Research paper

Design of a Discrete-time Sliding Mode Controller for Nonlinear Affine Systems based on Disturbance Estimation

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Article Info	Extended Abstract
	Background and Objectives: Nowadays, because of accuracy, speed, cost, and flexibility of digital control laws, control systems are implemented by computers, microprocessors or DSP chips. Therefore, many investigators have recently focused on the design of discrete-time controllers and computer-based control. **Methods:** In this paper, a sliding mode controller based on the disturbance estimation is designed for a class of discrete-time nonlinear affine systems. Based on two disturbance compensator schemes, static and dynamic, procedures of sliding mode controller design are proposed for the discrete-time system.
	Results: In the case of measurable state variables, the instantaneous value of disturbances can be estimated based on the value of states and control signals. In two proposed control laws, there is no switching expression to induce the problem of chattering. Moreover, based on the necessary and sufficient quasi-sliding mode condition proposed by Sarpturk, boundedness and robustness of the proposed controllers is evaluated. In the case of constant or slowly time-varying disturbances, the quasi-sliding mode band converges asymptotically to zero and in this case, the proposed method is converted to the ideal sliding mode. Finally, two examples are provided to verify the proposed control laws and to compare the performance of the proposed controllers.
	Conclusion: In this paper, a sliding mode controller based on the disturbance estimator was designed for a discrete-time nonlinear affine system. Due to the effectiveness of disturbance estimators in the performance of controllers, two kinds of disturbance estimators were considered.

Introduction

Nowadays, because of accuracy, speed, cost, and flexibility of digital control laws, control systems are implemented by computers, microprocessors or DSP chips [1]. Therefore, many investigators have recently focused on the design of discrete-time controllers and computer-based control. In general, the design of discrete-time control systems can be done in two approaches, namely, the direct discrete-time design and the emulation-based approach [2]. The direct discrete-time design is done directly in the discrete-time domain,

using a discrete-time model of the plant. In the emulation-based approach, the controller design is done in the continuous-time domain. Then, by using a discretization method, the control law will be digitalized to produce a discrete-time control law for digital implementation. However, this approach may result in an unstable system after digitization such as zero-order hold method [3].

The sliding mode control method is very famous among variable structure control systems. Although

designing of the discrete-time sliding-mode control is different from that of a continuous-time case, it clearly has the desired performance similar to the continuous-time case in the presence of disturbances. Unfortunately, the system states can only be approached to the sliding surface and remain around it. This band is well-known in the literature as the quasi-sliding-mode band. The first work of this control method was introduced in [4] which the emulation-based approach was considered. The design of sliding mode controllers for discrete-time systems is a topic of extensive research in various control applications, such as induction motors [5][6], Computer Numeric Control (CNC) servomechanism [7], piezoelectric actuators [8], heavy water reactor [9], wheeled mobile robot trajectory-tracking [10], boost converter [11], Voltage Source Converter based High Voltage Direct Current (VSC-HVDC) systems [12], pumping system [13], automotive electronic throttle body [14], and cart-inverted pendulum [15].

There are two approaches that can be found in the literature for the design of discrete-time sliding mode controllers [16]. One of the approaches in this regard is to assume a specified control and to prove the stability this proposed control law [17][18]. In the second one that also called reaching law approach, a desirable sliding surface is considered and then the control law will be determined. The reaching law approach has been introduced in [19] in which dynamics of the sliding surface are considered directly and the control law has nonlinear and linear parts. Moreover, other researchers widely developed this reaching law approach [20][21] [22] [23] [24]. Unfortunately, the chattering phenomenon always exists in this approach. To solve this problem, a multi-power reaching law has been introduced in [25] which has replaced the discontinuous function by a power term of the switching function. Moreover, Bartoszewicz has introduced a non-stationary sliding surface to reduce this problem [26].

One of the basic conditions in the discrete Lyapunov stability method has been proposed by Sarpturk [27]. It has been widely used for the stability of sliding mode controllers in discrete-time cases and to show the existence of a quasi-sliding motion. Furthermore, Furuta's sector control approach provided the quasi-sliding mode existence conditions by another discrete Lyapunov function [28]. Spurgeon [29] proposed a method of hyperplane design with disturbance rejection capability based on the discrete Lyapunov method. Note that due to the sampling process in the discrete-time case, a complete rejection of disturbances is not possible. In these cases, using a disturbance estimator can improve the robustness of the system [30].

