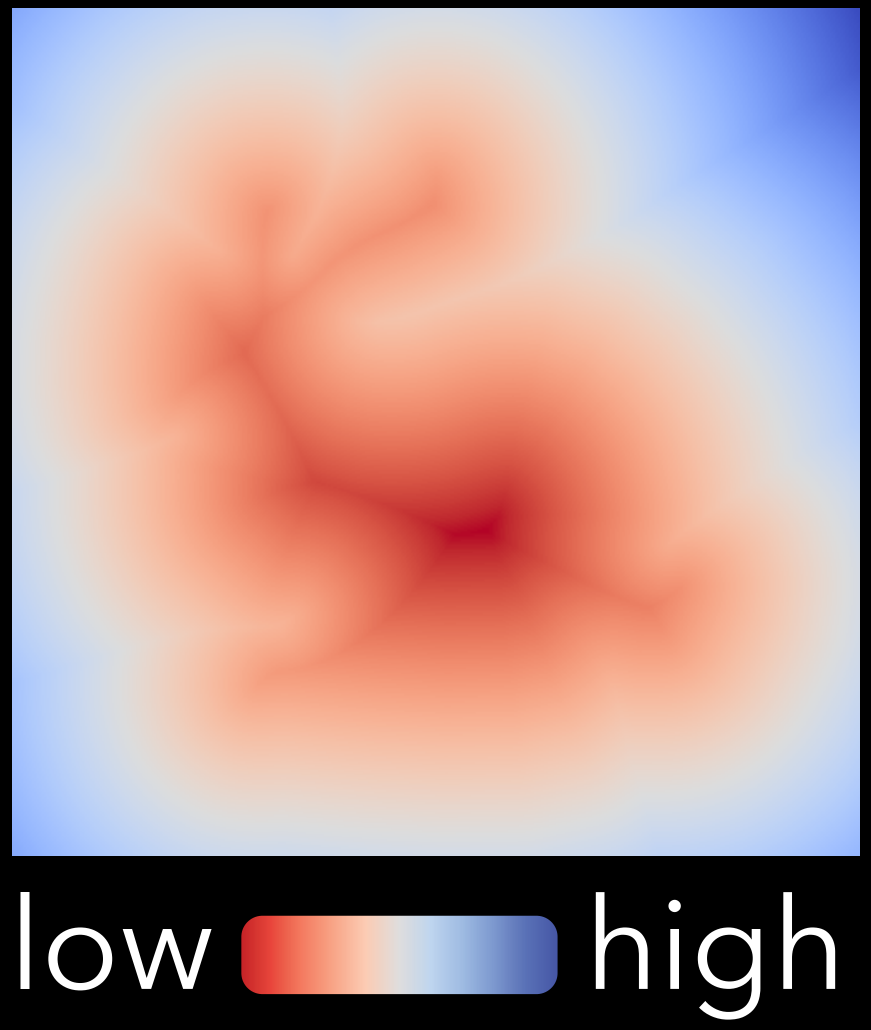
stochastic implicit $G(x)$

probabilistic vacancy

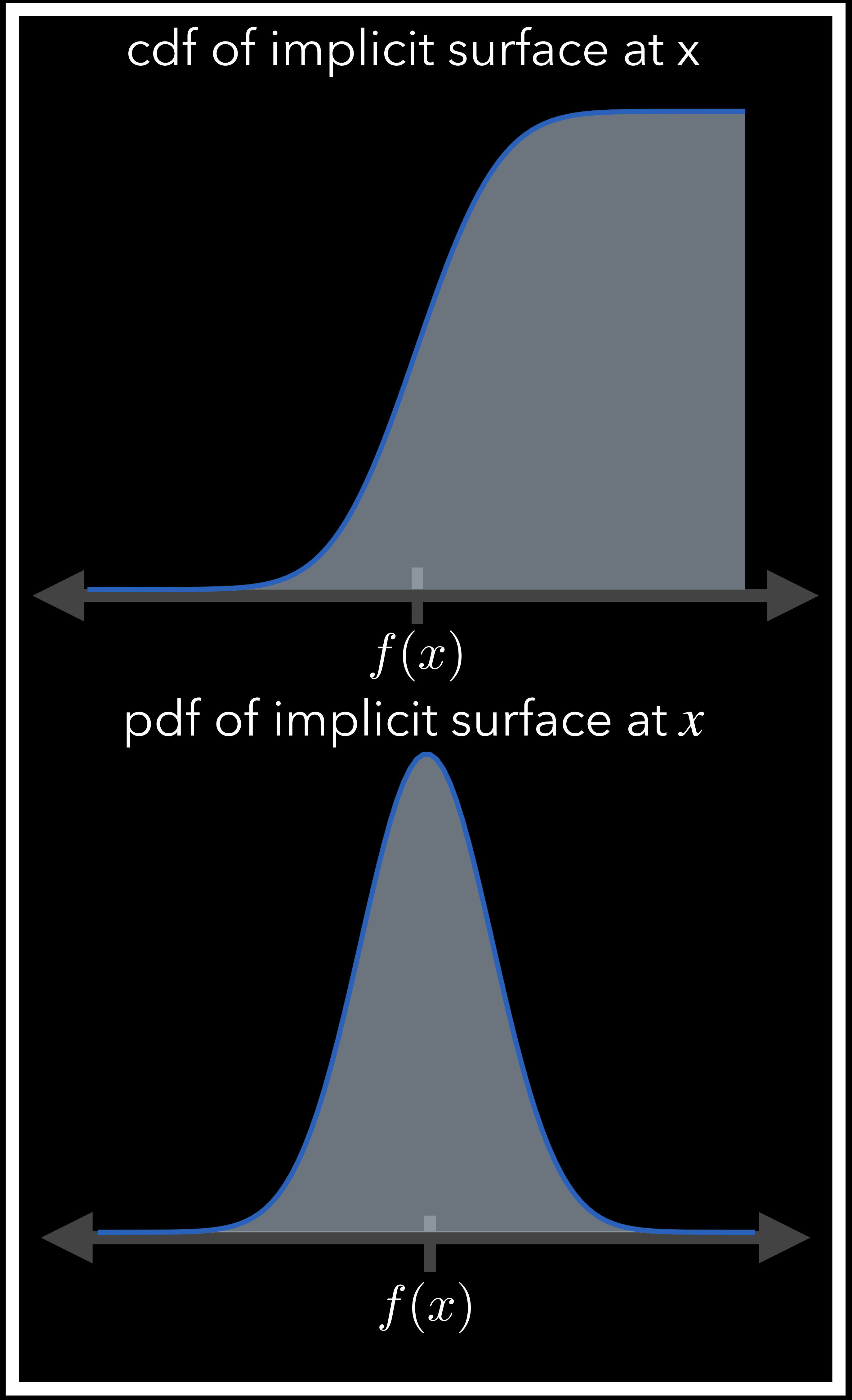


stochastic implicit surfaces

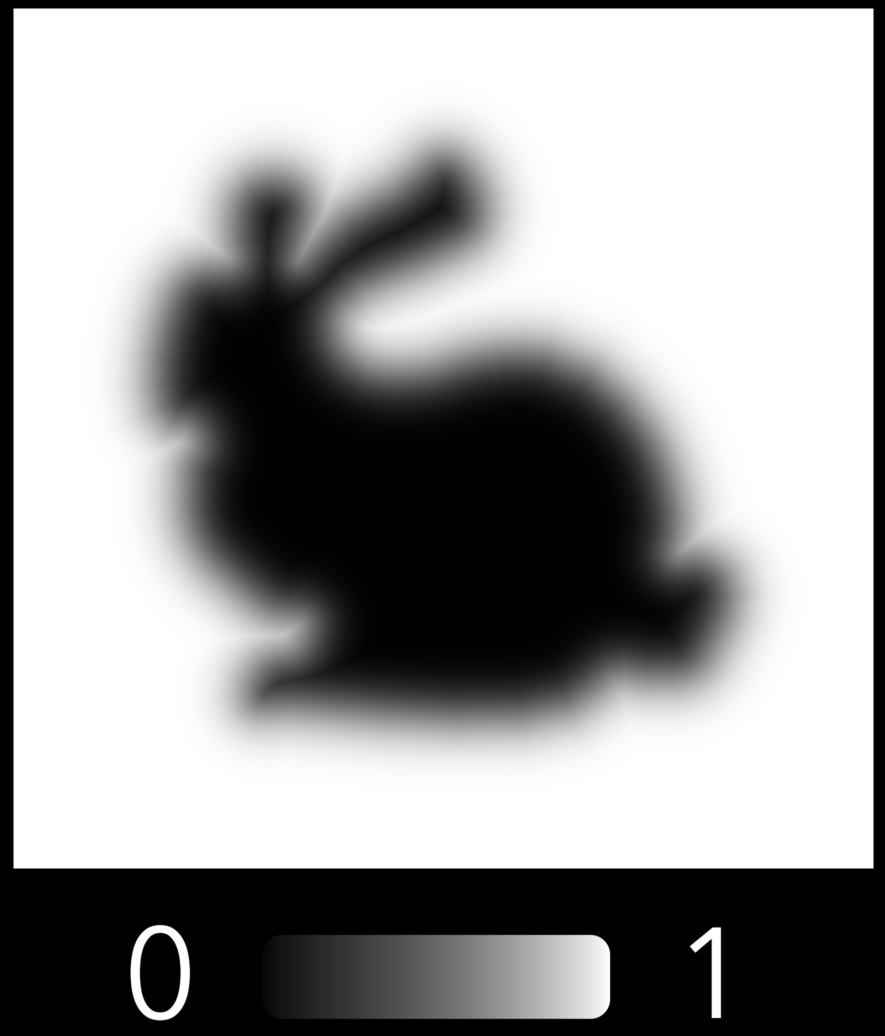
implicit surface



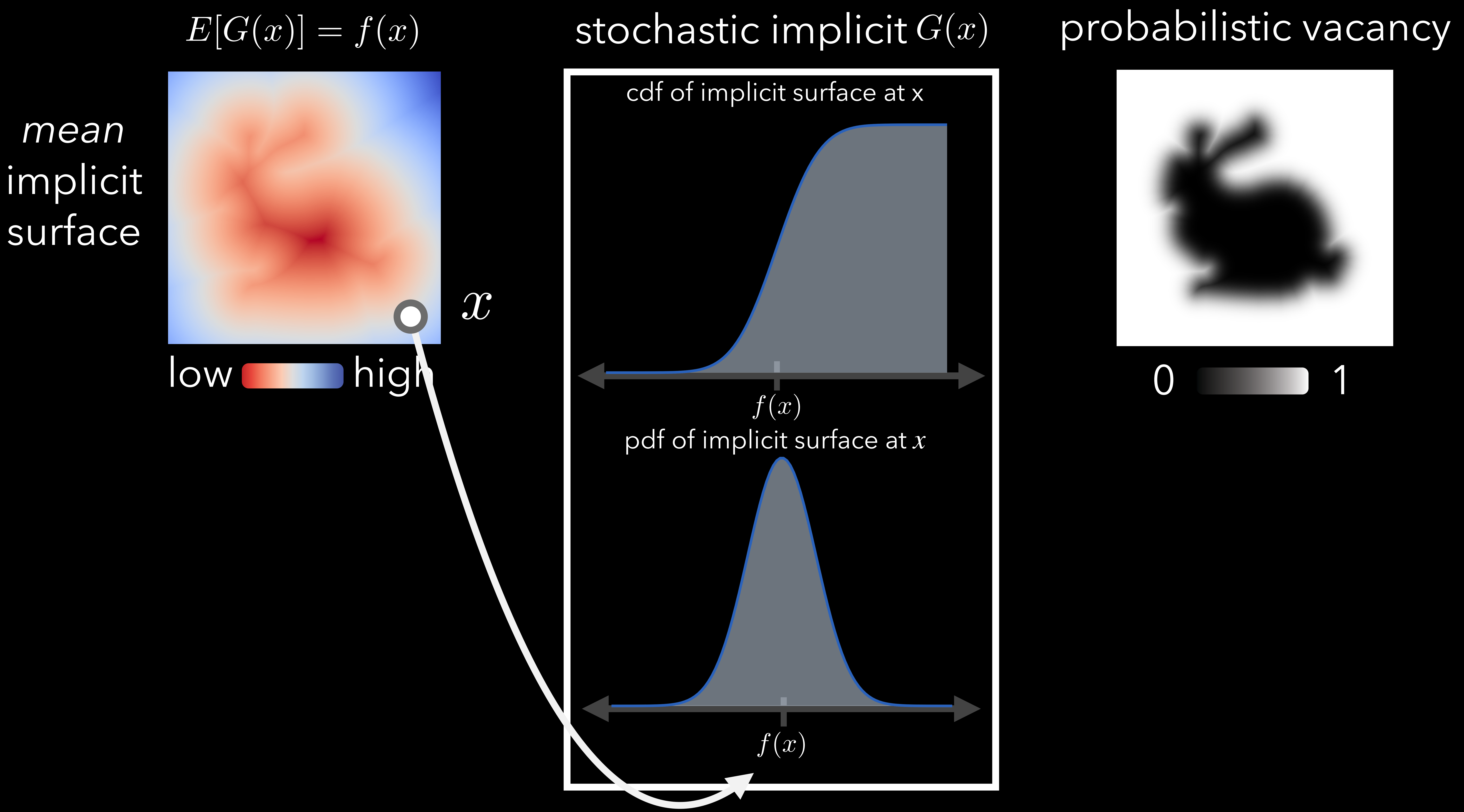
stochastic implicit $G(x)$



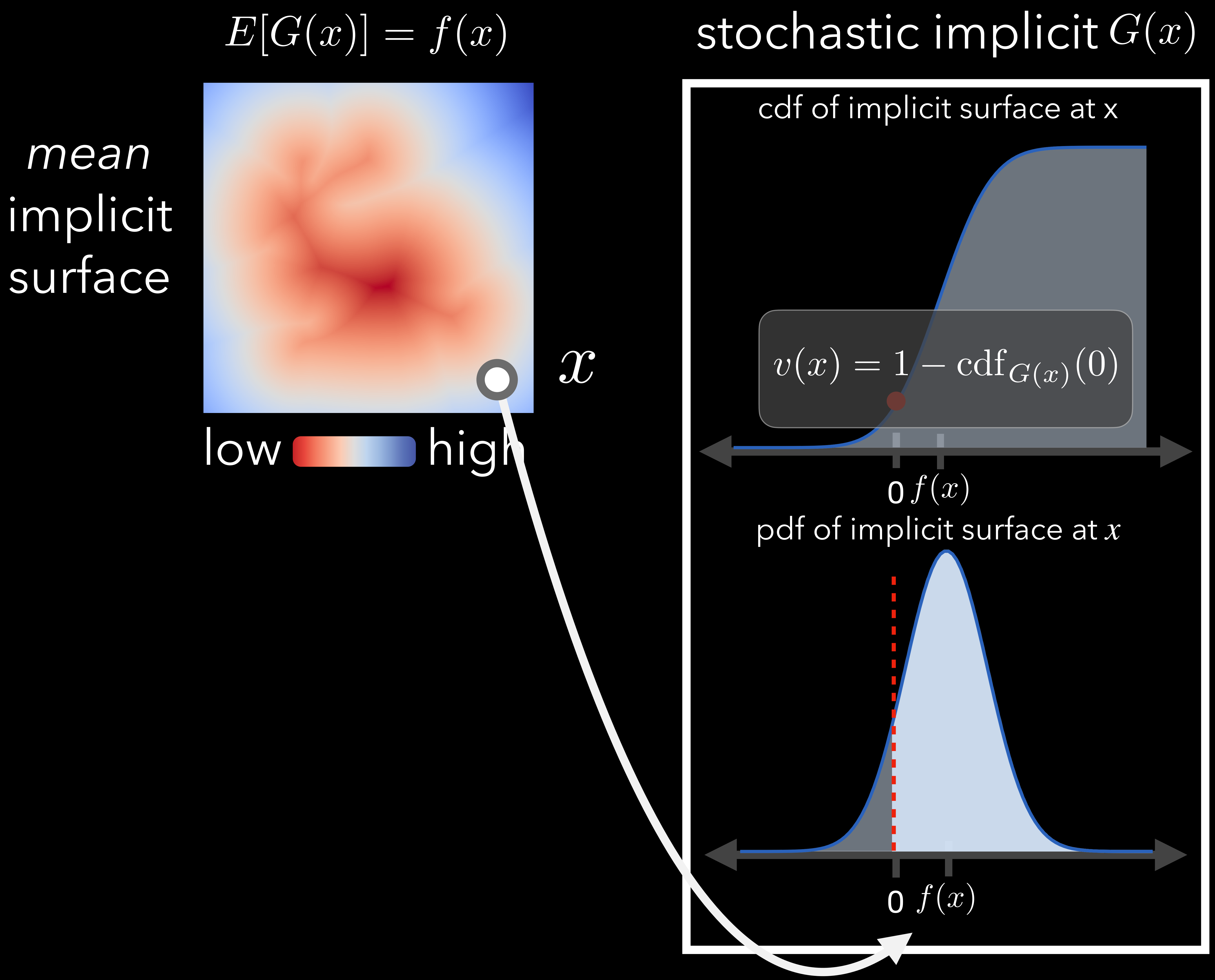
probabilistic vacancy

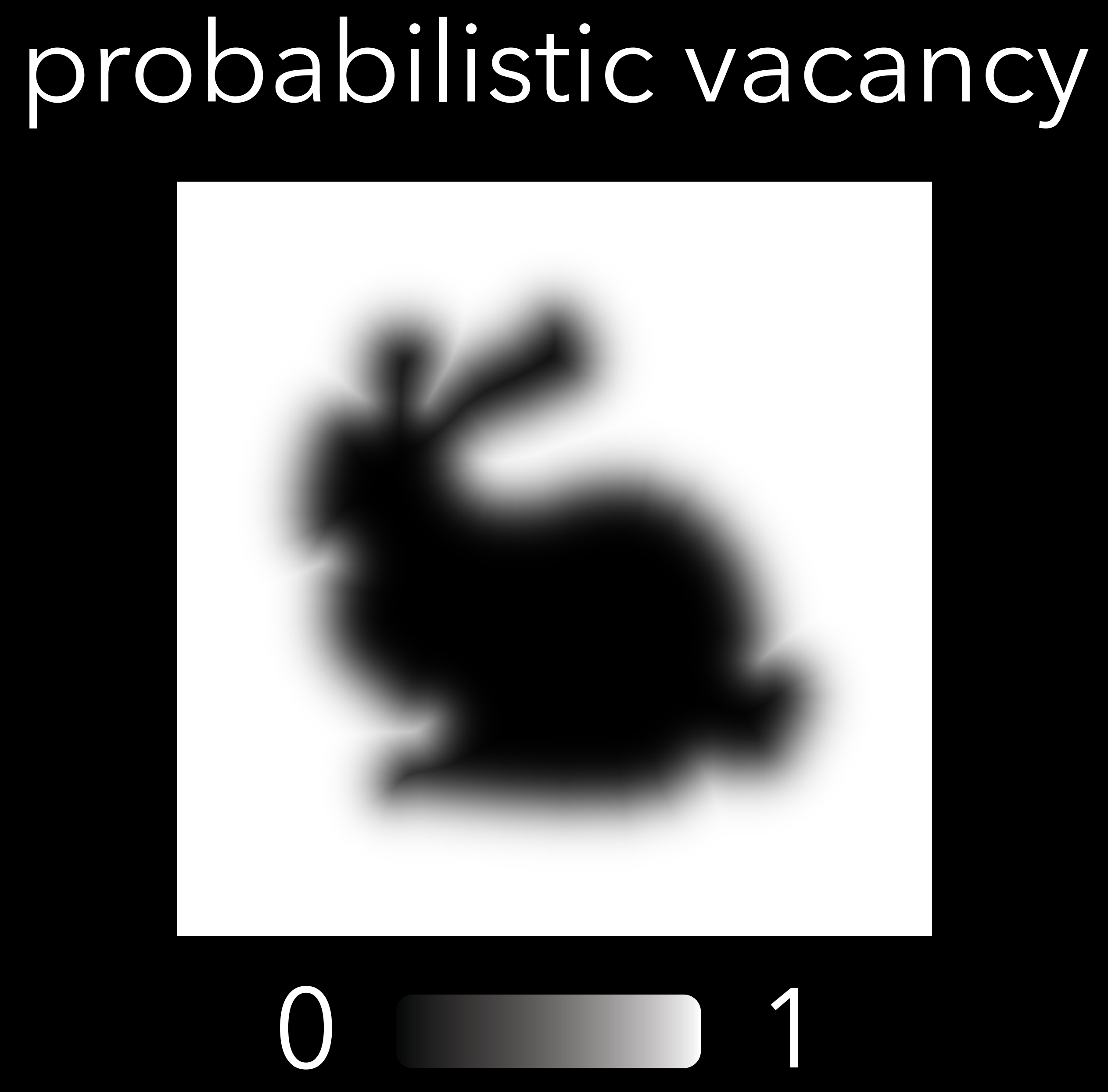


stochastic implicit surfaces



stochastic implicit surfaces





probability of vacancy?

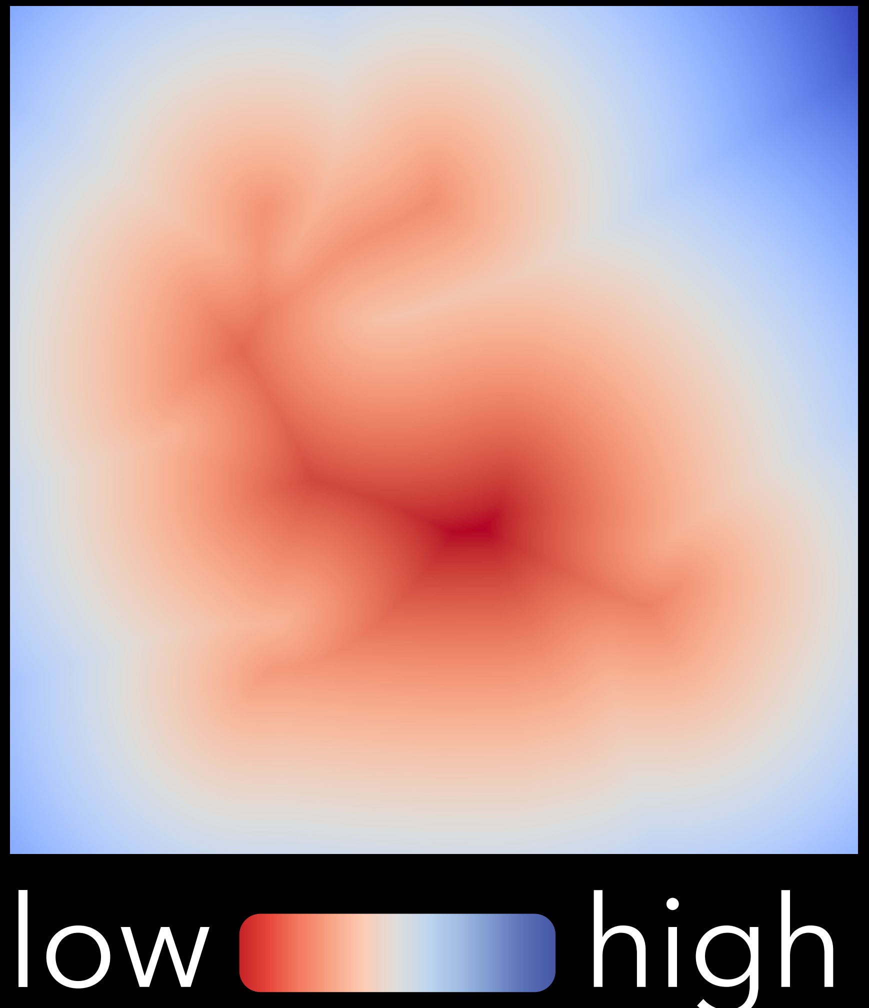
$$P(G(x) > 0)$$

(i.e. outside zero level set)

stochastic implicit surfaces

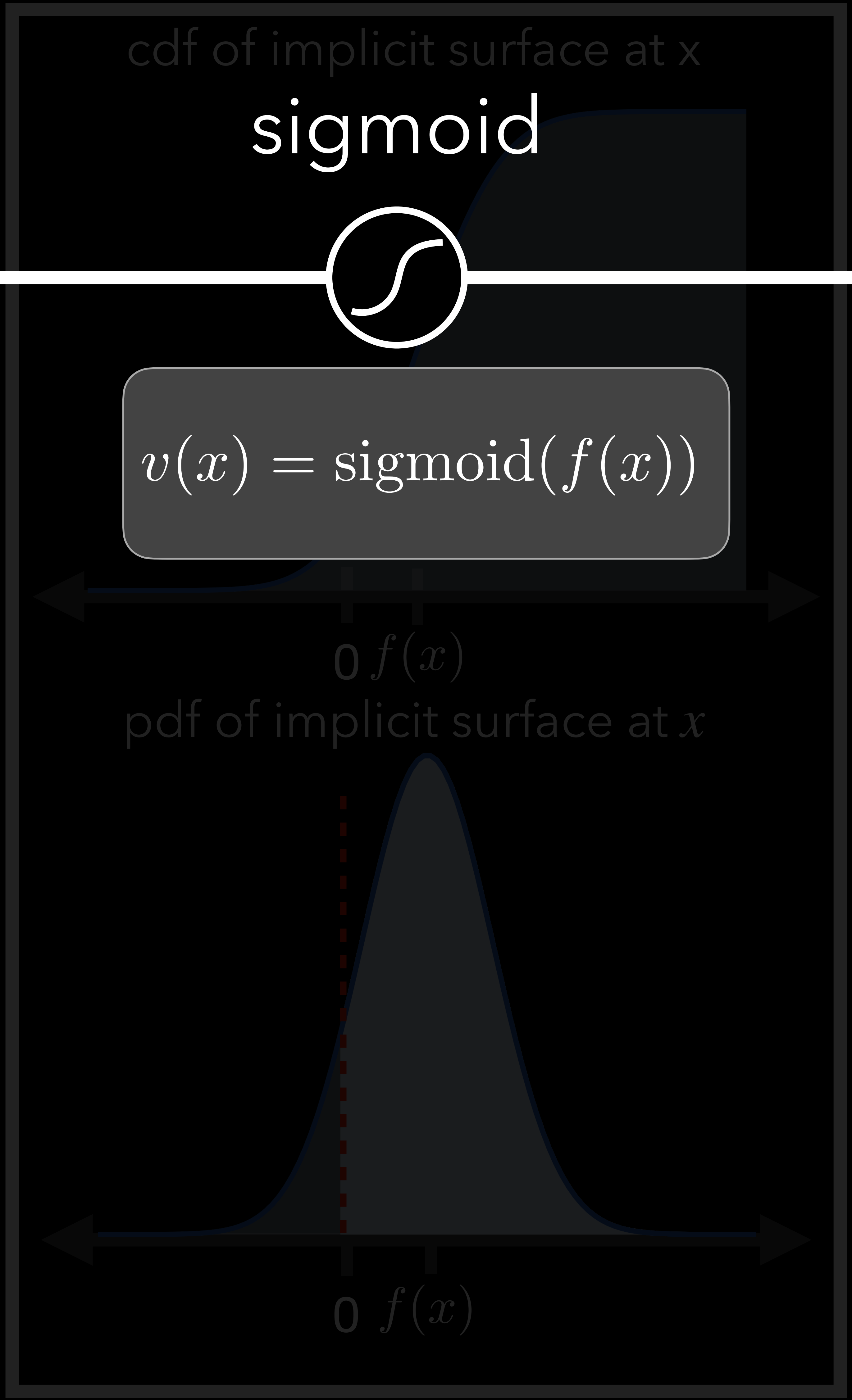
$E[G(x)] = f(x)$

mean implicit surface



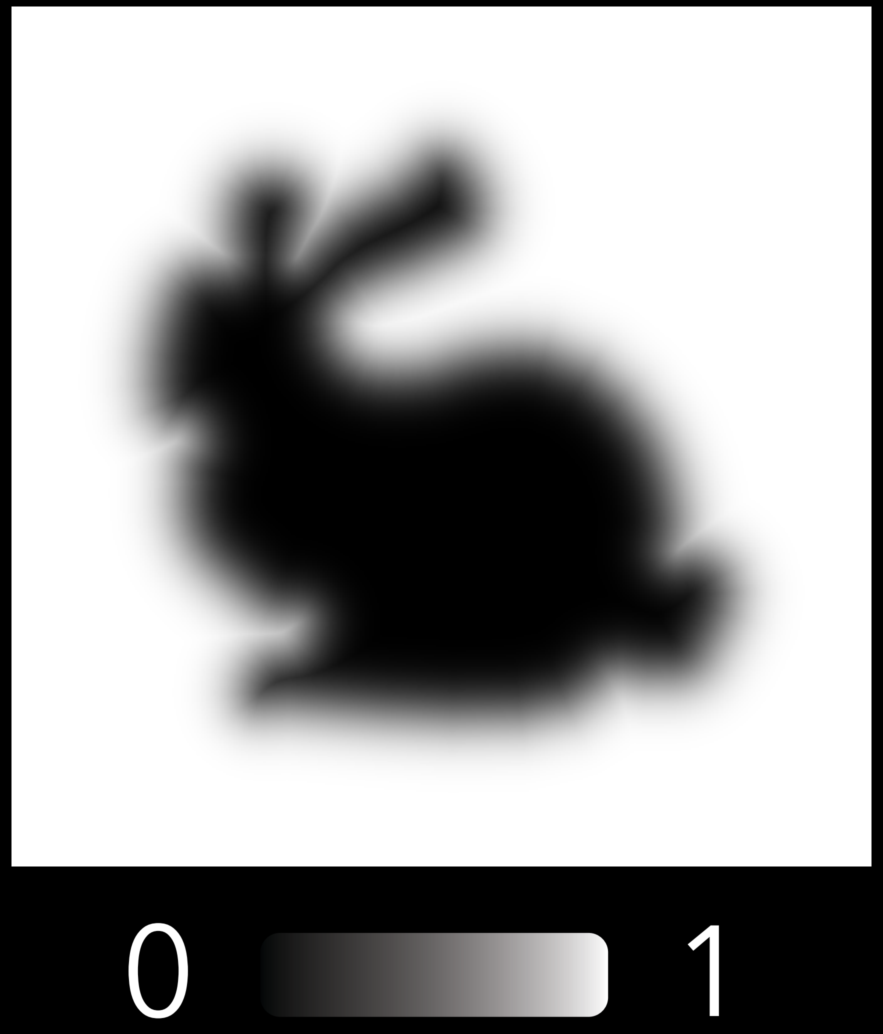
stochastic implicit $G(x)$

sigmoid



$v(x) = \text{sigmoid}(f(x))$

probabilistic vacancy



probability of vacancy?

