Conjunctive Choice Logic

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Abstract

Handling preferences in presence of constraints is an important concept in many applications. The main purpose of this paper is to provide a new non-classical logic for representing and reasoning with preferences and functional constraints in uncertain environments. This logic, called Conjunctive Choice Logic (CCL), aims to determine feasible solutions in such environments and evaluate their satisfaction degrees. Our logic extends the propositional logic with a new logical connective called ordered conjunction. This operator, viewed as a kind of prioritized conjunction, is used when there are preferences and constraints between pieces of information.

Introduction

In decision theory, handling human preferences in presence of constraints is a very important concept which has been extensively studied in different research areas (Chabakauri 2014; Schermann and Ennser-Jedenastik 2014; van Cranenburgh, Chorus, and van Wee 2014; Zou et al. 2014). The main purpose is to provide efficient and effective models for handling preferences and constraints in a compact way.

As an example, let us consider the travel reservation system "E-travel" [1] used by different companies, such as CNRS (French national center of scientific research)to buy or to obtain information about flights and trains. When users' queries concern information about direct flights between two cities, such as the price of travel tickets, the system is often satisfactory. The situation becomes more complex when one considers indirect flights which may involve other means of transportation like trains. In this case, the number of possible solutions becomes very significant and even increases exponentially. To deal with this situation, a user may purchase a ticket based not only on general flight parameters, but also on personal preferences such as departure/arrival date/time, airports, airlines, total travel time, connection airports and other preferences (Johnson, Hess, and Matthews 2014; Roman and Martin 2014). For example, a user may look for a trip between Paris and Mexico with a connection outside USA (a strong preference), a price $\leq$ 300 \$ and if possible duration $\leq$ 2 hours. Given these preferences, trips with an USA airport stop are unacceptable. For remaining solutions, those where both price $\leq$ 300 \$ and duration $\leq$ 2 hours are satisfied are the most preferred ones. Then solutions where only price $\leq$ 300 \$ is satisfied are more preferred to solutions where only duration $\leq$ 2 hours is satisfied (Benferhat and Boudjelida 2011).

Recently, a set of logical and graphical settings have been proposed for representing, learning and reasoning with preferences (Domshlak et al. 2011; Dubois and Prade 2014; Liu and Liao 2015; Pedersen, Dyrkolbotn, and Agotnes 2014; van Benthem, Girard, and Roy 2009). In (Boutilier 1992; Boutilier et al. 2004), conditional and qualitative preferences were expressed through a graphical structure called Conditional Preference networks (CP-nets). Preference elicitation in such a framework appears to be natural and intuitive and different extensions of CP-nets have been proposed (Brafman and Domshlak 2002; Li, Vo, and Kowalczyk 2015; Wang et al. 2012). Possibilistic logic is another framework for representing preferences (Dubois and Prade 2004a). It handles pairs of propositional logic formulas associated with priority levels.

A logic for representing choices and preferences called Qualitative Choice Logic (QCL) has been proposed in (Brewka, Benferhat, and Berre 2004) (see also (Benferhat and Sedki 2008; Bouzar-Benlabiod, Benferhat, and Bouabana-Tebibel 2015) for its variants). QCL is an extension of propositional logic for representing alternatives, or ranked options for problem solutions. QCL uses a disjunctive interpretation of preferences. If the first option, for instance, is satisfied then there is no need to consider other alternatives. This may be useful in some applications where all options are mutually exclusives. However in practice, one may consider other options even if the first option is satisfied.

In this paper, we propose a conjunctive interpretation of preferences. More precisely, we propose a new logic that we call Conjunctive Choice Logic (CCL) which can be viewed as a counterpart of QCL for representing conjunctive preferences. Our logic is also an extension of propositional logic. The non-standard part of CCL logic is a new logical connective called ordered conjunction and denoted by $\vec{\odot}$. Intuitively, if A and B are propositional formulas then A $\vec{\odot}$ B means: "if possible satisfy both A and B, but if not then falsifying B is preferred to falsifying A". Our operator extends propositional conjunction when there are preferences

[1] Amadeus e-Travel Management. http://www.amadeus.com.

and constraints between pieces of information. As will see later, even if intuitively our logic CCL looks like a counterpart of QCL, it cannot be simply defined from QCL. Indeed, handling ordered conjunctions raises new issues that are not encountered with standard QCL. In fact, QCL expresses weak preferences (a preference over disjunctions) while here CCL expresses strong preferences (a preference over conjunctions).

The rest of the paper is organized as follows. Section 2 introduces the syntax and semantics of formulas using CCL language. More precisely, we introduce basic conjunctive choice formulas which represent simple forms of ordered propositional conjunctions and general conjunctive choice formulas which can represent complex rules that involve preferences for arbitrary formulas. We also define models of a set of formulas and show how to determine the preferred ones. Section 3 presents the notion of equivalence between two CCL formulas and shows how to translate a set of general conjunctive choice formulas into a set of basic conjunctive choice formulas. Section 4 concludes the paper.

