Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2009, Article ID 617320, 12 pages
doi:10.1155/2009/617320

Research Article

Cooperative Detection for Primary User in Cognitive Radio Networks

Jia Zhu, Yulong Zou, and Baoyu Zheng

Institute of Signal Processing and Transmission, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu 210003, China

Correspondence should be addressed to Yulong Zou, zouyulong198412@126.com

Received 26 May 2009; Revised 3 August 2009; Accepted 5 November 2009

Recommended by Vincent Lau

We propose two novel cooperative detection schemes based on the AF (Amplify and Forward) and DF (Decode and Forward) protocols to achieve spatial diversity gains for cognitive radio networks, which are referred to as the AF-CDS, (AF-based Cooperative Detection Scheme) and DF-CDS (DF-based Cooperative Detection Scheme), respectively. Closed-form expressions of detection probabilities for the noncooperation scheme, AND-CDS (AND-based Cooperative Detection Scheme), AF-CDS and DF-CDS, are derived over Rayleigh fading channels. Also, we analyze the overall agility for the proposed cooperative detection schemes and show that our schemes can further reduce the detection time. In addition, we compare the DF-CDS with the AF-CDS in terms of detection probability and agility gain, depicting the advantage of DF-CDS at low SNR region and high false alarm probability region.

1. Introduction

Cognitive radio (CR), built on software-defined radio, has been proposed in [1] as a means to promote the efficient use of the precious radio spectrum resources. It is defined as an intelligent wireless communication system [2] that is aware of the surrounding environment and utilizes the methodology of understanding-by-building to learn from the environment. Spectrum detection technique (also referred to as spectrum sensing) enables CR networks to adapt to the environment by detecting spectrum holes, and the most efficient way to detect the spectrum holes is to detect the presence of primary users [3]. In reality, however, it is difficult for a cognitive radio to have a direct measurement of the channel between a primary receiver and a transmitter. Therefore, the most recent work focuses on the primary transmitter detection based on local observations of secondary users (see [4–7]). Generally speaking, the spectrum sensing schemes proposed in recent years can be classified as noncooperative detection and cooperative detection.

At present, three noncooperative transmitter detection methods, namely, the matched filter detection, the energy detection and the cyclostationary feature detection, have been presented for CR networks. In [4], Sahai et al. have investigated the matched filter detector that can achieve high processing gain by employing coherent reception. In [5], energy detector has been put forward as an optimal method for the occasion where the secondary users cannot gather sufficient information about the primary user signal such as the modulation type, the pulse shape and so on, but it cannot differentiate signal types, thus inclining to false detection triggered by some unintended signals. The cyclostationary feature detection, as an alternative method, has been further presented in [6, 7], which can differentiate the modulated signal from the additive noise. However, this scheme is computationally complex and requires long observation time.

As to the cooperative detection, a collaborative spectrum sensing method has been proposed by Ghasemi and Sousa in [8], where the hard decision about the presence of the primary user from each secondary user is pooled together to determine the presence of primary user by utilizing a majority logic rule without the consideration of cooperative technology that has been proven as an effective means to combat Rayleigh fading [9–12]. More recently, Ganesan and Li [13, 14] have applied the AF protocol to the detection of primary users, and shown that by allowing the secondary users to cooperate with each other the detection time can be reduced. However, this cooperative scheme needs a centralized controller to manage all secondary users, which is some unreasonable for a practical wireless communication system. Besides, how to detect the presence of primary users through cooperation in DF-based CR networks, where the cooperative user has the ability to decode its received signal, is an open challenge.

In this paper, we address the above mentioned issues and present (1) a more practical AF-based cooperative detection scheme without the assumption of centralized controller and (2) a totally new DF-based cooperative detection scheme for the occasion where the cooperative relay has decoding ability. Our main contributions can be described as follows. Firstly, we propose two new cooperative detection schemes, namely, AF-CDS and DF-CDS, to detect the presence of primary users more quickly and accurately. Secondly, we develop the closed-form expressions of detection probability and detection time for both AF-CDS and DF-CDS over Rayleigh fading channels. Thirdly, the performance analysis for the noncooperation scheme and the AND-CDS (AND-based cooperative detection scheme) is also presented for the purpose of comparison with our schemes.

The remainder of this paper is organized as follows. Section 2 describes the system model used throughout this paper and proposes two new cooperative detection schemes (i.e., AF-CDS and DF-CDS) to improve the detection performance of cognitive radio network. In Section 3, we derive the closed-form expressions of detection probabilities and agility gains for the AF-CDS and DF-CDS as well as the traditional noncooperation scheme and AND-CDS, followed by numerical results analysis in Section 4, where we show the superiority of the proposed AF-CDS and DF-CDS schemes in terms of detection performances. Finally, we make some concluding remarks in Section 5.

2. Proposed Cooperative Detection Schemes

In this section, we first describe the system model used in the paper, and then propose AF-CDS and DF-CDS to improve the detection performance of CR networks.

2.1. System Model. Consider a cognitive radio network with a primary user and two secondary users as shown in Figure 1, where the wireless link between the primary user and the secondary user $S1$ occurs shadowing fading, and $S2$ acts as a cooperative relay for $S1$. All users in the CR network are equipped with a single antenna, and the antenna at any user can be utilized for both transmission and reception. There is independent, additive complex white Gaussian noise with zero-mean and double-sided power spectral density N_0 at each receiver.

Without loss of generality, let s be the primary user indicator, namely, $s = Es$ implies the presence of primary user and $s = 0$ implies its absence. Therefore, the signals received by secondary users $S1$ and $S2$ can be expressed as

$$r_1(k) = h_{p1}(k)s + n_1(k), \tag{1}$$

$$r_2(k) = h_{p2}(k)s + n_2(k), \tag{2}$$

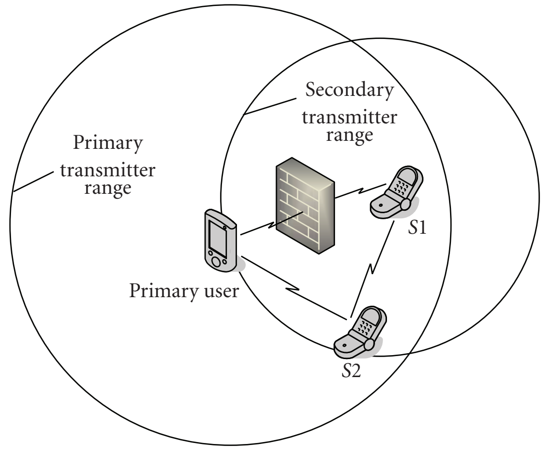

Figure 1: Cooperative spectrum sensing model in cognitive radio networks.

where the subscripts $p1$ and $p2$ denote the transmission from the primary user to $S1$ and that from the primary user to $S2$, respectively. Besides, $h_{p1}(k)$, $h_{p2}(k)$ and $h_{21}(k)$ are the fading coefficients of the wireless channel from the primary user to $S1$, from the primary user to $S2$, and from $S2$ to $S1$, respectively. Note that the variances of the three random variables (RVs) $h_{p1}(k)$, $h_{p2}(k)$ and $h_{21}(k)$ are $\sigma_{p1}^2, \sigma_{p2}^2$ and σ_{21}^2, respectively. Throughout the paper, we make following assumptions: (1) All wireless channels are independent from each other in space; (2) The relaying protocols (i.e., the AF and the DF) employ full duplex mode, meaning that the relay can perform signal reception when transmits.