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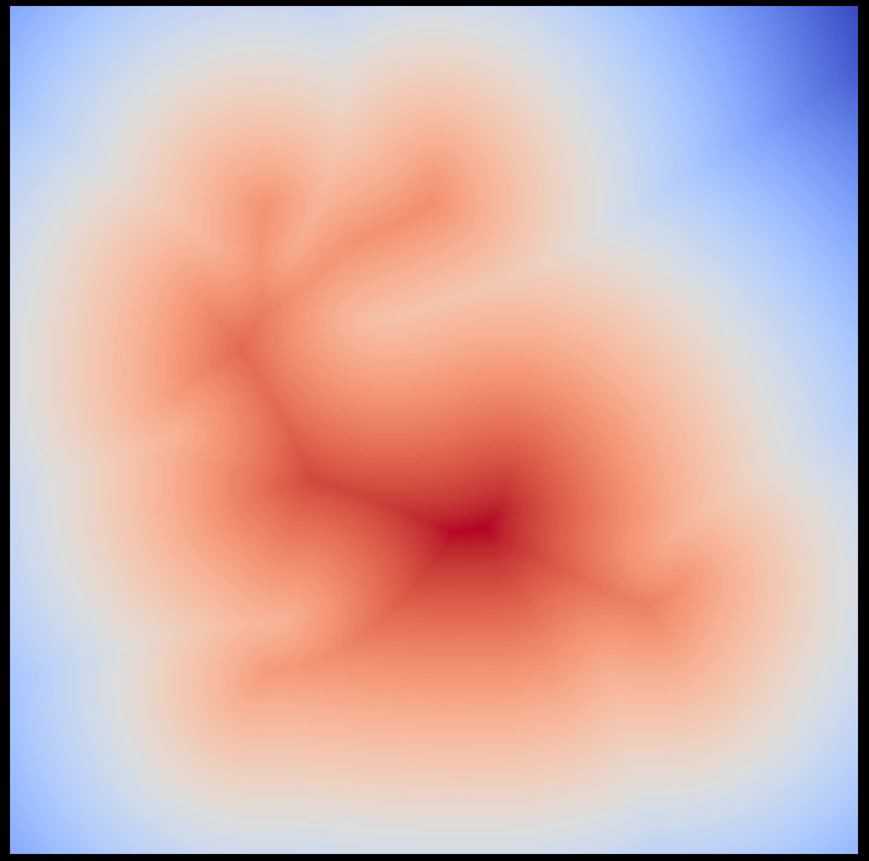


low ▬ high

stochastic implicit $G(x)$

sigmoid

$$v(x) = \text{sigmoid}(f(x))$$

Each symmetric PDF corresponds to a different type of sigmoid (i.e. Gaussian –> Erf)

Provides common stochastic lens for prior work.

probabilistic vacancy

0 ▬ 1

probability of vacancy?

$$P(G(x) > 0)$$

(i.e. outside zero level set)

density

projected area

$$\sigma(x, \omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

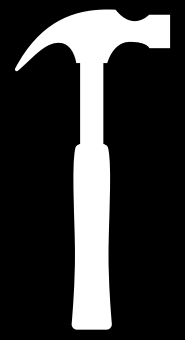
$$v(x) = 1 - \mathrm{cdf}_{G(x)}(0)$$

$G(x)$

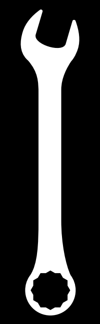
implicit distribution

$D_x(m)$

distribution of normals



build your own attenuation coefficient



density

projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

$$v(x) = 1 - \mathrm{cdf}_{G(x)}(0)$$

$G(x)$

$D_x(m)$

implicit distribution

distribution of normals

reciprocal?