The CCL language

As advocated in the introduction, CCL is an extension of propositional logic. The non-standard part of the CCL language is a new logical connective $\vec{\odot}$. Intuitively, if A and B are propositional formulas then $A \vec{\odot} B$ means: if possible satisfy both A and B, but if not then falsifying B is preferred to falsifying A. Namely, solutions where both A and B are true are more acceptable and preferred to solutions where only A is true. Solutions where A is false are considered as unacceptable and should be rejected. Hence, the first option is reserved for integrity constraints that should absolutely be satisfied. When there is no integrity constraints, the first option is simply represented by a tautology.

We call the new connective *ordered conjunction* , denoted by $\vec{\odot}$. It is viewed as a kind of prioritized conjunction. In particular, it is not symmetric $A \vec{\odot} B$ is different from $B \vec{\odot} A$. However, it is associative namely $(A \vec{\odot} B) \vec{\odot} C = A \vec{\odot} (B \vec{\odot} C)$.

Basic Conjunctive Choice Formulas (BCCF)

We will follow the same structure as the one used in (Brewka, Benferhat, and Berre 2004) for describing the language. We first present a simple form of preferences called basic conjunctive choice formulas where the new connective operator $\vec{\odot}$ can only be applied between propositional formulas. Then we introduce the general language where the operator $\vec{\odot}$ can appear everywhere in a formula.

Syntax We denote by PS the set of propositional symbols and by $PROP_{PS}$ the set of propositional formulas that can be built using classical logical connectives $(\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow)$ over PS.

Given a set of propositional formulas $A_1, \ldots, A_n$, the formula $A_1 \vec{\odot} \ldots \vec{\odot} A_n$, called *Basic Conjunctive Choice Formulas (BCCF)*, is used to express basic forms of ordered conjunctions. Such formulas are constructed in the following way:

Definition 1 *The language composed of basic conjunctive choice formulas, denoted by $BCCF_{PS}$, is the smallest set of formulas defined inductively as follow:*

1. *if $F \in PROP_{PS}$, then $F \in BCCF_{PS}$.*
2. *if $F_1, F_2 \in BCCF_{PS}$ then $(F_1 \vec{\odot} F_2) \in BCCF_{PS}$.*
3. *Every basic conjunctive choice formula is only obtained by applying the two rules above a finite number of times.*

$BCCF_{PS}$ can be viewed as simple forms for representing prioritized information. As we will see later, a basic conjunctive choice formula $A_1 \vec{\odot} \ldots \vec{\odot} A_n$ induces an ordering on possible solutions or interpretations in a very natural way. Intuitively, solutions where $A_1 \wedge \ldots \wedge A_n$ is true are the preferred ones. If $A_1 \wedge \ldots \wedge A_n$ is false then falsifying A_n is preferred to falsifying A_{n-1} and falsifying A_{n-1} is preferred to falsifying A_{n-2}. More generally falsifying A_i (while $A_1, \ldots, A_{i-1}$ are satisfied) is preferred to falsifying A_j (while $A_1, \ldots, A_{j-1}$ are satisfied) when $i > j$.

Example 2 *Let us consider the example given in the introduction. We define the $BCCF_{PS}$ language for representing simple preferences and queries of a travel reservation system such E-travel. We will denote by:*

- *$\varphi = \{V_1, V_2, ..., V_n\}$ the set of variables representing E-travel system queries attributes.*
- *$D_{V_i} = \{v_i^1, v_i^2, ..., v_i^{m_i}\}$ the domain that each variable V_i can take.*

As an example of attributes that E-travel uses to compose a query we can list :

- *$\varphi = \{$ DA (departure airport), AA (arrival airport), DD (departure date), AD (arrival date), CO (travel company), CL (travel class) $\}$*

As an example of values that these attributes can take :

- *$D_{DA} = \{$L (Lille), P (Paris), Q (Quebec City)$\}$*
- *$D_{CO} = \{$AF (Air France), AC(Air Canada)$\}$*
- *$D_{CL} = \{$E (Economy), F (First)$\}$*

We represent by $V_i = v_i^j$, with $V_i \in \varphi$ and $v_i^j \in D_{V_i}$ an atomic formula in the $BCCF_{PS}$ language. Formally, given a set of symbols : $v_1, v_2, ..., v_n$ representing values (or instances) associated with the set of attributes $V_1, V_2, ..., V_n$. An E-travel query is a conjunction of (attributes, values) of the form : $V_1 = v_1 \wedge \ldots \wedge V_n = v_n$.

