10 Appendix

10.1 Programs

Below we show all benchmark programs. Only the model is shown, the remainder of the program is simply an array with the data points and a for loop iterating over each one to update the posterior. In these models the prior is denoted as `param`.

```
param := Uniform(250,350);
p := Beta(277,param);
n := 4000;
np := (n*p);
half := (0.5*n);
Votes := Binomial(n,p);
if (Votes > half){
        win := 1;
}
else {
        win := 0;
};
return win;
```

Election: A model of an election. Taken from [27]. We continualize the Binomial to a Gaussian, and infer the scale parameter, describing the probability of winning.

```
param := DiscUniform(35,55);
ethnicity := Normal(0,10);
colRank := Binomial(param,0.6);
yExp := Binomial(20,0.5);
if (ethnicity > 10){
        colRank := (colRank + 5);
}
expRank := (yExp - colRank);
if ((colRank <= 5) or (expRank > -5)) {
    hire := 1;
}
else {
    hire := 0;
};
return hire;
```

Fairness [2]: A decision making program that chooses whether to hire an individual sampled from a given population distribution. We continualize the Binomial to Gaussian (which was used in the original).

```
education_num := Binomial(30,0.33); // prior on education level
age := Normal(38.5816, 186.0614);
sex := Bernoulli(0.33);
capital_gain := Normal(1077.6488, 54542539.1784);
capital_loss := Normal(87.3038, 162376.9378);
hours_per_week := Normal(40.4374, 152.4589);

N_age := ((age - 17.0) / 62.0);
N_education_num := ((education_num - 3.0) / 13.0);
N_capital_gain := ((capital_gain - 0.0) / 22040.0);
N_capital_loss := ((capital_loss - 0.0) / 1258.0);
N_hours_per_week := (hours_per_week - 4.0) / 73.0);
```

```

t := (-0.0106 * N_age) + (-0.0194 * N_education_num) + (-5.7412 * N_capital_gain)
    + (0.0086 * N_capital_loss) + (-0.0123*N_hours_per_week) + 1.0257;

if (sex > 0) {
     t := t + -0.0043;
}
return t;
```

Fairness SVM Example [2]. An SVM classifier

```
skillA := Poisson(100); //poisson prior on skill
skillB := 100;
perfA1 := Binomial(skillA,0.9);
perfB1 := Binomial(skillB,0.9);
if (perfA1 > perfB1) {
    res := 1;
}
else {
    res := 0;
};
return res;
```

TrueSkill [30]: A popular standard rating system for online gaming matches. We continualize the Poisson to a Gaussian (which was used in the original) and estimate the player's skill given multiple matches.

```
param := Uniform(0,10) //prior on position
which := Uniform(0,1);
if (which < 0.9){
    x := Normal(param,1);
}
else {
    if (which < 0.95){
        x := Uniform(0,10);
    }
    else {
        x := 10;
    };
};
return x;
```

SVE [58]: A Discrete-Continuous mixture modeling a robot localizing itself from observed sensor readings, hence we estimate the position which has a uniform prior. We continualize the constant (the original had a near-constant triangular distribution).

```
param := Beta(1,1)
successes := Binomial(100,param);
return successes;
```

Beta Binomial Rate Inference [39]: A Beta Binomial from the Cognitive Science literature, where we must infer the success rate given the number of trials.

```
param := DiscUniform(500,1500);
NumMet := Binomial(param,0.5);
NumInfected := Binomial(NumMet,0.3);
return NumInfected;
```

Discrete Disease Model: a chain double Binomial model (based on [4]) of an infectious disease spread, where we estimate the amount of contacts having observed the number of sick patients.

```
param := DiscUniform(10,50)
PlanktonCount := Binomial(param,0.5);
return PlanktonCount;
```

Plankton: An ecological model taken directly from [10] that highlights the difficulties in estimating n for discrete models of plankton populations.

```
param := Uniform(0.5,1) //prior on the probability of getting question
    right
percentile := Uniform(0,1);
if (percentile > 0.5) //top half of the class
    k := Binomial(20,param)
else:
    k := Binomial(20,0.5)   //0.5 is guessing probability
return k;
```

Exam: Another Cognitive Science model taken from [39] that has a latent mixture of students (those who studied vs. those who did not) and a different probability of getting a question right for each group (the group who did not study simply has 0.5 chance of getting a T/F question correct, while the group who studied could have their probability lie between 0.5 and 1)

10.2 WebPPL MCMC

Below are the inference times in WebPPL for 3500 samples for multiple β.

