Computing multiple c-revision using OCF knowledge bases

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Abstract

Belief revision is the process of revising epistemic states in the light of new information. In this paper epistemic states are represented in the framework of Spohn's Ordinal Conditional Functions (OCF). The input is a consistent set of propositional formulas issued from different and independent sources.

We focus on the so-called multiple iterated belief c-revision recently proposed by Kern-Isberner. We propose a computation of c-revision when epistemic states are compactly represented by weighted propositional knowledge bases, called OCF knowledge bases.

Introduction

Intelligent agents need to update their knowledge, or epistemic states, on the basis of new observations on their environment. This problem is known as the one of belief revision and is axiomatically characterized by the well known AGM postulates (Alchourrón, Gärdenfors, and Makinson 1985; Hansson 1997).

There are at least three issues that need to be addressed when modeling a belief revision problem.

The first issue concerns the representation of initial epistemic states. An epistemic state should at least contain a set of accepted beliefs. It also contains some meta-information which is very useful for defining a meaningful belief revision operation. A simple representational format of an epistemic state is a closed set of propositional formulas, called a belief set. However, epistemic states have in general a complex structure. They can be represented by a total preorder over a set of propositional interpretations (Katsuno and Mendelzon 1991), a probability distribution, a possibility distribution (Dubois and Prade 1988), an ordinal conditional function or simply an OCF function (Spohn 2012; Williams 1995; Beierle, Hermsen, and Kern-Isberner 2014; Benferhat et al. 2002), a set of conditionals (Kern-Isberner 2004; 2001; Kern-Isberner and Eichhorn 2014; Benferhat, Dubois, and Prade 1999; Goldszmidt and Pearl 1997), a partial pre-order over a set of interpretations (Benferhat, Largue, and Papini 2005; Touazi, Cayrol, and Dubois 2015; Ma, Benferhat, and Liu 2012).

The second issue concerns the representation of the input or the new information. The input can be a simple observation over a static world, a result of an action or even a result of an intervention (external action) that forces some variables to take specific values (Pearl 2000). The input can be simply a propositional formula, a set of propositional formulas, a partial or total pre-order on a set of interpretations, a set of uncertain and mutually exclusive formulas inducing a partition of a set of interpretations (Jeffrey 1965; Dubois and Prade 1997), etc.

The third issue concerns the definition of a revision operation where from an initial epistemic state and the input produces a new epistemic state. This new epistemic state should at least satisfy two requirements. Firstly, it should accept the input. Secondly, it should be as close as possible to the initial epistemic state. Depending on the representation of epistemic states and on a the nature of the input, a large number of revision operations (e.g. (Benferhat et al. 2000; Falappa et al. 2012; Hansson 1998; Konieczny, Grespan, and Pérez 2010; Papini 1992)) or contraction operations (e.g. (Adaricheva et al. 2012; Booth, Meyer, and Varzinczak 2009; Delgrande and Wassermann 2010)) has been proposed in the literature. Other approaches that deal with inconsistency have also been proposed in (e.g. (Konieczny, Lang, and Marquis 2005; Qi, Liu, and Bell 2006; Benferhat, Dubois, and Prade 1993; 1998)).

The framework considered in this paper for representing epistemic states is the one of ordinal conditional functions OCF (Spohn 2012). We will analyze the so-called multiple iterated propositional c-revision recently proposed in (Kern-Isberner and Huvermann 2015). An important feature of c-revision is that it can be iterated since initial and revised epistemic states are both represented by OCF distributions. The iterated belief c-revision has as input a consistent set of propositional sentences S assumed to be provided by different and independent sources. The revised ordinal conditional distribution should accept all sentences of the input. Independence relations between information sources are represented by the fact that counter-models of S are ranked with respect to the number of falsified propositions in S. Namely, the more an interpretation falsifies formulas in S, the less it is a preferred interpretation.

Multiple iterated belief c-revision has been shown to satisfy all natural and rational properties given in (Kern-Isberner and Huvermann 2015). However, c-revision is only defined at the semantic level (over the set of interpretations) which is impossible to be provided in pratice if the set of variables is important. This paper addresses this issue by providing an equivalent characterization of c-revision defined on a compact representation of ordinal conditional functions over the set of interpretations. An OCF over a set of interpretations will be compactly represented by a set of weighted formulas called OCF knowledge bases.

