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Abstract

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The contributor was a member of a research group that developed a new line of research, called *Formal Concept Analysis*, in the 1980s. To our own surprise, this field is still active, though it never became mainstream. Publications occur on an almost daily basis, there are international conferences and productive research groups.

The core idea of Formal Concept Analysis is a formalization of "concept" and of "concept hierarchies". Initially however, it was not at all clear to us what "formalization" meant. Understanding this became an important, time consuming part of our development, but perhaps also an ingredient of its success.

The motivation for introducing Formal Concept Analysis did not come from the quest for specific applications. In the late 1970s, many young mathematicians (in Germany and perhaps beyond) became interested in the meaning and significance of their scientific work. Working groups on topics such as modeling or mathematization were set up in many universities, and these provided impetus, some of which continues to have an impact today. At the University of Darmstadt, Rudolf Wille initiated a seminar which, over months and quite emotional, discussed and investigated the meanings of mathematical order theory. Ordered sets and in particular lattices were a central topic of Wille's mathematical research.

A distinctive feature of this group was that Wille was philosophically trained and brought this into the discussions. Very gradually, this led to an understanding of how meaning arises in mathematics. We learnt to understand individual applications as instances of more general principles and to reveal those. And then, one approach that immediately struck us was the description of concept hierarchies by lattices (i.e., algebraic structures with join and meet). As a result of the discussions, Wille had derived a mathematical definition for "concept" that was mathematically convincing because it was both elegant and effective. The group immediately picked up on this, producing examples, algorithms, computer

programs that allowed them to delve deeper and deeper into the field. We even started to understand the mathematical properties of lattices better because we had additional intuition and visualization.

It was Rudolf Wille again who made sure that our enthusiasm, which was mainly mathematically motivated in the beginning, slowly became more realistic. He made sure that we regularly dealt with other views. In his (daily) research seminar, psychologists, linguists and philosophers now presented their understanding of "concepts". It became clearer to us that this formalization was not a psychological or biological model of human conceptualization. Indeed, that never was our intention. A formal system had been found which was particularly useful because it was formulated in terms of human thought. It is mathematical enough to build a substantial theory on it, with many nontrivial mathematical theorems, algorithms, and connections to other mathematical subfields. At the same time, it is close enough to our thinking to be intuitive.

Our way of speaking has become more cautious. In the beginning, we spoke of "concepts" and "context" to refer to mathematically defined things. Today it is standard to speak of "formal concepts" and "formal contexts". This emphasizes what is meant: not a modeling, but mathematical language suitable for descriptions. This approach develops an amazing power. The mathematical apparatus is now much more extensive than anything previously used in applications. And therein lies the strength: a formalization sophisticated in this way also allows actual applications to be worked out much more flexibly.