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THE MEANING OF AVERAGE COMPRESSIVE AND TENSILE STRENGTH FOR HOEK-BROWN m_i CONSTANT DETERMINATION

One of the most widely used failure criteria for rocks in the world is the Hoek-Brown failure criterion. For its use, the m_i empirical parameter for a specific rock type is needed. The triaxial compression test is recommended for its determination; however, the full stress path for every rock comprises confined tension as well. This affects the course of the Hoek-Brown envelope, which is non-linear and starts at uniaxial tension. Fifty-one series of tests were carried out for three rock types: sandstone, claystone and limestone, to show the difference between the results of the m_i determination, using two different approaches – so-called linear and non-linear. Moreover, the consistency between the developed simplified methods of constant determination and m_i were checked. These comprised the UCS-based method, R-index method, TS-based method and advanced regression functions of compressive and tensile strength. The relationship between m_i constant and the internal friction angle was checked as well. The analysis of the results showed that the consistency with the regression models developed by researchers depends on the chosen estimator. If it is derived from the triaxial test only, the results are closer to a linear determination of m_i constant and have a good correlation with internal friction angle. If both tensile and compressive strength are used for its determination, the non-linear value correlates better with the advanced regression functions, but quite poor with the average compressive strength (R-index method) and tensile strength (TS-based method). Taking into account that every rock retained next to the geotechnical or mining object is not only compressed but also tensed, the non-linear m_i interpretation seems to be more correct. The interlayers and discontinuities inside sedimentary rocks increase the scatter of lab results and reduce the accuracy of m_i determination.

Keywords: Hoek-Brown failure criterion; rock strength; m_i constant; m_i constant interpretation; laboratory tests on rocks

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Introduction

One of the most widely used failure criteria for rocks in the world is the Hoek-Brown failure criterion [1,2]. It can be used for both intact and fractured rock. Its first form was described by Eq. (1). The researchers agree that the original criterion can be applied successfully for most rocks in which the rock mass strength is controlled by tightly interlocking angular rock pieces [3]. So, although the exponent at the brackets is now defined as "a", the form of Eq. (1) is still used.

$$\sigma_1 = \sigma_3 + \sigma_c \cdot \left(m_b \frac{\sigma_3}{\sigma_c} + s \right)^{0.5} = \sigma_3 + \sigma_c \sqrt{m_b \frac{\sigma_3}{\sigma_c} + s} \tag{1}$$

where, for intact rock [1] and fractured rock [2], m_b is accordingly:

$$m_b = m_i \cdot e^{\frac{GSI-100}{28}} \qquad m_b = m_i \cdot e^{\frac{GSI-100}{14}} \tag{2}$$

So, in both cases, the m_i empirical parameter for a specific rock type is needed, no matter what form of Hoek-Brown criterion is used – the original or the improved one. This parameter plays a significant role in rock engineering analyses and should be determined as best as possible. This paper shows the meaning of tensile strength for its determination.

The meaning of m_i is often underestimated. It is very often treated as the simple constant which has to be used in the Hoek-Brown failure criterion, but the real influence on the rock mass is not analysed. This is an incorrect approach because, for a specific rock, it can change in a very wide range, irrespective of the origin of the rock [4]. This research [4] proved that, for example, the range of m_i for sandstone is from 2 to 38 and for quartzite from 9 to 41. This shows how difficult it is to guide engineers in this matter if they have no data from laboratory tests.

In mining or tunnelling practice, the influence of the chosen m_i value is crucial for stress analysis in weak rocks. Then the range of damage zone increases considerably, because of the changes of the m_b constant value, which depends on m_i. Moreover in mining engineering, every mining operations lead to rock mass damage, what immediately changes the parameters of Hoek-Brown failure criterion [5].

To show the meaning of m_i constant in geotechnical analyses if you want to determine the range of damage zone the simple calculations using RS2 software were carried out. Assumed that the 12-meter diameter circular tunnel was driven in the homogenous rock at the depth of 100 m. First, two rock bodies were considered: in the first case – a weak rock, with compressive strength of 25 MPa, Young's modulus E = 3000 MPa and GSI = 25, while in the second – a medium rock – σ_c = 35 MPa, E = 5000 MPa and GSI = 50. The different damage zone ranges were obtained (Fig. 1), when changing the m_i value from 4 to 24. The range of the damage zone for weak rock varied from 2.66 m to 4.90 m, and for medium rock from 0.96 m to 2.21 m (TABLE 1). But if in the third case the above parameters were increased to: σ_c = 50 MPa, E = 7000 MPa and GSI = 65 (a hard rock), the range of damage zone was zero, no matter what m_i was used in the failure criterion (for m_i = 4 the range of damage zone was 4 cm, so nearly zero). One can say then, that a rock mass 'forgives' a wrong m_i assessment for hard rocks in geomechanical calculations. The same observation did Zenah and Görög [6] underlined that if the rock is weak and there are two parallel tunnels, the displacements for rocks of GSI equals 30 and low m_i immediately will cause the tunnel collapse. For sedimentary rocks m_i constant is certainly more important, and that is why authors focus on chosen m_i determination for sedimentary rocks in this paper.

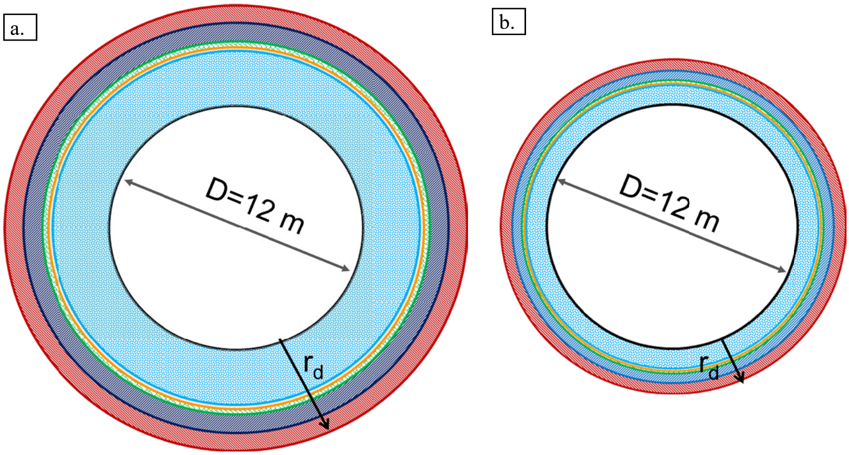

Fig. 1. The range of damage zone (rd) around the 12-metre circular tunnel in (a) weak rock and (b) medium rock; (m_i = 8 – red colour, m_i = 12 – dark blue colour, m_i = 16 – green colour, m_i = 20 – orange colour, m_i = 24 – blue colour)

TABLE 1

The range of damage zone around the 12-metre diameter tunnel for weak and medium rocks changing the m_i value

m_i value	Damage zone range r_d [m]	
	Weak rock	**Medium rock**
4	4.90	2.21
8	4.04	1.66
12	3.51	1.36
16	3.15	1.19
20	2.87	1.06
24	2.66	0.96

1. Interpretation of m_i constant in Hoek-Brown failure criterion

Some scholars simplistically interpret the m_i constant. For example, Villeneuve et al. [7] wrote that: "…*m_i controls the steepness and curvature of the failure envelope, and is derived from curve-fitting the failure criterion to triaxial test data*." Yes, this is true (Fig. 2), because the stress path of different rocks changes its course, but it is obvious that the meaning of the m_i parameter has a physical basis. It is strictly connected with a rock structure, so the type of rock, as it is shown in Fig. 2. The m_i itself should reflect the shear resistance on the cracks inside the rock being in a triaxial state of stress. If one wants to know the origin of m_i, he has to come back to the authors of the criterion – Dr Hoek and Dr Brown [1,2]. They developed the criterion in a trial-and-error process using the Griffith theory as a starting point, seeking an empirical relationship that fitted observed shear failure in rock being in triaxial compression [8]. They claimed that, for small confining pressure, the crack initiation occurs usually at 40-60% of ultimate stress. But this range can, of course, change and it is typical for the specific rock from the analysed stratigraphic section of a geological column.