The control methods based on a disturbance estimator are very effective in compensating disturbances [31] [32] [33]. This scheme of control has been studied in the past several decades and has been utilized in many practical applications [34][35][36]. Disturbance estimators may be used with other controllers to overcome disturbances. In the case of measurable state variables, the instantaneous value of disturbances can be estimated based on the value of states and control signals. Unfortunately, to the best of our knowledge, few results are available to design discrete-time sliding mode controllers based on the disturbance estimation. For instance, [37][38] have proposed an anti-disturbance control method for a class of Multi-Input Multi-Output (MIMO) linear systems with some nonlinear terms.

In [39], an output feedback- based sliding mode controller has been proposed for a class of linear systems in the presence of matched disturbances. However, these control schemes are only applicable to linear systems.

In this paper, the design procedure of a sliding mode controller based on the disturbance estimation for discrete-time nonlinear affine systems is considered. The main objective of this paper is to derive a control law to guarantee the quasi-sliding mode condition of Sarpturk. The proposed control law consists of a sliding mode controller and a disturbance estimator to compensate disturbances for discrete-time systems.

The contributions of the present paper mainly lie in two aspects. First, the direct discrete-time design is considered using two disturbance compensator schemes, static and dynamic types for a nonlinear affine system. Moreover, the necessary and sufficient condition of quasi-sliding mode method is investigated to evaluate the boundedness and robustness of proposed controllers. It is proved that the presented control scheme provides the robustness against external disturbance.

In this regard, the boundedness of disturbance estimation error and the sliding variable is assured. Moreover, in the case of constant or slowly time-varying disturbances, the quasi-sliding mode band converges asymptotically to zero and in this case, the proposed method is converted to the ideal sliding mode controller.

Preliminaries

The following model shows the class of nonlinear discrete-time systems considered in this paper:

$$
\begin{aligned}
x_1(k+1) &= f_1(x_1(k), x_2(k)) \\
x_2(k+1) &= f_2(x_1(k), x_2(k)) \\
&\quad + G(x_1(k), x_2(k))u(k) \\
&\quad + d(k)
\end{aligned}
\tag{1}
$$

where $x_1(k) \in \mathbb{R}^n$ and $x_2(k) \in \mathbb{R}$ are the state vectors, $u(k) \in \mathbb{R}$ denotes the system input, $d(k)$ represents the

disturbances, f_1, f_2, and G are differentiable functions where $f_1(0,0) = 0$ and $f_2(0,0) = 0$. The main objective is to find a discrete-time controller based on the sliding mode method to converge the states of the system (1) to zero in the presence of disturbances. In this paper, the following assumptions are considered:

Assumption 1: All state variables are available.

Assumption 2: The matrix Function $G(.)$ is non-singular for all values of state variables.

Assumption 3: There exists a function $x_2(k) = \phi(x_1(k))$, with the property $\phi(0) = 0$, such that the reduced order system with the dynamic $x_1(k + 1) = f_1(x_1(k), \phi(x_1(k)))$ is asymptotically stable at the origin.

Assumption 4: The changing rate of the disturbance $d(k)$ is considered bounded as,

$$|d(k) - d(k - 1)| \leq \eta, \quad \forall k > 0.$$

Discrete-time controllers, which are based on the sliding mode method, have an inherent difference with the continuous-time case. In these systems, only quasi-sliding modes will appear. In other words, the state variables approach the sliding surface but cannot stay on it.

In order to prepare for the main result, we consider the following definition. This definition clearly shows the quasi-sliding mode.

Definition 1 [26]**:** The quasi-sliding mode is the motion of the state variables in a band around the sliding surface $s(k) = 0$, with a predefined width ε (the width of the quasi sliding mode band). Mathematically, we have $|s(k)| \leq \varepsilon$, where ε is a positive constant. Therefore, the states of the system remain always in a small band.

Results and Discussion

The design procedure of the proposed controller is composed of two parts; First, design a sliding surface which represents a desired stable dynamics and performance of the plant.

Then, a state-feedback control law, $u(k)$, is designed to guarantee the finite-time reaching of the state variables to the sliding surface and remaining on it.

First, let us define a sliding surface according to Assumption 3 as follows:

$$s(k) = x_2(k) - \phi(x_1(k)) \tag{2}$$

Clearly, the origin of the dynamics of the system on the surface according to assumption 3 is asymptotically stable.

Our purpose is to reach the sliding surface from any arbitrary state $x(0)$ in a finite number of sampling-time steps. In an ideal sliding motion, a discrete-time equivalent control can be proposed using the equality $s(k + 1) = s(k) = 0, \forall k > k_s$, where $k_s > 0$ denotes the sampling instant in which the sliding motion starts [40][41].