Now, in some situations, a user may express a preference over attributes. In this case, an initial query in E-travel can be represented as basic conjunctive choice formula (BCCF) as follows : $V_1 = v_1 \wedge \ldots \wedge V_k = v_k \vec{\odot} \ldots \vec{\odot} V_n = v_n$. A query of a user who looks for a simple flight between Lille and Quebec City for the '31/12/2015' with Air France in first class as options will take the following basic conjunctive choice form :

$Q_1 : DA = "L" \wedge AA = "Q" \wedge DD = "31/12/2015" \vec{\odot} CO = "AF" \vec{\odot} CL = "F"$

In this example, travels where :

$DA = "L" \wedge AA = "Q" \wedge DD = "31/12/2015" \wedge CO = "AF" \wedge CL = "F" \ldots (1)$

are the preferred ones.

If (1) is not satisfied, then falsifying $CL = "F"$ is preferred to falsifying $CO = "AF"$.

Semantics A propositional interpretation I will be represented by the set of its satisfied atoms. Ω denotes the set of all possible interpretations. The semantic of $BCCF_{PS}$ formulas is based on the degree of satisfaction of a formula in a particular model I. Intuitively, this degree can be viewed as a degree of acceptability associated with each interpretation or solution: the lower the satisfaction degree of an interpretation the more preferred it is.

Given a basic conjunctive choice formula : $F_1 = A_1 \vec{\odot} \ldots \vec{\odot} A_n$, an interpretation or a solution I satisfies F_1 to degree 1 if it satisfies all options of F_1. This represents an ideal solution. Now, interpretations that satisfy $(A_1, \ldots, A_{n-1})$ and only falsify A_n are considered as the second best solutions. They will get the degree 2. The third best solutions (having degree 3) are those that satisfy $(A_1, \ldots, A_{n-2})$ but falsify A_{n-2}. And so on. More generally, an interpretation satisfies F_1 to a degree $n-k+2$, if it falsifies the k^{th} option of F_1 (namely A_k) and satisfies the first $(k-1)$ options of F_1.

Lastly, the first option is viewed as a constraint that should be satisfied. Hence, interpretations where the first option is falsified is considered as unacceptable, and get the highest possible value $(+\infty)$.

For propositional formulas F_1, there is only one degree of satisfaction (namely 1) obtained when F_1 is satisfied by I. If F_1 is not satisfied, then again the satisfaction degree will be simply equal to infinity.

Definition 3 *Satisfaction degree of interpretations with respect to $BCCF_{PS}$ formulas*

1. *Let $F = A_1 \vec{\odot} \ldots \vec{\odot} A_n \in BCCF_{PS}$, and let I be an interpretation which satisfies F to a degree k, then $I \models_k F$ and :*

$$k = \begin{cases} 1 & \textit{iff} \quad I \models (A_1 \wedge \ldots \wedge A_n) \\ n - min\{j \mid I \not\models A_j\} + 2 & \textit{iff} \quad j > 1 \\ \infty & \textit{iff} \quad I \not\models A_1 \end{cases}$$

2. *Let $F \in PROP_{PS}$, $I \models_1 F$ iff $I \models F$, and $I \models_\infty F$ iff $I \not\models F$.*

Example 4 *The following table gives satisfaction degrees obtained by applying all possible interpretations to query Q_1 given in example 1. These satisfaction degrees are obtained after assigning truth values True(T), False(F), to the formulas composing the query Q_1. The symbol (*) in the table represents all possible values. Table 1 shows that Q_1 has one preferred solution (satisfaction degree = 1) when all atomic formulas are true. Q_1 is not satisfied if the first choice formula is false.*

Table 1: Example of Satisfaction degrees of a $BCCF_{PS}$ formula

$DA = "L" \wedge AA = "Q" \wedge DD = "31/12/2015"$	$CO = "AF"$	$CL = "F"$	Satisfaction Degree
F	*	*	∞
T	F	*	3
T	T	F	2
T	T	T	1

General Conjunctive Choice Formulas ($GCCF$)

The $GCCF$ language not only allows to represent simple and basic forms of preferences, but can also represent complex rules that involve conjunctive preferences for arbitrary formulas. *General Conjunctive Choice Formulas* represent any formulas that can be obtained from PS using connectors $\neg, \vee, \wedge, \vec{\odot}$.

Definition 5 *The language composed of general conjunctive choice formulas, denoted by $GCCF_{PS}$, is defined inductively as follow:*

1. *if $F \in BCCF_{PS}$, then $F \in GCCF_{PS}$.*
2. *if $F_1, F_2 \in GCCF_{PS}$ then $\neg F_1 \in GCCF_{PS}, F_1 \vee F_2 \in GCCF_{PS}, F_1 \wedge F_2 \in GCCF_{PS}, F_1 \vec{\odot} F_2 \in GCCF_{PS}$.*
3. *Every general conjunctive choice formula is only obtained by applying the two rules above a finite number of times.*

Items (1) simply says that basic conjunctive choice formulas are also general conjunctive choice formulas. Item (2) states that general conjunctive choice formulas can be combined using conjunction $\wedge$, disjunction $\vee$, negation $\neg$ and ordered conjunction $\vec{\odot}$.