2.2. AF-Based Cooperative Detection Scheme. An important requirement of a cognitive radio network is to detect the presence of primary users as quickly as possible. Suppose that the primary user starts using the spectrum band. Then, the two secondary users need to sense the unavailability of the band as soon as possible to avoid collision with primary user. However, when the wireless link between the primary user and $S1$ encounters shadowing fading, the signal received by $S1$ from the primary user is so weak that $S1$ takes a long time to detect its presence. We show that by cooperation with $S2$ the detection probability of $S1$ can be increased, thus reducing the overall detection time of the CR network.

Throughout the paper, we allow the secondary user $S2$ to act as a cooperative relay for $S1$. Figure 1 describes a scenario where two secondary users $S1$ and $S2$ are engaged in detecting the presence of primary user in a particular band. The whole implementation process of AF-CDS can be separated into two consecutive phases, that is, (1) in odd time slot $2k-1$, both $S1$ and $S2$ receive the signals transmitted from primary user; (2) in even time slot $2k, S2$ starts relaying its received information to $S1$ in accordance with AF protocol, and thus, in this time slot, $S1$ would receive two signal copies simultaneously from the primary user and $S2$, respectively.

In the time slot $2k-1$, the signals received by $S1$ and $S2$ can be expressed as

$$r_1(2k-1) = h_{p1}(2k-1)s + n_1(2k-1), \tag{3}$$

$$r_2(2k-1) = h_{p2}(2k-1)s + n_2(2k-1), \tag{4}$$

where $n_1(2k-1)$ and $n_2(2k-1)$ are the additive complex Gaussian noise with zero mean and double-sided power spectral density N_0, and independent from each other. In the even slot $2k$, the cooperative user, $S2$, relays the message from primary user to $S1$. According to AF protocol, the received signal $r_2(2k-1)$ as defined in (3) will be multiplied by a relay gain G, and then forwarded to $S1$ without any sort of decoding. Meanwhile, in the slot $2k, S1$ simultaneously receives the signal from primary user. Therefore, the signal received by $S1$ in even time slot $2k$ can be written as

$$r_{1,AF}(2k) = h_{p1}(2k)s + Gh_{21}(2k)r_2(2k-1) + n_1(2k), \tag{5}$$

where $h_{21}(2k)$ denotes the instantaneous fading coefficient of the wireless channel from $S2$ to $S1$, and $n_2(2k)$ denotes the zero-mean additive complex Gaussian noise with double-sided power spectral density N_0. Substituting $r_2(2k-1)$ from (4) into (5) gives

$$\begin{aligned} r_{1,\mathrm{AF}}(2k) = {} & h_{p1}(2k)s + Gh_{21}(2k)h_{p2}(2k-1)s \\ & + Gh_{21}(2k)n_2(2k-1) + n_1(2k). \end{aligned} \tag{6}$$

For the convenience of theoretical analysis, consider relay gain $G = 1/h_{21}(2k)$ to compensate the fading distortion from $S2$ to $S1$. Substituting this result into (6) yields

$$r_{1,\mathrm{AF}}(2k) = h_{p1}(2k)s + h_{p2}(2k-1)s + n_2(2k-1) + n_1(2k). \tag{7}$$

Now, the detection problem of $S1$ under AF-CDS can be stated as follows: given the observation

$$r_1(2k-1) = h_{p1}(2k-1)s + n_1(2k-1) \tag{8}$$

in the odd time slot $2k-1$, *and*

$$r_{1,\mathrm{AF}}(2k) = h_{p1}(2k)s + h_{p2}(2k-1)s + n_2(2k-1) + n_1(2k) \tag{9}$$

in the even time slot $2k$, the detector decides on

$$\begin{aligned} H_1 &: s = Es \\ H_0 &: s = 0. \end{aligned} \tag{10}$$

This is a standard detection problem for which there are many choices of detector available such as energy detector, matched filter detector and cyclostationary feature detector in the literatures [4–7]. In this paper, we use the energy detector (ED) [15] to show the advantage of the proposed cooperative scheme. The reasons for choosing ED are two-fold [13]: (1) We want to show the effect of user cooperation on detection of primary user in CR networks. Hence, the choice of detector is not critical; (2) We model the signal as a random variable with known power, and thus ED is optimal [15]. Let $T_1(H_0)$ and $T_1(H_1)$ be the output power of the ED of the secondary user $S1$ in the odd time slot $2k-1$ under hypothesis H_0 and H_1, respectively. Thus, from (8), we can easily obtain

$$\begin{aligned} T_1(H_0) &= |n_1(2k-1)|^2, \\ T_1(H_1) &= \left|h_{p1}(2k-1)\right|^2 Es + N_0. \end{aligned} \tag{11}$$

Similarly, define $T_{1,\mathrm{AF}}(H_0)$ and $T_{1,\mathrm{AF}}(H_1)$ as the output power of $S1$'s ED in the even slot $2k$ under hypothesis H_0 and H_1, respectively. Hence, from (9), we can get

$$T_{1,\mathrm{AF}}(H_0) = |n_1(2k)|^2 + |n_2(2k-1)|^2, \tag{12}$$

$$T_{1,\mathrm{AF}}(H_1) = \left|h_{p1}(2k)\right|^2 Es + \left|h_{p2}(2k-1)\right|^2 Es + 2N_0. \tag{13}$$

We will use (11)–(13) listed above to analyze the detection probability and detection time for the proposed AF-CDS in Section 3.

2.3. DF-Based Cooperative Detection Scheme. In this subsection, we present DF-CDS to achieve a better detection performance for CR networks. Consider the same scenario as AF-CDS with two secondary users $S1$ and $S2$ operating in a fixed TDMA mode for detecting the presence of primary user (see Figure 1). Here, according to DF protocol, the cooperative user $S2$ should regenerate the primary user indicator $\hat{s}$ based on its received signals, and then transmit the estimated indicator $\hat{s}$ to $S1$. Consequently, the detailed process of DF-CDS can be described as follows: (1) in the odd time slot $2k-1$, both $S1$ and $S2$ receive the signal from the primary user; (2) in the even time slot $2k, S2$ decodes its received information and forwards the decoding result to $S1$. Clearly, in odd time slots, the process of DF-CDS is the same as AF-CDS, implying that the output power of energy detector of $S1$ for DF-CDS in odd time slots can also be expressed as (11). Besides, in the even time slot $2k$, the signal received by $S1$ can be given by

$$r_{1,\mathrm{DF}}(2k) = h_{p1}(2k)s + h_{21}(2k)\hat{s} + n_1(2k). \tag{14}$$

One can see that there are two possible cases for the decoding result of the primary user indicator, namely, the correct and the wrong decisions. Without loss of generality, let cases $\theta = 0$ and $\theta = 1$ denote the estimated indicator $\hat{s} = s$ and $\hat{s} \neq s$, respectively. Let $T_{1,\mathrm{DF}}(\theta = 0, H_0)$ and $T_{1,\mathrm{DF}}(\theta = 0, H_1)$ be the output power of $S1$'s ED in even time slots for case $\theta = 0$ under hypothesis H_0 and H_1, respectively. Thus, from (14), it is easy to obtain

$$\begin{aligned} T_{1,\mathrm{DF}}(\theta = 0, H_0) &= |n_1(2k)|^2, \\ T_{1,\mathrm{DF}}(\theta = 0, H_1) &= \left|h_{p1}(2k)\right|^2 Es + |h_{21}(2k)|^2 Es + N_0. \end{aligned} \tag{15}$$

Similarly, define $T_{1,\mathrm{DF}}(\theta = 1, H_0)$ and $T_{1,\mathrm{DF}}(\theta = 1, H_1)$ as the output power of $S1$'s ED for case $\theta = 1$ under hypothesis H_0 and H_1, respectively. Thus, we can easily get

$$T_{1,\mathrm{DF}}(\theta = 1, H_0) = |h_{21}(2k)|^2 Es + N_0,$$
$$T_{1,\mathrm{DF}}(\theta = 1, H_1) = \left|h_{p1}(2k)\right|^2 Es + N_0. \tag{16}$$

Now, we have described the system model and formulated the primary user detection problems for AF-CDS and DF-CDS, based on which a detailed performance analysis will be presented in the following.