Table 4: Inference Times (s) and Error Ratios for each model, $\beta = 0.001$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Election	-	-	3.053	0.039	0.628	0.04
SVE	-	-	0.514	0.089	0.505	0.095
TrueSkill	3.668	0.01	0.524	0.044	0.592	0.107
DiscreteDiseaseModel	4.944	0.009	1.352	0.016	0.495	0.014
SVMfairness	-	-	0.634	0.503	0.994	0.361
Exam	3.973	0.087	0.511	0.122	0.513	0.147
Fairness	4.396	0.057	0.574	0.058	0.602	0.103
GPAExample	0.806	0.09	0.719	0.097	0.594	0.066
BetaBinomial	1.224	0.028	0.553	0.022	0.461	0.015
Plankton	0.57	0.017	0.454	0.065	0.481	0.04

Table 5: Inference Times (s) and Error Ratios for each model, $\beta = 0.05$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Election	-	-	3.306	0.043	0.613	0.068
SVE	-	-	0.532	0.135	0.507	0.12
TrueSkill	3.668	0.01	0.516	0.055	0.588	0.11
DiscreteDiseaseModel	4.944	0.009	1.38	0.024	0.484	0.015
SVMfairness	-	-	0.626	0.471	1.106	0.309
Exam	3.973	0.087	0.509	0.119	0.502	0.12
Fairness	4.396	0.057	0.55	0.052	0.598	0.078
GPAExample	0.806	0.09	0.679	0.096	0.59	0.083
BetaBinomial	1.224	0.028	0.541	0.011	0.458	0.016
Plankton	0.57	0.017	0.445	0.073	0.456	0.032

Table 6: Inference Times (s) and Error Ratios for each model, $\beta = 0.1$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Election	-	-	3.232	0.051	0.616	0.036
SVE	-	-	0.522	0.045	0.516	0.092
TrueSkill	3.668	0.01	0.494	0.059	0.587	0.053
DiscreteDiseaseModel	4.944	0.009	1.351	0.013	0.49	0.009
SVMfairness	-	-	0.626	0.454	0.98	0.261
Exam	3.973	0.087	0.504	0.126	0.527	0.133
Fairness	4.396	0.057	0.563	0.056	0.603	0.093
GPAExample	0.806	0.09	0.631	0.07	0.605	0.058
BetaBinomial	1.224	0.028	0.564	0.024	0.459	0.013
Plankton	0.57	0.017	0.457	0.08	0.454	0.042

Table 7: Inference Times (s) and Error Ratios for each model, $\beta = 0.25$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Election	-	-	3.212	0.04	0.666	0.053
SVE	-	-	0.503	0.099	0.525	0.076
TrueSkill	3.668	0.01	0.502	0.058	0.602	0.064
DiscreteDiseaseModel	4.944	0.009	1.38	0.016	0.504	0.01
SVMfairness	-	-	0.643	0.527	1.055	0.361
Exam	3.973	0.087	0.514	0.118	0.509	0.13
Fairness	4.396	0.057	0.554	0.056	0.591	0.064
GPAExample	0.806	0.09	0.625	0.098	0.598	0.075
BetaBinomial	1.224	0.028	0.554	0.027	0.469	0.015
Plankton	0.57	0.017	0.47	0.08	0.466	0.035

Table 8: Inference Times (s) and Error Ratios for each model, $\beta = 0.5$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Election	-	-	2.967	0.044	0.602	0.045
SVE	-	-	0.534	0.063	0.504	0.094
TrueSkill	3.668	0.01	0.502	0.04	0.594	0.091
DiscreteDiseaseModel	4.944	0.009	1.405	0.01	0.527	0.008
SVMfairness	-	-	0.637	0.372	1.015	0.331
Exam	3.973	0.087	0.504	0.122	0.516	0.104
Fairness	4.396	0.057	0.602	0.054	0.601	0.079
GPAExample	0.806	0.09	0.697	0.087	0.609	0.117
BetaBinomial	1.224	0.028	0.544	0.024	0.462	0.019
Plankton	0.57	0.017	0.452	0.057	0.473	0.043

Table 9: Inference Times (s) and Error Ratios for each model, $\beta = 1$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Election	-	-	2.94	0.048	0.61	0.065
SVE	-	-	0.513	0.06	0.506	0.115
TrueSkill	3.668	0.01	0.515	0.054	0.606	0.076
DiscreteDiseaseModel	4.944	0.009	1.27	0.023	0.489	0.012
SVMfairness	-	-	0.63	0.381	1.024	0.296
Exam	3.973	0.087	0.556	0.113	0.52	0.117
Fairness	4.396	0.057	0.566	0.062	0.586	0.054
GPAExample	0.806	0.09	0.646	0.083	0.594	0.086
BetaBinomial	1.224	0.028	0.546	0.022	0.499	0.015
Plankton	0.57	0.017	0.461	0.064	0.469	0.031

10.3 Pyro Variational Inference

Below are the inference times in Pyro for multiple β.