NeuS [Wang et al. 2021]

VolSDF [Yariv et al. 2021]

Ours

density

projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

$$v(x) = 1 - \mathrm{cdf}_{G(x)}(0)$$

	$G(x)$	$D_x(m)$	
	implicit distribution	**distribution of normals**	**reciprocal?**
NeuS [Wang et al. 2021]	Logistic		
VolSDF [Yariv et al. 2021]	Laplace		
Ours	**Gaussian**		

density

projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2}$$

$$v(x) = 1 - \mathrm{cdf}_{G(x)}(0)$$

$$G(x)$$

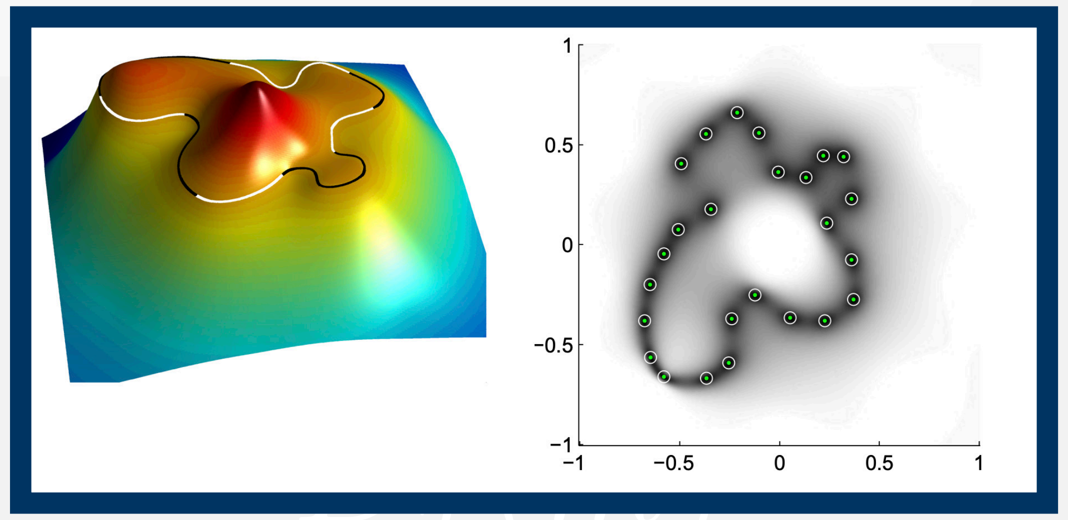
implicit distribution

NeuS [Wang et al. 2021]	Logistic
VolSDF [Yariv et al. 2021]	Laplace
Ours	**Gaussian**

reciprocal?

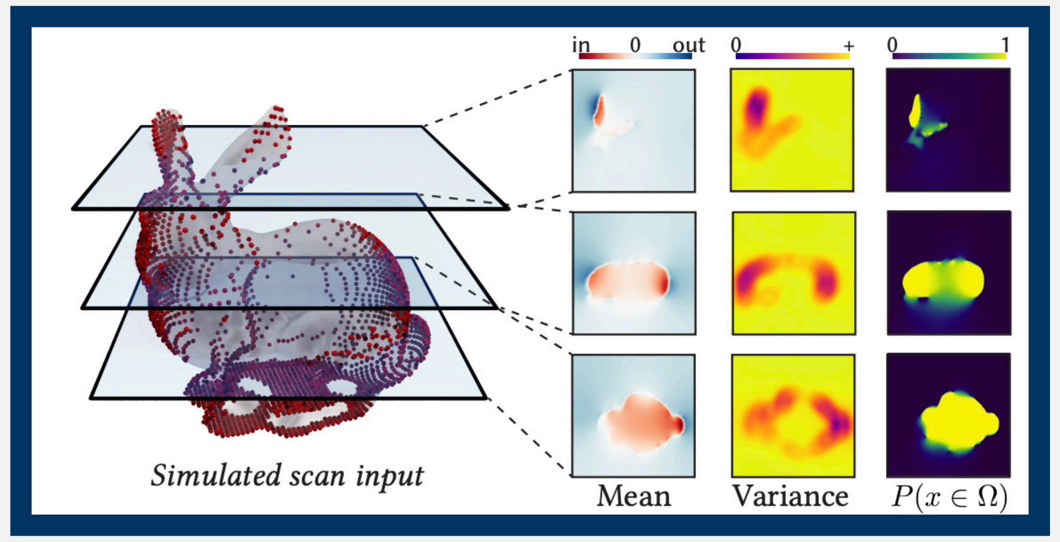
Gaussian process implicit surfaces

[Williams et al. 2006]



[Dragiev et al. 2011]

[Sellán and Jacobson 2022]



density — projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

$$v(x) = 1 - \mathrm{cdf}_{G(x)}(0)$$

	$G(x)$	$D_x(m)$	
	implicit distribution	**distribution of normals**	**reciprocal?**
NeuS [Wang et al. 2021]	Logistic	boundary-like	
VolSDF [Yariv et al. 2021]	Laplace	interior-like	
Ours	**Gaussian**	both (spatially varying)	

density · projected area

$$\sigma(x,\omega) = \frac{\|\nabla v(x)\|}{v(x)} \cdot \int_{S^2} |\omega \cdot m| D_x(m) \mathrm{d}m$$

$$v(x) = 1 - \mathrm{cdf}_{G(x)}(0)$$

	$G(x)$	$D_x(m)$	
	implicit distribution	**distribution of normals**	**reciprocal?**
NeuS [Wang et al. 2021]	Logistic	boundary-like	✗
VolSDF [Yariv et al. 2021]	Laplace	interior-like	✓
Ours	**Gaussian**	both (spatially varying)	✓

Goal: understand the design space for the attenuation coefficient

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- We specifically compare only the **attenuation coefficients**

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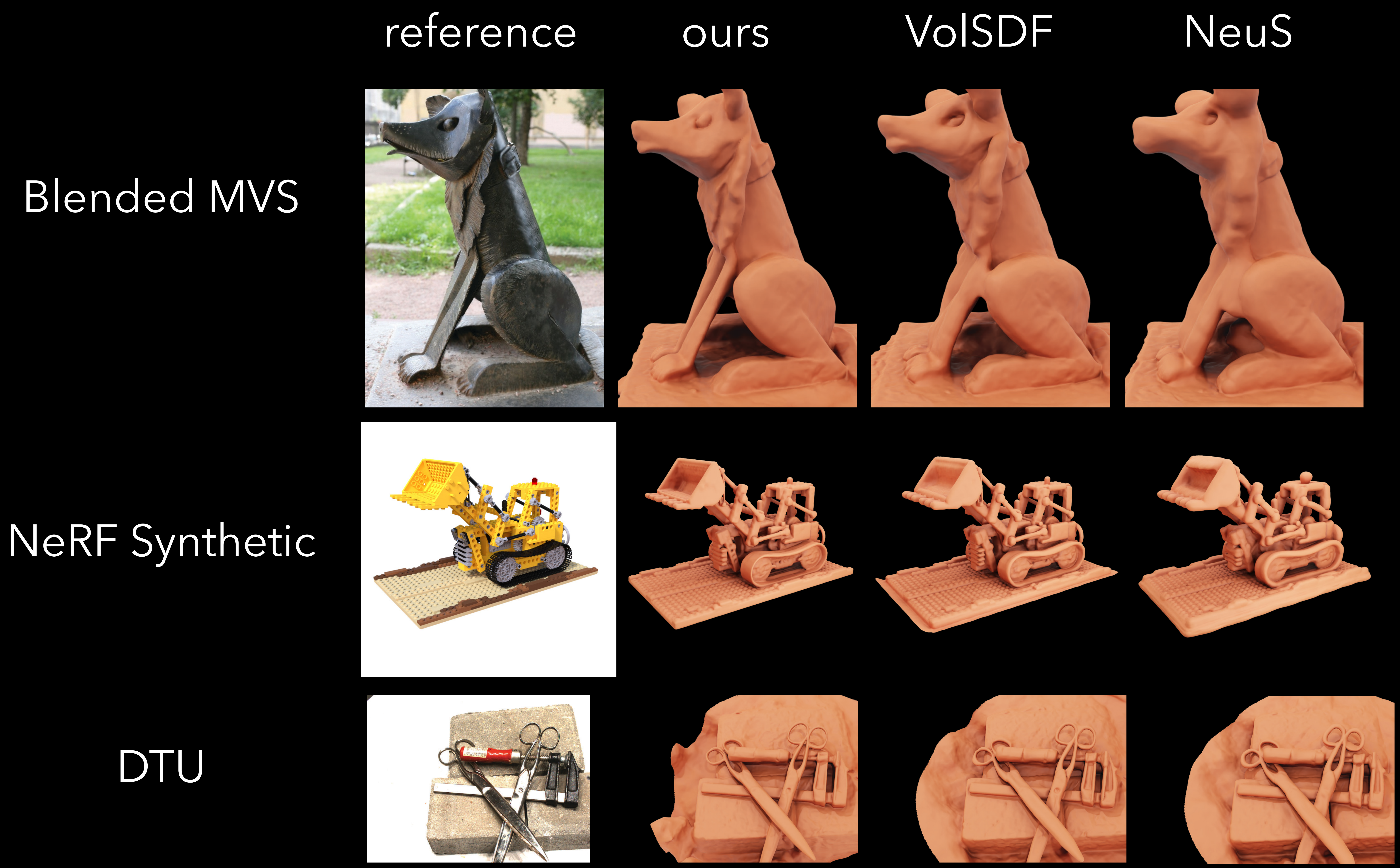
- We specifically compare only the **attenuation coefficients**
- We run all experiments using **same reconstruction pipeline**
- We consider only **basic quadrature** for transmittance estimation

consistently improves surface reconstruction



Blended MVS
reference

Blended MVS

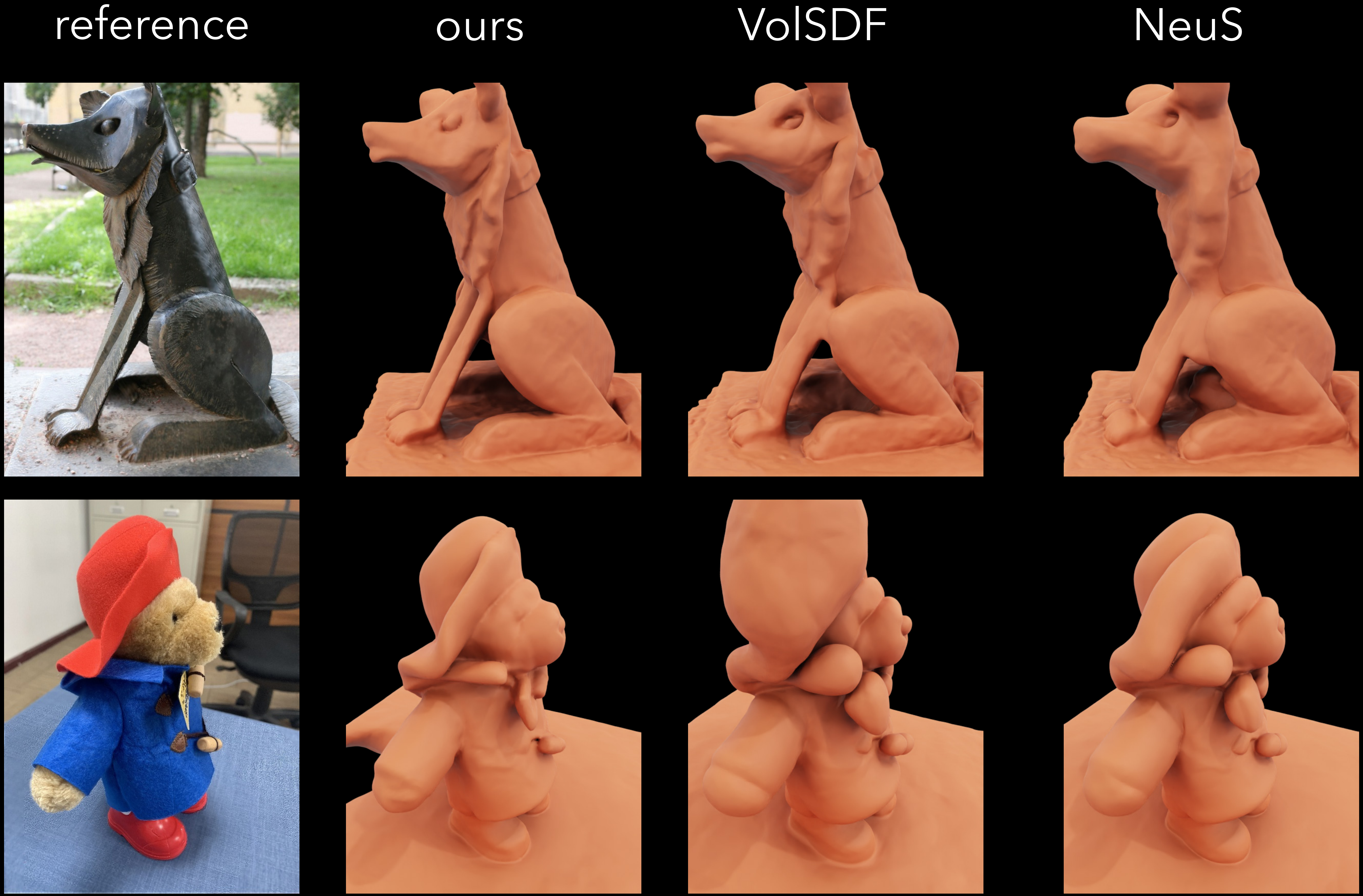
VolSDF

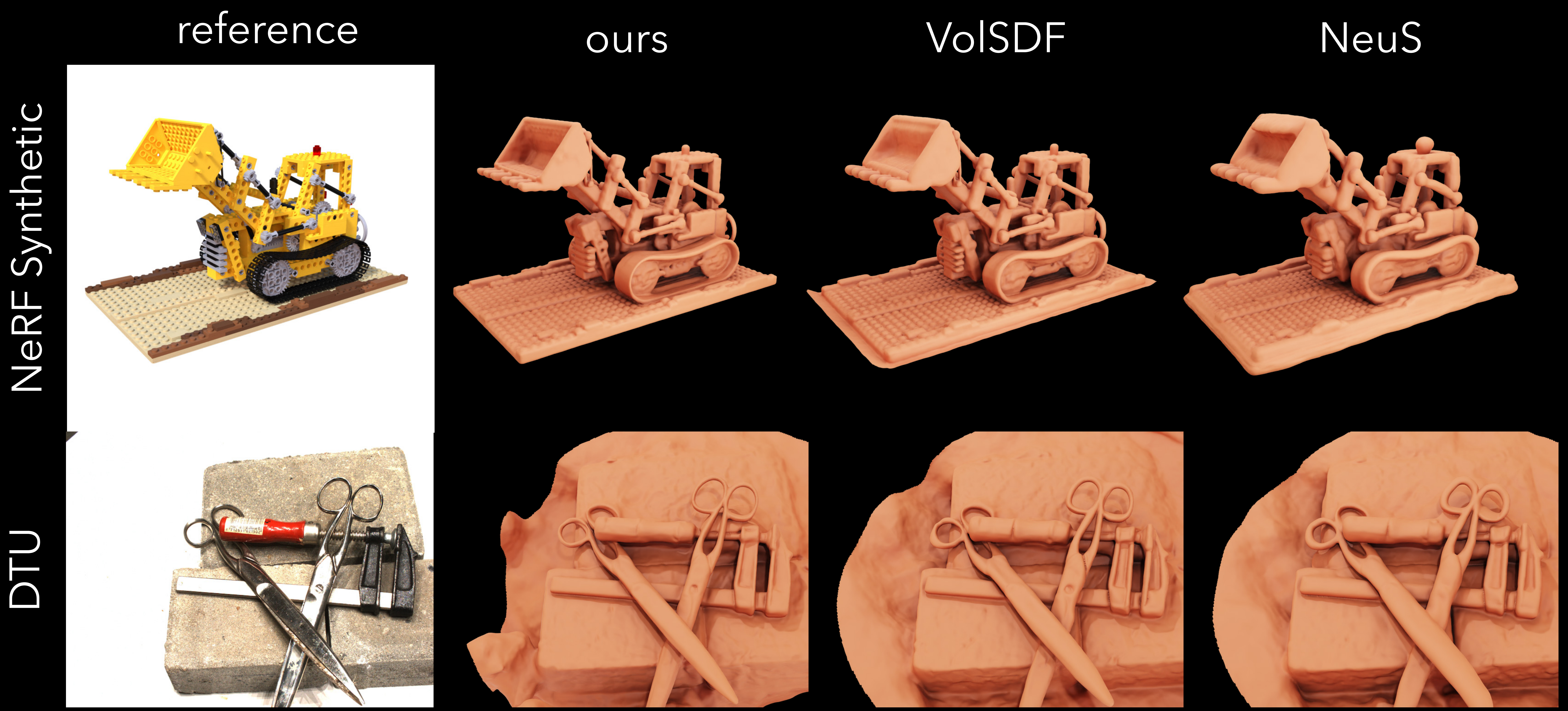
Blended MVS

NeuS

Blended MVS
ours

Blended MVS





mean Chamfer distance

	ours	VolSDF	NeuS
NeRF Synthetic	**0.133**	0.252	0.201
DTU	**1.57**	1.84	2.17

several ablations to design our attenuation coefficient

Table 8. Chamfer distances on the DTU dataset when using different implicit function distributions Ψ for the density $\sigma^{\parallel}$.

Ψ model	logistic	Laplace	**Gaussian**
24	2.73	**1.92**	1.99
37	3.56	3.65	**3.08**
40	**1.94**	2.32	2.28
55	1.77	1.65	**1.64**
63	1.86	1.76	1.76
65	2.67	2.60	**2.45**
69	1.73	1.58	**1.31**
83	1.85	1.94	**1.69**
97	**1.82**	2.13	1.83
105	1.90	2.04	**1.74**
106	1.09	0.98	0.98
110	1.98	1.92	**1.76**
114	1.29	1.43	**0.96**
118	**1.39**	1.54	1.67
122	2.11	1.89	**1.59**
mean	1.98	1.96	**1.78**
median	1.86	1.92	**1.74**

Table 9. Chamfer distances on the DTU dataset when using different distributions of normals D for the projected area $\sigma^{\perp}$.