3. Performance Analysis of the Proposed Schemes over Rayleigh Fading Channels

In this section, we investigate the detection probability and detection time for the proposed AF-CDS and DF-CDS in Rayleigh fading channels. For the purpose of comparison, let us consider first the noncooperative detection scheme and the existing AND-CDS (AND-based cooperative detection scheme) proposed in [8]. Given the transmitted signal s with power Es, the signal received by the user $S1$ can be expressed as

$$r_1(k) = h_{p1}(k)s + n_1(k). \tag{17}$$

Hence, according to the energy detector principles, we can easily calculate the probability of detection of primary user by $S1$ under noncooperative scheme as

$$P_{1,n} = \Pr\left\{\left|h_{p1}(k)\right|^2 > \frac{\lambda - N_0}{Es}\right\}, \tag{18}$$

where λ is energy detection threshold that is determined by false alarm probability, which will be illustrated in the following. Note that the PDF of RV $X = |h_{p1}(k)|^2$ can be given by

$$f_X(x) = \frac{1}{\sigma_{p1}^2}\exp\left(-\frac{x}{\sigma_{p1}^2}\right)U(x), \tag{19}$$

where $U(\cdot)$ is a unit step function. Thus, (18) can be further calculated by

$$P_{1,n} = \exp\left(-\frac{1}{\sigma_{p1}^2}\max\left(\frac{\lambda - N_0}{Es}, 0\right)\right). \tag{20}$$

Furthermore, from (17), the false alarm probability $\alpha(0 \le \alpha \le 1)$ can be given by

$$\alpha = \Pr\left\{|n_1(k)|^2 > \lambda\right\}. \tag{21}$$

Noting that $n_1(k)$ is zero-mean complex Gaussian noise with double-sided spectral power density N_0, we can easily obtain $|n_1(k)|^2 \sim \varepsilon(1/2N_0)$. Thus, the false alarm probability is calculated as

$$\alpha = \exp\left(-\frac{\max(\lambda, 0)}{2N_0}\right) \tag{22}$$

which results in

$$\lambda = -2N_0 \ln\alpha \ge 0. \tag{23}$$

Substituting the threshold λ from (23) into (20) yields

$$P_{1,n} = \exp\left(-\frac{1}{\sigma_{p1}^2\gamma_s}\max(-2\ln\alpha - 1, 0)\right), \tag{24}$$

where $\gamma_s = Es/N_0$ is the transmitting signal-to-noise ratio (T-SNR). In a similar way, the detection probability of $S2$ under noncooperative scheme can be given by

$$P_{2,n} = \exp\left(-\frac{1}{\sigma_{p2}^2\gamma_s}\max(-2\ln\alpha - 1, 0)\right). \tag{25}$$

We now investigate the overall detection probability for the cognitive radio network scenario depicted as Figure 1, that is, the probability that the presence of primary user is detected by both $S1$ and $S2$. Since the two users independently detect the primary user in a distributed way, it can be easily shown that the overall detection probability is given by

$$P_n = P_{1,n}P_{2,n}, \tag{26}$$

where $P_{1,n}$ and $P_{2,n}$ are defined in (24) and (25), respectively. Besides, let $\tau_{i,n}(i = 1, 2)$ be the number of slots taken by $S1$ (and $S2$) to detect the presence of primary user under the noncooperation scheme. This detection time $\tau_{i,n}$ can be modeled as a geometric random variable, that is,

$$\Pr\{\tau_{i,n} = k\} = (1 - P_{i,n})^{k-1}P_{i,n}. \tag{27}$$

It can be easily shown that the overall detection time τ_n is given by

$$\tau_n = \max(\tau_{1,n}, \tau_{2,n}). \tag{28}$$

Thus, from (28), we can calculate the average overall detection time T_n as

$$\begin{aligned}\mathrm{T}_n &= \sum_{k=1}^{+\infty} k\Pr(\tau_{1,n} = k, \tau_{2,n} \le k) + \sum_{k=1}^{+\infty} k\Pr(\tau_{1,n} < k, \tau_{2,n} = k)\\ &= \frac{1}{1 - \overline{P}_{1,n}} + \frac{1}{1 - \overline{P}_{2,n}} - \frac{1}{1 - \overline{P}_{1,n}\overline{P}_{2,n}},\end{aligned} \tag{29}$$

where $\overline{P}_{1,n} = 1 - P_{1,n}$ and $\overline{P}_{2,n} = 1 - P_{2,n}$. Then, we focus on the detection performance analysis for the existing cooperative sensing as presented in [8], where an AND-based cooperative detection scheme (AND-CDS) is proposed in order to combat the fading environments. Similarly to the sense protocols of the AF-CDS and DF-CDS, the implementation process of the AND-CDS is also divided into two phases for the convenience of making a fair comparison with our scheme, that is, (1) in odd time slot $2k - 1$, $S1$ and $S2$ detect independently whether or not the primary user is active; (2) in subsequent even time slot $2k$, $S2$ forwards its detected result to $S1$ who would then use the AND rule to fuse the detection results of $S1$ and $S2$. Clearly, it is easy to show the probability of detection of primary user by $S1$ in odd time slots being given by

$$P_{1,\mathrm{AND}}^{(o)} = \exp\left(-\frac{1}{\sigma_{p1}^2\gamma_s}\max(-2\ln\alpha - 1, 0)\right), \quad (30)$$

where α is the false alarm probability. In even time slots, $S1$ would fuse the decision results by using AND rule, and thus, the corresponding detection probability and false alarm probability are calculated as (see [8] for details)

$$\mathrm{P}_{1,\mathrm{AND}}^{(e)} = \Pr\left\{\left|h_{p1}(k)\right|^2 Es + N_0 > \lambda, \left|h_{p2}(k)\right|^2 Es + N_0 > \lambda\right\}, \quad (31)$$

$$\alpha = \Pr\left\{|n_1(k)|^2 > \lambda, |n_2(k)|^2 > \lambda\right\}. \quad (32)$$

Combining (31) and (32) gives

$$P_{1,\mathrm{AND}}^{(e)} = \exp\left(-\frac{\max(-\ln\alpha - 1, 0)}{\sigma_{p1}^2\gamma_s}\right)\exp\left(-\frac{\max(-\ln\alpha - 1, 0)}{\sigma_{p2}^2\gamma_s}\right). \quad (33)$$

Hence, for any given time slot k, we can calculate the probability of detection of primary user by $S1$ using AND-based cooperation scheme as

$$P_{1,\mathrm{AND}} = P(k = o)P_{1,\mathrm{AND}}^{(o)} + P(k = e)P_{1,\mathrm{AND}}^{(e)}, \quad (34)$$

where $P(k = o)$ and $P(k = e)$ are the probabilities of time slot k belonging to odd and even, respectively. Generally, the probability events $\{k = o, k = e\}$ follow an equal probability distribution, giving $P(k = o) = P(k = e) = 1/2$. Substituting this result into (34) yields

$$P_{1,\mathrm{AF}} = \frac{P_{1,\mathrm{AND}}^{(o)} + P_{1,\mathrm{AND}}^{(e)}}{2}. \quad (35)$$

In addition, the detection probability of $S2$ is easily given by

$$P_{2,\mathrm{AND}} = \exp\left(-\frac{\max(-2\ln\alpha - 1, 0)}{\sigma_{p2}^2\gamma_s}\right). \quad (36)$$