The rest of this paper is organized as follows. Section 2 gives a refresher on OCF distributions and on their compact representations OCF knowledge bases. Section 3 presents the multiple iterated belief c-revision defined on OCF distributions. Section 4 shows how multiple iterated belief c-revision can be directly defined on OCF knowledge bases. Section 5 concludes the paper.

OCF-based representations of epistemic states

Let $\mathcal{L}$ denote a finite propositional language and Ω be the set of propositional interpretations. We will denote by ω an element of Ω. Greek letters $\phi,\ \psi,\ ...$ represent propositional formulas. $\vDash$ denotes a propositional logic satisfaction relation.

In belief revision, epistemic states represent a set of all available beliefs. There are different representations of epistemic states: an uncertainty (probability, possibility, etc) distributions, a total pre-order over Ω, a partial pre-order over Ω, etc. In this paper, we use ordinal conditional functions (OCF) to represent epistemic states (Spohn 2012; Touazi, Cayrol, and Dubois 2015; Williams 1995).

An OCF distribution can be simply viewed as a function that assigns to each interpretation ω of Ω an integer denoted by $k(\omega)$. $k(\omega)$ represents the degree of surprise of having ω as being the real world. $k(\omega) = 0$ means that nothing prevents ω for being the real world. $k(\omega) = 1$ means that ω is somewhat surprising to be the real world. $k(\omega) = +\infty$ simply means that it is impossible for ω to be the real world.

Example 1: Let a and b be two propositional symbols. Table 1 gives an example of an epistemic state represented by an OCF distribution k:

ω	$k(\omega)$
ab	4
$\neg ab$	1
$a\neg b$	1
$\neg a\neg b$	0

Table 1: An example of an OCF distribution

From Table 1, the most normal state of world is the one where both a and b are false. A surprising world (with a degree of surprise 1) is the one where either a or b is true. A more surprising world (with a degree of surprise 4) is the one where both a and b are true.

From an OCF distribution k, one can induce a degree of surprise over formulas ϕ of $\mathcal{L}$, simply denoted by $k(\phi)$ and defined by:

$$k(\phi) \ = \ min\{k(\omega) : \omega \in \Omega, \omega \vDash \phi\}. \qquad (1)$$

For example, from Table 1 we have $k(\neg a \vee \neg b) = min(k(\neg a\neg b), k(\neg ab), k(a\neg b)) = 0$ while $k(a \vee b) = 1$.

Given an OCF distribution, a set of accepted beliefs is a propositional formulas such that its models are those having minimal surprise degrees in k. In Example 1, the set of accepted beliefs is represented by the propositional formulas $\neg a \wedge \neg b$.

In practice, an OCF distribution k cannot be provided over a set of interpretations Ω (except if the number of propositional variables is small). A compact representation may be provided using for instance the concept of OCF networks (Kern-Isberner and Eichhorn 2013; Benferhat and Tabia 2010; Darwiche and Goldszmidt 1994; Eichhorn and Kern-Isberner 2014) or the concept of weighted propositional knowledge bases.

In this paper, we only consider weighted propositional knowledge bases, simply called OCF knowledge bases and denoted by $\mathcal{K}$. An OCF knowledge base is a set of weighted formulas of the form $\mathcal{K} = \{(\phi_i, \alpha_i) : \ i = 1, .., n\}$ where ϕ_i's are propositional formulas and α_i's are positive integers. The higher is the certainty degree α_i, the more certain is the formula ϕ_i. When $\alpha_i = +\infty$ this means that ϕ_i represents an integrity constraint that should absolutely be satisfied. Formulas with a certainty degree equal to '0' are not explicitly stated in $\mathcal{K}$. Weighted or prioritized knowledge bases have been intensively used in the literature for handling uncertainty (such as in a possibilistic logic framework (Lang 2001; Benferhat 2010)) or for handling inconsistency.