Example 6 *The initial queries language used in Example 1 does not offer the possibility to express different users constraints. Clearly there is a need to use a richer language combining all possible attributes to form a general query. Since the initial language only contains basic conjunctive choice formulas, one may need to use a language which will contain negation, conjunction and disjunction operations. Formally, a user query on a set of attributes $V_1, V_2, ..., V_n$ can be expressed in the following way :*

$$((V_1 = v_1^1 \wedge ... \wedge V_k = v_k^1) \vec{\odot} ... \vec{\odot} V_n = v_n^1) \vee ... \vee ((V_1 = v_1^{m_1} \wedge ... \wedge V_k = v_k^{m_k}) \vec{\odot} ... \vec{\odot} V_n = v_n^{m_n})$$

Each literal v_i^j represents the value that an attribute V_i can take.

For example a user can launch a query with a constraint of having either travels to Quebec City on the '30/08/2015' with Lille as departure airport or travels from Paris to Quebec City on the '31/08/2015'. In the first case he prefers to travel with Air France in the economic class and in the second case he prefers to travel with Air Canada in the first class.

Thus the query in $GCCF_{PS}$ answering to these constraints will be: $Q_2 : ((DA = "L" \wedge AA = "Q" \wedge DD = "30/08/2015") \vec{\odot} CO = "AF" \vec{\odot} CL = "E") \vee ((DA = "P" \wedge AA = "Q" \wedge DD = "31/08/2015") \vec{\odot} CO = "AC" \vec{\odot} CL = "F")$

Semantics The semantic of $GCCF_{PS}$ formulas depends on the satisfaction degree of this formula in a particular model I. Consider $F' = (F_1 \vec{\odot} F_2) \in GCCF_{PS}$ and I an interpretation. Two situations arise here. The first one is that I satisfies F_1 to a degree 1 and F_2 to a degree k. In this case, the satisfaction degree of F' depends only on the satisfaction degree of F_2 and it will be k. The second situation occurs when I satisfies F_1 to a degree $k \neq 1$, then the satisfaction degree of F' in this case, depends on the number of possible satisfaction degrees or options that F_2 admits. Hence, if we assume there are j such options for F_2, F' will be satisfied in the $(j+k)^{th}$ best possible way which will represent its satisfaction degree.

In the following, we denote by $npsd(F_2)$ the number of possible satisfaction degrees of $F2$. Intuitively, if $npsd(F2) = n$, then there may be n^{th} best way of satisfying $F2$. There is only one way to satisfy propositional formulas, hence they all have an $npsd$ equal to 1.

For conjunction and disjunction options we obtain the maximum number of possible satisfaction degrees of the sub-formulas. For instance, if $npsd(F_1) = j$ and $npsd(F_2) = k$ with $j < k$, then $F_1 \vee F_2$ and $F_1 \wedge F_2$ cannot have more then k options.

Definition 7 *The $npsd$ of a formula indicates the number of satisfaction degrees that a formula can have. Let F_1 and F_2 be two formulas in $GCCF_{PS}$ and A a propositional atom.*

$npsd(A) = 1$
$npsd(\neg F) = 1$
$npsd(F_1 \vee F_2) = max(npsd(F_1), npsd(F_2))$
$npsd(F_1 \wedge F_2) = max(npsd(F_1), npsd(F_2))$
$npsd(F_1 \vec{\odot} F_2) = npsd(F_1) + npsd(F_2)$

It easy to check that $npsd$ is associative. In particular, the two formulas $((F_1 \vec{\odot} F_2) \vec{\odot} F_3)$ and $(F_1 \vec{\odot} (F_2 \vec{\odot} F_3))$ have the same $npsd$, namely :
$npsd((F_1 \vec{\odot} F_2) \vec{\odot} F_3) = npsd(F_1 \vec{\odot} (F_2 \vec{\odot} F_3)) = npsd(F_1) + npsd(F_2) + npsd(F_3)$.

The following definition gives the CCL satisfaction relation denoted by $\mid\!\sim^{CCL}$. The relation is indexed according to the degree of satisfaction of a formula in a model.