Therefore, the overall detection probability of the cognitive system using the AND-CDS can be given by

$$P_{\mathrm{AND}} = P_{1,\mathrm{AND}}P_{2,\mathrm{AND}}, \quad (37)$$

where the parameters $P_{1,\mathrm{AND}}$ and $P_{2,\mathrm{AND}}$ are defined in (35) and (36), respectively. Besides, the detection time slots, $\tau_{1,\mathrm{AND}}$ and $\tau_{2,\mathrm{AND}}$, taken by the secondary users $S1$ and $S2$ using AND-CDS, respectively, can be modeled as

$$\begin{aligned}
&\Pr\{\tau_{1,\mathrm{AND}} = m\} \\
&= \begin{cases} \left(\overline{P}_{1,\mathrm{AND}}^{(o)}\right)^{k-1}\left(\overline{P}_{1,\mathrm{AND}}^{(e)}\right)^{k-1}P_{1,\mathrm{AND}}^{(o)}, & m = 2k - 1, \\ \left(\overline{P}_{1,\mathrm{AND}}^{(o)}\right)^{k}\left(\overline{P}_{1,\mathrm{AND}}^{(e)}\right)^{k-1}P_{1,\mathrm{AND}}^{(e)}, & m = 2k, \end{cases} \\
&\Pr\{\tau_{2,\mathrm{AND}} = k\} = \left(\overline{P}_{2,\mathrm{AND}}\right)^{k-1}P_{2,\mathrm{AND}},
\end{aligned} \quad (38)$$

where $\overline{P}_{1,\mathrm{AND}}^{(o)} = 1 - P_{1,\mathrm{AND}}^{(o)}$, $\overline{P}_{1,\mathrm{AND}}^{(e)} = 1 - P_{1,\mathrm{AND}}^{(e)}$ and $\overline{P}_{2,\mathrm{AND}} = 1 - P_{2,\mathrm{AND}}$. Thus, the overall detection time τ_{AND} of the CR network by using AND-CDS can be given by

$$\tau_{\mathrm{AND}} = \max(\tau_{1,\mathrm{AND}}, \tau_{2,\mathrm{AND}}), \quad (39)$$

from which the corresponding average detection time T_{AND} can be calculated as (see Appendix A for details)

$$\begin{aligned}
\mathrm{T}_{\mathrm{AND}} = &\sum_{k=1}^{+\infty}(2k-1)\left(\overline{P}_{1,\mathrm{AND}}^{(o)}\right)^{k-1}\left(\overline{P}_{1,\mathrm{AND}}^{(e)}\right)^{k-1}P_{1,\mathrm{AND}}^{(o)} \\
&\times\left[1 - \left(\overline{P}_{2,\mathrm{AND}}\right)^{2k-1}\right] \quad \text{(I)} \\
&+ \sum_{k=1}^{+\infty}(2k-1)\left[1 - \left(\overline{P}_{1,\mathrm{AND}}^{(o)}\overline{P}_{1,\mathrm{AND}}^{(e)}\right)^{k-1}\right] \\
&\times\left(\overline{P}_{2,\mathrm{AND}}\right)^{2k-2}P_{2,\mathrm{AND}} \quad \text{(II)} \\
&+ \sum_{k=1}^{+\infty}2k\left(\overline{P}_{1,\mathrm{AND}}^{(o)}\right)^{k}\left(\overline{P}_{1,\mathrm{AND}}^{(e)}\right)^{k-1}P_{1,\mathrm{AND}}^{(e)} \\
&\times\left[1 - \left(\overline{P}_{2,\mathrm{AND}}\right)^{2k}\right] \quad \text{(III)} \\
&+ \sum_{k=1}^{+\infty}2k\left[1 - \overline{P}_{1,\mathrm{AND}}^{(o)}\left(\overline{P}_{1,\mathrm{AND}}^{(o)}\overline{P}_{1,\mathrm{AND}}^{(e)}\right)^{k-1}\right] \\
&\times\left(\overline{P}_{2,\mathrm{AND}}\right)^{2k-1}P_{2,\mathrm{AND}}, \quad \text{(IV)}
\end{aligned} \quad (40)$$

where the closed-form expressions of items (I), (II), (III), and (IV) can be found in Appendix A. Based on (40), we define a agility gain of the AND-CDS over the noncooperation scheme as

$$G_{\mathrm{AND}} = \frac{T_n}{T_{\mathrm{AND}}}, \quad (41)$$

where the parameters T_n and T_{AND} are given in (29) and (40), respectively. In the following, we focus on deriving the closed-form expressions of detection probabilities and agility gains for the AF-CDS and DF-CDS.

3.1. Detection Performance of AF-CDS. Obviously, from (11), we can show that the probability of detection of primary user by $S1$ in odd time slots is given by

$$P_{1,\mathrm{AF}}^{(o)} = \exp\left(-\frac{1}{\sigma_{p1}^2 \gamma_s} \max(-2\ln\alpha - 1, 0)\right). \tag{42}$$

In addition, from (9), we can calculate the probability of detection of primary user by $S1$ in even slots as

$$P_{1,\mathrm{AF}}^{(e)} = \Pr\{T_{1,\mathrm{AF}}(H_1) > \lambda\}, \tag{43}$$

where $T_{1,\mathrm{AF}}(H_1)$ is given in (13) and λ is determined by

$$\alpha = \Pr\{T_{1,\mathrm{AF}}(H_0) > \lambda\}, \tag{44}$$

where $T_{1,\mathrm{AF}}(H_0)$ is given in (12). Thus, substituting $T_{1,\mathrm{AF}}(H_1)$ from (13) into (43) yields

$$P_{1,\mathrm{AF}}^{(e)} = \Pr\left\{ \left|h_{p1}(2k)\right|^2 + \left|h_{p2}(2k-1)\right|^2 > \frac{\lambda - 2N_0}{Es} \right\}. \tag{45}$$

Note that RVs $|h_{p1}(2k)|^2$ and $|h_{p2}(2k-1)|^2$ follow the exponential distribution with parameters $1/\sigma_{p1}^2$ and $1/\sigma_{p2}^2$, respectively, and independent from each other. Consequently, the joint PDF of $(x = |h_{p1}(2k)|^2, y = | h_{p2}(2k-1)|^2)$ can be written as

$$f(x, y) = \begin{cases} \dfrac{1}{\sigma_{p1}^2 \sigma_{p2}^2} \exp\left(-\dfrac{x}{\sigma_{p1}^2} - \dfrac{y}{\sigma_{p2}^2}\right) & x > 0, y > 0, \\ 0 & \text{others}, \end{cases} \tag{46}$$

combining (45) and (46) yields (see Appendix B for details)

$$P_{1,\mathrm{AF}}^{(e)} = \begin{cases} \left(1 + \dfrac{1}{\sigma_{p1}^2} \max\left(\dfrac{\lambda - 2N_0}{Es}, 0\right)\right) \\ \quad \times \exp\left(-\dfrac{1}{\sigma_{p1}^2} \max\left(\dfrac{\lambda - 2N_0}{Es}, 0\right)\right); \\ \qquad\qquad \sigma_{p1}^2 = \sigma_{p2}^2 \\ \dfrac{\sigma_{p1}^2}{\sigma_{p1}^2 - \sigma_{p2}^2} \exp\left(-\dfrac{1}{\sigma_{p1}^2} \max\left(\dfrac{\lambda - 2N_0}{Es}, 0\right)\right) \\ \quad + \dfrac{\sigma_{p2}^2}{\sigma_{p2}^2 - \sigma_{p1}^2} \exp\left(-\dfrac{1}{\sigma_{p2}^2} \max\left(\dfrac{\lambda - 2N_0}{Es}, 0\right)\right); \\ \qquad\qquad \sigma_{p1}^2 \neq \sigma_{p2}^2. \end{cases} \tag{47}$$