If the following control law is used, the state variables reach the sliding surface in one sampling period (one-step reaching):

$$\begin{aligned} s(k + 1) &= x_2(k + 1) - \phi(x_1(k + 1)) \\ &= f_2(x_1(k), x_2(k)) \\ &\quad + G(x_1(k), x_2(k))u(k) \\ &\quad + d(k) - \phi(f_1(x_1(k), x_2(k))) \end{aligned} \tag{3}$$

By solving $s(k + 1) = 0$, the so-called equivalent control law can be obtained as,

$$\begin{aligned} u(k) = G^{-1}(x_1(k), x_2(k))[\phi(f_1(x_1(k), x_2(k))) \\ - f_2(x_1(k), x_2(k)) - d(k)] \end{aligned} \tag{4}$$

Since the disturbance $d(k)$ is unknown, the control law (4) cannot be actually implemented. In real systems, measuring the disturbance, $d(k)$, is impossible.

Therefore, the value of $d(k)$ in (4) should be replaced with its estimation which is called $\hat{d}(k)$. To do this, two disturbance estimator schemes are utilized to estimate unknown disturbance.

From the discrete-time model of (1) and Assumption 1, disturbance $d(k)$ can be predicted by its previous value. Figure 1 shows the situation of the proposed method in the control field:
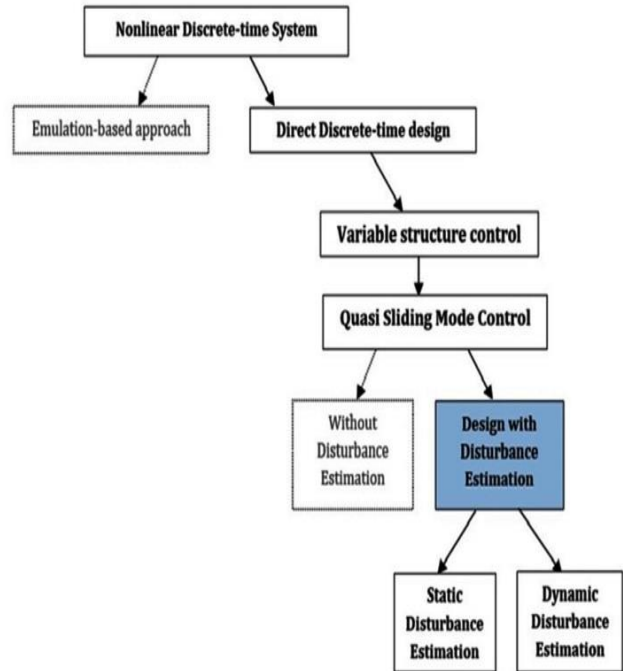

Fig. 1: The situation of the proposed method.

A. Design Based on a Static Disturbance Estimator

Now, a discrete-time disturbance estimator by the one-step delay is proposed [41]. The disturbance can be predicted as,

$$\hat{d}(k) = d(k-1)$$
$$= x_2(k) - f_2(x_1(k-1), x_2(k-1))$$
$$-G(x_1(k-1), x_2(k-1))u(k-1) \tag{5}$$

where $\hat{d}(k)$ is the disturbance estimation of $d(k)$. Convergent analysis of disturbance estimator can be investigated by the following expression. Substituting the control law (4) into (3) leads to

$$s(k+1) = f_2(x_1(k), x_2(k))$$
$$+ \phi(f_1(x_1(k), x_2(k)))$$
$$- f_2(x_1(k), x_2(k)) - \hat{d}(k)$$
$$+ d(k)$$
$$- \phi(f_1(x_1(k), x_2(k))) \tag{6}$$
$$= d(k) - \hat{d}(k)$$

Using disturbance estimator (5), we have

$$s(k+1) = d(k) - d(k-1) \tag{7}$$

It is clear from Assumption 4 and (7) that the quasi-sliding mode band can be achieved

$$|s(k+1)| = |d(k) - d(k-1)| \le \eta \tag{8}$$

Thus, the system states will enter a quasi-sliding mode band around the sliding surface and remain in it. The constant η is the width of that band.

For the stability of the closed-loop discrete-time system and to have a desired quasi-sliding mode motion, it is necessary to force the state variables to this band, from any initial condition $x(0)$, and to steer the state in the ε-vicinity of $s(k)$, regardless of the action of any bounded disturbance.

In what follows, the necessary and sufficient condition is considered to evaluate the boundedness and robustness of the closed-loop discrete-time system.

Lemma 1 [27]**:** A necessary and sufficient condition to guarantee the sliding motion and convergence of a discrete closed-loop control system is as,

$$|s(k+1)| < |s(k)| \tag{9}$$

This is equivalent to the following inequalities:

$$|s(k+1) - s(k)|sgn(s(k)) < 0$$
$$|s(k+1) + s(k)|sgn(s(k)) \ge 0 \tag{10}$$

where $sgn()$ is the sign function. The first inequality is the necessary sliding mode existence condition and the second one gives sufficient condition for the convergence of the quasi-sliding mode.