Table 10: Variational Inference Times (s) and Error Ratios for each model, $\beta = 0.01$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Exam	-	-	-	-	0.602	0.201
SVE	0.677	0.684	1.455	2.458	1.461	0.697
TrueSkill	-	-	-	-	1.797	0.064
BetaBinomial	-	-	-	-	1.589	0.583
DiscreteDisease	-	-	-	-	1.736	0.471
Fairness	-	-	-	-	1.836	0.792
SVMfairness	-	-	-	-	1.803	0.33
Election	-	-	-	-	1.747	0.071
GPAExample	-	-	-	-	3.282	0.244
Plankton	-	-	-	-	3.412	0.903

Table 11: Variational Inference Times (s) and Error Ratios for each model, $\beta = 0.1$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Exam	-	-	-	-	0.603	0.222
SVE	0.677	0.684	1.478	3.095	1.471	0.587
TrueSkill	-	-	-	-	1.809	0.119
BetaBinomial	-	-	-	-	1.605	0.834
DiscreteDisease	-	-	-	-	1.734	0.248
Fairness	-	-	-	-	1.813	0.722
SVMfairness	-	-	-	-	1.8	0.2
Election	-	-	-	-	1.762	0.07
GPAExample	-	-	-	-	3.111	0.207
Plankton	-	-	-	-	3.432	0.297

Table 12: Variational Inference Times (s) and Error Ratios for each model, $\beta = 0.25$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Exam	-	-	-	-	0.603	0.351
SVE	0.677	0.684	1.386	2.33	1.463	0.557
TrueSkill	-	-	-	-	1.768	0.085
BetaBinomial	-	-	-	-	1.6	0.694
DiscreteDisease	-	-	-	-	1.748	0.452
Fairness	-	-	-	-	1.805	0.738
SVMfairness	-	-	-	-	1.801	0.3
Election	-	-	-	-	1.762	0.118
GPAExample	-	-	-	-	3.213	0.217
Plankton	-	-	-	-	3.421	0.833

Table 13: Variational Inference Times (s) and Error Ratios for each model, $\beta = 0.5$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Exam	-	-	-	-	0.602	0.213
SVE	0.677	0.684	1.374	2.941	1.46	0.566
TrueSkill	-	-	-	-	1.802	0.062
BetaBinomial	-	-	-	-	1.596	0.708
DiscreteDisease	-	-	-	-	1.731	0.471
Fairness	-	-	-	-	1.827	0.769
SVMfairness	-	-	-	-	1.806	0.293
Election	-	-	-	-	1.755	0.11
GPAExample	-	-	-	-	3.341	0.241
Plankton	-	-	-	-	3.427	0.763

Table 14: Variational Inference Times (s) and Error Ratios for each model, $\beta = 0.75$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Exam	-	-	-	-	0.603	0.285
SVE	0.677	0.684	1.393	1.48	1.448	0.348
TrueSkill	-	-	-	-	1.79	0.09
BetaBinomial	-	-	-	-	1.587	0.497
DiscreteDisease	-	-	-	-	1.747	0.553
Fairness	-	-	-	-	1.83	0.753
SVMfairness	-	-	-	-	1.804	0.301
Election	-	-	-	-	1.764	0.064
GPAExample	-	-	-	-	3.435	0.321
Plankton	-	-	-	-	3.434	0.53

Table 15: Variational Inference Times (s) and Error Ratios for each model, $\beta = 1.0$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Exam	-	-	-	-	0.602	0.236
SVE	0.677	0.684	1.461	2.382	1.458	0.733
TrueSkill	-	-	-	-	1.789	0.083
BetaBinomial	-	-	-	-	1.595	0.615
DiscreteDisease	-	-	-	-	1.768	0.379
Fairness	-	-	-	-	1.826	0.749
SVMfairness	-	-	-	-	1.807	0.362
Election	-	-	-	-	1.759	0.107
GPAExample	-	-	-	-	3.374	0.199
Plankton	-	-	-	-	3.437	0.599

Table 16: Variational Inference Times (s) and Error Ratios for each model, $\beta = 2.0$

Program	Orig. Time	Orig. Error	Naive Time	Naive Error	Leios Time	Leios Error
Exam	-	-	-	-	0.604	0.3
SVE	0.677	0.684	1.301	3.458	1.473	0.509
TrueSkill	-	-	-	-	1.79	0.073
BetaBinomial	-	-	-	-	1.584	0.732
DiscreteDisease	-	-	-	-	1.747	0.606
Fairness	-	-	-	-	1.828	0.693
SVMfairness	-	-	-	-	1.815	0.307
Election	-	-	-	-	1.753	0.079
GPAExample	-	-	-	-	3.365	0.229
Plankton	-	-	-	-	3.44	0.791