Besides, from (44), the corresponding false alarm probability α is given by

$$\alpha = \Pr\left\{|n_1(2k)|^2 + |n_2(2k-1)|^2 > \lambda\right\}, \tag{48}$$

from which we can obtain (see Appendix C for details)

$$\lambda = -2[W(-\alpha e^{-1}) + 1]N_0, \tag{49}$$

where $W(\cdot)$ is Lambert's W function. Substituting λ from (49) into (47) gives

$$P_{1,\mathrm{AF}}^{(e)} = \begin{cases} \left(1 + \dfrac{1}{\sigma_{p1}^2 \gamma_s} \max(-2W(-\alpha e^{-1}) - 4, 0)\right) \\ \quad \times \exp\left(-\dfrac{1}{\sigma_{p1}^2 \gamma_s} \max(-2W(-\alpha e^{-1}) - 4, 0)\right); \\ \qquad\qquad \sigma_{p1}^2 = \sigma_{p2}^2 \\ \dfrac{\sigma_{p1}^2}{\sigma_{p1}^2 - \sigma_{p2}^2} \exp\left(-\dfrac{1}{\sigma_{p1}^2 \gamma_s} \max(-2W(-\alpha e^{-1}) - 4, 0)\right) \\ \quad + \dfrac{\sigma_{p2}^2}{\sigma_{p2}^2 - \sigma_{p1}^2} \exp\left(-\dfrac{1}{\sigma_{p2}^2 \gamma_s} \max(-2W(-\alpha e^{-1}) - 4, 0)\right). \end{cases} \tag{50}$$

Similarly to (35), we can calculate the probability of detection of primary user by $S1$ under AF-CDS as

$$P_{1,\mathrm{AF}} = \frac{P_{1,\mathrm{AF}}^{(o)} + P_{1,\mathrm{AF}}^{(e)}}{2}. \tag{51}$$

Meanwhile, the detection probability of $S2$ under AF-CDS can be easily given by

$$P_{2,\mathrm{AF}} = P_{2,n} = \exp\left(-\frac{1}{\sigma_{p2}^2 \gamma_s} \max(-2\ln\alpha - 1, 0)\right). \tag{52}$$

Hence, we can obtain the overall detection probability for the AF-CDS as

$$P_{\mathrm{AF}} = P_{1,\mathrm{AF}} P_{2,\mathrm{AF}}, \tag{53}$$

where $P_{1,\mathrm{AF}}$ and $P_{2,\mathrm{AF}}$ are defined in (51) and (52), respectively. Also, the detection time slots, $\tau_{1,\mathrm{AF}}$ and $\tau_{2,\mathrm{AF}}$, taken by the secondary users $S1$ and $S2$, respectively, can be modeled as

$$\Pr\{\tau_{1,\mathrm{AF}} = m\} = \begin{cases} \left(\overline{P}_{1,\mathrm{AF}}^{(o)}\right)^{k-1} \left(\overline{P}_{1,\mathrm{AF}}^{(e)}\right)^{k-1} P_{1,\mathrm{AF}}^{(o)}, & m = 2k-1, \\ \left(\overline{P}_{1,\mathrm{AF}}^{(o)}\right)^{k} \left(\overline{P}_{1,\mathrm{AF}}^{(e)}\right)^{k-1} P_{1,\mathrm{AF}}^{(e)}, & m = 2k, \end{cases}$$

$$\Pr\{\tau_{2,\mathrm{AF}} = k\} = \left(\overline{P}_{2,\mathrm{AF}}\right)^{k-1} P_{2,\mathrm{AF}}, \tag{54}$$

where $\overline{P}_{1,\mathrm{AF}}^{(o)} = 1 - P_{1,\mathrm{AF}}^{(o)}, \overline{P}_{1,\mathrm{AF}}^{(e)} = 1 - P_{1,\mathrm{AF}}^{(e)}$ and $\overline{P}_{2,\mathrm{AF}} = 1 - P_{2,\mathrm{AF}}$. Similarly, the overall detection time τ_{AF} of the CR network under AF-CDS can be given by

$$\tau_{\mathrm{AF}} = \max(\tau_{1,\mathrm{AF}}, \tau_{2,\mathrm{AF}}), \tag{55}$$

from which the corresponding average detection time T_{AF} can be calculated as

$$\begin{aligned} T_{\mathrm{AF}} = \sum_{k=1}^{+\infty}&(2k-1)\left(\overline{P}_{1,\mathrm{AF}}^{(o)}\right)^{k-1}\left(\overline{P}_{1,\mathrm{AF}}^{(e)}\right)^{k-1}P_{1,\mathrm{AF}}^{(o)} \\ &\times\left[1-\left(\overline{P}_{2,\mathrm{AF}}\right)^{2k-1}\right] \qquad \text{(I)} \\ &+\sum_{k=1}^{+\infty}(2k-1)\left[1-\left(\overline{P}_{1,\mathrm{AF}}^{(o)}\overline{P}_{1,\mathrm{AF}}^{(e)}\right)^{k-1}\right] \\ &\times\left(\overline{P}_{2,\mathrm{AF}}\right)^{2k-2}P_{2,\mathrm{AF}} \qquad \text{(II)} \\ &+\sum_{k=1}^{+\infty}2k\left(\overline{P}_{1,\mathrm{AF}}^{(o)}\right)^{k}\left(\overline{P}_{1,\mathrm{AF}}^{(e)}\right)^{k-1}P_{1,\mathrm{AF}}^{(e)} \\ &\times\left[1-\left(\overline{P}_{2,\mathrm{AF}}\right)^{2k}\right] \qquad \text{(III)} \\ &+\sum_{k=1}^{+\infty}2k\left[1-\overline{P}_{1,\mathrm{AF}}^{(o)}\left(\overline{P}_{1,\mathrm{AF}}^{(o)}\overline{P}_{1,\mathrm{AF}}^{(e)}\right)^{k-1}\right] \\ &\times\left(\overline{P}_{2,\mathrm{AF}}\right)^{2k-1}P_{2,\mathrm{AF}}, \qquad \text{(IV)} \end{aligned} \tag{56}$$

where the closed-form expressions of the summations of infinite series can be obtained in a similar way as shown in (40). We can use (56) as a performance evaluation to show the advantage of the proposed AF-CDS. In addition, the computational complexity of (56) is moderate since the corresponding closed-form expressions involve simple arithmetic operations only. Define the agility gain of the AF-CDS over the noncooperation scheme as

$$G_{\mathrm{AF}} = \frac{\mathrm{T}_n}{\mathrm{T}_{\mathrm{AF}}}, \tag{57}$$

where T_n and T_{AF} are given in (29) and (56), respectively.

3.2. Detection Performance of DF-CDS. As has been said in Section 2.3, in odd time slots, the implementation process of DF-CDS is the same as AF-CDS, and thus the detection probability of $S1$ in odd time slots can be given by

$$P_{1,\mathrm{DF}}^{(o)} = P_{1,\mathrm{AF}}^{(o)} = \exp\left(-\frac{1}{\sigma_{p1}^2\gamma_s}\max(-2\ln\alpha-1,0)\right). \tag{58}$$

Without loss of generality, let case $\theta = 0$ and $\theta = 1$ denote the estimated indicator $\hat{s} = s$ and $\hat{s} \neq s$, respectively.