Now, we will demonstrate that the proposed control law ensures the convergence of $s(k)$ to the ε-band of the sliding surface.

Theorem 1: Consider the discrete-time nonlinear system (1) with the control law (4), the sliding surface (2) and the disturbance estimator (5). Then, the quasi-sliding

mode condition (9) or its equivalent conditions (10) are satisfied outside the following region which is named as Ω_B:

$$\Omega_B = \{x(k) : |s(k)| \le \eta\} \tag{11}$$

Proof: The proof will be presented into two parts:
Part I: This part is related to the sliding condition. According to (7), one has

$$s(k+1) - s(k) = [d(k) - d(k-1)] - s(k) \tag{12}$$

Post-multiplying (12) by $sgn(s(k))$

$$[s(k+1) - s(k)]sgn(s(k))$$
$$= [d(k)$$
$$- d(k-1)]sgn(s(k))$$
$$- |s(k)| \tag{13}$$

Now, we suppose that $|s(k)| > \eta$ is satisfied. equation (13) may be rewritten as

$$[s(k+1) - s(k)]sgn(s(k))$$
$$< |d(k) - d(k-1)|$$
$$- |s(k)| < -(|s(k)| - \eta) \tag{14}$$
$$< 0$$

Thus, we can conclude that if $|s(k)| > \eta$ then $[s(k+1) - s(k)]sgn(s(k)) < 0$, which guarantees the sliding condition.

Part II: This part is related to the convergence condition. From (7), the term $s(k+1) + s(k)$ obeys from the following relation:

$$s(k+1) + s(k) = [d(k) - d(k-1)] + s(k) \tag{15}$$

Post-multiplying (15) by $sgn(s(k))$

$$[s(k+1) + s(k)]sgn(s(k))$$
$$= [d(k) - d(k$$
$$- 1)]sgn(s(k)) + |s(k)| \tag{16}$$

Similarly, we suppose that $|s(k)| > \eta$ is satisfied and thus, (16) can be rewritten as

$$[s(k+1) + s(k)]sgn(s(k))$$
$$\ge -|d(k) - d(k-1)| + |s(k)| \tag{17}$$
$$\ge (|s(k)| - \eta) \ge 0$$

Thus, we can conclude that if $|s(k)| > \eta$, then $[s(k+1) + s(k)]sgn(s(k)) \ge 0$, which guarantees the convergence condition.

According to these parts and Lemma 1, if $|s(k)| > \eta$, it concludes that $|s(k+1)| < |s(k)|$, which means that $s(k)$ is decreasing outside the region Ω_B. Consequently, the system trajectories will approach the sliding surface in a finite number of sampling-time steps. The proof is complete. ∎

Remark 1: The proposed control law (4) when $d(k)$ has been replaced with its estimation, $\hat{d}(k)$ presented in (5), contains no switching expression. Therefore, the chattering phenomenon will not appear in the closed-loop system. Moreover, in the proposed control law,

knowing the upper bound of the disturbance $d(k)$ is not necessary. These benefits lead to the wide applicability of the proposed controller.

Remark 2: As can be seen from the above result, when the disturbance is constant or slowly time-varying, disturbance estimator can yield to the exact estimation. Therefore, the width of the mentioned band (quasi-sliding mode band) converges asymptotically to zero. Thus, in this case, $s(k)$ goes asymptotically to zero and we have an ideal sliding mode.

B. Design Based on a Dynamic Disturbance Estimation

In this section, a dynamic disturbance estimator is proposed. Because of the lack of knowledge of disturbance, the preceding equivalent control (4) cannot be actually implemented. Therefore, in the following, we propose a dynamic disturbance estimator inspired by the method in [42].

The disturbance estimator is suggested as

$$
\begin{aligned}
\hat{d}(k) =& \ px_2(k) - z(k) \\
z(k+1) =& \ z(k) + p[f_2(x_1(k), x_2(k)) \\
&+ G(x_1(k), x_2(k))u(k) \\
&+ \hat{d}(k) - x_2(k)]
\end{aligned}
\tag{18}
$$

where $z(k) \in \mathbb{R}$ is a new state and $0 < p < 1$ is an arbitrary positive constant.