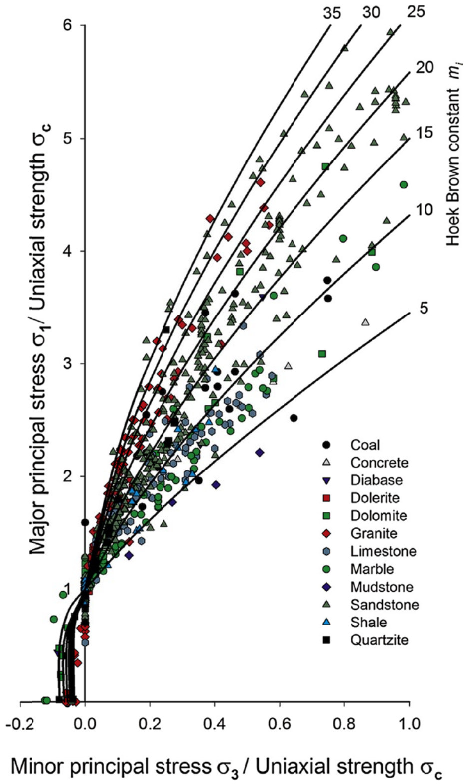

Fig. 2. Stress path for different types of rocks based on triaxial test results [8]

1.1. m_i as the proportion between compressive and tensile strength

For solid rocks, one can say that m_i shows the proportion between the compressive and tensile strength, because if one assumes $\sigma_1 = 0$, $\sigma_3 = -\sigma_t$ and $m_b = m_i$, $s = 1$ (intact rock, where GSI = 100) and consider the original Hoek-Brown criterion:

$$0 = \sigma_t + \sigma_c \sqrt{m_i \frac{\sigma_t}{\sigma_c} + 1} \tag{3}$$

one obtains:

$$m_i = \frac{\sigma_c}{\sigma_t} - \frac{\sigma_t}{\sigma_c} \tag{4}$$

Taking into consideration that $\sigma_c = (8\text{-}20)\ \sigma_t$, one can say that:

$$m_i \approx \frac{\sigma_c}{\sigma_t} \tag{5}$$

Using Griffith's theory [9] directly and assuming uniaxial load on a rock, so that $\sigma_1 = \sigma_c$ and $\sigma_3 = 0$, it can be said that σ_c/σ_t is 8. Using the developed Griffith theory given by Paterson and Wang [10], who considered the developing cracks in a rock, so-called 'penny-shaped cracks', the proportion σ_c/σ_t should also be fixed and equals 12. In practice, the stress path of every rock is different and the proportion between compressive and tensile strength can vary in a wide range, depending on its structure [8,11,12]. The hundreds of laboratory tests conducted by authors on carboniferous rocks: coal, mudstone, siltstone and sandstone, show that the minimum proportion between compressive and tensile strength is ca. 4-6, while the maximum reaches ca. 40 for mudstone and siltstone, 51 for sandstone and as much as 65 for coal. The reason is that there is usually a big problem with the homogeneity of sedimentary rocks, and the average values depend not only on the rock structure but also on the laminae or other thin mineral layers as well, which weaken a rock and cause a high variation in its strength. Comparing the results obtained with the m_i values recommended by Douglas [4] and Hoek [12], it is clear that the average proportion between compressive and tensile strength can be quite far from the recommended value of m_i constant. In other words, this can reflect the real m_i value incorrectly and it will then give the wrong results if the Hoek-Brown failure criterion is employed. It should be noted also that, when considering sedimentary rocks, it is sometimes exceptionally difficult to describe unequivocally the type of rock due to the many intrusions, laminae and contribution of other minerals (Fig. 3). This causes untypical rock identification in the form of e.g. 'sandy shale' or 'mudstone with quartz laminae', and such rock types are not listed in the tables that guide the m_i values.

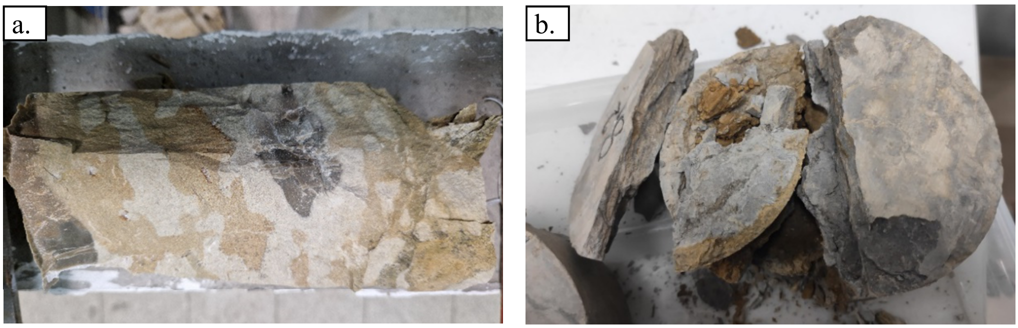

Fig. 3. Disturbed geological structure of sedimentary rocks, a. limestone with silt and flints, b. shale with clay laminae

All in all, local cracks and discontinuities in every rock mean that the m_i value should be determined individually. This individualism shows the range of the studied proportion σ_c/σ_t, which is between 35 (mudstone) and 59 (coal), as shown in TABLE 2, where results of laboratory tests carried out by authors were performed. The presented ranges are larger than those published by Davarpanah et al. [13] because the number of studied data sets is very high, especially for mudstone and sandstone, and the number of different structural cases is higher too. For a more bedded and laminated structure (coal), the difference between compressive and tensile strength is higher. That is why this ratio is often referred to as 'brittleness', indicating the tendency of the rock to break easily into small pieces.

TABLE 2

Proportion between compressive and tensile strength for carboniferous rocks obtained in laboratory tests

Type of rock	Number of tests	Proportion between compressive and tensile strength σ_c/σ_t				Recommended average m_i value		
		Minimum	Maximum	Average	Standard deviation	Douglas [4]	Hoek et al. [3]	Hoek [12]
Coal	39	6.6	65.5	**24.5**	12.6	—	15	—
Mudstone	298	3.9	38.7	**13.2**	5.3	9	4	4
Siltstone	150	4.9	43.3	**12.6**	5.2	8	9	7
Sandstone	252	4.4	51.6	**15.1**	6.8	14	19	17

1.2. m_i dependent on the friction between cracked surfaces

The approach to the m_i as the parameter involved in friction between the two surfaces of a joint in the fractured rock was shown by Zuo et al. [14]. They studied the growth of microcracks in rocklike materials based on considerations of fracture mechanics. The authors assumed a sliding-crack model and the Griffith's rock model developed by Paterson and Wong [10]. They considered the generated wing cracks with the close tips and claimed that the frictional strength of the sliding surfaces is then overcome. They found that the failure initiation criterion can be expressed by Eq. (6):

$$\sigma_1 = \sigma_3 + \sqrt{\frac{\mu}{\kappa}\frac{\sigma_c}{|\sigma_t|}\sigma_3\sigma_c + \sigma_c{}^2} \tag{6}$$

where μ is the friction coefficient, so $\mu = \mathrm{tg}\phi$, and κ is the coefficient that can be derived from various approximations based on a maximum stress criterion or a maximum energy release criterion. Zuo et al. [14] found that for typical internal friction angles from 35° to 55°, the κ coefficient is equal to 1.

Of course, if assume that:

$$\frac{\mu}{\kappa}\frac{\sigma_c}{|\sigma_t|} = m_i \tag{7}$$

then

$$\sigma_1 = \sigma_3 + \sigma_c\sqrt{m_i\sigma_3\sigma_c + \sigma_c{}^2} = \sigma_3 + \sigma_c\sqrt{m_i\frac{\sigma_3}{\sigma_c} + \sigma_c} \tag{8}$$

So one gets the original Hoek-Brown failure criterion formula.

Even though m_i again depends on the tensile and compressive strength, it needs to be highlighted that m_i should also be proportional to the internal friction angle ϕ. Hence the determined m_i constant and internal friction angle ϕ should show a relationship. This is logical because the change of the m_i value immediately changes the steepness of the slope of the Hoek-Brown failure envelope.

2. The methods of m_i determination

2.1. Triaxial compressive test

The triaxial compressive test is the basic recommended test for m_i parameter determination. Then not only m_i but the average compressive strength σ_c, which matches the Mohr circles envelope, can be determined as well [1,2,15]. However, there are some limitations to the methodology. First, in deriving both values the confined stress σ_3 used during tests should be in a range of $0 < \sigma_3 < 0.5\sigma_c$ (Hoek and Brown, 1980, 1988). Then at least five or more triaxial test results should be obtained to get five well-spaced data points that can allow m_i and σ_c to be determined [15]. The coefficient of determination R^2 should be determined as well to check the validity of the data obtained. Hoek [15] underlines that high-quality triaxial test data usually gives a coefficient of determination R^2 greater than 0.9. All three parameters, m_i, σ_c and R^2, are functions of two variables: $x = \sigma_3'$ and $y = (\sigma_1' - \sigma_3')^2$, where σ_1' and σ_3' are the maximum and minimum effective principal stresses, in this case – they are breaking vertical and horizontal stress, respectively. For engineering practice, laboratory tests should be carried out on moisture contents as close as possible to those that occur in the field [15].