Case 1 ($[\theta = 0]$). This case corresponds to $\hat{s} = s$, implying

$$\left|h_{p2}(2k-1)\right|^2 Es + N_0 > \lambda. \tag{59}$$

Hence, the probability of occurrence of case $\theta = 0$ can be given by

$$\begin{aligned} \Pr(\theta=0) &= \Pr\left\{\left|h_{p2}(2k-1)\right|^2 > \frac{\lambda-N_0}{Es}\right\} \\ &= \exp\left(-\frac{1}{\sigma_{p2}^2}\max\left(\frac{\lambda-N_0}{Es},0\right)\right). \end{aligned} \tag{60}$$

Also, it can be shown that

$$\lambda = -2N_0\ln\alpha. \tag{61}$$

Substituting the threshold λ from (61) into (60) yields

$$\Pr(\theta=0) = \exp\left(-\frac{1}{\sigma_{p2}^2\gamma_s}\max(-2\ln\alpha-1,0)\right). \tag{62}$$

From (15), we can easily calculate the corresponding detection probability as

$$P_{1,\mathrm{DF}}^{(e)}(\theta=0) = \Pr\left\{\left|h_{p1}(2k)\right|^2 + |h_{21}(2k)|^2 > \frac{\lambda-N_0}{Es}\right\}. \tag{63}$$

Performing the probability integration in (63) yields

$$P_{1,\mathrm{DF}}^{(e)}(\theta=0) = \begin{cases} \left(1+\dfrac{1}{\sigma_{p1}^2\gamma_s}\max(-2\ln\alpha-1,0)\right) \\ \quad\times\exp\left(-\dfrac{1}{\sigma_{p1}^2\gamma_s}\max(-2\ln\alpha-1,0)\right); & \sigma_{p1}^2=\sigma_{12}^2 \\ \dfrac{\sigma_{p1}^2}{\sigma_{p1}^2-\sigma_{12}^2}\exp\left(-\dfrac{1}{\sigma_{p1}^2\gamma_s}\max(-2\ln\alpha-1,0)\right) \\ \quad+\dfrac{\sigma_{12}^2}{\sigma_{12}^2-\sigma_{p1}^2}\exp\left(-\dfrac{1}{\sigma_{12}^2\gamma_s}\max(-2\ln\alpha-1,0)\right); \\ & \sigma_{p1}^2\neq\sigma_{12}^2 \end{cases} \tag{64}$$

Case 2 ($[\theta = 1]$). This case corresponds to $\hat{s} \neq s$, meaning

$$\left|h_{p2}(2k-1)\right|^2 Es + N_0 < \lambda, \tag{65}$$

from which we can obtain

$$\Pr(\theta=1) = 1-\exp\left(-\frac{1}{\sigma_{p2}^2\gamma_s}\max(-2\ln\alpha-1,0)\right). \tag{66}$$

Therefore, from (16), it can be shown that

$$P_{1,\mathrm{DF}}^{(e)}(\theta=1) = \exp\left(\frac{\sigma_{21}^2}{\sigma_{p1}^2}\ln\alpha\right). \tag{67}$$

Hence, by using (62), (64), (66) and (67), the probability of detection of primary user by $S1$ in even time slots can be obtained as

$$P_{1,\mathrm{DF}}^{(e)} = \Pr(\theta=0)\cdot P_{1,\mathrm{DF}}^{(e)}(\theta=0) + \Pr(\theta=1)\cdot P_{1,\mathrm{DF}}^{(e)}(\theta=1). \tag{68}$$

According to (58) and (68), the probability of detection of primary user by $S1$ under DF-CDS is given by

$$P_{1,\mathrm{DF}} = \frac{P_{1,\mathrm{DF}}^{(o)}+P_{1,\mathrm{DF}}^{(e)}}{2}. \tag{69}$$

Besides, the detection probability of $S2$ under DF-CDS can be easily given by

$$P_{2,\mathrm{DF}} = P_{2,\mathrm{AF}} = \exp\left(-\frac{1}{\sigma_{p2}^2\gamma_s}\max(-2\ln\alpha - 1, 0)\right). \quad (70)$$

Thus, we can obtain the overall detection probability of the CR network under DF-CDS as

$$P_{\mathrm{DF}} = P_{1,\mathrm{DF}}P_{2,\mathrm{DF}}, \quad (71)$$

where $P_{1,\mathrm{DF}}$ and $P_{2,\mathrm{DF}}$ are given in (69) and (70), respectively. Also, we can model the detection time slots $\tau_{1,\mathrm{DF}}$ and $\tau_{2,\mathrm{DF}}$ (taken by the secondary users $S1$ and $S2$ under DF-CDS, resp.) as

$$\Pr\{\tau_{1,\mathrm{DF}} = m\} = \begin{cases} \left(\overline{P}_{1,\mathrm{DF}}^{(o)}\right)^{k-1}\left(\overline{P}_{1,\mathrm{DF}}^{(e)}\right)^{k-1}P_{1,\mathrm{DF}}^{(o)}, & m = 2k-1, \\ \left(\overline{P}_{1,\mathrm{DF}}^{(o)}\right)^{k}\left(\overline{P}_{1,\mathrm{DF}}^{(e)}\right)^{k-1}P_{1,\mathrm{DF}}^{(e)}, & m = 2k, \end{cases}$$

$$\Pr\{\tau_{2,\mathrm{DF}} = k\} = \left(\overline{P}_{2,\mathrm{DF}}\right)^{k-1}P_{2,\mathrm{DF}}. \quad (72)$$

In a similar way, we can obtain the average overall detection time T_{DF} as

$$\begin{aligned} \mathrm{T}_{\mathrm{DF}} = &\sum_{k=1}^{+\infty}(2k-1)\left(\overline{P}_{1,\mathrm{DF}}^{(o)}\right)^{k-1}\left(\overline{P}_{1,\mathrm{DF}}^{(e)}\right)^{k-1}P_{1,\mathrm{DF}}^{(o)}\left[1-\left(\overline{P}_{2,\mathrm{DF}}\right)^{2k-1}\right] \\ &+\sum_{k=1}^{+\infty}(2k-1)\left[1-\left(\overline{P}_{1,\mathrm{DF}}^{(o)}\overline{P}_{1,\mathrm{DF}}^{(e)}\right)^{k-1}\right]\left(\overline{P}_{2,DF}\right)^{2k-2}P_{2,\mathrm{DF}} \\ &+\sum_{k=1}^{+\infty}2k\left(\overline{P}_{1,\mathrm{DF}}^{(o)}\right)^{k}\left(\overline{P}_{1,\mathrm{DF}}^{(e)}\right)^{k-1}P_{1,\mathrm{DF}}^{(e)}\left[1-\left(\overline{P}_{2,\mathrm{DF}}\right)^{2k}\right] \\ &+\sum_{k=1}^{+\infty}2k\left[1-\overline{P}_{1,\mathrm{DF}}^{(o)}\left(\overline{P}_{1,\mathrm{DF}}^{(o)}\overline{P}_{1,\mathrm{DF}}^{(e)}\right)^{k-1}\right]\left(\overline{P}_{2,\mathrm{DF}}\right)^{2k-1}P_{2,\mathrm{DF}}. \end{aligned} \quad (73)$$

Similarly, the closed-form expressions of the summations of infinite series can be obtained as (40). Also, we will utilize (73) to show the merits of the proposed DF-CDS. Define the agility gain of the DF-CDS over the noncooperation scheme as

$$G_{\mathrm{DF}} = \frac{\mathrm{T}_n}{\mathrm{T}_{\mathrm{DF}}}, \quad (74)$$

where T_n and T_{DF} are given in (29) and (73), respectively.

4. Numerical Results and Analysis

Figure 2 shows the plots of (26), (37), (53) and (71) as a function of T-SNR γ_s with $\alpha = 0.01$, $\sigma_{p1}^2 = 1$ and $\eta = \mu = 10\,\mathrm{dB}$, where $\eta = 20\log_{10}(\sigma_{p2}/\sigma_{p1})$ and $\mu = 20\log_{10}(\sigma_{21}/\sigma_{p1})$. From Figure 2, one can see that, the detection probabilities of the cooperative detection schemes (i.e., AND-CDS, AF-CDS and DF-CDS) are always larger than the noncooperation scheme across the whole range of T-SNR γ_s. As observed from Figure 2, the proposed AF-CDS and DF-CDS outperform the traditional AND-CDS in terms of detection probability, showing the superiority of our schemes. Besides, it is shown from Figure 2 that the detection probability of DF-CDS is superior to the AF-CDS at low SNR region, but inferior to AF-CDS when $\gamma_s > 10\,\mathrm{dB}$.