Given an OCF knowledge base $\mathcal{K}$, one can induce a unique OCF distribution, denoted by $k_{\mathcal{K}}$ and defined by:

$\forall \omega \in \Omega$,

$$k_{\mathcal{K}}(\omega) = \begin{cases} 0 & if\ \forall(\phi_i, \alpha_i) \in \mathcal{K},\ \omega \vDash \phi_i \\ max\ \{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{K}, \omega \nvDash \phi_i\}\ otherwise. \end{cases} \qquad (2)$$

Namely, $k_{\mathcal{K}}(\omega)$ is associated with the highest certain formulas in $\mathcal{K}$ falsified by ω. Models of formulas in $\mathcal{K}$ are considered as the most normal interpretations (hence they have a surprise degree equal to 0). Clearly, the concepts of OCF distributions and OCF knowledge bases are very close to the concepts of possibility distributions and possibilistic knowledge bases used in a possibility theory framework (Dubois and Prade 1988), where instead of using a set of integers, the unit interval [0,1] is used.

Example 2: let $\mathcal{K} = \{(\neg a \vee \neg b, 4), (\neg a, 1), (\neg b, 1)\}$. This knowledge base means that we are somewhat certain that a and b are both false and we are even more confident if only one of them is false. One can easily check that applying Equation (2) to the knowledge base $\mathcal{K}$ will simply lead to the OCF distribution given in Table 1. For instance, $k_{\mathcal{K}}(a \wedge b) = max\ \{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{K}, a \wedge b \nvDash \phi_i\} = max\{4, 1, 1\} = 4$.

C-revision of OCF distributions

Several works have been proposed for revising OCF distributions. For instance, in (Williams 1995) a general form of changing OCF distributions, called transmutations (Williams 1994), has been proposed. In (Kern-Isberner

2001; Kern-Isberner and Eichhorn 2014) a revision of OCF distributions with a set of conditionals has also been proposed.

In this section, we focus on a so-called multiple iterated belief c-revision proposed in (Kern-Isberner and Huvermann 2015) for revising an OCF distribution with a consistent set of propositional formulas $\mathcal{S} = \{u_1, .., u_n\}$. In order to have a faithfull revision operation, each propositional formula u_i is associated with an integer β_i. These integers β_i's are not explicitly stated by the user, but they are implicitly constrainted as it will be shown below. More precisely:

Definition 1: Let k be an OCF distribution. Let $\mathcal{S} = \{u_1, .., u_n\}$ be a consistent finite set of propositional formulas. Then the propositional c-revision of k with $\mathcal{S}$, denoted by $k * \mathcal{S}$, is defined by:

$$\forall \omega \in \Omega, k * \mathcal{S}(\omega) = k(\omega) - k(u_1 \wedge .. \wedge u_n) + \sum_{i=1,\ \omega \vDash \neg u_i}^{n} \beta_i, \tag{3}$$

where $(\beta_1, .., \beta_n)$ are positive integers satisfying:

$$\forall i, \beta_i > k(u_1 \wedge .. \wedge u_n) - \min_{\omega \vDash \neg u_i} \{k(\omega) + \sum_{j \neq i,\ \omega \vDash \neg u_j}^{n} \beta_j\} \tag{4}$$

The revision process given in Equation (3) first consists in shifting up each interpretation ω with the sum of weights β_i's of propositional formulas u_i that it falsifies. The expression "$-k(u_1 \wedge .. \wedge u_n)$" is a normalization term that guarantees that $min\{k * \mathcal{S}(\omega) : \omega \in \Omega\}$ is equal to zero. Propositional formulas from $\mathcal{S}$ are assumed to be issued from independent sources. Hence, interpretations will be compared with respect to the number of falsified formulas. This is reflected by the expression "$\sum_{i=1,\ \omega \vDash \neg u_i}^{n} \beta_i$" in the definition of the resulted revised OCF $k * \mathcal{S}$.