2.2. m_i as a function of compressive or tensile strength

By analysing the origin of the m_i parameter and conducting mathematical calculations, it is obvious that the constant has to be related to the rock strength. Some scientists still try to determine it from a simple brittleness proportion, i.e. σ_c/σ_t (the so-called *R-index method*) but, as Sari [16] underlines, this is a simplified method of determining m_i. However it's worth noting that Aladajare and Wang [17] successfully built the regression model of m_i constant and UCS using Bayesian statistics and characterized this constant in probabilistic way used site-specific UCS data and ranges of m_i reported in Hoek's guideline chart. They recommend this approach if no triaxial compression test data are available.

Shen and Karakus [18], after testing five types of rock: coal, marble, limestone, sandstone and granite, represented by 12–35 data sets, developed another method of determining m_i based on uniaxial compressive strength (*UCS-based method*). They noticed that the results obtained for four of the rock types (excluding limestone) have trends of decreasing m_i with increasing σ_c. As a result, they implemented m_{in}, the so-called 'normalised m_i', to the Hoek-Brown criterion (the unit for m_{in} is 1/MPa). The relationship between normalised m_i and the compressive strength is shown by the formula:

$$m_{in} = a\sigma_c^{\,b} \tag{9}$$

where a and b are constants dependent on the rock type.

The Hoek-Brown criterion can then be rewritten as follows:

$$\sigma_1 = \sigma_3 + \sigma_c \sqrt{m_{in}\sigma_3 + 1} \tag{10}$$

Thanks to this approach for all rock types (including limestone), Shen and Karakus [18], after regression analysis, got very high coefficients of determination R^2 of 0.89-0.96. Also high R^2 equals 0.80 was obtained by Vasarhelyi scientific team [19] while comparing normalised m_i and the compressive strength for 44 samples of granitic rocks (monzonites and monzogranites). The results from uniaxial compression tests were always taken to the analyses.

Another solution was shown by Wang and Shen [20], who determined m_i from the tensile strength (*TS-based method*). Values of m_i were calculated from regression analysis over a confining stress range from 0 to $0.5\sigma_c$ as suggested by Hoek and Brown [15]. Thanks to their research, m_i constant can be determined from the formula:

$$m_i = A\sigma_t^B \tag{11}$$

Here A and B are also constants dependent on the rock type, but the authors more precisely describe A as a coefficient dependent on the friction coefficient of rock joints and B as the rock cracking parameter. Wang and Shen [20] tested the same type of rocks as Shen and Karakus [18] – coal, marble, limestone, sandstone and granite, and got very promising results with coefficients of determination of 0.93-0.98, strongly recommending this approach for m_i constant determination.

2.3. m_i in strength regression analyses

Since the simple proportion between a rock's compressive and tensile strength can only approximate m_i constant [16], many scholars have tried to find another way of determining the value of m_i. The best results were given by regression analyses which used both strength, σ_c and σ_t. The most interesting studies were delivered by Arshadnejad [21] and Davarpanah et al. [13]. They studied the m_i constant of varied rocks and grouped them according to their origin. Davarpanah et al. [13] studied 4 igneous, 5 sedimentary and 6 metamorphic rocks (they treated coal as a metamorphic rock) and Arshadnejad [21] – 13, 9 and 5, respectively. Thanks to the research and regression analysis carried out by the above authors, they suggested that the value of m_i can be derived as the function of x, where:

$$x = \frac{\sigma_c - 2.5\sigma_t}{\sigma_t} \tag{12) [21}$$

$$x = \frac{\sigma_c - 2\sigma_t}{\sigma_t} \tag{13) [13}$$

By sorting rocks according to their origin, both scholars obtained very high consistency between the developed formulae and the above variables. The coefficient of determination was 0.96-0.99 for Arshadnejad's [21] formula and ca. 0.99 for Davarpanah et al.'s [13] formula.

$$m_i = e^{10x \cdot cx^d} \tag{14}$$

There are no other publications in which researchers have proved such high consistency between m_i constant and the regression results.

However Mostyn and Douglas [22] who also underlined that m_i is closely related to the ratio of σ_c/σ_t, stated that also the method of m_i estimation has a large impact on parameters derived from experimental data. Using in their study artificially generated data they proved that depending on regression and fitting method m_i can vary from 4.1 even to 35.2. They showed that for actual data m_i is equal to 12.0, but if excluding both σ_c and σ_t results and using normal equations m_i rises to 21.7, even though both interpretations provide a reasonable fit for the data with high R-square. They also showed that using normal equations and excluding σ_c values gives m_i equals to 12.7, while excluding σ_t gives m_i equals to 5.1. Simultaneously they suggested using extended equations using all data to get the best results of the regression to actual data. It can be said that Arshadnejad [21] and Davarpanah et al. [13] followed this suggestion.

2.4. The meaning of confined tension in m_i interpretation

The most common state of stress inside a rock mass is triaxial. But there are some regions in a rock mass where there is no triaxial state of stress but a confined tensile state of stress or even uniaxial tension can occur. The tension regions in a rock mass are met in geologically disturbed areas as faults [23,24] and next to the mining working where confinement is removed [23,25]. The full stress path in a rock mass is shown in Fig. 4. The first possible case of stress is uniaxial tension and, starting from this point, the rock is under confined tension. Then uniaxial compression can take place and finally the rock is subjected to triaxial compression.

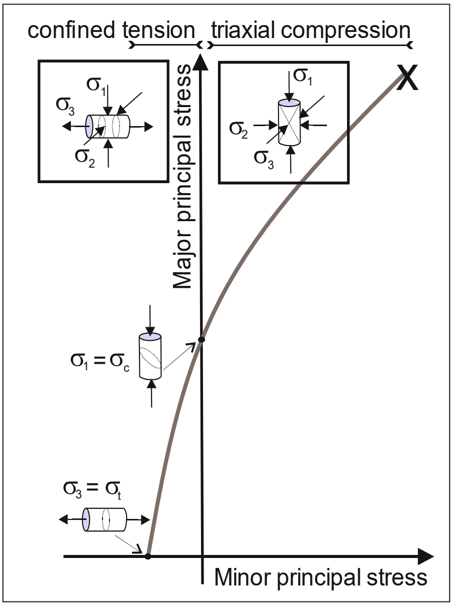

Fig. 4. Failure envelope of the rock

For m_i determination triaxial tests on rocks are recommended. Then using regression analysis, the compressive strength can be determined as well. But additionally, one can carry out a series of uniaxial compressive strength tests. So there are two possibilities to use compressive strength which is needed in methods of m_i determination.

You can also notice that the interpretation of the above results in $\sigma_1 - \sigma_3$ coordinate system gives the linear failure envelope (Fig. 5a) – Mohr circles and Mohr-Coulomb envelope. However, if the set of tension tests is carried out for the specific rock (usually Brazilian tests), then the interpretation of results gives the parabolic Hoek-Brown envelope (Fig. 5b). In the first case, if you haven't done the UCS test, the unknown compressive strength (estimated compressive strength σ_{cest}) is determined by the Mohr envelope, while the compressive strength is fixed as the average value (σ_{cav}) obtained from laboratory tests in the second case. But then, the course of the envelope and the σ_c value influence the value of m_i. Moreover, the envelope will cross the vertical axis at different points and its inclination will be changed (Fig. 5). So if you interpret the results in $\sigma - \tau$ coordinates, it also means another cohesion and internal friction angle of a rock. Summing up, conducting laboratory tests on rocks to determine the m_i constant, the results can be interpreted in two ways: 1) based on a triaxial tests only, 2) on the basis not only of triaxial tests, but also the average tensile strength and average compressive strength of the rock. These two different approach can affect a simple brittleness proportion σ_c/σ_t, internal friction angle of rock and last but not least – m_i constant value.