Definition 8 *Let F_1 and F_2 be two formulas in $GCCF_{PS}$ and A a propositional atom.*

1. $I \mid\!\sim_k^{CCL} A$ *and*
$$k = \begin{cases} 1 & \textit{iff} \quad A \in I \\ \infty & \textit{iff} \quad A \notin I \end{cases}$$
2. $I \mid\!\sim_k^{CCL} F_1 \wedge F_2$ *and*
$$k = \begin{cases} max(m,n) & \textit{iff} \quad I \mid\!\sim_m^{CCL} F_1 \textit{ and } I \mid\!\sim_n^{CCL} F_2 \\ & \textit{and } m \neq \infty \textit{ and } n \neq \infty \\ \infty & \textit{iff} \quad I \mid\!\sim_\infty^{CCL} F_1 \textit{ or } I \mid\!\sim_\infty^{CCL} F_2 \end{cases}$$
3. $I \mid\!\sim_k^{CCL} F_1 \vee F_2$ *and*
$$k = \begin{cases} min(m,n) & \textit{iff} \quad I \mid\!\sim_m^{CCL} F_1 \textit{ and } I \mid\!\sim_n^{CCL} F_2 \\ & \textit{and } m \neq \infty \textit{ or } n \neq \infty \\ \infty & \textit{iff} \quad I \mid\!\sim_\infty^{CCL} F_1 \textit{ and } I \mid\!\sim_\infty^{CCL} F_2 \end{cases}$$
4. $I \mid\!\sim_k^{CCL} F_1 \vec{\odot} F_2$ *and*
$$k = \begin{cases} m + npsd(F_2) & \textit{iff} \quad I \mid\!\sim_m^{CCL} F_1 \textit{ and } m \neq 1 \\ & \textit{and } m \neq \infty \\ m & \textit{iff} \quad I \mid\!\sim_1^{CCL} F_1 \textit{ and } I \mid\!\sim_m^{CCL} F_2 \\ \infty & \textit{iff} \quad I \mid\!\sim_\infty^{CCL} F_1 \end{cases}$$
5. $I \mid\!\sim_1^{CCL} \neg F_1$ *iff* $I \mid\!\sim_\infty^{CCL} F_1$.

Item (1) of Definition 5 states that propositional atoms that compose an interpretation I have degree 1 while the others (those that are not true in I) have the highest impossibility degree ∞.

Item (2) expresses that if I is unacceptable with respect to F_1 or F_2 then it remains unacceptable for their conjunction. This confirm the conjunctive understanding of '$\wedge$' where both F_1 and F_2 should be acceptable in I to declare that '$\vec{\odot}$' is also acceptable. Now, if I is somewhat acceptable to both F_1 and F_2, then the acceptability degree of $F_1 \wedge F_2$ should be equal to the maximal acceptability degree of F_1 and F_2. Clearly, if F_1 and F_2 are propositional formulas then '$\wedge$' recovers the propositional conjunction.

Item (3) is clearly dual to item (2) and again disjunction used in $GCCF_{PS}$ recovers propositional disjunction when both F_1 and F_2 are propositional formulas.

Item (4) states that if F_1 is unacceptable, then $F_1 \vec{\odot} F_2$ is also unacceptable. Intuitively, F_1 is unacceptable means that the first option (which reflects some strong preferences) is not satisfied. This also means that the first option in $F_1 \vec{\odot} F_2$ is also not satisfied, hence I is unacceptable. Now, if F_1 is fully satisfied then the satisfaction of $F_1 \vec{\odot} F_2$ is the same as the one of F_2. If I satisfies F_1 to a degree k and n is the number of options in F_1 then this means that I falsifies the $(n-k+2)^{th}$ option in F_1 and the $npsd(F_2) + (n-k+2)^{th}$ option in $F_1 \vec{\odot} F_2$. From item (4), one can check that the conjunctive preference operator $\vec{\odot}$ is associative.

There are different ways to define negated preference $\neg F_1$. The one used here is the absence of satisfaction of F_1 to some degree.

Preferred Models in $GCCF_{PS}$

In the following, a set of available preferences formulas (basic and general) will be denoted by T. Propositional formulas will be denoted by K.

Definition 9 *Let T be a set of formulas. An interpretation I is a model of T if it satisfies each formula in T with a degree $\neq \infty$. Otherwise it is called a counter model of T.*

Remark 10 *Sometimes preferences are considered as different from "flexible" constraints. In the sense that preferences should not exclude feasible solutions. Namely, for any preference formula $F_1 \in T$, there should be some k such that $I \models_k F_1$. A possible way to recover this interpretation of preferences without modifying our semantics is to replace each preference formula F_1 by $\top \vec{\odot} F_1$, which guarantee each preference to be satisfied to some degrees.*

When dealing with preferences, it may happen that there is no solution that satisfies all conjunctive preferences to a degree 1. In this case, it is important to rank-order all solutions to determine the preferred ones. We use a lexicographic ordering which is based on the number of formulas satisfied to a particular degree used also in QCL (Brewka, Benferhat, and Berre 2004). The preference relations between solutions is defined as follows:

Definition 11 *Let $M^k(T)$ denotes the subset of formulas of T satisfied by a model M to a degree k. A model M_1*

is $K \cup T$-preferred over a model M_2 if there is a k such that $|M_1^k(T)| > |M_2^k(T)|$ and for all $j < k$: $|M_1^j(T)| = |M_2^j(T)|$. M is a preferred model of $K \cup T$ if:

1. *M is a model of $K \cup T$,*
2. *M is a maximally $K \cup T$-preferred model.*

Intuitively, a preferred model of $K \cup T$ is a model of $K \cup T$ which satisfies the maximal number of best options of CCL formulas. This leads to an approach where solutions are preferred when they contain the highest number of most preferred options.

The lexicographic ordering (known also as cardinality-based ordering) has been used in different context such us in belief revision or in inconsistency handling of prioritized propositional knowledge.

Example 12 *Let F_1, F_2, F_3 be three CCL formulas with three options each : $F_1 = A_1 \vec{\odot} A_2 \vec{\odot} A_3, F_2 = B_1 \vec{\odot} B_2 \vec{\odot} B_3$ and $F_3 = C_1 \vec{\odot} C_2 \vec{\odot} C_3$.*

Let M_1 be a model falsifying A_3, B_3 and C_1. In this model F_1 and F_2 are satisfied with degree 2 and F_3 is not satisfied. Let M_2 be another model falsifying A_2, B_2 and C_3. In this model F_1 and F_2 are satisfied with degree 3 and F_3 is satisfied with degree 2. In this example, M_1 is preferred over M_2 because it contains the highest number of most preferred options. Namely, M_1 has two formulas satisfied with degree 2, whereas the number of formulas satisfied in M_2 with degree 2 is only one, although the rest of formulas in M_1 are not satisfied and the rest of formulas in M_2 are satisfied with degree 3.

Normalization form

In this section, we show that any set of $GCCF_{PS}$ formulas, can equivalently be transformed into a set of basic conjunctive choice formulas. This normalisation is useful for computational issues, however for representational purpose it is more convenient to use $GCCF_{PS}$ than $BCCF_{PS}$. We first need to introduce the notion of normal form function, which associates with each general conjunctive choice formulas in $GCCF_{PS}$, its corresponding basic conjunctive choice formulas. This normal form function is denoted by N_{CCL}.

Definition 13 *We define a normal function denoted by N_{CCL}, a function from $GCCF_{PS} \longrightarrow BCCF_{PS}$, such that :*

1. *Normal form of basic conjunctive choice formulas and propositional formulas are these formulas themselves:*

   (a) $\forall F_1 \in BCCF_{PS}, N_{CCL}(F_1) = F_1$.

2. *The normal form is decomposable with respect to negation, conjunction, disjunction and ordered conjunction of general conjunctive choice formulas:*

   (a) $\forall F_1 \in GCCF_{PS}$ *and* $F_1 \notin BCCF_{PS}$, $N_{CCL}(\neg F_1) = N_{CCL}(\neg N_{CCL}(F_1))$

   (b) $\forall F_1, F_2 \in GCCF_{PS}$ *and* ($F_1 \notin BCCF_{PS}$ *or* $F_2 \notin BCCF_{PS}$) ,
   $N_{CCL}(F_1 \wedge F_2) = N_{CCL}(N_{CCL}(F_1) \wedge N_{CCL}(F_2))$

   (c) $\forall F_1, F_2 \in GCCF_{PS}$ *and* ($F_1 \notin BCCF_{PS}$ *or* $F_2 \notin BCCF_{PS}$) ,
   $N_{CCL}(F_1 \vee F_2) = N_{CCL}(N_{CCL}(F_1) \vee N_{CCL}(F_2))$

   (d) $\forall F_1, F_2 \in GCCF_{PS}$ *and* ($F_1 \notin BCCF_{PS}$ *or* $F_2 \notin BCCF_{PS}$) ,
   $N_{CCL}(F_1 \vec{\odot} F_2) = N_{CCL}(N_{CCL}(F_1) \vec{\odot} N_{CCL}(F_2))$

3. *Normal form of negated, conjunction and disjunction of basic conjunctive choice formulas are : Let $F_1 = a_1 \vec{\odot} \dots \vec{\odot} a_n$ and $F_2 = b_1 \vec{\odot} \dots \vec{\odot} b_m$ be two formulas $\in BCCF_{PS}$ such that a_i's and b_i's are propositional formulas.*