Example 3: Let us continue our example and consider the OCF distribution given in Table 1 (which is the same distribution as the one given in (Kern-Isberner and Huvermann 2015)). Assume that $\mathcal{S} = \{a, b\}$. Let β_1 and β_2 the weights associated with a and b respectively. We have $k(a \wedge b) = 4$ and using Equation (3) we get:

ω	$k * \mathcal{S}(\omega)$
ab	0
$\neg ab$	$\beta_2 - 3$
$a\neg b$	$\beta_1 - 3$
$\neg a \neg b$	$\beta_1 + \beta_2 - 4$

Table 2: The result of revising k, given in Table 1, by $\mathcal{S} = \{a, b\}$

Using Table 1, Equation (4) gives: $\beta_1 > 4 - min(1 - \beta_2)$ and $\beta_2 > 4 - min(1 - \beta_1)$ which are equivalent to $\beta_1 > 3$ and $\beta_2 > 3$.

Clearly, the c-revision is characterized by a set of parameters (weight). Each set of parameters induces an OCF distribution. In (Kern-Isberner and Huvermann 2015) a so-called minimal c-revision has also been proposed. This is obtained by considering only vectors of weights $(\beta_1, .., \beta_n)$ that satisfy Equation (4) and which are pareto-optimal. In the above example, a minimal c-revision is obtained when β_1 and β_2 are both assigned the degree of 4.

Note that the input considered in multiple iterated belief c-revision is different from the notion of uncertain input proposed in (Jeffrey 1965) for conditioning probability distributions. Indeed, in Jeffrey's rule of conditioning the input represents a partition of the set interpretations Ω, while in c-revision the input is a consistent set of propositional formulas.

Syntactic representations of c-revision

The aim of this section is to describe the syntactic representations of multiple iterated belief c-revision when OCF distributions are encoded by means of OCF knowledge bases. More precisely, let $\mathcal{K}$ be an OCF knowledge base and $k_{\mathcal{K}}$ be its associated OCF distribution obtained using Equation (2). Let $\mathcal{S}$ be an input. The aim of this section is to compute, from $\mathcal{K}$ and $\mathcal{S} = \{u_1, .., u_n\}$, a new OCF knowledge base $\mathcal{K}'$ such that:

$$\forall \omega, \quad k_{\mathcal{K}'}(\omega) = k_{\mathcal{K}} * \mathcal{S}(\omega),$$

where $k_{\mathcal{K}'}$ and $k_{\mathcal{K}}$ are the OCF distributions associated with $\mathcal{K}'$ and $\mathcal{K}$ using Equation (2).

To achieve this aim, we proceed in four steps:

- Compute $k(u_1 \wedge .. \wedge u_n)$.
- Compute the syntactic counterpart of adding an OCF distribution k with a binary possibility distribution.
- Compute the syntactic counterpart of the c-revision of $\mathcal{K}$ with $\mathcal{S}$ for a fixed vector of integers $(\beta_1, .., \beta_n)$ associated with formulas of $\mathcal{S}$.
- Provide the syntactic counterpart of the set of inequalities that the weights β_i's should satisfy (see Equation (4)).

The following subsections provide details of each of the above steps.

Computing $k(u_1 \wedge .. \wedge u_n)$

The aim of this subsection is to compute $k(u_1 \wedge .. \wedge u_n)$ directly from an OCF knowledge base $\mathcal{K}$. As is it shown in the following proposition, computing $k(u_1 \wedge .. \wedge u_n)$ comes down to compute the highest rank α such that formulas of $\mathcal{K}$ having a weight higher than or equal to α are inconsistent with $u_1 \wedge .. \wedge u_n$. More precisely:

Proposition 1: Let $\mathcal{K}$ be an OCF knowledge base and $k_{\mathcal{K}}$ be its associated OCF distribution using Equation (2). Let $\mathcal{S} = \{u_1, .., u_n\}$ be a consistent set of propositional formulas. Let $\mathcal{K}_{\geq \alpha}$ be the α-cut of $\mathcal{K}$ defined by $\mathcal{K}_{\geq \alpha} = \{\phi_j : (\phi_j, \gamma_j) \in \mathcal{K}, \gamma_j \geq \alpha\}$. Then:

$$k_{\mathcal{K}}(u_1 \wedge .. \wedge u_n) = max\{\alpha_i : \mathcal{K}_{\geq \alpha_i} \wedge (u_1 \wedge .. \wedge u_n)\ is\ inconsistent\}.$$