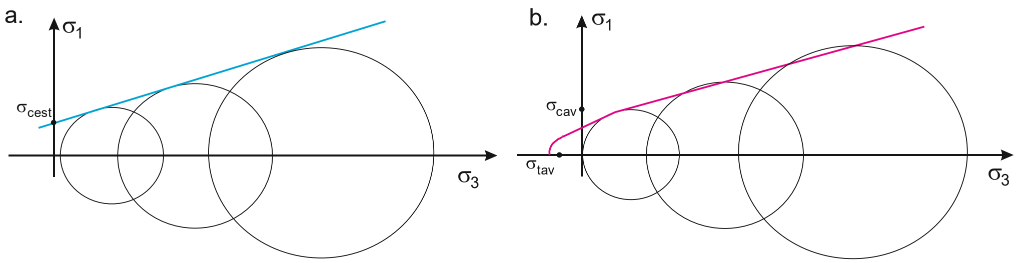

Fig. 5. Failure envelope based on triaxial compressive test results: a. not considering tensile and compressive strength Mohr-Coulomb envelope; b. considering average tensile and compressive strength – Hoek-Brown envelope

To check the difference in both approaches to m_i determination, 51 series of tests on three rock types were carried out for: sandstone (10), mudstone (29) and limestone (12). There were Carboniferous sandstones and mudstones from Silesian coal mines and Triassic limestones from the Kielce region. The number of samples met the recommendation – minimum 10, given in [17] where the effect of the data quantity on average value of m_i and its standard deviation were studied.

Four or five triaxial tests carried out on the same rock were considered to find m_i in a linear result interpretation as stated by Hoek (Fig. 5a) – then called m_i *linear* and triaxial tests results with the average value of tensile strength and average value of compressive strength were considered in a non-linear interpretation as in Fig. 5b, then called m_i *non-linear*. For the both interpretations 50 triaxial tests for sandstone, 157 for mudstone and 48 for limestone were carried out respectively and 337 compressive tests (98 for sandstones, 203 for mudstones and 36 for limestones) and 354 Brazilian tensile tests (96 for sandstones, 222 for mudstones and 36 for limestones). The average m_i values obtained for the chosen rocks were compared with the recommendations given by Carter and Marinos [26] and results scatter was discussed. Then the possibility of using the simple proportion of compressive and tensile strength (brittleness) for m_i determination was checked. Finally, the formulae derived by Shen and Karakus [18], Wang and Shen [20], Arshadnejad [21] and Davarpanah et al. [13] were used to show their usefulness for m_i determination, and how much the m_i constant obtained by using their different approaches matched the proposed formulae. The internal friction angle value derived from two different m_i values – linear and nonlinear were compared as well. The point of the conducted study was not to find another empirical formula for m_i constant determination but try to answer if the tensile strength is needed for m_i calculation and/or how much adding a tensile strength to the data set can change the m_i value.

3. Rock strength and m_i constant

3.1. Strength of tested rocks

To get the average values of compressive strength and tensile strength 337 compressive tests and 354 Brazilian tensile tests were done. The results are presented in TABLES 3 and 4. To show discrepancies between the values of single tests for the rock, the minimum, maximum and standard deviation (*SD*) values were given. To show the difference between the estimated rock compressive strength (σ_{cest}) obtained by drawing the Mohr circle envelope and the tested rock compressive strength (σ_c), the coefficient of determination R^2 was calculated, as well as the absolute average relative error percentage (*AAREP*) suggested by Wang and Shen [20], which is described by Eq. (15). The smaller the *AAREP*, the more reliable the estimation of the compressive strength is. The *AAREP* is based on the discrepancy percentage D_p, which shows here, in turn, the difference between the predicted and tested values of the compressive strength (Eq. 16).

$$AAREP = \frac{\sum_{i=1}^{N} \left| D_p \right|}{N} \tag{15}$$

$$D_p = \frac{\sigma_{c\,est-}^{i}\sigma_c^{i}}{\sigma_c^{i}} \times 100\% \tag{16}$$

For the tested rocks the minimum values of compressive strength are 5-7 times lower than the maximum (TABLE 3). This proportion for the estimated σ_c value is lower. But even though the scatter of results is the highest for limestone, the standard deviation is the lowest and, what is more important, the consistency with the estimated compressive strength value obtained from the Mohr circles envelope is very good. In this case, the coefficient of determination is 84%, *AAREP* reached only 8.8%, and the difference between both strengths is only 2.6 MPa. On other hand, an insignificant difference between the estimated and tested compressive strength was obtained for the mudstone, which is only 0.7 MPa, giving *AAREP* equal to 15.5 and R^2 of only 38%. The worst consistency between results was found for the sandstone. However, it should be underlined that the estimated compressive strength value calculated based on triaxial strength tests is very similar to the average σc obtained for lab tests for every rock.

Analyzing the scatter of tensile strength results, a 10-fold difference for the individual limestone samples was observed, nearly 7-fold for mudstone and only 4-fold for sandstone (TABLE 4). The R-index calculated for σ_c tested and σ_{cest} estimated was similar for mudstone, varied by 8% for limestone and by about 15% for sandstone. So, on one hand, having over

200 data for mudstone the difference of R-index using σ_c or σ_{cest} is neglected (what is expected), but on other hand nearly 100 data for sandstone still can give 11% different results. This simple analysis proves how difficult is quantify heterogenic sedimentary rocks, even the number of using data seems to be sufficient.

Based on this analysis, the similar difference of m_i results for chosen rocks can be expected to obtain (determined in the two different ways) if using compressive and tensile strength to its calculation. Simultaneously, this analysis showed that, although all three rock types represent sedimentary rocks, the different *AAREP* and different R^2 values cannot give high consistency between all the results and the derived m_i constant.

TABLE 3

Tested and estimated compressive strength for chosen rocks

Rock type	**No of tests**	**Compressive strength σ_c [MPa]**				**Estimated σ_{cest} [MPa]**				**AAREP [%]**	R^2
		Min.	**Max.**	**Av.**	**SD**	**Min.**	**Max.**	**Av.**	**SD**		
Mudstone	203	22.3	116.7	**66.6**	17.7	31.9	101.0	**67.3**	15.7	**15.5**	0.380
Sandstone	98	31.2	151.8	**97.5**	20.3	23.5	134.6	**85.4**	31.9	**19.4**	**0.547**
Limestone	36	11.1	74.9	**35.0**	15.7	15.5	51.5	**32.4**	10.8	**8.8**	**0.837**

TABLE 4

Tensile strength for chosen rocks and R-index

Rock type	**No of tests**	**Tensile strength σ_t [MPa]**				**R-index**		
		Min.	**Max.**	**Av.**	**SD**	**With σ_c (tested)**	**With σ_{cest} (estimated)**	**Difference [%]**
Mudstone	222	2.03	13.81	**6.19**	2.32	**10.8**	**10.9**	0.9
Sandstone	96	4.06	17.74	**8.00**	2.70	**12.1**	**10.7**	11.0
Limestone	36	0.79	8.17	**3.61**	1.69	**9.7**	**9.0**	7.8

3.2. m_i value for chosen rocks in two different lab results interpretations

The m_i constant was derived in two ways: 1) based solely on triaxial test, 2) based on triaxial tests along with the average tensile strength and average compressive strength of the rock. The average values of m_i for all the chosen rocks (51 tests) are presented in TABLE 5. Here we can see, that the absolute difference between the non-linear and linear approach to interpretation of the results is quite small between 5.6% and 12.2%. However, it is worth noting the very wide range of

TABLE 5

Average values of m_i determined for chosen type of rocks

Type of rock	**Number of test series**	**m_i non-linear**			**m_i linear**		
		Average	**Stand. deviation**	**Standard error (α = 0.05)**	**Average**	**Stand. deviation**	**Standard error (α = 0.05)**
Mudstone	29	8.8	2.2	0.8	7.5	4.4	1.6
Sandstone	10	10.4	5.8	3.9	9.6	6.1	4.1
Limestone	12	7.5	2.9	1.9	7.9	3.5	2.2

m_i values for the specific rock set, which is higher if we consider linear m_i values. It reaches 23 for mudstone, 20.5 for sandstone and 12 for limestone, while the range of the non-linear m_i values for chosen rocks is 8, 15 and 10, respectively. Hence coefficient of m_i variation CV (standard deviation divided by average value) is high, but also lower if interpret the laboratory results in non-linear manner for every type of rock were tested. Then CV for sandstone is 56.3%, for mudstone 25.0% and for limestone 39.3%, while for linear interpretation is 63.3%, 58.3% and 45.1% respectively.