Remark 3 [42]: When the initial value of the disturbance is known as $\hat{d}(0)$, it is possible to choose the initial value of the newly defined state variable as $z(0) = px(0) - \hat{d}(0)$. In the case of unknown initial values of the disturbance, only set $z(0) = px(0)$, which is equivalent to considering $\hat{d}(0) = 0$. These activities reduce the undesirable transient response of the proposed observer. Now, convergent analysis of dynamic disturbance estimator can be investigated by the following expression. Substituting the control law (4) into (3) leads to

$$
\begin{aligned}
s(k+1) =& \ f_2(x_1(k), x_2(k)) \\
&+ \phi(f_1(x_1(k), x_2(k))) \\
&- f_2(x_1(k), x_2(k)) - \hat{d}(k) \\
&+ d(k) \\
&- \phi(f_1(x_1(k), x_2(k))) \\
=& \ d(k) - \hat{d}(k)
\end{aligned}
\tag{19}
$$

Using disturbance estimator in equation (18), we have

$$
\begin{aligned}
s(k+1) =& \ d(k) - px_2(k) + z(k) \\
=& \ d(k) - p[f_2(x_1(k-1), x_2(k-1)) \\
&+ G(x_1(k-1), x_2(k-1))u(k-1) \\
&+ d(k-1)] + z(k-1) \\
&+ p[f_2(x_1(k-1), x_2(k-1)) \\
&+ G(x_1(k-1), x_2(k-1))u(k-1) \\
&+ \hat{d}(k-1) - x_2(k-1)] \\
=& \ d(k) - p[d(k-1) - \hat{d}(k-1)] + z(k-1) \\
&- Kx_2(k-1) \\
=& \ d(k) - p[d(k-1) - \hat{d}(k-1)] - \hat{d}(k-1)
\end{aligned}
\tag{20}
$$

Let us add and subtract $d(k-1)$ to it as

$$
\begin{aligned}
s(k+1) =& \ d(k) - d(k-1) - p[d(k-1) \\
&- \hat{d}(k-1)] + d(k-1) \\
&- \hat{d}(k-1)
\end{aligned}
\tag{21}
$$

Using (19), we have,

$$
s(k+1) = (1-p)s(k) + (d(k) - d(k-1))
\tag{22}
$$

It is clear from Assumption 4 and equation (22) that if $0 < p < 1$, the quasi-sliding mode band is as,

$$
\begin{aligned}
|s(k+1)| \le& \ (1-p)^{k+1}|s(0)| + \eta \sum_{n=0}^{k} (1-p)^n \\
\le& \ (1-p)^{k+1}\{|s(0)| - \frac{\eta}{p}\} + \frac{\eta}{p}
\end{aligned}
\tag{23}
$$

Consequently, the state variables will enter the sliding band and will not leave it. Now, similar to the procedures in the previous section, we will demonstrate that the proposed control law with disturbance estimator (18) ensures the convergence of $s(k)$ to the ε-band of the sliding surface.

Theorem 2: Consider the discrete-time nonlinear system (1) with the control law (4), the sliding surface (2) and the disturbance estimator (18). Then, the quasi-sliding mode condition (9) or its equivalent conditions (10) are satisfied outside the following region:

$$
\Omega_B = \left\{ x(k) : \ |s(k)| \le \frac{\eta}{p} \right\}
\tag{24}
$$

Proof: Similar to the procedures in the previous theorem, the proof will be presented into two parts:

Part I: This part is related to the sliding condition. according to (22), one has

$$
s(k+1) - s(k) = [d(k) - d(k-1)] - ps(k)
\tag{25}
$$

Post-multiplying (25) by $sgn(s(k))$

$$
\begin{aligned}
[s(k+1) - s(k)]sgn(s(k)) \\
= [d(k) - d(k-1)]sgn(s(k)) \\
- p|s(k)|
\end{aligned}
\tag{26}
$$

Now, we suppose that $|s(k)| > \eta/p$ is satisfied. Equation (26) may be rewritten as

$$
\begin{aligned}
[s(k+1) - s(k)]sgn(s(k)) \\
< |d(k) - d(k-1)| - p|s(k)| \\
< -(p|s(k)| - \eta) < 0
\end{aligned}
\tag{27}
$$

Thus, we can conclude that if $|s(k)| > \eta/p$ then $[s(k+1) - s(k)]sgn(s(k)) < 0$, which guarantees the sliding condition.

Part II: This part is related to the convergence condition. The term $s(k+1) + s(k)$ obeys from the following relation:

$$
\begin{aligned}
s(k+1) + s(k) \\
= [d(k) - d(k-1)] + (2-p)s(k)
\end{aligned}
\tag{28}
$$

Post-multiplying (28) by $sgn(s(k))$

$$[s(k+1) + s(k)]sgn(s(k))$$
$$= [d(k) - d(k-1)]sgn(s(k)) \quad (29)$$
$$+ (2-p)|s(k)|$$

Similarly, we suppose that $|s(k)| > \eta/(2-p)$ is satisfied. (29) can be expressed as

$$[s(k+1) + s(k)]sgn(s(k))$$
$$\geq -|d(k) - d(k-1)| + |s(k)| \quad (30)$$
$$\geq ((2-p)|s(k)| - \eta) \geq 0$$

Thus, we can conclude that if $|s(k)| > \eta/(2-p)$ then $[s(k+1) + s(k)]sgn(s(k)) \geq 0$, which guarantees the convergence condition.

