

# A flow-based formulation for parallel machine scheduling problems using decision diagrams.

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We present a new flow-based formulation for identical parallel machine scheduling with a regular objective function and no idle time. We use a decision diagram containing all the possible sequences of jobs that follow some ordering rules to construct the new formulation. These rules do not exclude the optimal solution of a given instance but constrain the optimal solution to some canonical form. To define these rules, we need to partition the planning horizon into non-uniform periods. The new formulation will have numerous variables and constraints, and hence we apply a Dantzig-Wolfe decomposition to compute the linear programming relaxation of the new flow-based formulation in a reasonable amount of time. Moreover, we will see that the lower bound will be stronger than the lower bound that one can calculate with the classical time-indexed formulation. We use a Branch-and-Price framework to solve the new formulation. Several instances from the literature will be solved for the first time.

*Key words:* parallel machine scheduling, weighted tardiness, column generation, decision diagrams

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