The ranges of obtained values of m_i for both approaches were shown in Fig. 6. The coloured frames show the range of m_i recommended by Carter and Marinos [26], but ca. 30% of the obtained results are beyond these frames. The same observation was made by Douglas [4], who stated that typically, more than 50% of the test data falls outside the recommended range indicated by the box for each rock type. Nevertheless, it can be seen that the non-linear m_i values change in a narrower range. As a consequence, considering the standard deviation (SD) of the m_i value using the linear and non-linear approach (TABLE 3), it is easily noticed that, if one takes the tensile strength in the analysis, then the scatter of results is lower. This is the case for every tested type of rock. While the SD is not very distinct for m_i calculated for limestone, for mudstone it is nearly two times higher if we determine m_i using only the results from the triaxial test. The standard error of the determined m_i value (for a confidence level of 5%) is also lower if we study non-linear m_i.

These analyses show that we get more concise results of the m_i value with lower variation for the non-linear method of the constant determination.

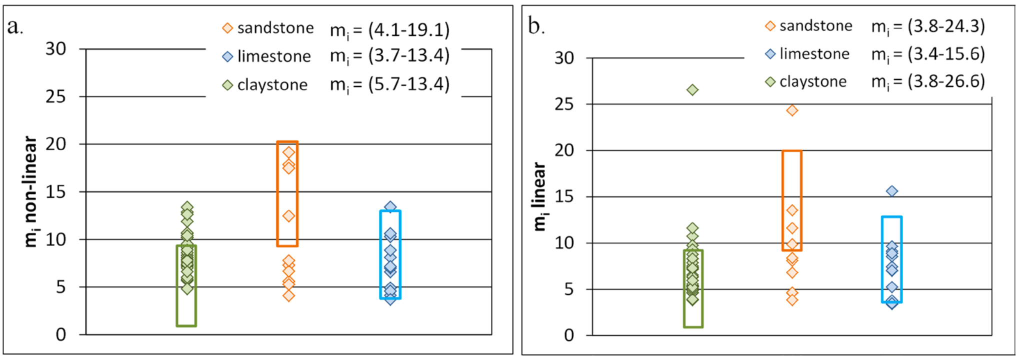

Fig. 6. Average m_i values for chosen rocks: a. non-linear condition; b. linear condition

Examples of how much the m_i value can vary if we employ different data interpretations using the non-linear and linear approach to lab results are shown in Figs. 7 and 8. In these cases, m_i for mudstone is 8.3 and 5.3, respectively, and for sandstone – 17.8 and 13.5. So the m_i constant determined differently for a specific set of samples of the same rock can vary in a wide range. The analysis of the results shows that usually the non-linear interpretation gives the higher m_i value, however there are also several samples where linear interpretation of laboratory tests gives higher results. The difference between average m_i and m_i determined for the set of samples from one site is usually no more than 6 (what suggested Richards and Read [27]). In 10% of cases the difference exceeded 6 and in one case even reached 15 (Fig. 9).

Analysing the results of m_i determination for sandstones collected from different sites, the heterogeneity of sedimentary rocks is clearly visible (Fig. 10). m_i constant values are in the other

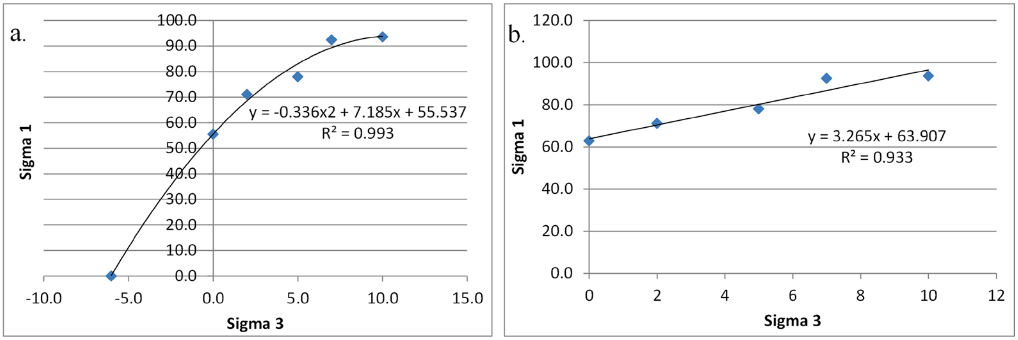

Fig. 7. Course of the stress path for sandy mudstone: a. including average compressive and tensile strength – m_i = 8.33; b. including only triaxial test results and one uniaxial test – m_i = 5.29

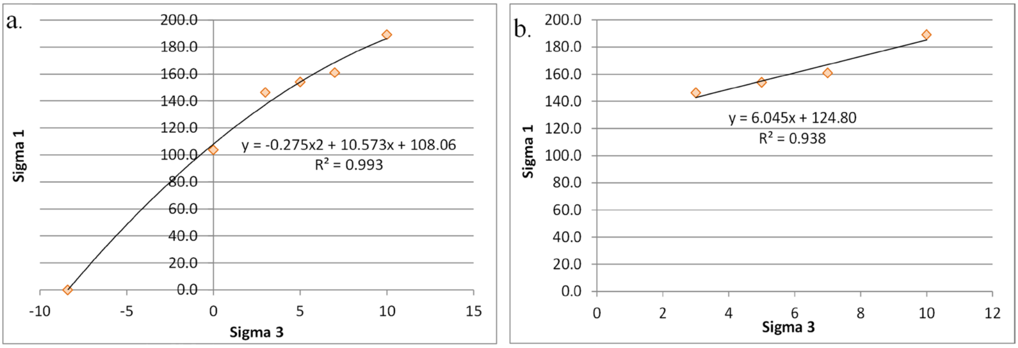

Fig. 8. Course of the stress path for sandstone: a. including average compressive and tensile strength – m_i = 17.84; b. including only triaxial test results – m_i = 13.53

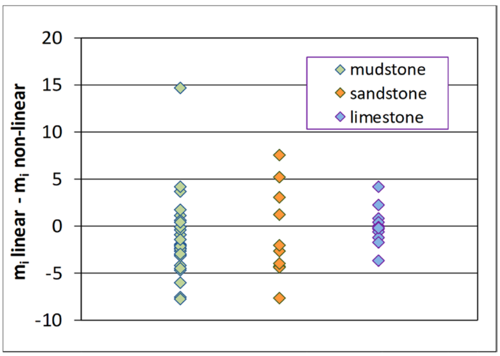

Fig. 9. The absolute difference between m_i linear and m_i non-linear for chosen rocks

range then. Moreover, there is a considerable difference in the m_i range taking into account linear and non-linear way of laboratory tests interpretation, however more consistent values were obtained for non-linear m_i.

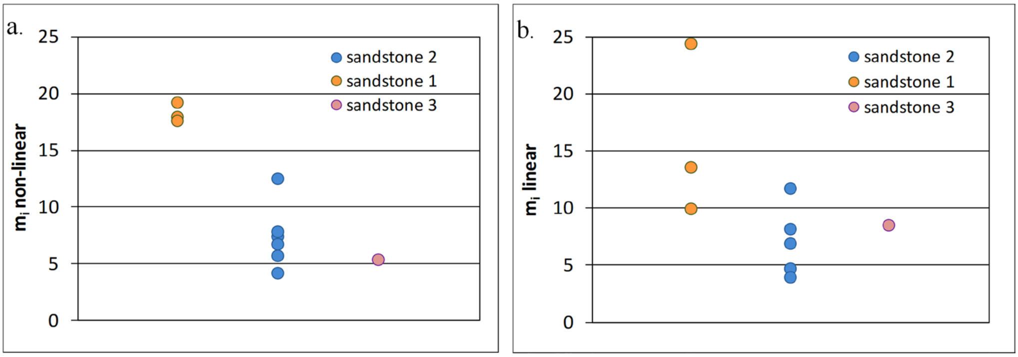

Fig. 10. m_i values for sandstones collected from different sites: a. in non-linear interpretation; b. in linear interpretation

The same tendency we can observe during the analysis of m_i constant for mudstones of different structures (Fig. 11). Very often sedimentary rocks are interbedded, with lamianae or other rock intrusions, so in fact we don't have the same type of rock, even though we use the same name. The consequence of the heterogeneity is its different behavior under a load, so rocks taken from different sites, even of the same geological origin, usually demonstrate varied physical parameters. Also m_i constant value depends on the rock structure and on the laboratory tests interpretation, where more consistent values were obtained for non-linear m_i. This analysis also demonstrates the importance of accurately identifying geological rock formation.