According to these parts, Lemma 1 and the equality $max_{0<P<1} \left\{ \frac{\eta}{p}, \frac{\eta}{2-p} \right\} = \frac{\eta}{p}$, it concludes $|s(k+1)| < |s(k)|$, which indicates that $s(k)$ is decreasing outside Ω_B and consequently, the system trajectories will approach the sliding surface in a finite number of sampling-time steps. The proof is complete. ∎

Remark 4: Note that the p variable indicates the convergent rate of observer internal state variable of disturbance estimator. This variable affects the rate of convergence and the width of the quasi sliding mode band.

C. Numerical Example

In this section, to illustrate the performance of the proposed controllers two simulations are presented.
Example 1: Consider the following discrete-time nonlinear system:

$$x_1(k+1) = 2x_1(k) - x_2(k)$$
$$x_2(k+1) = \frac{x_1(k)x_2(k)}{1 + x_2^2(k)} + u(k) + d(k) \quad (31)$$

A time-varying disturbance is considered as $d(k) = sin(0.1\,k)$ to investigate the robustness of the system. Two proposed methods in the previous sections will be applied to this example.

The function ϕ given in (2) can be considered as $\phi(x_1(k)) = 1.5x_1(k)$. Clearly, the origin of the dynamic $x_1(k+1) = 0.5x_1(k)$ is asymptotically stable. Accordingly, the sliding surface is designed as,

$$s(k) = x_2(k) - 1.5x_1(k) \quad (32)$$

Therefore, the equivalent control law is as,

$$u(k) = 1.5\big(2x_1(k) - x_2(k)\big) - \frac{x_1(k)x_2(k)}{1 + x_2^2(k)} - \hat{d}(k)$$
$$(33)$$

where $\hat{d}(k)$ is the disturbance estimation of $d(k)$ can be predicted by (5) or (18). We consider $p = 0.9$ for observer internal state variable of disturbance estimator in (18).

Numerical simulations are given in Figs. 2-5. Figs. 2-4 show the state variables and control input variable for two proposed methods, respectively. It is observed from these figures that the proposed methods stabilize the system and attenuate the disturbance.

Furthermore, the sliding surface as shown in Fig. 5 is stable with small fluctuations. The quasi-sliding mode widths are $|s(k)| < 0.1$ for the proposed control law 1 and $|s(k)| < 0.11$ for the proposed control law 2, which agree with the theoretical results.
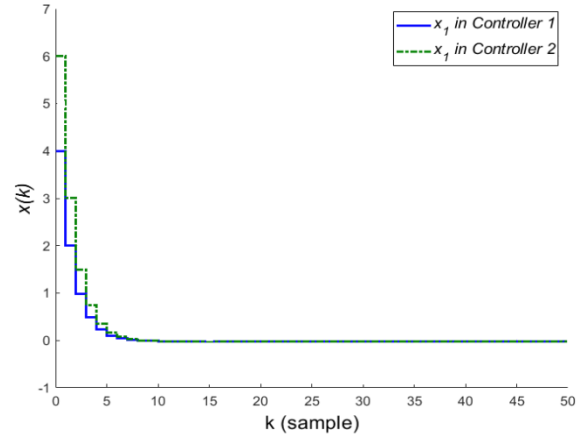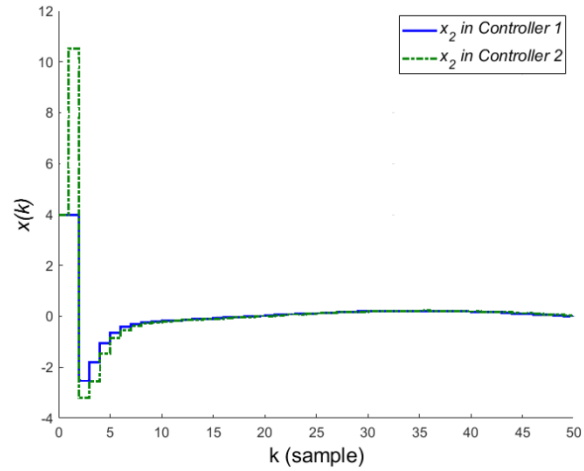