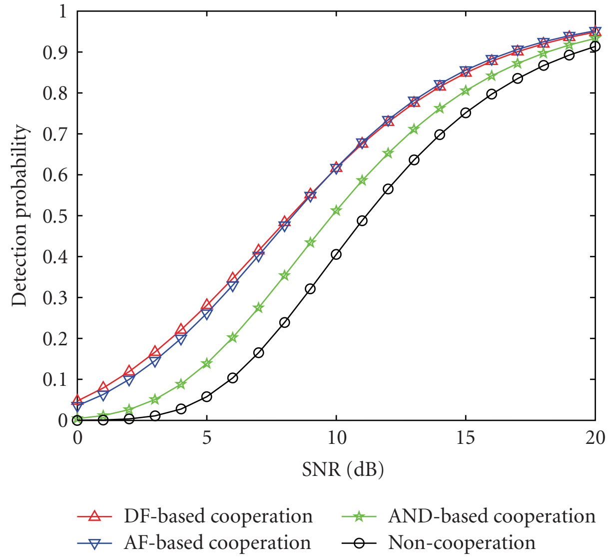

Figure 2: Detection probability performance versus T-SNR γ_s with $\alpha = 0.01$, $\sigma_{p1}^2 = 1$ and $\eta = \mu = 10\,\mathrm{dB}$, where $\eta = 20\log_{10}(\sigma_{p2}/\sigma_{p1})$ and $\mu = 20\log_{10}(\sigma_{21}/\sigma_{p1})$.

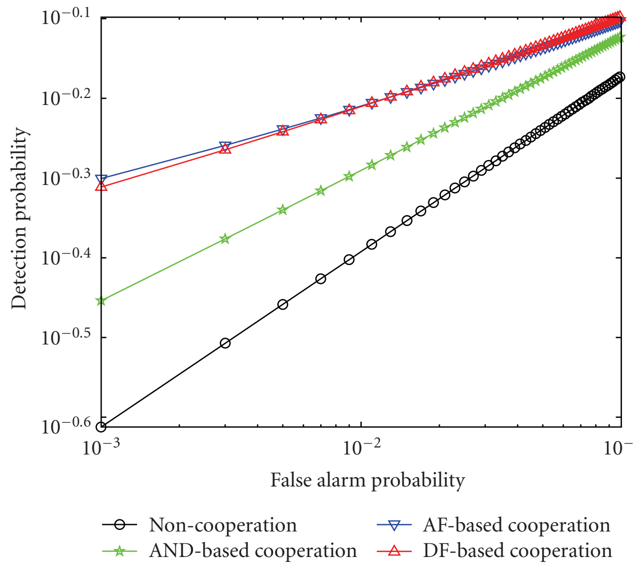

Figure 3: Detection probability versus false alarm probability with $\gamma_s = 10\,\mathrm{dB}$, $\sigma_{p1}^2 = 1$ and $\eta = \mu = 10\,\mathrm{dB}$, where $\eta = 20\log_{10}(\sigma_{p2}/\sigma_{p1})$ and $\mu = 20\log_{10}(\sigma_{21}/\sigma_{p1})$.

In Figure 3, we plot (26), (37), (53) and (71) as a function of false alarm probability α with $\gamma_s = 10\,\mathrm{dB}, \sigma_{p1}^2 = 1$ and $\eta = \mu = 10\,\mathrm{dB}$, where $\eta = 20\log_{10}(\sigma_{p2}/\sigma_{p1})$ and

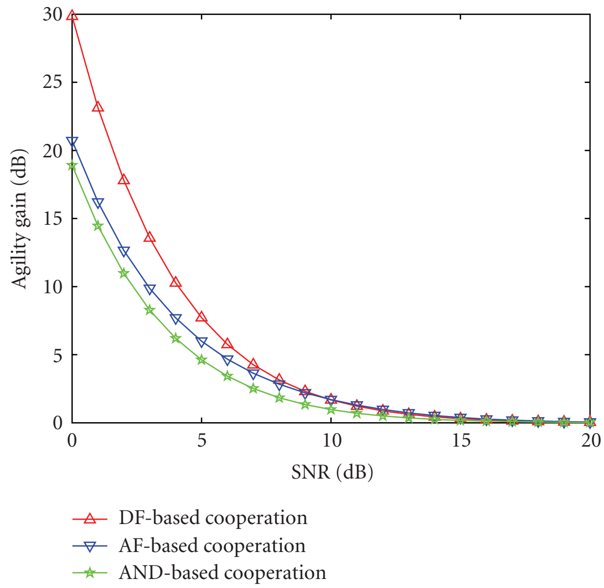

FIGURE 4: Agility gain versus T-SNR with $\alpha = 0.001$, $\sigma_{p1}^2 = 1$, $\eta = 5$ dB and $\mu = 10$ dB, where $\eta = 20\log_{10}(\sigma_{p2}/\sigma_{p1})$ and $\mu = 20\log_{10}(\sigma_{21}/\sigma_{p1})$.

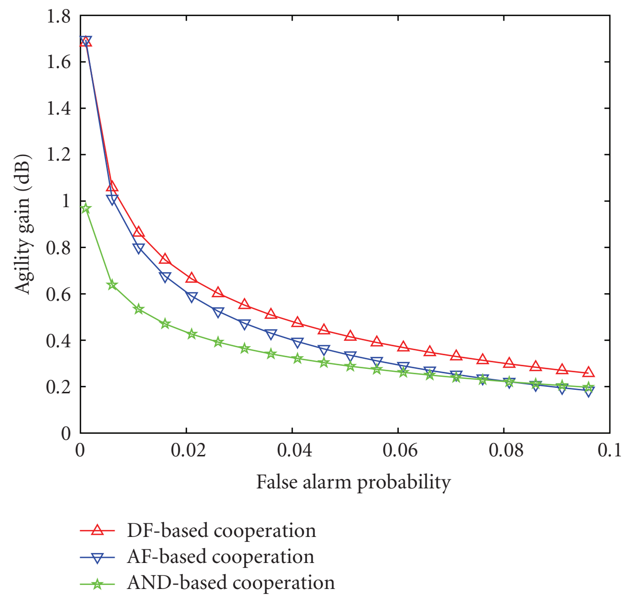

FIGURE 5: Agility gain versus false alarm probability with $\gamma_s = 10$ dB, $\sigma_{p1}^2 = 1$, $\eta = 5$ dB and $\mu = 10$ dB, where $\eta = 20\log_{10}(\sigma_{p2}/\sigma_{p1})$ and $\mu = 20\log_{10}(\sigma_{21}/\sigma_{p1})$.

$\mu = 20\log_{10}(\sigma_{21}/\sigma_{p1})$. It is seen from Figure 3 that the detection probability performances of the three cooperative schemes are much better than the noncooperation scheme. Also, Figure 3 shows that the detection performances of our schemes (i.e., AF-CDS and DF-CDS) are superior to the known AND-CDS scheme. In addition, one can see from Figure 3 that DF-CDS can achieve more performance gains than AF-CDS when $\alpha > 0.02$. Therefore, DF-CDS outperforms AF-CDS at low SNR region or high false alarm probability region.