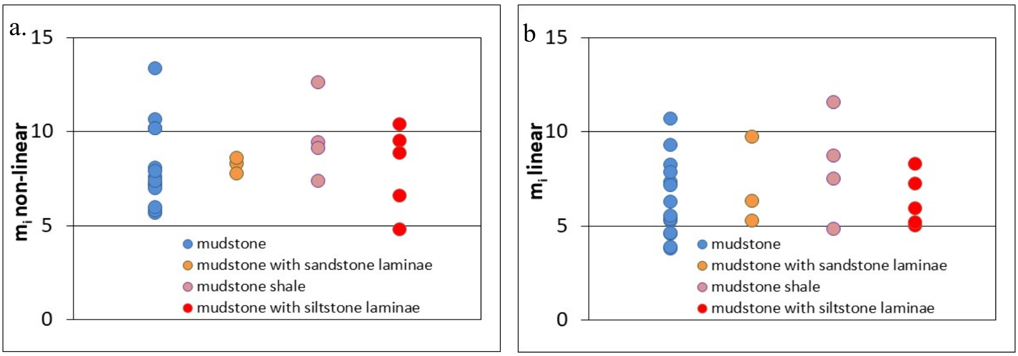

Fig. 11. m_i values for mudstones of different structure: a. in non-linear results interpretation; b. in linear results interpretation

4. m_i constant as the function of rock strength

4.1. Relationship between m_i and a rock strength

After conducting the laboratory tests, two different values of m_i were obtained, depending on whether the average compressive and tensile strength of the rock was used or not. Both strengths are independent parameters, which are determined based on the laboratory tests carried out on other samples. Hence we can perform a regression analysis to check how the use of the above strength values matches m_i constant versus interpreting m_i from triaxial tests only. First we checked the relationship between m_i and the functions of compressive and tensile strength, which are recommended for m_i determination – the R-index method, UCS-based method and TS-based method.

Combining the results of all the experiments, one can see no visible trend between the R-index (the proportion between compressive and tensile strength) and m_i constant. The results are a cloud of points (Fig. 12 and TABLE 6). The same can be said about the TS-based method, which proves that using only the tensile strength for the Hoek–Brown m_i constant is problematic and is not recommended (Fig. 13 and TABLE 6). Moreover, in some cases the reverse trend of m_i change was observed during analyses employing both approaches (m_i decreases with the strength of rock), which shows too much discrepancy between results. All in all, the low coefficient of determination does not allow to state unequivocally which m_i correlates better with the strength functions.

TABLE 6

Coefficient of determination R^2 of correlation between m_i and chosen strength function

Rock type	R-index method		UCS-based method		TS-based method	
	m_i linear	m_i non-lin.	m_i linear	m_i non-lin.	m_i linear	m_i non-lin.
Mudstone	0.004*	0.070*	0.872	0.742	0.001*	0.001
Sandstone	0.145	0.056	0.566	0.646	0.071*	0.065
Limestone	0.100	0.180	0.278	0.267	0.110	0.038

* reverse trend

The results of the analyses also show that the strength discrepancy for each type of rock (TABLES 3 and 4) and the higher or lower R-index do not influence the accuracy of m_i constant determination. And simultaneously the low or high *AAREP* does not indicate how high the consistency between the UCS-based method function and m_i constant is.

However, using the normalised m_i value and UCS-based method developed by Shen and Karakus [18], can observe a noticeable relationship (Fig. 14). In this case for the m_i linear function the estimated compressive strength was used in the regression analysis, while the tested compressive strength was used for the non-linear m_i function. The coefficient of determination R^2 for the recommended power function is 0.45 if we consider m_i constant derived from interpreting the linear triaxial test results, and 0.22 if we add the interpretation results of the average compressive and tensile strength, so taking into consideration the non-linear way of m_i determination. Hence the consistency between the m_i and normalised m_i for all data is not high, and this approach does not give satisfactory results. However, for a specific type of rock, the coefficient of determination R^2 is equal to 0.57-0.65 for sandstone and 0.74-0.87 for mudstone (TABLE 6). This is an interesting result if we take into account the usually inhomogeneous structure of mudstone, which is very often laminated with varying amounts of quartz minerals. Moreover, if apply the linear function of regression in the analysis, then obtain a higher coefficient of determination for every rock type, though sometimes the consistency between the results was greater if considered the non-linear value of m_i. This observation is surprising if take into account that m_i linear is determined from the Mohr envelope which crosses the σ_1 axis where the compressive strength is estimated.

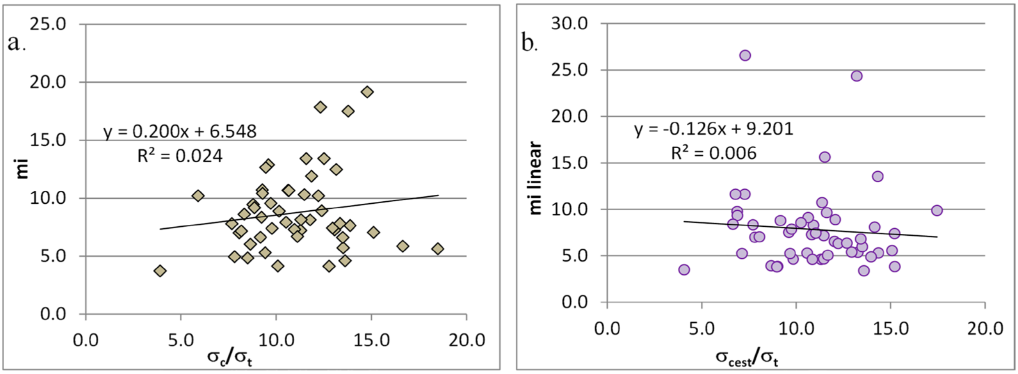

Fig. 12. Relationship between R-index and m_i constant derived in different ways for rocks, a. non-linear interpretation of results with average compressive and tensile strength, b. linear interpretation of results with estimated compressive strength

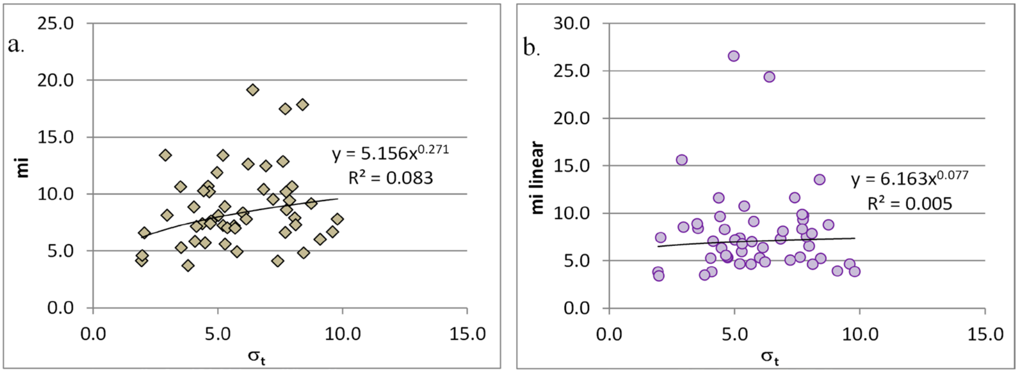

Fig. 13. Relationship between the TS-based method and m_i constant derived in different ways for rocks, a. non-linear interpretation of results, b. linear interpretation of results

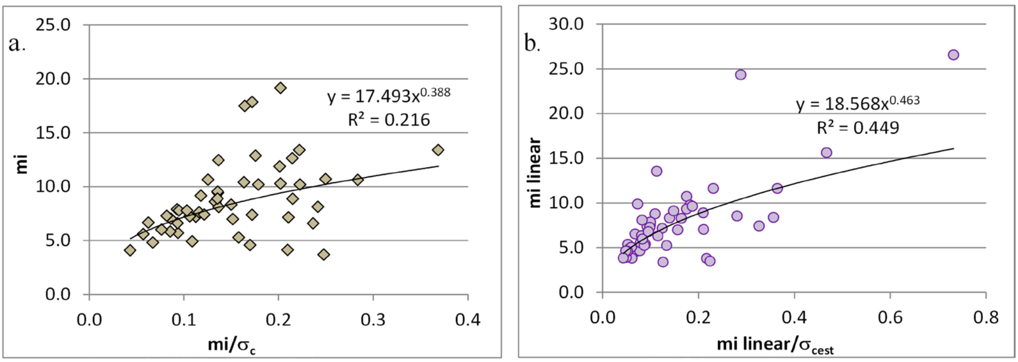

Fig. 14. Relationship between m_{in} (UCS-based method) and m_i constant derived in different ways for rocks, a. non-linear interpretation of results with average compressive strength, b. linear interpretation of results with estimated compressive strength

The conducted study showed also that, for the set of rock samples with no other rock interlayers taken from the specific site, the use of the UCS-based method with normalised m_{in} (m_i/σ_c) can give satisfactory results. In rock engineering practice, all rock properties are strictly dependent on the rock structure, so collecting quasi-homogeneous samples from the same field can give very similar results. As an example, we present this method applied for six groups of sandstone taken from the roof of a roadway from one of the Polish coal mines – Pniowek (Fig. 15). The relationship between normalised m_{in} and m_i is nearly linear and, if we apply the recommended power function R^2, it is equal to ca. 90%, while for m_i non-linear it is slightly higher and in this case all the points of the determined m_i constant are inside the field of the 95% level of confidence.