   (a) $N_{CCL}((a_1 \vec{\odot} \dots \vec{\odot} a_n) \wedge (b_1 \vec{\odot} \dots \vec{\odot} b_m)) = c_1 \vec{\odot} \dots \vec{\odot} c_k$
   where k = max (m, n) and :

   i. *If* $m = n$: $c_i = a_i \wedge b_i$

   ii. *If* $m < n$:

   $$c_i = \begin{cases} a_i \wedge b_1 & \textit{iff} \quad i \leq n-m \\ a_i \wedge b_{i-n+m} & \textit{iff} \quad i > n-m \end{cases}$$

   iii. *If* $m > n$:

   $$c_i = \begin{cases} a_1 \wedge b_i & \textit{iff} \quad i \leq m-n \\ a_{i-m+n} \wedge b_i & \textit{iff} \quad i > m-n \end{cases}$$

   (b) $N_{CCL}((a_1 \vec{\odot} \dots \vec{\odot} a_n) \vee (b_1 \vec{\odot} \dots \vec{\odot} b_m)) = c_1 \vec{\odot} \dots \vec{\odot} c_k$
   where k = max (m, n) and :

   i. *If* $m = n$: $c_i = ((a_1 \wedge \dots \wedge a_i) \vee b_i) \wedge (a_i \vee (b_1 \wedge \dots \wedge b_i))$

   ii. *If* $m < n$:

   $$c_i = \begin{cases} ((a_1 \wedge \dots \wedge a_i) \vee b_1) \wedge (a_i \vee b_1) & \textit{iff} \\ \qquad i \leq n-m & \\ ((a_1 \wedge \dots \wedge a_i) \vee b_{i-n+m}) \wedge & \\ (a_i \vee (b_1 \wedge \dots \wedge b_{i-n+m})) & \textit{iff} \\ \qquad i > n-m & \end{cases}$$

   iii. *If* $m > n$:

   $$c_i = \begin{cases} (a_1 \vee (b_1 \wedge \dots \wedge b_i)) \wedge (a_1 \vee b_i) & \textit{iff} \\ \qquad i \leq m-n & \\ (a_{i-m+n} \vee (b_1 \wedge \dots \wedge b_i)) \wedge & \\ ((a_1 \wedge \dots \wedge a_{i-m+n}) \vee b_i) & \textit{iff} \\ \qquad i > m-n & \end{cases}$$

   (c) $N_{CCL}(\neg(a_1 \vec{\odot} \dots \vec{\odot} a_n)) = \neg a_1$

Repeated application of this definition rules moves $\vec{\odot}$ outside (or eliminates it) until we obtain a basic conjunctive choice formula.

Property (1) of definition 8 says that the normal form of a basic conjunctive choice formula F_1, is the formula F_1.

Property $2(a, b, c, d)$ expresses that the normal form function is decomposable with respect to negation, conjunction, disjunction and ordered conjunction.

Property $3(a, b, c)$ gives the definition of conjunction, disjunction and negation applied to basic conjunctive choice formulas.

Proposition 14 *Let F_1 be a formula in $GCCF_{PS}$ and $N_{CCL}(F_1)$ be its normal form using Property 3 of Definition 8. Let I be an interpretation. Then :*

$$I \mid\!\sim_k^{CCL} F_1 \text{ iff } I \models_k N_{CCL}(F_1)$$

Where $\mid\!\sim_k^{CCL}$ is given by Definition 5 and $\models_k$ is given by Definition 2.

Example 15 *Let us compute the normal form of the query Q_2 given in example 3 using Definition 8. The query Q_2 is a disjunction of two basic choice formulas. For simplicity of writing, we will consider the following general form of Q_2 : $(a_1 \vec{\odot} a_2 \vec{\odot} a_3) \vee (b_1 \vec{\odot} b_2 \vec{\odot} b_3)$ where : $a_1 = (DA = "L" \wedge AA = "Q" \wedge DD = "30/08/2015")$, $a_2 = (CO = "AF")$, $a_3 = (CL = "E")$, $b_1 = (DA = "P" \wedge AA = "Q" \wedge DD = "31/08/2015")$, $b_2 = (CO = "AC")$, $b_3 = (CL = "F")$.*

We first apply item $3-(b)$ of Definition 8 : $N_{CCL}((a_1 \vec{\odot} a_2 \vec{\odot} a_3) \vee (b_1 \vec{\odot} b_2 \vec{\odot} b_3)) = N_{CCL}((a_1 \vee b_1) \vec{\odot} (((a_1 \wedge a_2) \vee b_2) \wedge (a_2 \vee (b_1 \wedge b_2))) \vec{\odot} (((a_1 \wedge a_2 \wedge a_3) \vee b_3) \wedge (a_3 \vee (b_1 \wedge b_2 \wedge b_3))))$