D model	delta (ReLU)	delta	mixture (const.)	**mixture (var.)**	SGGX (var.)
24	3.57	2.73	2.43	2.16	**2.10**
37	4.02	3.56	4.16	3.40	**3.32**
40	1.99	1.94	1.94	**1.76**	1.83
55	1.71	1.77	1.85	**1.43**	1.64
63	2.04	1.86	1.85	**1.60**	1.80
65	2.37	2.67	2.19	**1.97**	2.34
69	1.70	1.73	1.57	**1.54**	1.43
83	2.33	1.85	1.79	1.55	**1.49**
97	2.38	**1.82**	2.25	1.91	2.20
105	3.17	1.90	1.85	**1.53**	1.82
106	1.07	1.09	0.99	1.32	**0.89**
110	1.90	1.98	1.89	**1.59**	1.79
114	1.16	1.29	1.37	1.26	**1.15**
118	1.37	1.39	1.75	**1.31**	1.35
122	1.83	2.11	**1.73**	1.85	1.95
mean	2.17	1.98	1.97	**1.75**	1.81
median	1.99	1.86	1.85	**1.59**	1.80

several ablations to design our attenuation coefficient

Table 8. Chamfer distances on the DTU dataset when using different implicit function distributions Ψ for the density $\sigma^{\parallel}$.

Ψ model	logistic	Laplace	**Gaussian**
24	2.73	**1.92**	1.99
37	[illegible]	[illegible]	[illegible]
40	**1.94**	[illegible]	2.28
55	[illegible]	[illegible]	**1.64**
63	1.86	1.76	1.76
65	[illegible]	[illegible]	[illegible]
69	1.73	1.58	**1.31**
83	1.85	1.94	**1.69**
97	**1.82**	2.13	1.83
105	[illegible]	[illegible]	[illegible]
106	[illegible]	[illegible]	[illegible]
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63	2.04	1.86	1.85	**1.60**	1.80
65	[illegible]	[illegible]	[illegible]	[illegible]	2.34
69	1.70	1.73	1.57	**1.54**	1.43
83	2.33	1.85	1.79	1.55	**1.49**
97	2.38	**1.82**	2.25	1.91	2.20
105	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
106	1.07	1.09	0.99	1.32	**0.89**
110	1.90	1.98	1.89	**1.59**	1.79
114	1.16	1.29	1.37	1.26	**1.15**
118	1.37	1.39	1.75	**1.31**	1.35
122	1.83	2.11	**1.73**	1.85	1.95
mean	2.17	1.98	1.97	**1.75**	1.81
median	1.99	1.86	1.85	**1.59**	1.80

Gaussian has 10% lower Chamfer distance than logistic (NeuS) and 9% lower than Laplace (VolSDF)

Linear mixture has 19% lower Chamfer distance than delta w/ ReLU (NeuS)