Fig. 2: State $x_1(k)$ in example 1
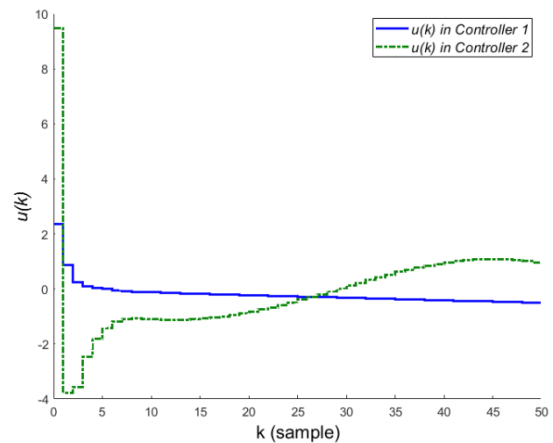

Fig. 3: State $x_2(k)$ in example 1

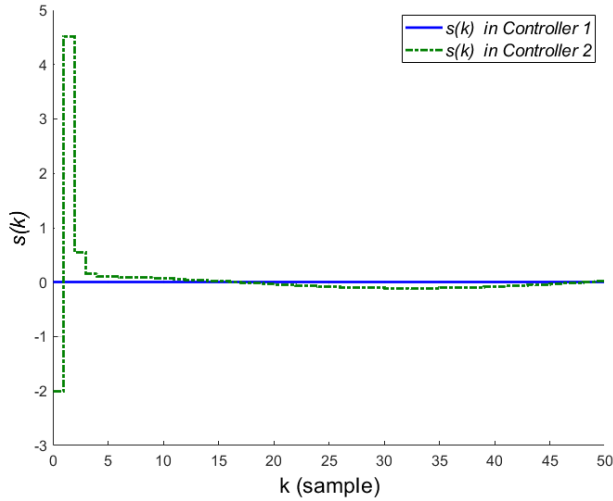Fig. 4: The control signal in example 1

Fig. 5: The sliding surface in example 1

Example 2: In this example, the proposed method is compared with the method of [43].
Consider a trailer-truck model [43]:

$$x_1(k+1) = x_1(k) - \frac{\eta T}{L}\sin(x_1(k)) + \frac{\eta T}{l}\tan(u(k))$$
$$+ d(k)$$

$$x_2(k+1) = x_2(k) + \frac{\eta T}{L}\sin(x_1(k))$$

$$x_3(k+1) = x_3(k)$$

$$+\eta T\cos(x_1(k))\sin\left(\frac{x_2(k)+x_2(k+1)}{2}\right)$$

(34)

where, $x_1(k)$ and $x_2(k)$ are respectively the angle between the axis of trailer and truck, and the angle of the trailer. Also, $x_3(k)$ denotes the vertical position of a definite point on the trailer.

The input variable (steering angle) is denoted by $u(k)$ and $d(k)$ is the model of disturbances. Moreover, L and l are the lengths of the trailer and the truck, respectively, and T denotes the sampling period, and η is a constant specifying the speed of the vehicle in a backward movement. By defining the state vector as $x_1'(k) = (x_2(k), x_3(k))^T$, $x_2'(k) = x_1(k)$ and $u'(k) = tan(u(k))$, system (34) is rewritten into the form of (1).

Backward movement control of trailer-truck has been used as a nonlinear benchmark control problem. Difficulties of this control problem are caused because of its nonlinearity and the jack-knife phenomenon. The task is the design of the control input variable to move the trailer-truck in the backward direction and along the horizontal line.

This means that for any initial condition, the trailer to be placed on the origin and also the alignments of the truck and its trailer are along the x-axis. In the simulation, parameters and initial conditions have been chosen as: $L = 0.13\,m, l = 0.087\,m, h = \frac{20.1m}{s}, T = 0.5\,s, x_1'(k) = (1.571, 1)^T$ and $x_2'(0) = 0$. Moreover, the function ϕ has been chosen as a linear function as $\phi(x_1'(k)) = c^T x_1'(k)$, where $c = (1.3325, 23.9)^T$. Finally, a time-varying disturbance as $d(k) = 0.1\sin(0.1\,k)$, has been considered, which is similar to [43].

The results of numerical simulations are illustrated in Figs. 6-8.

Here, Quasi-Sliding Mode Controller (QSMC) means the modified quasi-sliding mode controller, (6) in [43], and the Proposed Law is the equivalent control law obtained using (4) with disturbance estimator (5). Figs. 6, 7 and 8 show the system states variable, control input and the sliding surface variable, respectively.

Due to the effect of the disturbance, the states are oscillating. It is observed from these figures that the disturbance attenuation is improved in the presence of disturbances. Moreover, the proposed method has an improved transient response compared to the QSMC controller.