Proof. By definition, we have:

$$k_{\mathcal{K}}(u_1 \wedge .. \wedge u_n) = min_{\omega \vDash u_1 \wedge .. \wedge u_n}\ k_{\mathcal{K}}(\omega)$$
$$= min_{\omega \vDash u_1 \wedge .. \wedge u_n}\ max\{\alpha_i : (\phi_i, \alpha_i) \in$$

$$\mathcal{K},\ \omega \not\models \phi_i\}$$
$$= min_{\omega \models u_1 \wedge .. \wedge u_n}\ max\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{K},\ \omega \models \neg\phi_i \wedge u_1 \wedge .. \wedge u_n\}$$
$$= max\{\alpha_i : \mathcal{K}_{\geq \alpha_i} \wedge (u_1 \wedge .. \wedge u_n)\ is\ inconsistent\}.$$

□

From computational point of view, computing $k(u_1 \wedge .. \wedge u_n)$ needs $O(\log_2 m)$ calls to a satisfiability test of a set of clauses, where m is the number of different degrees used in $\mathcal{K}$.

Example 4: Let us continue our example. Recall that $\mathcal{K} = \{(\neg a \vee \neg b, 4), (\neg a, 1), (\neg b, 1)\}$ and that its associated OCF distribution is given in Table 1. Let $\mathcal{S} = \{a, b\}$. From Table 1, we have $k(a \wedge b) = 4$. One can easily check that:

$$max\{\alpha_i : \mathcal{K}_{\geq \alpha_i} \wedge (a \wedge b)\ is\ inconsistent\} = k(a \wedge b) = 4.$$

Next subsection is devoted to a syntactic computation of the result of adding an OCF distribution with a binary OCF distribution. A binary OCF distribution k' is an OCF distribution where the degree of surprise of each interpretation ω is either equal to 0 (namely, $k'(\omega) = 0$) or is equal to some constant β ($k'(\omega) = \beta$). Intuitively, a binary distribution will represent a weighted formula (u_i, β_i) of the input (models of u_i will have 0 degree, while counter-models of u_i will have a surprise degree equal to β_i).

Syntactic computations of adding an OCF distribution with a binary OCF distribution

The aim of this section is to provide a syntactic counterpart of:

$$\forall \omega \in \Omega, \qquad k'(\omega) = k(\omega) + \sum_{i=1, \omega \not\models \neg u_i}^{n} \beta_i, \qquad (5)$$

where β_i's are weights associated with each propositional formula u_i of $\mathcal{S}$.

More precisely, our aim is to compute a new knowledge base $\mathcal{K}'$, from $\mathcal{K}$ and $\mathcal{S} = \{u_1, .., u_n\}$, such that

$$\forall \omega \in \Omega, \qquad k_{\mathcal{K}'}(\omega) = k'(\omega) = k(\omega) + \sum_{i=1, \omega \not\models \neg u_i}^{n} \beta_i.$$

Equation (5) is clearly a part of the definition of c-revision given by Equation (3).

Let us first denote k_{u_i} the binary OCF distribution associated with (u_i, β_i) and defined by:

$$\forall \omega \in \Omega,\ k_{u_i}(\omega) = \begin{cases} 0 & if\ \omega \models u_i \\ \beta_i & otherwise. \end{cases}$$

Clearly, Equation (5) can be rewritten as:

$$\forall \omega \in \Omega, \qquad k'(\omega) = k(\omega) + k_{u_1}(\omega) + ... + k_{u_n}(\omega). \quad (6)$$

The following proposition gives the counterpart of combining $k(\omega)$ with some individual and binary distribution k_{u_i}:

Proposition 2: Let $\mathcal{K}$ be an OCF knowledge base. Let (u_i, β_i) be a weighted propositional formula. Let $\mathcal{K}' = \{(u_i, \beta_i)\} \cup \mathcal{K} \cup \{(u_i \vee \phi_j, \alpha_j + \beta_i) : (\phi_j, \alpha_j) \in \mathcal{K}\}$. Then:

$$\forall \omega \in \Omega,\ k_{\mathcal{K}'}(\omega) = k(\omega) + k_{u_i}(\omega),$$

where k and $k_{\mathcal{K}'}$ are the OCF distributions associated with $\mathcal{K}$ and $\mathcal{K}'$ using Equation (2).