Reciprocity has a 8% lower Chamfer distance than non-reciprocal delta

learned fields

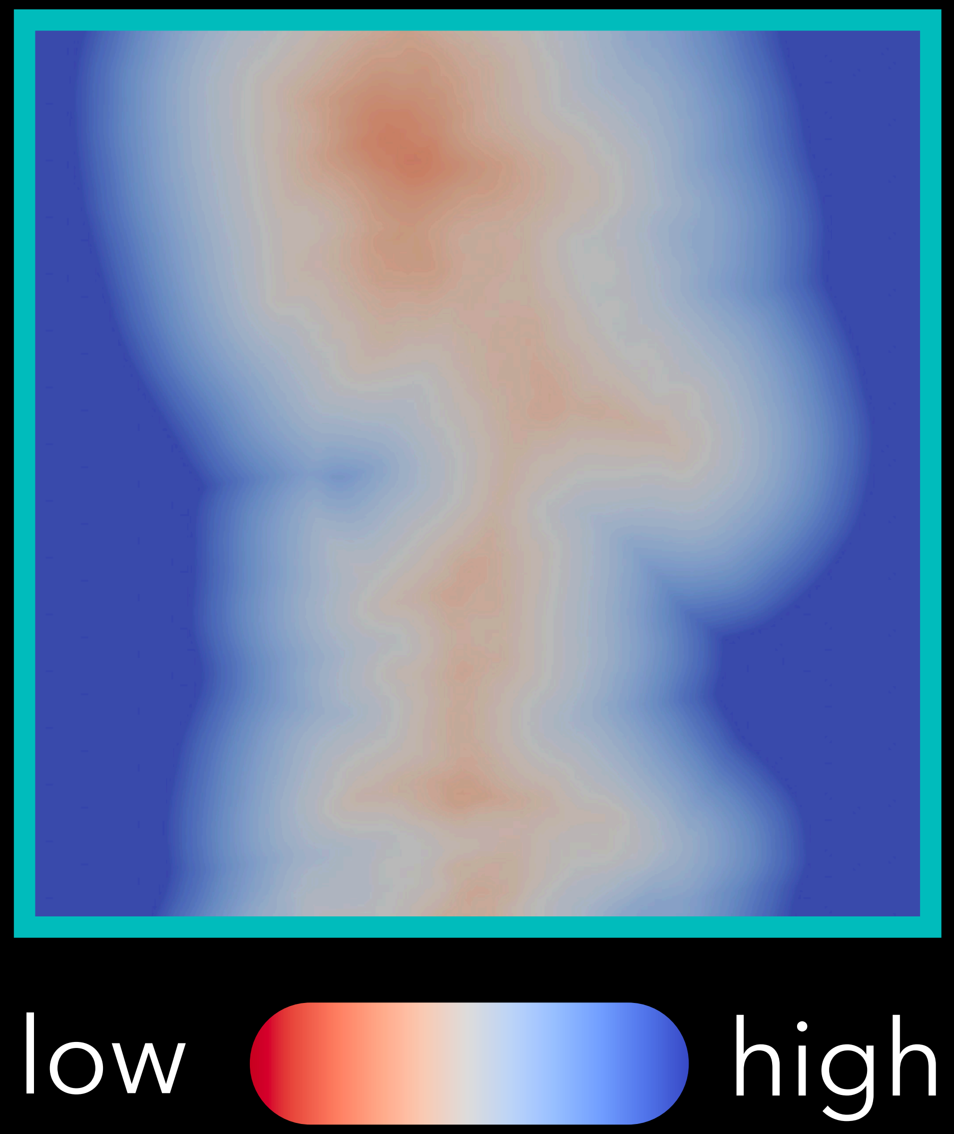
reconstructed surface

learned fields

reconstructed surface

$f(x)$

mean implicit



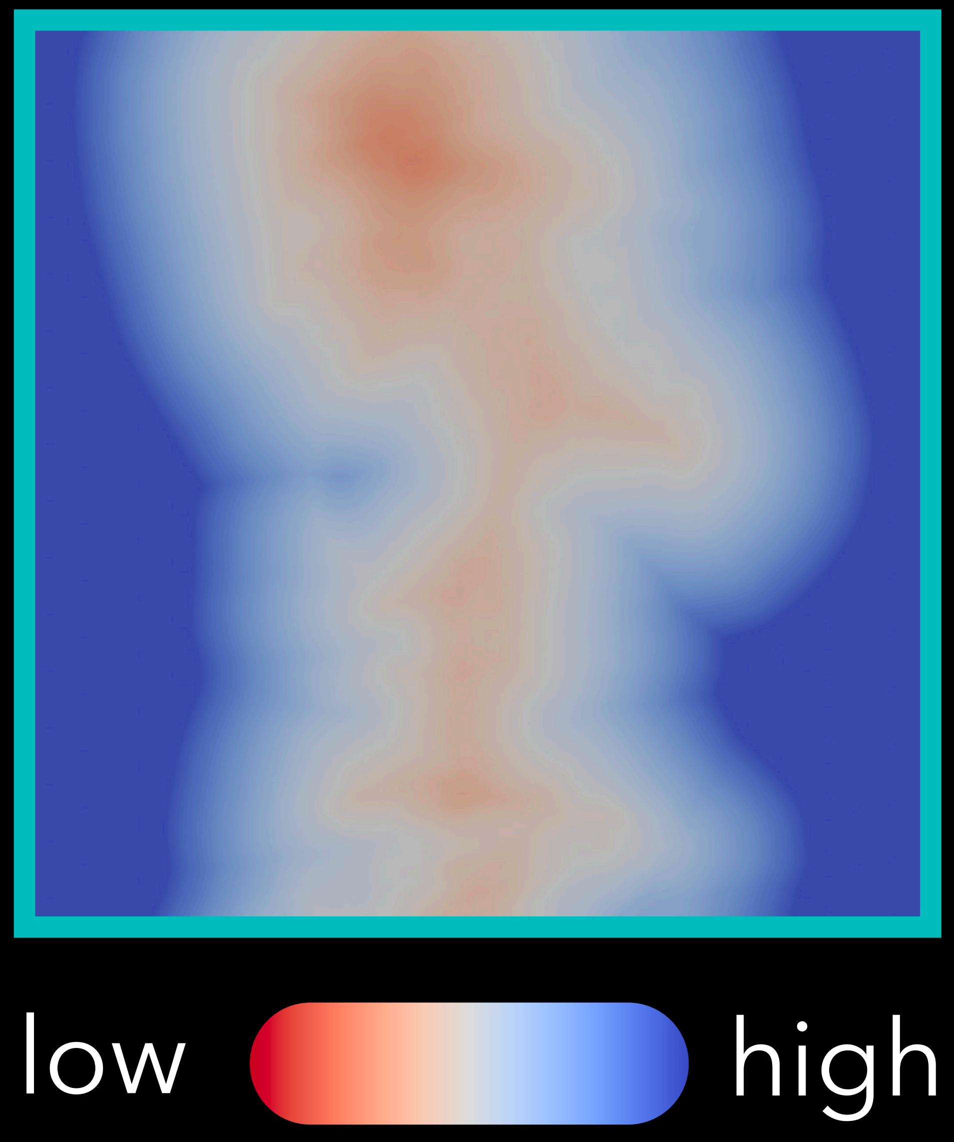
low high

learned fields

reconstructed
surface

$f(x)$

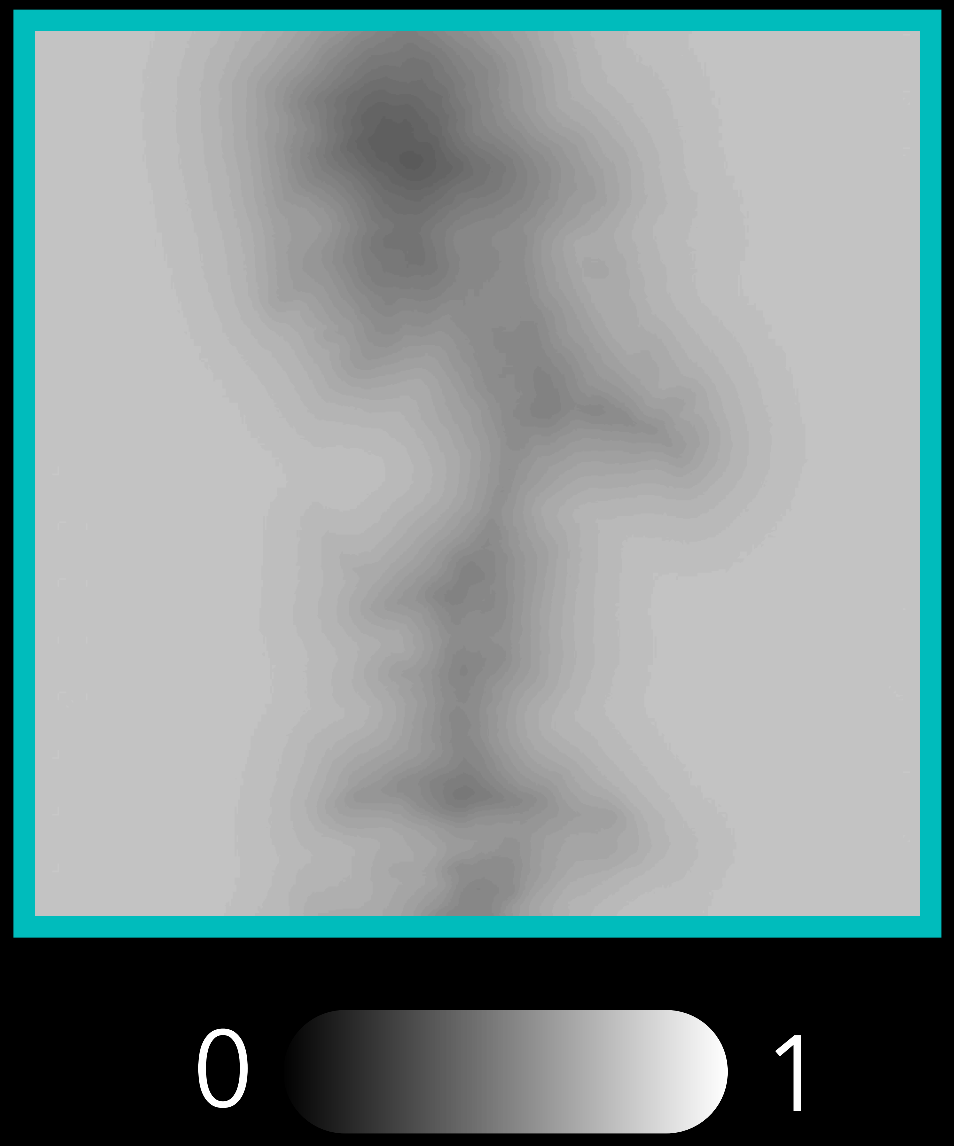
mean implicit



low high

$v(x)$

vacancy



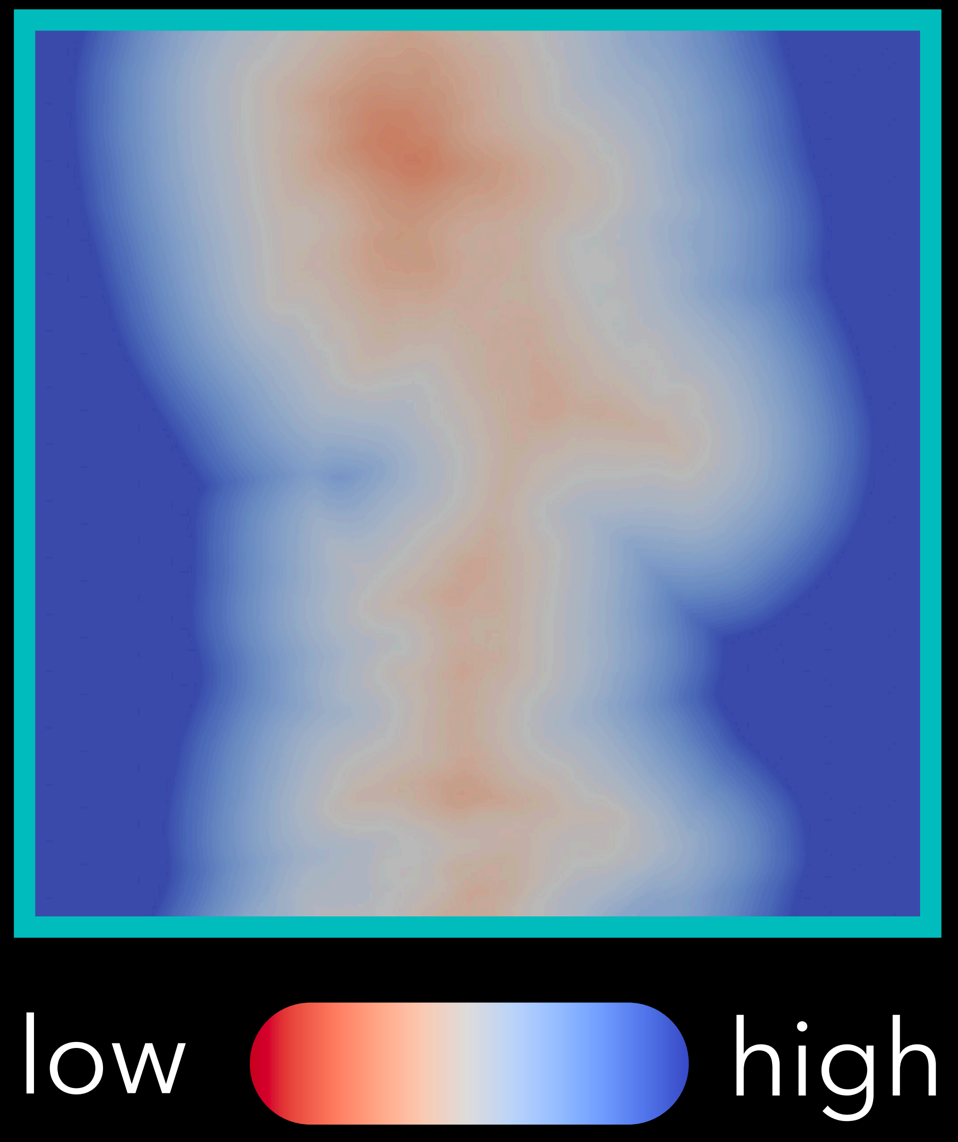
0 1

learned fields

reconstructed surface

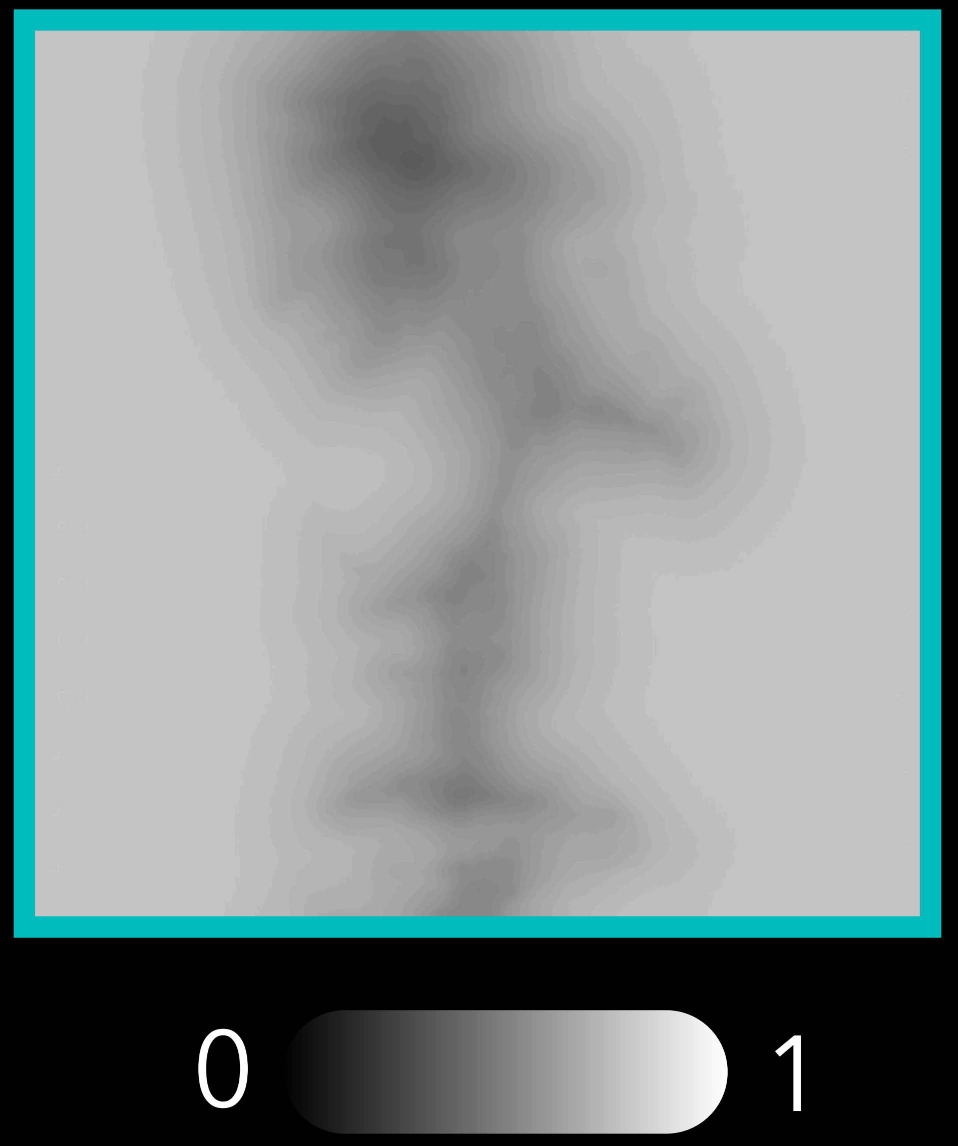
$f(x)$

mean implicit



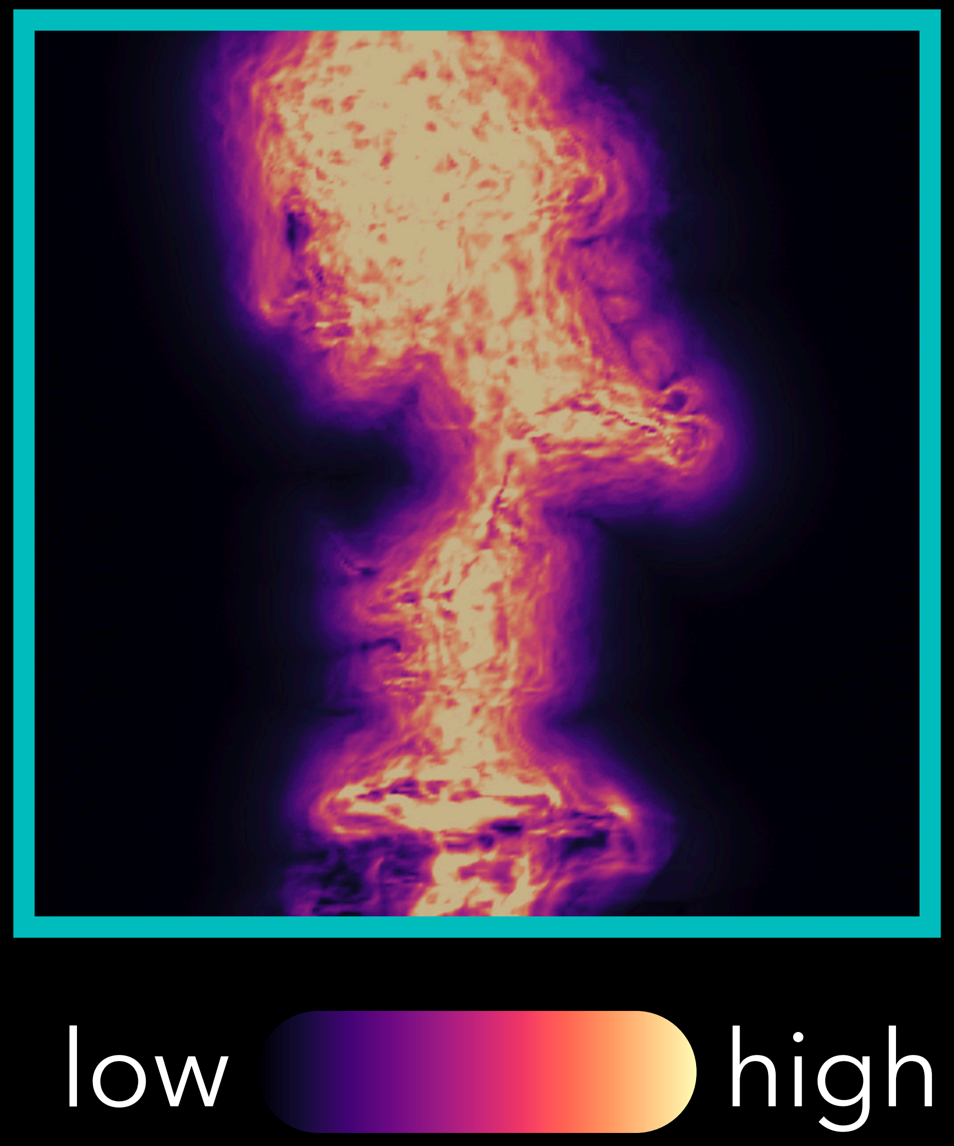
$v(x)$

vacancy



$$\frac{\|v(x)\|}{v(x)}$$

density



learned fields

learned fields

linear mixture distribution of normals

$$D_x(m) = \alpha(x)\delta_{n(x)}(m) + (1 - \alpha(x))\frac{1}{2\pi}$$

boundary interior

Objects as volumes: A stochastic geometry view of opaque solids – interactive supplement

We provide different visualizations on scenes from three datasets.

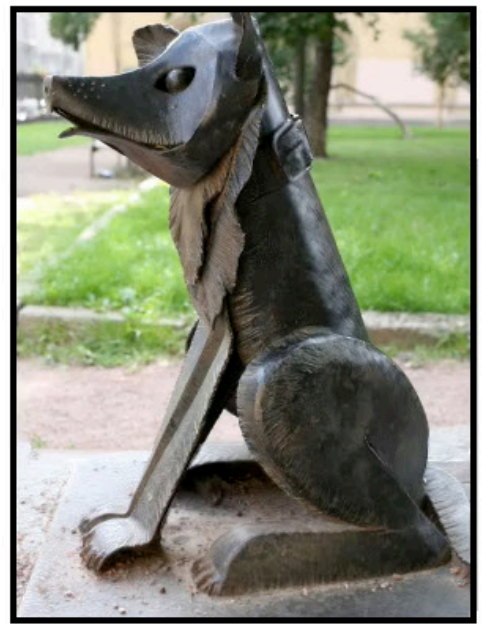
Datasets: BlendedMVS NeRF Realistic Synthetic DTU

project page

BlendedMVS

bear

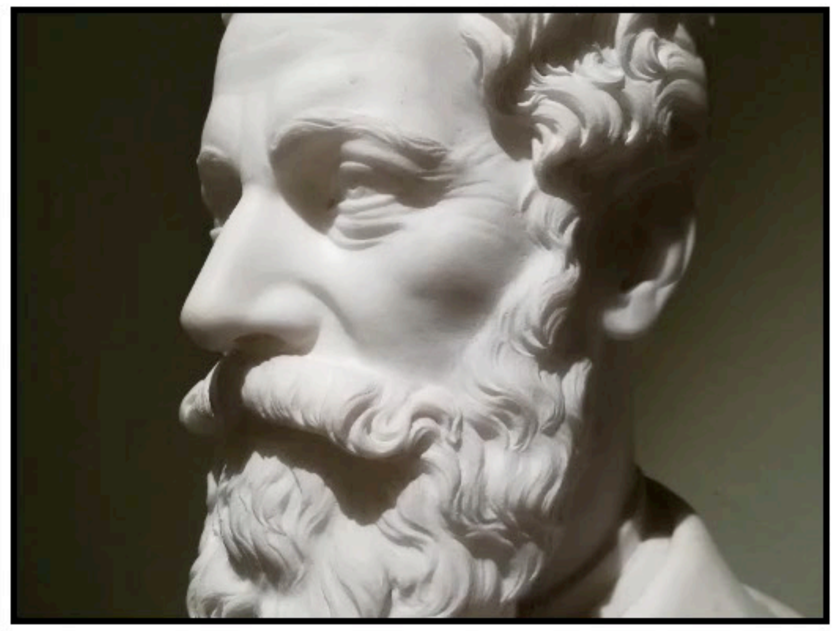
clock



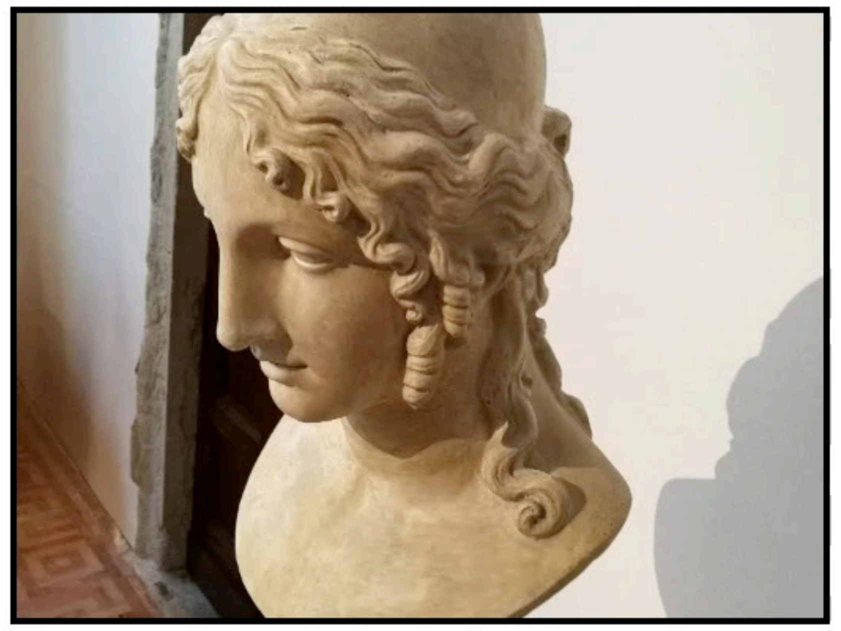
dog

durian

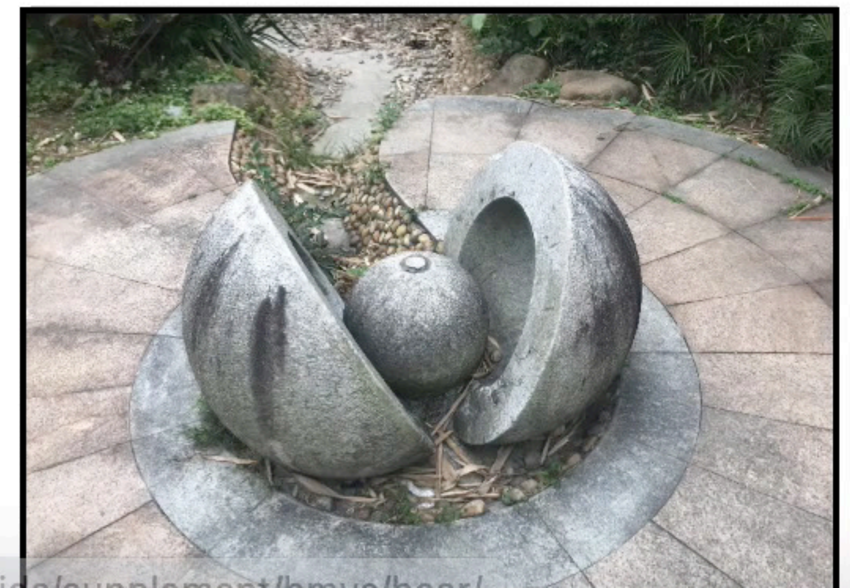
jade



man



sculpture



Objects as volumes: A stochastic geometry view of opaque solids – interactive supplement

We provide different visualizations on scenes from three datasets.

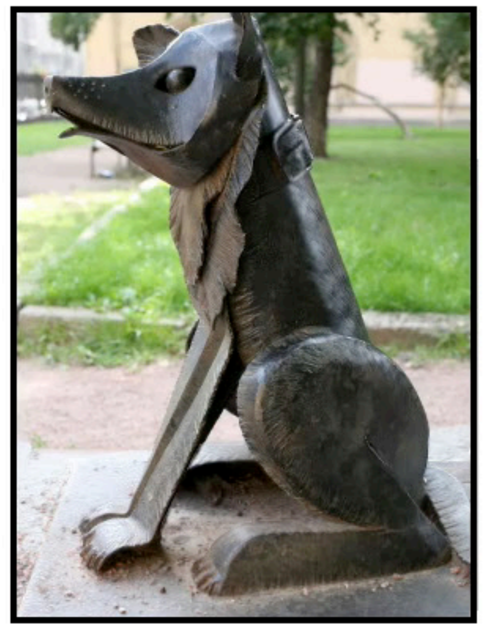
Datasets: BlendedMVS NeRF Realistic Synthetic DTU