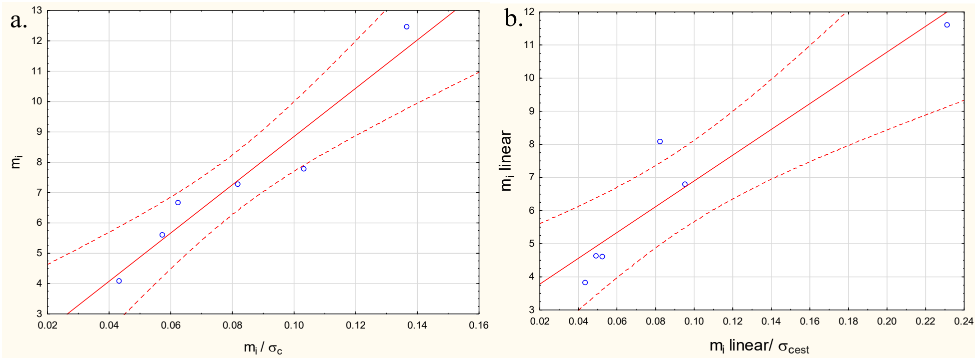

Fig. 15. UCS-based method applied for sandstone from the specific site, a. non-linear interpretation of results with average compressive strength, b. linear interpretation of results with estimated compressive strength

The use of the power function of the normalised m_i with the involved UCS for m_i determination causes the results to match each other generally better than if we use linear m_i, which seems logical, if the UCS-based method does not use the tensile strength for m_i determination. So this cannot be an indicator of which m_i – linear or non-linear – better describes the rock properties.

4.2. m_i constant vs. internal friction angle

The study by Zuo et al. [14] proved that m_i constant depends on the friction between joint surfaces in the fractured rock. As Barton [28] comments, during shearing the friction is mobilised at larger strain and remains to the end of the shear deformation. The research shows that the residual friction angle was only about 2 to 4 degrees less than the peak friction angle [28-30]. Its peak value is close to the internal friction angle and then decreases together with surface polishing. Hence the m_i can be treated as a function of the internal friction angle.

The regression analysis is conducted using the power function, as in earlier cases. The results show a medium but clear relationship between both parameters (Fig. 16). The higher consistency was obtained when considering m_i linear, when R^2 was 0.58 (for non-linear m_i – 0.44), but here again it must be mentioned that the internal friction angle was found based on linear interpretation of the Mohr-Coulomb envelope in normal-shear stress (p-q) coordination. So again, if the variable (in this case the friction angle) is dependent only on the compressive strength, then linear m_i better matches the regression and the coefficient of determination is higher.

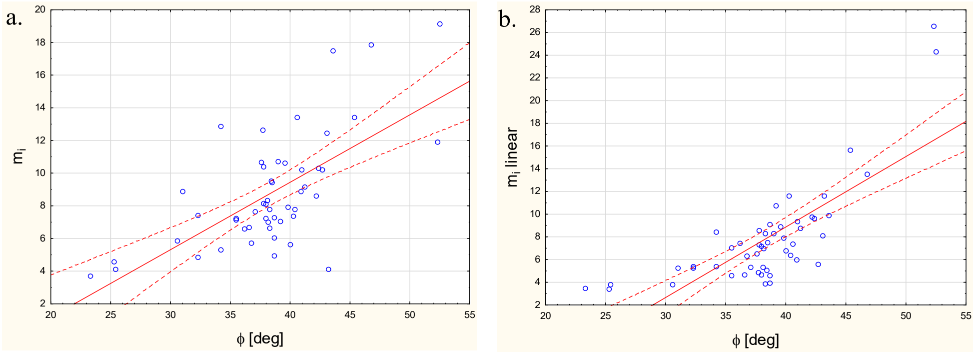

Fig. 16. Relationship between the internal friction angle and function of m_i constant derived in different ways for all data, a. non-linear interpretation of results, b. linear interpretation of results

In TABLE 7 the constants a and b of the power function and the coefficient of determination R^2 for specific rocks are presented if m_i is determined based on the internal friction angle. Using the power function gives the best match between the studied parameters. There is then an extremely high relationship for limestone (94%) and a high one for sandstone (66%) if we consider linear m_i. So the power function of the internal friction angle can reflect the m_i constant value, but an individual function is needed to get the precise value of m_i.

TABLE 7

Constants a and b of power function and coefficient of determination R^2 if m_i constant is derived in different ways for chosen rocks on the basis of internal friction angle

Type of rock	m_i linear			m_i non-linear		
	a	***b***	R^2	***a***	***b***	R^2
Mudstone	0.0013	2.345	0.346	0.2376	0.981	0.155
Sandstone	0.00001	3.644	0.655	0.00001	3.246	0.518
Limestone	0.0046	2.067	0.939	0.0317	1.525	0.675

5. Empirical methods of m_i determination

To check how much the linear or non-linear m_i values match the derived more advanced functions where compressive and tensile strength are involved, the authors employed two functions performed by Arshadnejad [21] and Davarpanah et al. [13], which were described in section 3 and whose formulae were presented in Eqs. (12)-(13).

The results of the analysis are shown in TABLES 8 and 9 and in Figs. 17 and 18. In these cases using the recommended power function, a strong relationship between the chosen expressions $\frac{\sigma_c - 2\sigma_t}{\sigma_t}$ or $\frac{\sigma_c - 2.5\sigma_t}{\sigma_t}$ and m_i values are observed. The coefficient of determination R^2 is equal to 0.65-0.75 for the first expression and 0.70-0.78 for the second one for all data. The coefficient is slightly higher if the analysis considers the non-linear value of m_i, i.e., is determined together with the average compressive and tensile strength of the tested rock. The obtained coefficient can be acknowledged as extremely high taking into consideration heterogeneous sedimentary rocks.

The constants a and b in formula (14) for all chosen rock types and the coefficient of determination R^2 are shown in TABLES 8 and 9. One can see that the function of $\frac{\sigma_c - 2.5\sigma_t}{\sigma_t}$ used can quite acceptably calculate the Hoek-Brown constant m_i, and using the non-linear way of m_i determination definitely gives a better match with the derived function. The R^2 coefficient is then 0.50-0.87, the highest for mudstone, where 29 sets of laboratory tests were considered. Taking into account all the data of sedimentary rocks, the function of:

$$m_i = e^{10\frac{\sigma_c - 2.5\sigma_t}{\sigma_t}\cdot 0.162\frac{\sigma_c - 2.5\sigma_t}{\sigma_t}^{-0.88}}$$

can be recommended for calculation of the m_i constant.

Comparing these results with the results of m_i determination using the UCS-based method, we can see that the level of fitting is more or less the same, but using the function of both compressive and tensile strength [13] gives better consistency with non-linear m_i, while using the compressive strength function only [18] gives better consistency with linear m_i.