Proof. Let $\omega \in \Omega$. We distinguish four cases depending whether ω is a model (or not) of u_i and formulas in $\mathcal{K}$:

a) $\omega \models u_i$ and $\forall (\phi_j, \alpha_j) \in \mathcal{K}\ \omega \models \phi_j$.Namely, ω is a model of u_i and satisfies all formulas in $\mathcal{K}$. In this case $k_{\mathcal{K}'}(\omega) = 0$ since $k(\omega) = 0$ and $k_{u_i}(\omega) = 0$.

b) $\omega \models u_i$ (hence, $\omega \models u_i \vee \phi_j$ for each $(\phi_j, \alpha_j) \in \mathcal{K}$) and $\exists (\phi_j, \alpha_j) \in \mathcal{K}$ such that $\omega \not\models \phi_i$. In this case, $k_{\mathcal{K}'}(\omega) = k_{\mathcal{K}}(\omega)$ since $k_{u_i}(\omega) = 0$.

c) $\omega \not\models u_i$ and $\forall (\phi_j, \alpha_i) \in \mathcal{K}$ we have $\omega \models \phi_j$ (hence $\omega \models u_i \vee \phi_j$ for each $(\phi_j, \alpha_j) \in \mathcal{K}$). Hence, $k_{\mathcal{K}'}(\omega) = k_u(\omega)$ since $k(\omega) = 0$.

d) $\omega \not\models u_i$ and $\exists (\phi_j, \alpha_j) \in \mathcal{K}$ such that $\omega \not\models \phi_j$. Namely, ω is neither a model of u_i nor a model of all propositional formulas in $\mathcal{K}$. Then by definition:

$$k_{\mathcal{K}'}(\omega) = max\{\beta_i, max\{\alpha_j : (\phi_j, \alpha_j) \in \mathcal{K}\ \omega \not\models \phi_j\}, max\{\alpha_j + \beta_i : (\phi_j, \alpha_j) \in \mathcal{K}\ \omega \not\models \phi_j\}\}$$
$$= max\{\alpha_j : (\phi_j, \alpha_j) \in \mathcal{K},\ \omega \not\models \phi_j\} + \beta_i$$
$$= k_{\mathcal{K}}(\omega) + k_u(\omega).$$

□

Note that Proposition 2 is similar to the syntactic fusion mode proposed in (Benferhat, Dubois, and Prade 1997) in a possibility theory framework. From Proposition 2, trivially in the worst case the size of $\mathcal{K}'$ is $2 * |\mathcal{K}| + 1$, and the computation of the OCF knowledge base $\mathcal{K}'$ is done in linear time with respect to the size of $\mathcal{K}$.
Clearly, the repetitive application of Proposition 2 on each propositional formula u_i of $\mathcal{S}$ allows us to provide the syntactic counterpart of Equation (6).

Example 5: Let us continue our example. We recall that $\mathcal{K} = \{(\neg a \vee \neg b, 4), (\neg a, 1), (\neg b, 1)\}$ and $\mathcal{S} = \{a, b\}$.
Applying Proposition 2 on $\mathcal{K}$ and (a, β_1) gives:

$$\mathcal{K}' = \{(\neg a \vee \neg b, 4), (\neg a, 1), (\neg b, 1)\} \cup \{(a, \beta_1)\} \cup \{(a \vee \neg b, 1 + \beta_1)\}$$

Again, applying Proposition 2 on $\mathcal{K}'$ and (b, β_2) gives:

$$\mathcal{K}'' = \{(\neg a \vee \neg b, 4), (\neg a, 1), (\neg b, 1), (a, \beta_1), (a \vee \neg b, 1 + \beta_1)\} \cup \{(b, \beta_2)\} \cup \{(\neg a \vee b, 1 + \beta_2), (a \vee b, \beta_1 + \beta_2)\}$$

Namely

$$\mathcal{K}'' \equiv \{(\neg a \vee \neg b, 4), (a, \beta_1), (b, \beta_2), (a \vee \neg b, 1 + \beta_1), (\neg a \vee b, 1 + \beta_2), (a \vee b, \beta_1 + \beta_2)\}$$