project page

BlendedMVS

bear

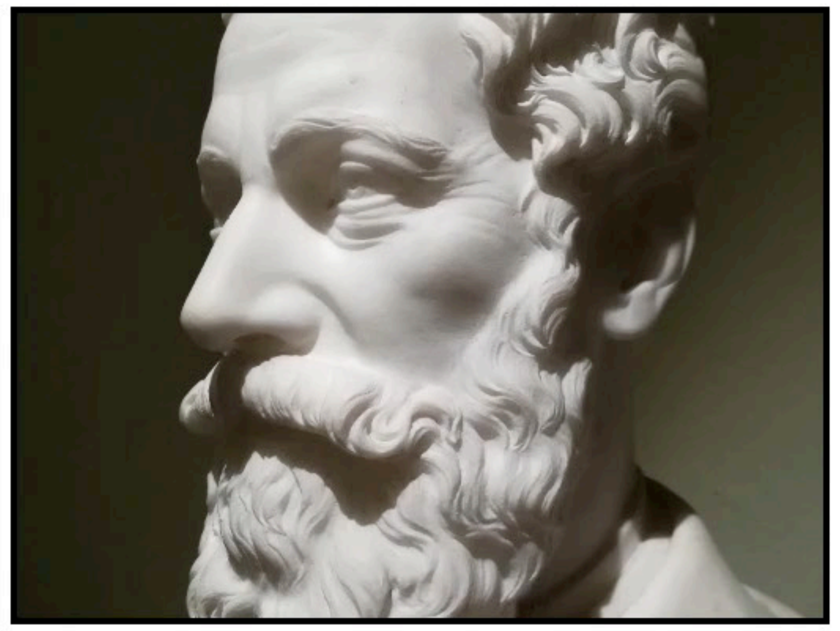
clock



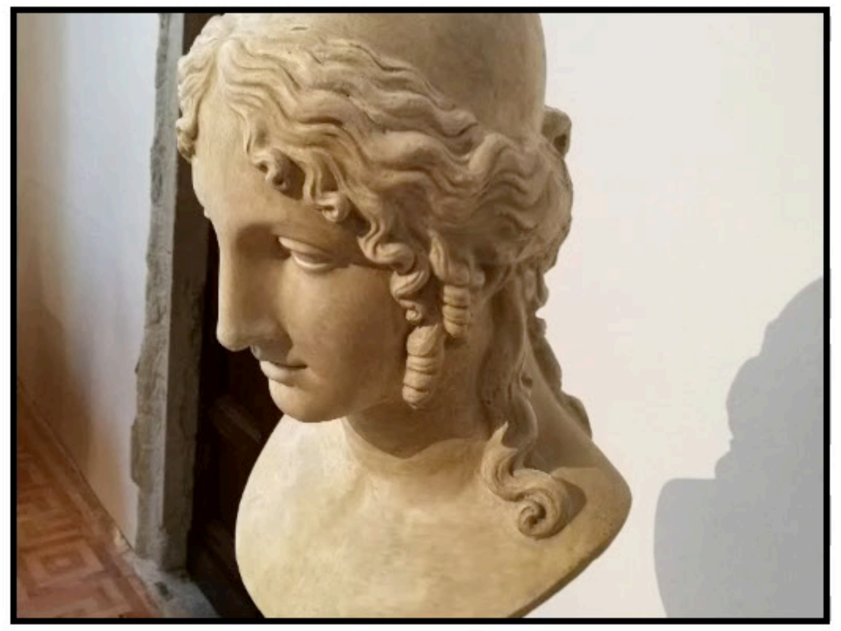
dog

durian

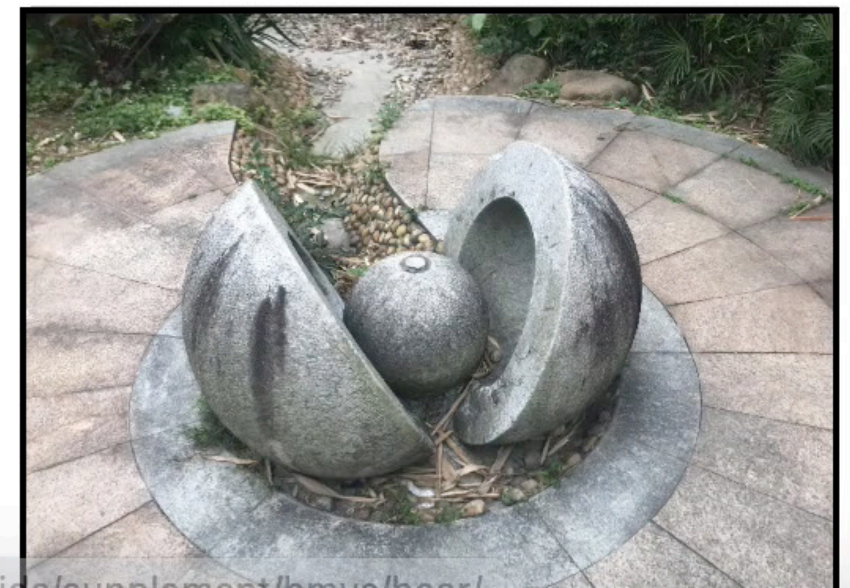
jade

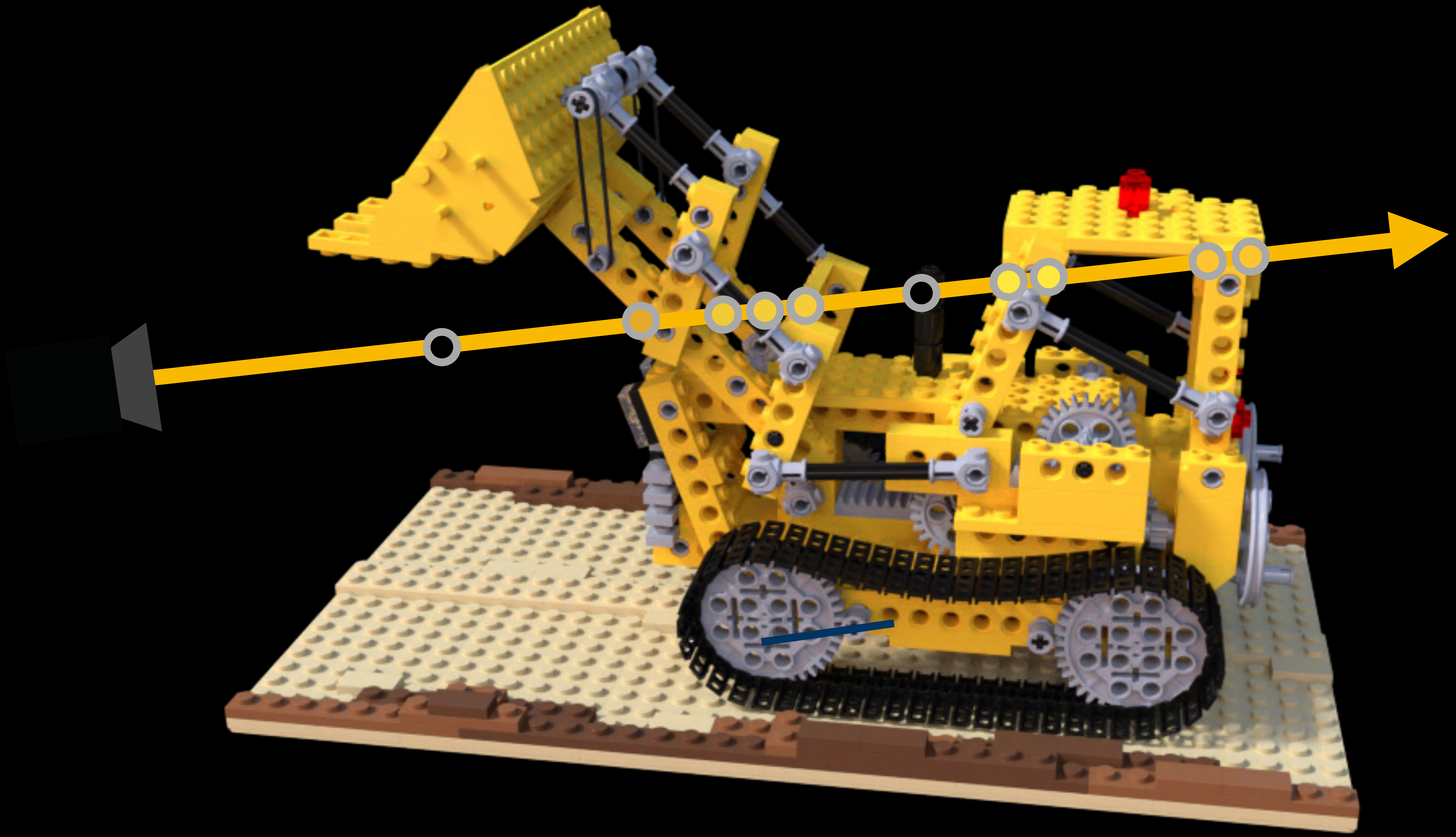


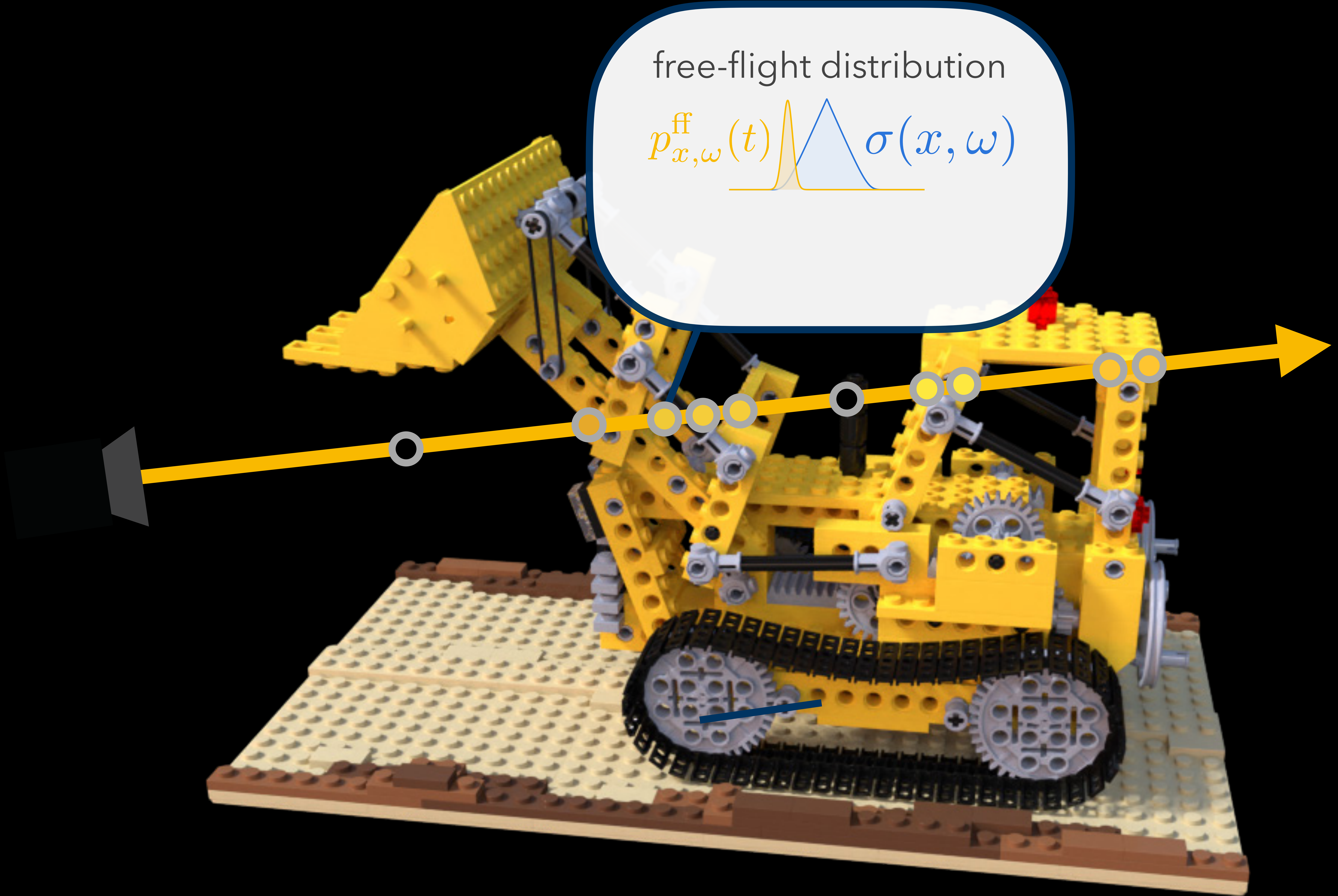
man

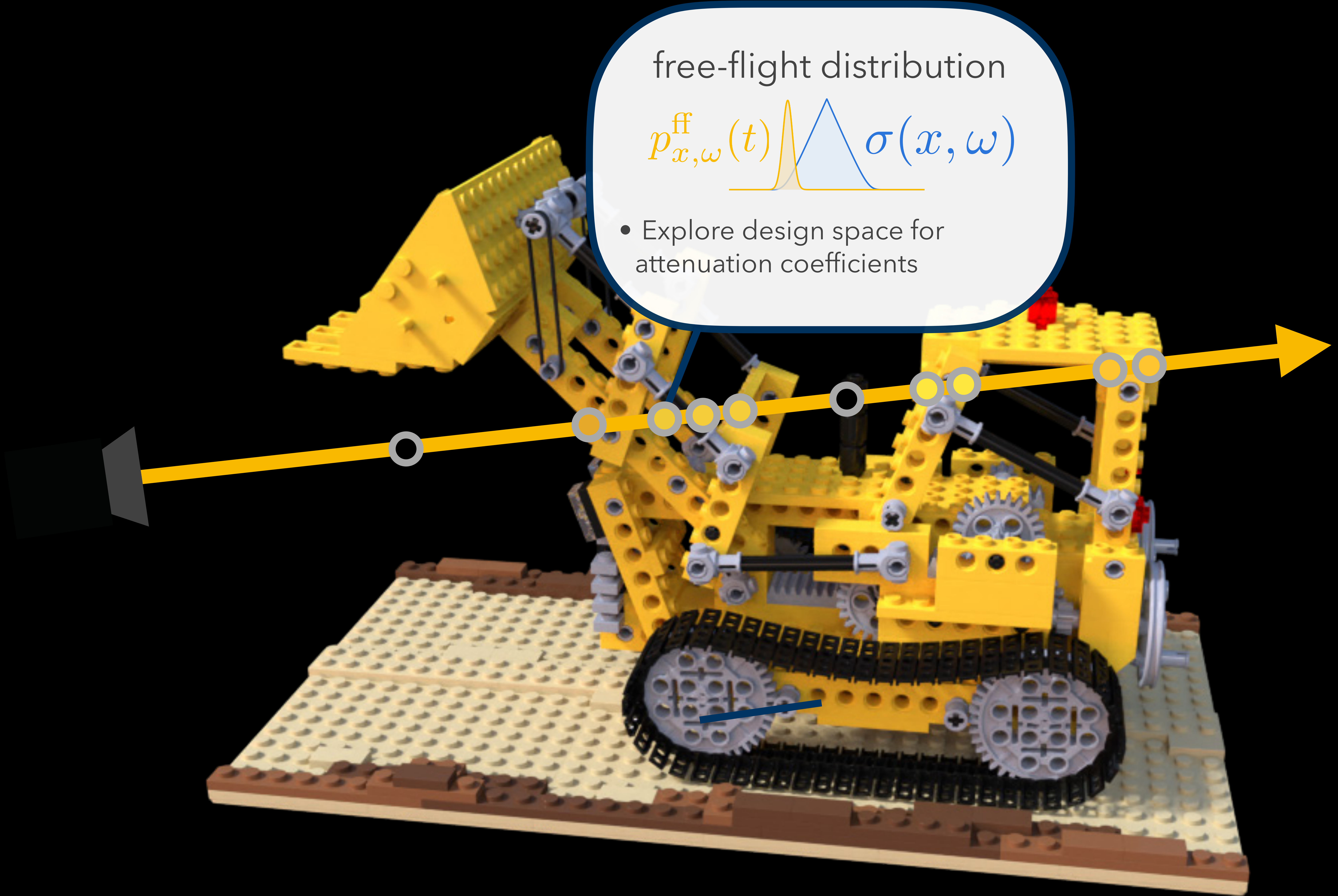


sculpture

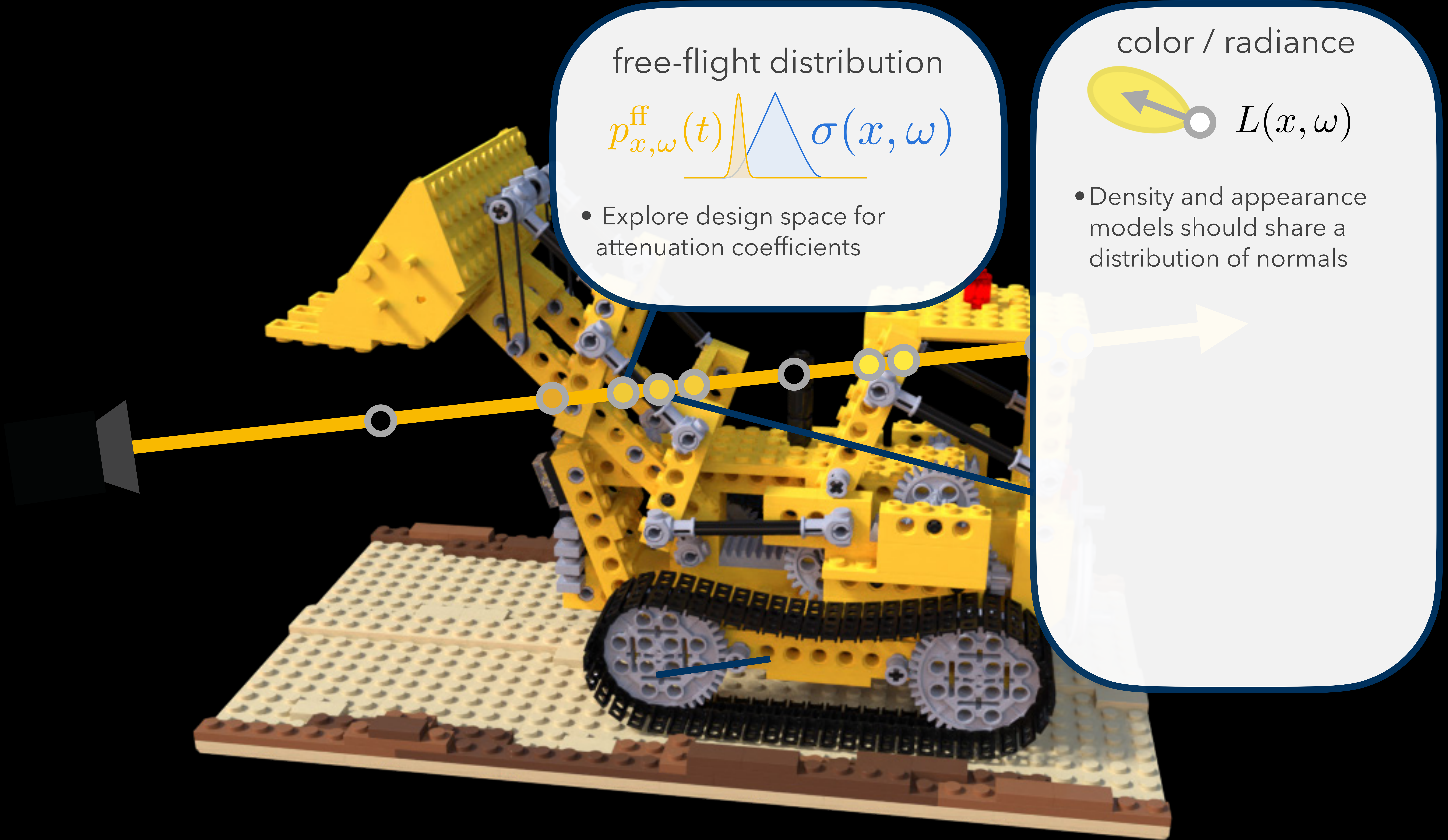





free-flight distribution
$p^{\mathrm{ff}}_{x,\omega}(t)$
$\sigma(x,\omega)$


free-flight distribution
$p^{\text{ff}}_{x,\omega}(t)$
$\sigma(x, \omega)$
• Explore design space for attenuation coefficients


free-flight distribution
$p^{\text{ff}}_{x,\omega}(t)$
$\sigma(x,\omega)$
• Explore design space for attenuation coefficients
color / radiance
$L(x,\omega)$


free-flight distribution
$p^{\text{ff}}_{x,\omega}(t)$
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• Explore design space for attenuation coefficients
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• Density and appearance models should share a distribution of normals