The performance of the controlled system in our method is $J = 13.76$ while for QSMC controller is $J = 14.12$ where the performance measured as $J = \sum_{k=0}^{100}|u(k)|$. Furthermore, the sliding surface is more stable with fewer fluctuations. The quasi-sliding mode width of the proposed method is $|s(k)| < 0.01$, which approve the theoretical results.

Conclusions

In this paper, a sliding mode controller based on the disturbance estimator was designed for a discrete-time nonlinear affine system. Due to the effectiveness of disturbance estimators in the performance of controllers, two kinds of disturbance estimators were considered.

The proposed control laws guaranteed the quasi-sliding mode condition which is one of the basic conditions in the stability of sliding mode controllers for discrete-time systems. The necessary and sufficient quasi-sliding mode conditions were derived for two disturbance estimators, and the width of the convergence band in quasi-sliding mode method was calculated.

It was shown that, when the disturbance was constant or slowly time-varying, the width of this band converges asymptotically to zero. Moreover, the control laws that suggested had no switching expression, which prevents the chattering phenomenon. Finally, two examples were presented to illustrate the effectiveness of the main result.
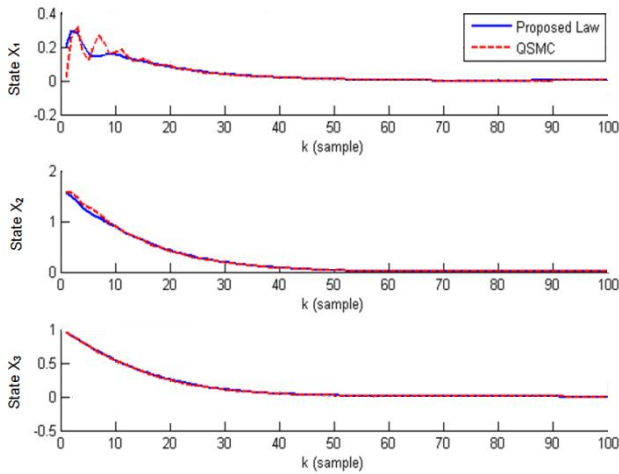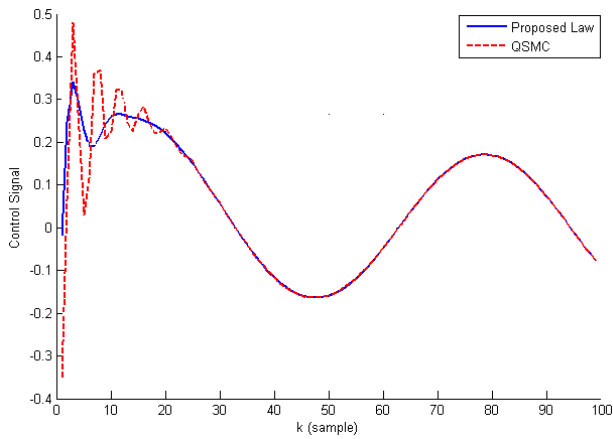
Fig. 6: States of the trailer-truck controller.

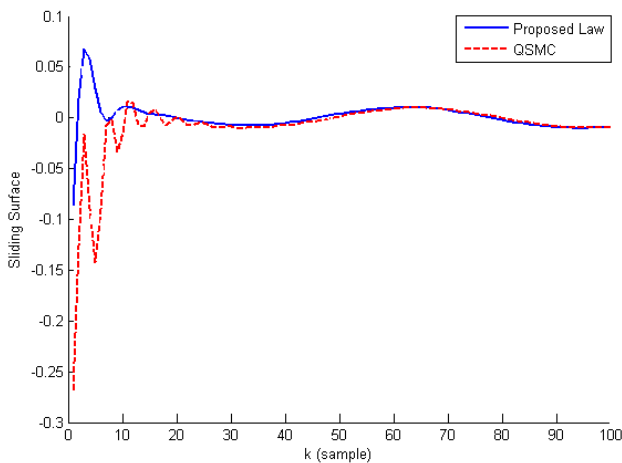Fig. 7: The control signal for the trailer-truck controller.

Fig. 8: Sliding surface for the trailer-truck controller.

Author Contributions

N. Azam Baleghi and M. H. Shafiei designed the experiments, collected the data and carried out the data analysis. They also interpreted the results and wrote the manuscript.

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Conflict of Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript.

Abbreviations

CNC	Computer Numeric Control
MIMO	Multi-Input Multi-Output
QSMC	Quasi-Sliding Mode Controller
VSC-HVDC	Voltage Source Converter based High Voltage Direct Current

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