(since (a, β_1) and $(\neg a \vee b, 1 + \beta_2)$ leads to $(b, \min(\beta_1, \beta_2 + 1))$ and (b, β_2) and $(a \vee \neg b, 1 + \beta_1)$ leads to $(a, \min(\beta_2, \beta_1 + 1))$)

$$\mathcal{K}'' \equiv \{(\neg a \vee \neg b, 4), (a \vee \neg b, 1 + \beta_1), (\neg a \vee b, 1 + \beta_2), (a \vee b, \beta_1 + \beta_2)\}.$$

From Table 3, we have:

$$\forall\omega \in \Omega \quad k_{\mathcal{K}''}(\omega) = k_{\mathcal{K}}(\omega) + k_a(\omega) + k_b(\omega).$$

Ω	k_a	k_b	$k_{\mathcal{K}}$	$k_{\mathcal{K}''}$
ab	0	0	4	4
$a\neg b$	0	β_2	1	1 + β_2
$\neg ab$	β_1	0	1	1 + β_1
$\neg a\neg b$	β_1	β_2	0	β_1 + β_2

Table 3: The resulting of adding OCF $k_{\mathcal{K}}$ with two binary distributions k_a and k_b

The last step is to compute the syntactic computation of c-revision. Namely, to compute the counterpart of Equation (3). This is the aim of next subsection.

Computing c-revision

The following Lemma will help us in providing the syntactic computation of multiple iterated belief c-revision operation.

Lemma 1: Let $\mathcal{K}$ be an OCF knowledge base and $\mathcal{S} = \{u_1, .., u_n\}$ be a consistent set of propositional formulas. Let $\mathcal{K}' = \{(\phi_i, \alpha_i - k(u_1 \wedge .. \wedge u_n)) : (\phi_i, \alpha_i) \in \mathcal{K}\}$. Then:

$$\forall\omega \in \Omega, \qquad k_{\mathcal{K}'}(\omega) = k(\omega) - k(u_1 \wedge .. \wedge u_n),$$

where k and $k_{\mathcal{K}'}$ are the OCF distributions associated with $\mathcal{K}$ and $\mathcal{K}'$ respectively.

The proof is immediate since by definition:

$$\begin{aligned}\forall\omega \in \Omega,\ \ k_{\mathcal{K}'}(\omega) &= max\{\alpha_i - k(u_1 \wedge .. \wedge u_n) : (\phi_i, \alpha_i) \in \mathcal{K}, \omega \not\models \phi_i\} \\ &= max\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{K}, \omega \not\models \phi_i\} - k(u_1 \wedge .. \wedge u_n) \\ &= k(\omega) - k(u_1 \wedge .. \wedge u_n).\end{aligned}$$

Now, to get the syntactic computation of $k * \mathcal{S}$, (the result of applying multiple iterated belief c-revision on k and $\mathcal{S}$ using Equation (3)) it is enough to apply Proposition 2 on each element of $\mathcal{S}$, and then apply Lemma 1.

Clearly, the computation of $\mathcal{K}'$ in Lemma 1 is done in linear time with respect to the size of k (once $k(u_1 \wedge .. \wedge u_n)$ is already computed).

Example 6: Let us continue our example. Recall that from Example 5 we have:

$$\mathcal{K}'' = \{(\neg a \vee \neg b, 4), (a \vee \neg b, 1 + \beta_1), (\neg a \vee b, 1 + \beta_2), (a \vee b, \beta_1 + \beta_2)\}.$$

Recall that $\mathcal{S} = \{a, b\}$ and $k(a \wedge b) = 4$.
Applying Lemma 1 on $\mathcal{K}''$ we get:

$$\begin{aligned}\mathcal{K}^* &= \{(\neg a \vee \neg b, 0), (a \vee \neg b, \beta_1 - 3), (\neg a \vee b, \beta_2 - 3), (a \vee b, \beta_1 + \beta_2 - 4)\} \\ &\equiv \{(\neg a \vee b, \beta_2 - 3), (a \vee \neg b, \beta_1 - 3), (a \vee b, \beta_1 + \beta_2 - 4)\}\end{aligned}$$

One can easily check that:

$$\forall\omega, \qquad k_{\mathcal{K}^*}(\omega) = k * \mathcal{S}(\omega),$$

where $k * \mathcal{S}(.)$ is given in Table 2 and $k_{\mathcal{K}^*}$ is the OCF distribution associated with $\mathcal{K}^*$ using Equation (2).