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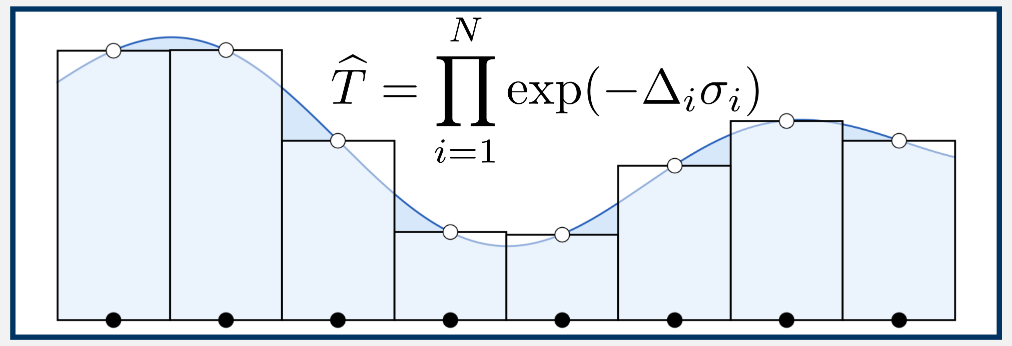
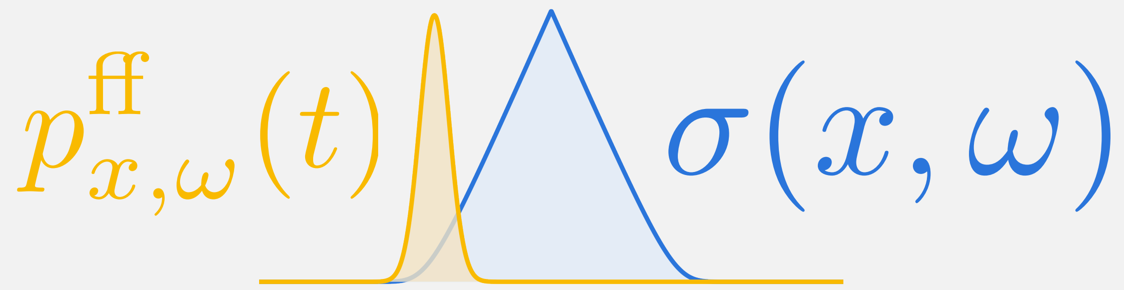
• Density and appearance models should share a distribution of normals
• Some preliminary results in supplement where we extend the attenuation in Ref-NeRF with a Phong NDF
reference
ours
Ref-NeRF

transmittance estimation



free-flight distribution



- Explore design space for attenuation coefficients

color / radiance

- Density and appearance models should share a distribution of normals
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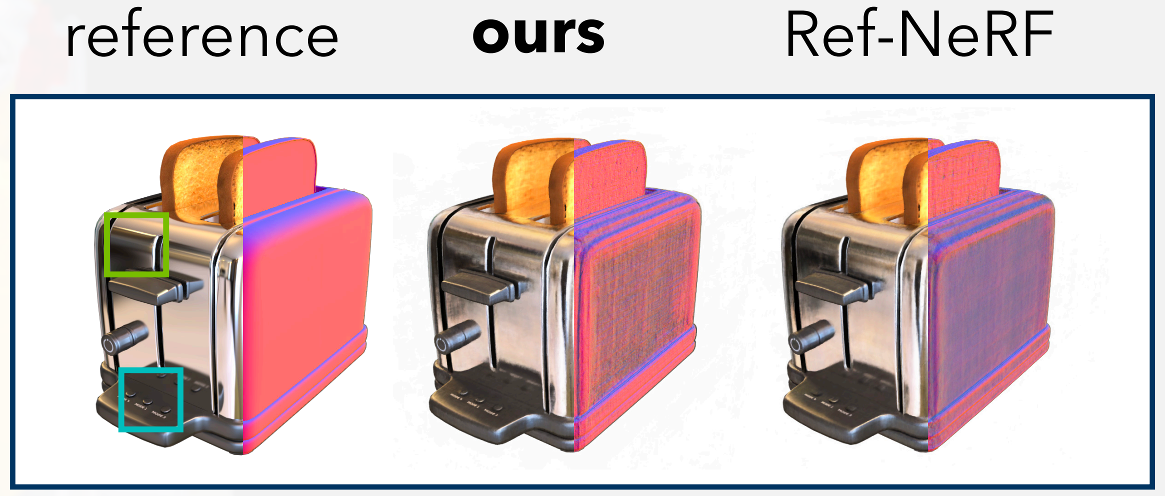

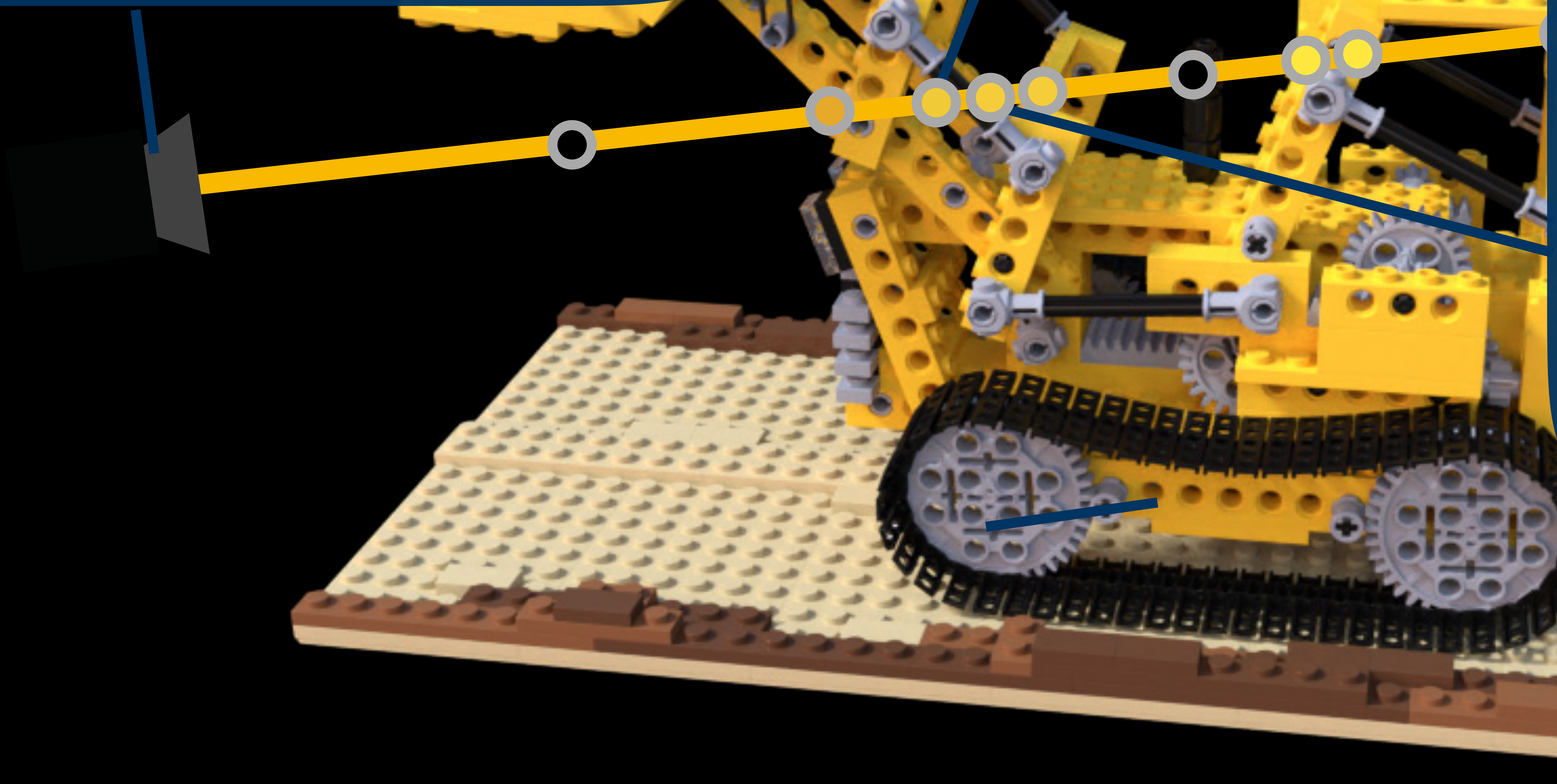


transmittance estimation
$\widehat{T} = \prod_{i=1}^{N} \exp(-\Delta_i \sigma_i)$
• Transmittance est. has influence on performance
free-flight distribution
$p^{\text{ff}}_{x,\omega}(t)$
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• Most methods still use NeRF style quadrature
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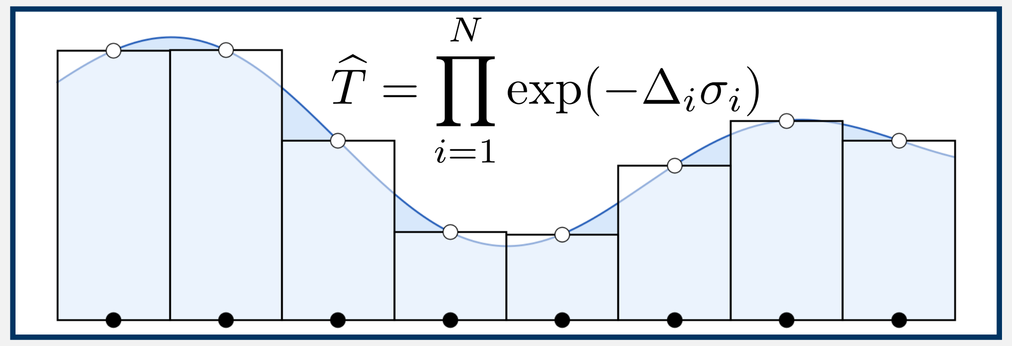
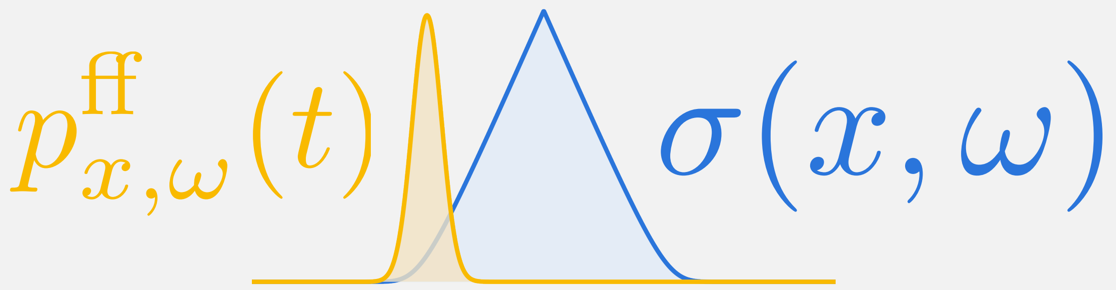


- Transmittance est. has influence on performance
- Most methods still use NeRF style quadrature
- Graphics offers unbiased tracking-based alternatives

free-flight distribution



- Explore design space for attenuation coefficients

color / radiance

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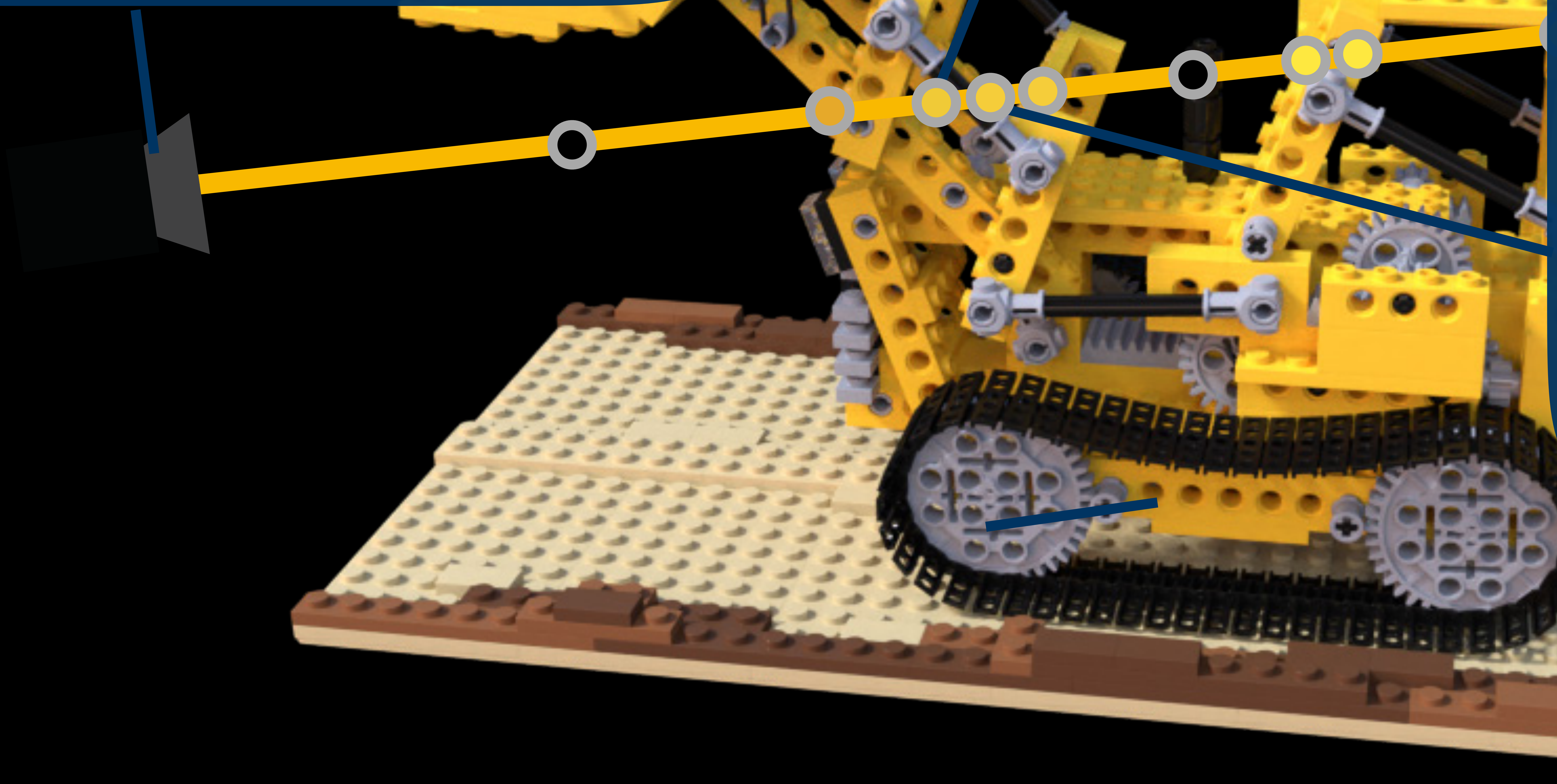
reference **ours** Ref-NeRF



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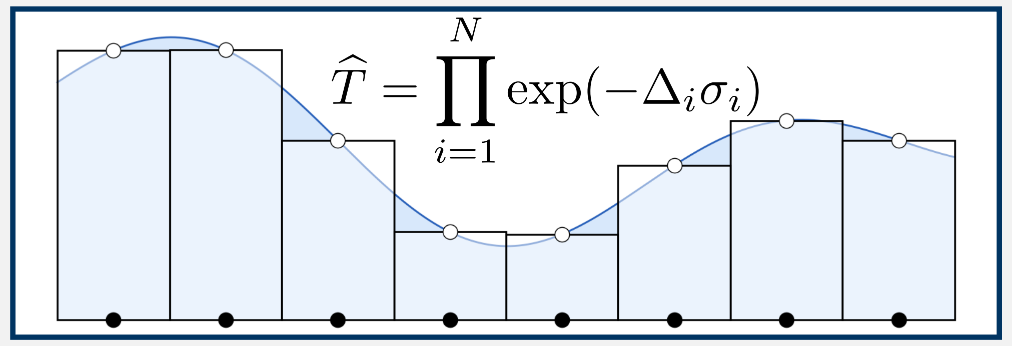

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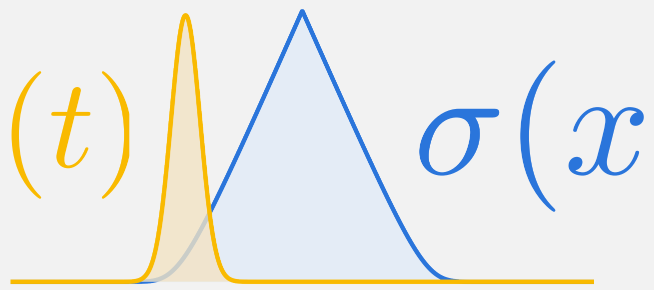
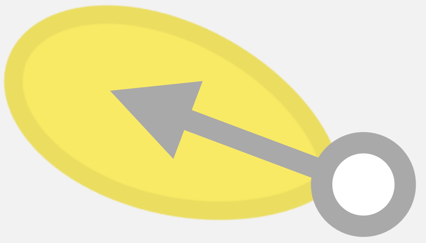