Then we apply item $2-(d)$ of Definition 8 (decomposition with respect to ordered conjunction): $N_{CCL}(Q_2) = N_{CCL}(N_{CCL}(a_1 \vee b_1) \vec{\odot} N_{CCL}(((a_1 \wedge a_2) \vee b_2) \wedge (a_2 \vee (b_1 \wedge b_2))) \vec{\odot} N_{CCL}(((a_1 \wedge a_2 \wedge a_3) \vee b_3) \wedge (a_3 \vee (b_1 \wedge b_2 \wedge b_3))))$

Finally, we can apply item $1-(a)$ of Definition 8 to normalize the obtained result (normal form of propositional formulas): $N_{CCL}(Q_2) = (a_1 \vee b_1) \vec{\odot} ((a_1 \wedge a_2) \vee b_2) \wedge (a_2 \vee (b_1 \wedge b_2)) \vec{\odot} (((a_1 \wedge a_2 \wedge a_3) \vee b_3) \wedge (a_3 \vee (b_1 \wedge b_2 \wedge b_3)))$

Table 2 gives satisfaction degrees obtained after assigning truth values True(T), False(F), to the formulas composing the query $N_{CCL}(Q_2)$. Note that (*) represents all possible values. We can check that satisfaction degrees of $N_{CCL}(Q_2)$ are equal to those of Q_2 after applying the same interpretations.

Table 2: Example of Satisfaction degrees of normal form of $GCCF_{PS}$ formulas

$F_1 : a_1 \vee b_1$

$F_2 : ((a_1 \wedge a_2) \vee b_2) \wedge (a_2 \vee (b_1 \wedge b_2))$

$F_3 : (((a_1 \wedge a_2 \wedge a_3) \vee b_3) \wedge (a_3 \vee (b_1 \wedge b_2 \wedge b_3)))$

S.D. : Satisfaction Degree

a_1	a_2	a_3	b_1	b_2	b_3	F_1	F_2	F_3	S. D. Q_2	S. D. N_{CCL} (Q_2)
F	*	*	F	*	*	∞	*	*	∞	∞
F	*	*	T	F	*	T	F	*	3	3
F	*	*	T	T	F	T	T	F	2	2
F	*	*	T	T	T	T	T	T	1	1
T	F	*	F	*	*	T	F	*	3	3
T	F	*	T	F	*	T	F	*	3	3
T	F	*	T	T	F	T	T	F	2	2
T	F	*	T	T	T	T	T	T	1	1
T	T	F	F	*	*	T	T	F	2	2
T	T	F	T	F	*	T	T	F	2	2
T	T	F	T	T	F	T	T	F	2	2
T	T	F	T	T	T	T	T	T	1	1
T	T	T	F	*	*	T	T	T	1	1
T	T	T	T	F	*	T	T	T	1	1
T	T	T	T	T	F	T	T	T	1	1
T	T	T	T	T	T	T	T	T	1	1

Related works and concluding discussions

In this paper, we provided a new non-classical logic for handling user's basic and complex preferences in presence of constraints. Our logic CCL shares some features of the so-called Qualitative Choice Logic (QCL) proposed in (Brewka, Benferhat, and Berre 2004). However, the difference between CCL and QCL is somewhat similar to the difference between propositional conjunction and propositional disjunction. CCL adopts a conjunctive interpretation of preferences while QCL follows a disjunctive interpretation of preferences. Hence, in a presence of "A is preferred to B" with A and B are propositional symbols, then CCL and QCL will induce two different rankings over the set of interpretations as showed in table 3.

Table 3: Comparison between CCL and QCL rankings

A	B	CCL ranking	QCL ranking
F	F	∞	∞
F	T	∞	2
T	F	2	1
T	T	1	1

CCL views A (the first option) as an integrity constraint that should be satisfied, while CCL only requires that one of the options should be satisfied (otherwise the solution is unacceptable).

Our logic also differs from graphical representation of preferences such as CP-nets (Boutilier 1992; Boutilier et al. 2004). In addition to the fact that our logic does not assume the ceteris paribus assumption (contrarily to CP-nets), our logic is not restricted to contextual preferences, where for each node one has to provide a preference over this node in the contexts of its parent. In our logic, general preferences can be easily expressed.

Lastly, a common point between possibilistic logic (Dubois and Prade 2004a) and CCL is that both of them induces a total ordering over interpretations based on falsified formulas or preference options. Possibilistic logic uses certainty degrees (a positive real number of $[0, 1]$) associated with a formula. Contrarily to our logic, nested preferences cannot be directly expressed in possibilistic logic.

In some situations, user's preferences may be in conflict with some organizational constraints. Thus, future work consists in providing operators enabling the fusion of these conflicting preferences and constraints (Papini 2010). These fusion operators can take advantage of the previous works on possibilistic logic to handle inconsistent parts of information (Dubois and Prade 2004a; 2004b; Schockaert and Prade 2011).

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