TABLE 8

Constants a and b in formula (14) for chosen rocks and coefficient of determination R^2 if $\frac{\sigma_c - 2\sigma_t}{\sigma_t}$, when m_i constant is derived in different ways

Type of rock	m_i linear			m_i non-linear		
	a	***b***	R^2	***a***	***b***	R^2
Mudstone	0.264	–1.555	73.7	0.296	–1.151	90.4
Sandstone	0.062	–0.486	29.4	0.144	–0.830	47.9
Limestone	0.024	–0.583	23.9	0.022	–0.507	38.2

TABLE 9

Constants a and b in formula (14) for chosen rocks and coefficient of determination R^2 if $\frac{\sigma_c - 2.5\sigma_t}{\sigma_t}$, when m_i constant is derived in different ways

Type of rock	m_i linear			m_i non-linear		
	a	***b***	R^2	***a***	***b***	R^2
Mudstone	0.106	–1.266	59.7	0.113	–1.195	86.6
Sandstone	0.068	–0.512	34.1	0.148	–0.838	51.2
Limestone	0.027	–0.620	31.8	0.026	–0.562	50.1

6. Conclusions and discussion

The m_i constant is indispensable for using the Hoek-Brown failure criterion. It is derived based on triaxial compression tests. However, every rock in a rock mass can be subjected not only to triaxial compression but also to confined tension, so the full stress path, from uniaxial ten-

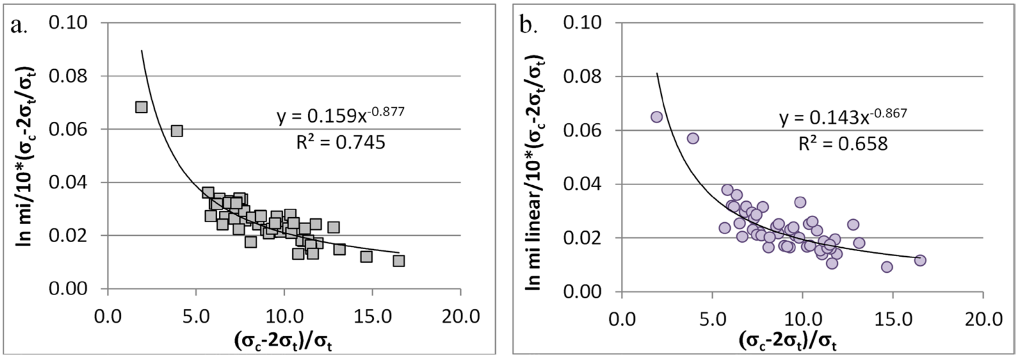

Fig. 17. Relationship between strength function $\dfrac{\sigma_c - 2\sigma_t}{\sigma_t}$ and function of m_i constant derived in different ways for all data, a. non-linear interpretation of results, b. linear interpretation of results

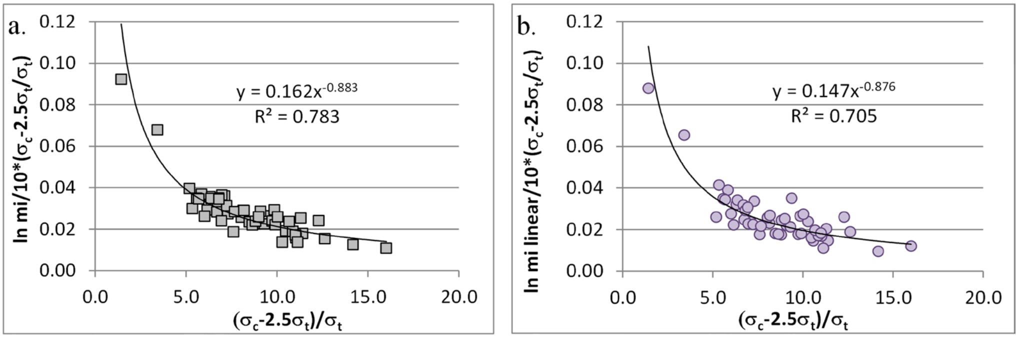

Fig. 18. Relationship between strength function $\dfrac{\sigma_c - 2.5\sigma_t}{\sigma_t}$ and function of m_i constant derived in different ways for all data, a. non-linear interpretation of results, b. linear interpretation of results

sion to theoretically infinite triaxial compression, should be considered for it. As a consequence, the interpretation of m_i constant can be settled based on a linear envelope of Mohr circles or a non-linear envelope – adding the average compressive and tensile strength to the other points. To check which approach to the results gives more reasonable effects, the study of the origin of m_i and its relationship to the developed methods of m_i constant determination was carried out. The conclusions of the analyses are as follows:

1. The different methods of m_i determination (linear and non-linear) give different results, however if one considers a tensile strength in the analysis (non-linear m_i determination), then the scatter of results is lower. The m_i values can then vary considerably, even by 5-6 points (by 100%) for the same sets of triaxial data. This has crucial consequences in geotechnical analyses, because the range of damage zones around the tunnel can change by 2-3 meters if a rock mass is weak.
2. The estimated uniaxial compressive strength value calculated based on triaxial strength tests is similar to the average σc obtained from several lab tests for each rock. The co-

efficient of determination (R^2) for mudstone, sandstone and limestone was: 0.38, 0.55 and 0.84, respectively, and the absolute average relative error percentage (*AAREP*) was 15.5%, 19.4% and 8.8%.

3. Assuming intact rock and solving the Hoek-Brown failure criterion equation, m_i depends theoretically only on the compressive and tensile strength. However, having a partly fractured rock structure, the fixed proportion cannot be equal to m_i. The study conducted shows that this proportion, often called "brittleness", changes for individual sets of rock samples in a wide range. This is the reason why the R-index method of m_i constant determination does not give satisfactory results, no matter what kind of derived m_i we consider.
4. Zuo et al. [14] analysis proved that m_i constant is dependent on the internal friction angle of rock. Because the residual value of the internal friction angle is a maximum 2-4 degrees lower than for intact rock, the value of the friction angle determined for a specific rock can explain its m_i constant. In this case, the linear interpretation of ϕ causes the relationship with the linear value of m_i to be higher, and for a specific rock the consistency can reach as much as 0.94 (R^2 for the power function).
5. The UCS-based method with the normalised m_i (m_i/σ_c) can give satisfactory results in m_i determination. The coefficient of determination (R^2) for the tested rocks varies from 0.27 to 0.87 and the relationship with the linear value of m_i is then higher. It should be stressed that, for a specific rock with no interlayers collected from the same area, the m_i constant can be precisely determined with the help of the UCS-based method with a 95% level of confidence. Nearly the same consistency can then be obtained for linear and non-linear m_i.
6. There is no relationship between the tensile strength (Brazilian test) and m_i constant (the so-called TS-based method). Employing only the tensile strength for m_i determination, use the rock parameter, most sensitive to the presence of cracks, laminae and discontinuities, which immediately causes the rock to break during the load test. So the tensile strength cannot be used individually in regression analyses considering sedimentary rocks.
7. The developed methods for determining m_i constant, which used compressive and tensile strength, can give good results. The best effect is given by the expression $\dfrac{\sigma_c - 2.5\sigma_t}{\sigma_t}$. The regression analysis showed that the non-linear m_i value better matches the derived formulae of the power function suggested by Arshadnejad [21] and the coefficient of determination then exceeds 0.78 (R = 0.88) for all data and exceeds 0.50 for every type of rock. This is a very high value for rock analysis and environmental sciences, especially if we take into account the inherent rock heterogeneity. However the formula of mi determination is complex.

Summarising, the study conducted showed that the most universal method for approximating m_i constant is the UCS-based method. However, the general formula for the specific rock type is needed to obtain satisfactory accuracy with the real m_i. In this case, the lab results approach does not consider the tensile strength, so the results of m_i constant are closer to the linear interpretation of m_i.

The m_i constant determined based on only triaxial tests (m_i linear) is also very close to the power function of the tangent of the internal friction angle. The inclination of the envelope does not take into account the tensile strength, so again linear interpretation of the lab results affects the relationship between ϕ and m_i. The developed formula can be helpful in geotechnical analysis if it is desired to change the failure criterion from Hoek-Brown to Mohr-Coulomb.

However, in our opinion, the most correct approach to determining m_i is by considering the full path of the rock strength: from uniaxial tensile strength, through confined tension up to triaxial compression. Then both forms of uniaxial strength – tensile and compressive – are responsible for rock damage and both should be involved in the determination of m_i constant. So the non-linear m_i better reflects the shear resistance on the cracks inside the fractured rock, and this is the significance of this parameter. Both strengths should be used also in the simplified formulae that can allow to find m_i constant, but their form should be advanced, as shown, for example, by Arshadnejad [21] and Davarpanah et al. [13]. And, of course, it should be borne in mind that m_i constant is a parameter related to the specific rock structure and local geological conditions and should not be taken from books and tables (especially consider far geological regions), because there are only average recommended values for a rock type. The most consistent results can undoubtedly be obtained for intact rock with no interlayers and discontinuities. In the case of nature of sedimentary rocks, a very high consistency between lab results and m_i constant is impossible to obtain.

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Contribution to the paper: research concept – P.M and Z.N., design – P.M., collection and/or assembly of data – P.M and Z.N., data analysis and interpretation – P.M and Z.N., writing the manuscript – P.M.

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