$p^{\mathrm{ff}}_{x,\omega}(t)$ $\sigma(x,\omega)$

- Explore design space for attenuation coefficients

color / radiance



$L(x,\omega)$

- Density and appearance models should share a distribution of normals
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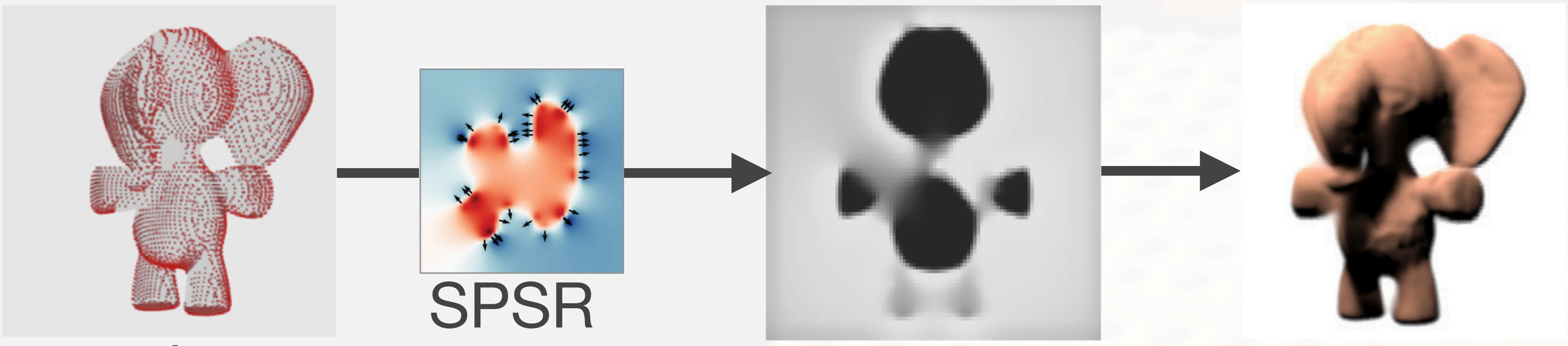
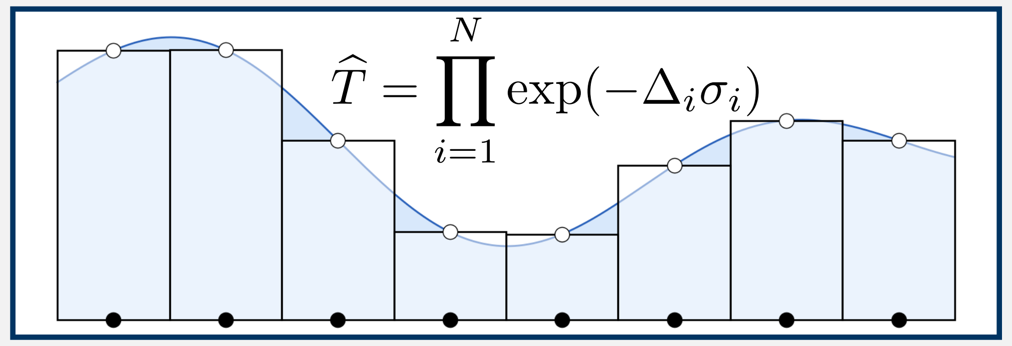
geometry representation



transmittance estimation



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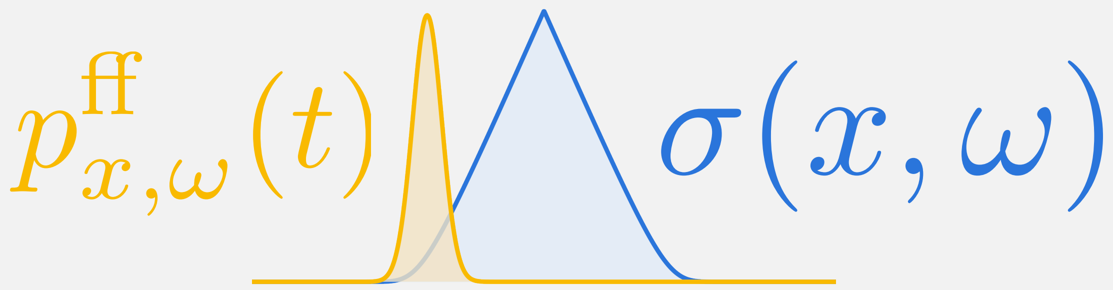
free-flight distribution



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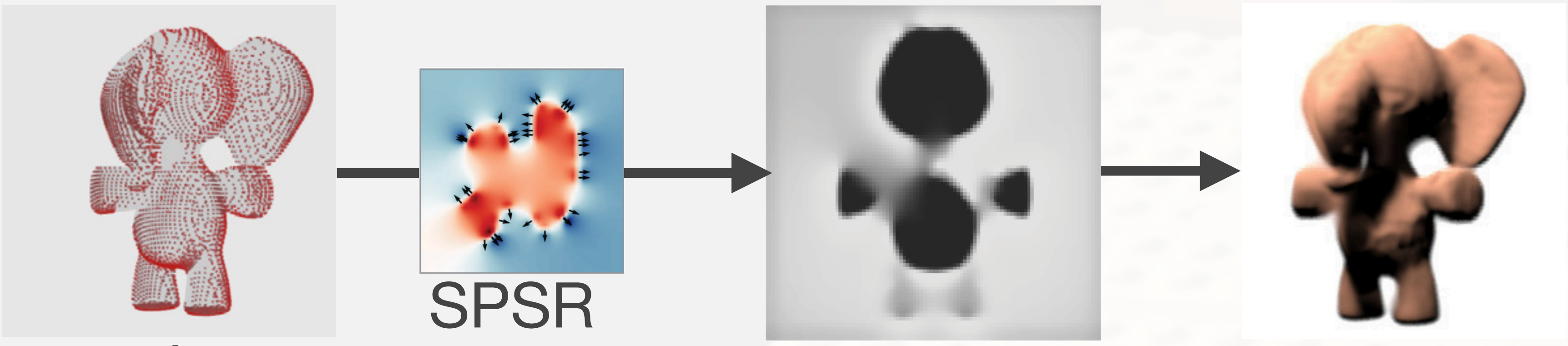
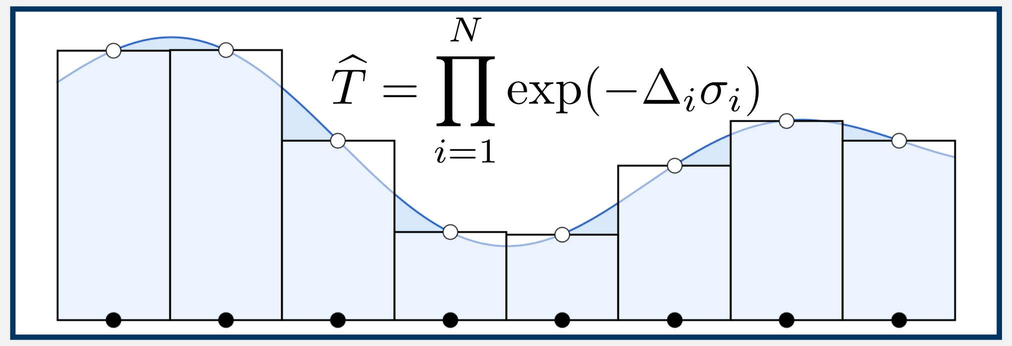


- 3DGS starting to be used for surface reconstruction

transmittance estimation



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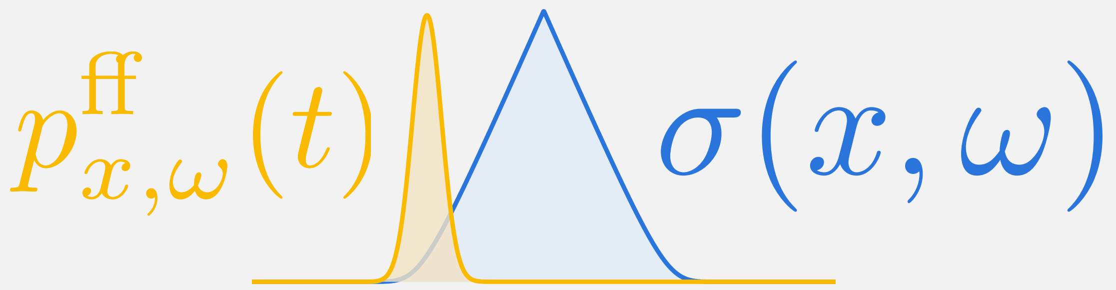

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color / radiance

- Density and appearance models should share a distribution of normals
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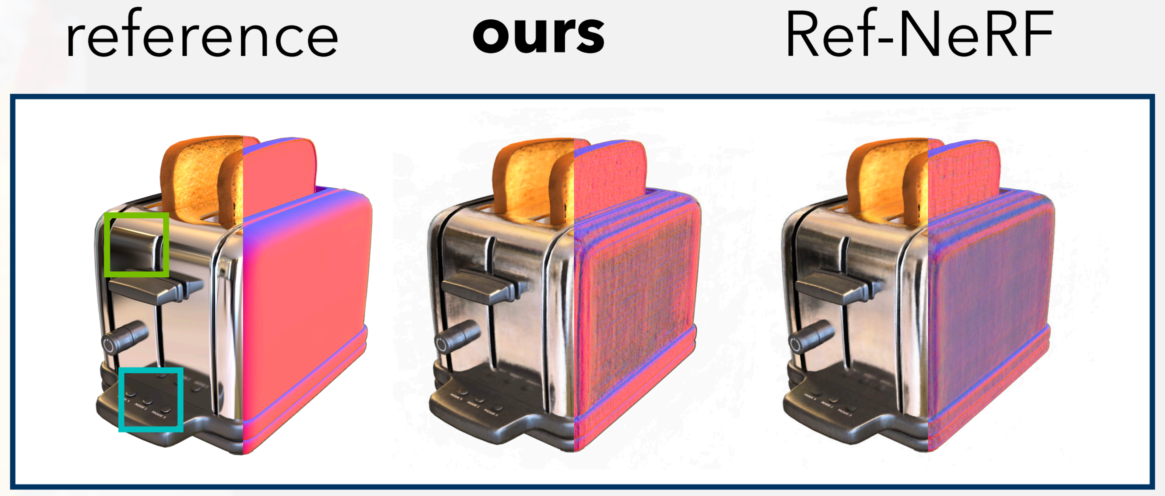
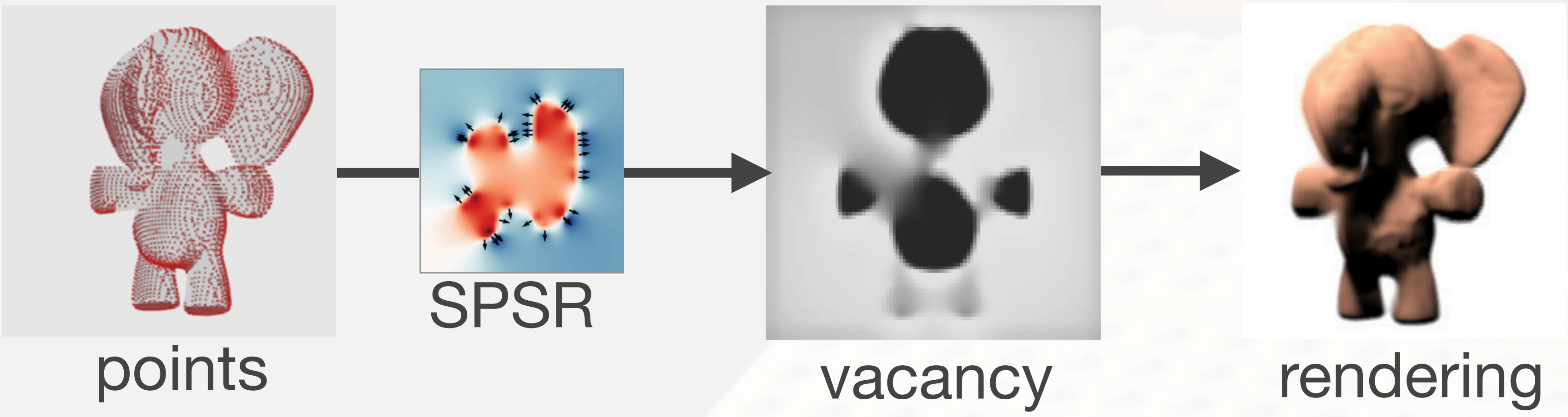
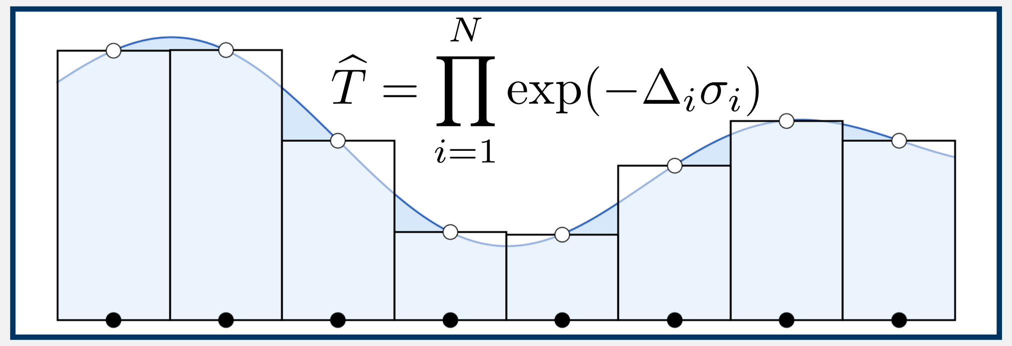


geometry representation



- 3DGS starting to be used for surface reconstruction
- Our method supports point based representations

transmittance estimation



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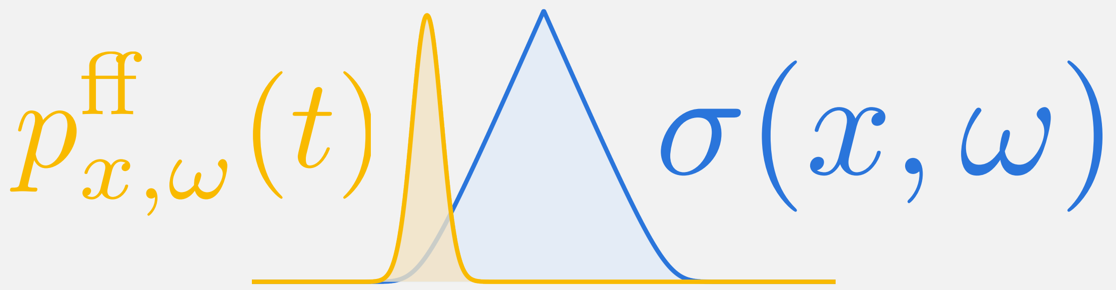

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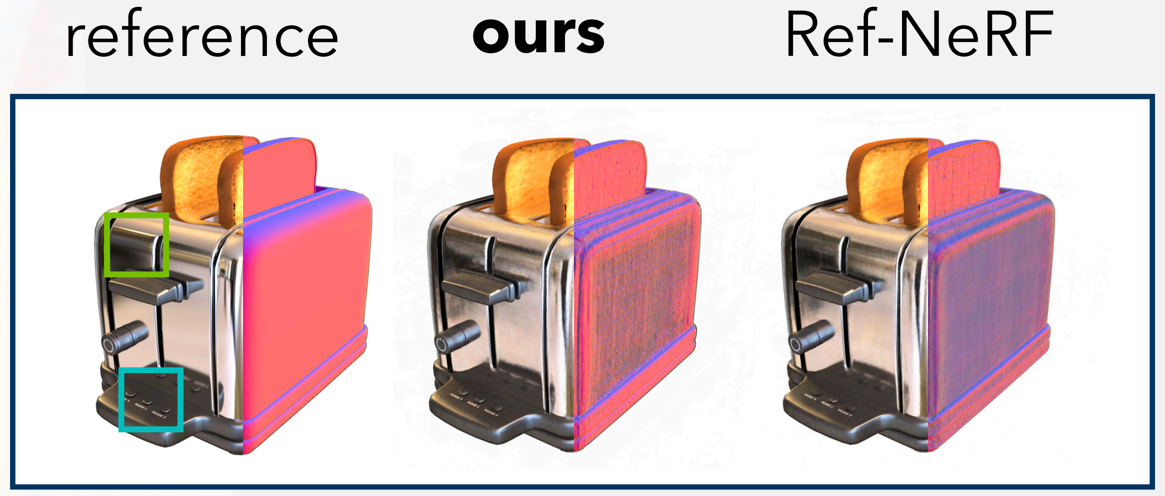
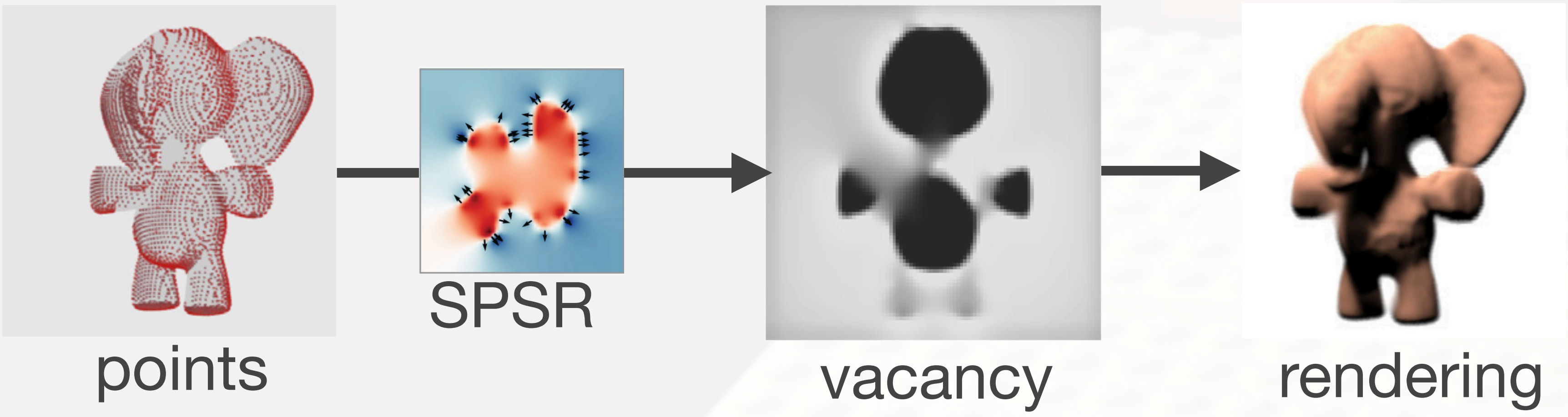


geometry representation



- 3DGS starting to be used for surface reconstruction
- Our method supports point based representations
- Missing probabilistically meaningful + differentiable vacancy for point cloud (some insights from Sellán and Jacobson [2022])

Thank you!

reference | ours | VolSDF | NeuS

project page

project: imaging.cs.cmu.edu/volumetric_opaque_solids

code: github.com/cmu-ci-lab/volumetric_opaque_solids

Carnegie Mellon University