On the characterization of inequality constraints

It remains now to characterize the constraints bearing on β_i's. Namely, our aim is to directly characterize from $\mathcal{K}$ and $\mathcal{S} = \{u_1, .., u_n\}$ the set of inequalities:

$$\forall i = 1, .., n,\ \beta_i > k(u_1 \wedge .. \wedge u_n) - \min_{\omega \not\models u_i}\{k(\omega) + \sum_{j \neq i, \omega \not\models \beta_j} \beta_j\}$$

The computation of such inequalities is possible thanks to Propositions (1,2) and to Lemma 1. Indeed, Proposition 1 allows us to compute $k(u_1 \wedge .. \wedge u_n)$. Proposition 2 allows us, for each i, to compute the syntactic counterpart of:

$$\forall\omega, k'(\omega) = k(\omega) + \sum_{i=1, j \neq i}^{n} k_{u_j}(\omega).$$

Using similar steps as in Proposition 1, we get:

$$\min_{\omega \not\models u_i} k'(\omega) = max\{\alpha_i : \mathcal{K}'_{\geq \alpha_i} \wedge \neg u_i \ is\ inconsistent\},$$

where $\mathcal{K}'$ is an OCF knowledge base associated with

$$\forall\omega \in \Omega, \qquad k'(\omega) = k(\omega) + \sum_{j=1, j \neq i}^{n} k_{u_j}(\omega).$$

Example 7: Let us finish our example where we have $\mathcal{K} = \{(\neg a \vee \neg b, 4), (\neg a, 1), (\neg b, 1)\}$ and $\mathcal{S} = \{a, b\}$. Recall that we already computed $k(a \wedge b)$ which is equal to 4. Let us now give the inequality relations associated with β_1 and β_2.
For β_1, our aim is to characterize:

$$\beta_1 > k(a \wedge b) + \min_{\omega \models \neg a}\{k(\omega) + k_b(\omega)\}$$

The knowledge base associated with $k(.) + k_b(.)$ is obtained using Proposition 2:

$$\begin{aligned}\mathcal{K}' &= \{(\neg a \vee \neg b, 4), (\neg a, 1), (\neg b, 1), (b, \beta_2), (\neg a \vee b, 1 + \beta_2)\} \\ &= \{(\neg a \vee \neg b, 4), (\neg b, 1), (b, \beta_2), (\neg a \vee b, 1 + \beta_2)\}\end{aligned}$$

Now,

$$\begin{aligned}\min_{\omega \models \neg a}\{k(\omega) + k_b(\omega)\} &= max\{\alpha_i : \mathcal{K}'_{\geq \alpha_i} \wedge \neg a \ is\ inconsistent\} \\ &= min(1, \beta_2).\end{aligned}$$

Similar result for β_2. Hence, β_1 and β_2 should satisfy:

$$\beta_1 > 4 - min(1, \beta_2),$$

and

$$\beta_2 > 4 - min(1, \beta_1).$$

Which are equivalent to $\beta_1 > 3$ and $\beta_2 > 3$.

These inequalities are the same as the ones given in Example 2 but obtained here using OCF knowledge bases.

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Conclusion

The multiple iterated propositional c-revision, defined in (Kern-Isberner and Huvermann 2015), is a revision operator that takes into account the independence relations that may exist between propositional formulas of the input.

This paper shows that c-revision can be equivalently defined using OCF knowledge bases. In particular, we provide an explicit computation of the inequalities associated with certainly degrees attached with formulas of the input.

In this paper, OCF distributions are obtained from OCF knowledge bases using a translation function similar to the one used in possibility theory. A future work is to redefine c-revision when OCF distributions are obtained from OCF knowledge bases using penalty logic (Darwiche and Marquis 2004; Bannay, Lang, and Schiex 1994).

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