Figure 4 illustrates the agility gain versus T-SNR γ_s of the AND-CDS, AF-CDS and DF-CDS with $\alpha = 0.001$, $\sigma_{p1}^2 = 1$, $\eta = 5$ dB and $\mu = 10$ dB, where the three curves are the plots of (41), (57) and (74), respectively. From the figure, we find that at this scenario, both the AF-CDS and the DF-CDS can achieve more agility gains than the AND-CDS across the whole T-SNR range, implying the advantages of the proposed schemes over the existing AND-CDS. For completeness sake, in Figure 5, we also plot the agility gains of the AND-CDS, AF-CDS and DF-CDS as a function of the false alarm probability α. As can be seen from Figure 5, the agility gains of the three cooperation schemes are larger than zero, showing the effectiveness of applying cooperation technology to spectrum sensing. In addition, from Figure 5, one can see that the proposed AF-CDS and DF-CDS outperform the known AND-CDS in terms of agility gain, which further confirms the merits of our schemes.

5. Conclusion

In this paper, we have presented two novel cooperative detection schemes (i.e., AF-CDS and DF-CDS) to improve the detection performance of cognitive radios. We have developed closed-form expressions of detection probability and agility gain for both AF-CDS and DF-CDS over Rayleigh fading channels. For the purpose of comparison, we have also analyzed the detection performances for the noncooperation and the AND-based cooperation schemes. Through conducting numerical experiments, it has been shown that both AF-CDS and DF-CDS are superior to the noncooperation and the AND-base cooperation schemes in terms of the detection probability and the agility gain. Furthermore, we have shown that DF-CDS outperforms AF-CDS at low SNR region or high false alarm probability region.

Appendices

A. Calculation of (40)

From (39), we can calculate the corresponding average detection time $\mathrm{T}_{\mathrm{AND}}$ as

$$\begin{aligned}\mathrm{T}_{\mathrm{AND}} = &\sum_{k=1}^{+\infty}(2k-1)\Pr[\max(\tau_{1,\mathrm{AND}},\tau_{2,\mathrm{AND}}) = 2k-1] \\ &+ \sum_{k=1}^{+\infty}2k\Pr[\max(\tau_{1,\mathrm{AND}},\tau_{2,\mathrm{AND}}) = 2k].\end{aligned} \tag{A.1}$$

Since RVs $\tau_{1,\mathrm{AND}}$ and $\tau_{2,\mathrm{AND}}$ are independent from each other, (A.1) can be further expressed as

$$\begin{aligned}\mathrm{T}_{\mathrm{AND}} = &\sum_{k=1}^{+\infty}(2k-1)\Pr(\tau_{1,\mathrm{AND}} = 2k-1)\Pr(\tau_{2,\mathrm{AND}} \le 2k-1) \\ &+ \sum_{k=1}^{+\infty}(2k-1)\Pr(\tau_{1,\mathrm{AND}} < 2k-1)\Pr(\tau_{2,\mathrm{AND}} = 2k-1)\end{aligned}$$

$$+\sum_{k=1}^{+\infty} 2k \Pr(\tau_{1,\text{AND}} = 2k) \Pr(\tau_{2,\text{AND}} \le 2k)$$
$$+\sum_{k=1}^{+\infty} 2k \Pr(\tau_{1,\text{AND}} < 2k) \Pr(\tau_{2,\text{AND}} = 2k), \tag{A.2}$$

where

$$\begin{aligned}
\Pr(\tau_{1,\text{AND}} = 2k-1) &= \left(\overline{P}^{(o)}_{1,\text{AND}}\right)^{k-1}\left(\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1} P^{(o)}_{1,\text{AND}},\\
\Pr(\tau_{1,\text{AND}} < 2k-1) &= \sum_{m=1}^{k-1}\left(\overline{P}^{(o)}_{1,\text{AND}}\right)^{m-1}\left(\overline{P}^{(e)}_{1,\text{AND}}\right)^{m-1} P^{(o)}_{1,\text{AND}}\\
&\quad+\sum_{m=1}^{k-1}\left(\overline{P}^{(o)}_{1,\text{AND}}\right)^{m}\left(\overline{P}^{(e)}_{1,\text{AND}}\right)^{m-1} P^{(e)}_{1,\text{AND}}\\
&= 1-\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1},\\
\Pr(\tau_{1,\text{AND}} < 2k) &= \Pr(\tau_{1,\text{AND}} < 2k-1)\\
&\quad+\Pr(\tau_{1,\text{AND}} = 2k-1)\\
&= 1-\overline{P}^{(o)}_{1,\text{AND}}\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1},\\
\Pr(\tau_{2,\text{AND}} = 2k-1) &= \left(\overline{P}_{2,\text{AND}}\right)^{2k-2} P_{2,\text{AND}}\\
\Pr(\tau_{2,\text{AND}} \le 2k-1) &= \sum_{m=1}^{2k-1}\left(\overline{P}_{2,\text{AF}}\right)^{m-1} P_{2,\text{AND}}\\
&= 1-\left(\overline{P}_{2,\text{AND}}\right)^{2k-1}.
\end{aligned} \tag{A.3}$$

Combining (A.2) and (A.3) gives

$$\begin{aligned}
T_{\text{AND}} &= \sum_{k=1}^{+\infty}(2k-1)\left(\overline{P}^{(o)}_{1,\text{AND}}\right)^{k-1}\left(\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1} P^{(o)}_{1,\text{AND}}\\
&\quad\times\left[1-\left(\overline{P}_{2,\text{AND}}\right)^{2k-1}\right] && \text{(I)}\\
&\quad+\sum_{k=1}^{+\infty}(2k-1)\left[1-\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1}\right]\\
&\quad\times\left(\overline{P}_{2,\text{AND}}\right)^{2k-2} P_{2,\text{AND}} && \text{(II)}\\
&\quad+\sum_{k=1}^{+\infty}2k\left(\overline{P}^{(o)}_{1,\text{AND}}\right)^{k}\left(\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1} P^{(e)}_{1,\text{AND}}\\
&\quad\times\left[1-\left(\overline{P}_{2,\text{AND}}\right)^{2k}\right] && \text{(III)}\\
&\quad+\sum_{k=1}^{+\infty}2k\left[1-\overline{P}^{(o)}_{1,\text{AND}}\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1}\right]\\
&\quad\times\left(\overline{P}_{2,\text{AND}}\right)^{2k-1} P_{2,\text{AND}}, && \text{(IV)}
\end{aligned} \tag{A.4}$$

where

$$\begin{aligned}
\text{I} &= \sum_{k=1}^{+\infty}2k\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1} P^{(o)}_{1,\text{AND}} - \sum_{k=1}^{+\infty}\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1}\\
&\quad\times P^{(o)}_{1,\text{AND}} - \sum_{k=1}^{+\infty}2k\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1} P^{(o)}_{1,\text{AND}}\left(\overline{P}_{2,\text{AND}}\right)^{2k-1}\\
&\quad+\sum_{k=1}^{+\infty}\left(\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^{k-1} P^{(o)}_{1,\text{AND}}\left(\overline{P}_{2,\text{AND}}\right)^{2k-1}\\
&= \frac{2P^{(o)}_{1,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^2} - \frac{P^{(o)}_{1,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)}\\
&\quad-\frac{2P^{(o)}_{1,\text{AND}}\overline{P}_{2,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\overline{P}^{2}_{2,\text{AND}}\right)^2}\\
&\quad+\frac{P^{(o)}_{1,\text{AND}}\overline{P}_{2,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\overline{P}^{2}_{2,\text{AND}}\right)}.
\end{aligned} \tag{A.5}$$

Similarly, we can obtain

$$\begin{aligned}
\text{II} &= \frac{2P_{2,\text{AND}}}{\left(1-\overline{P}^{2}_{2,\text{AND}}\right)^2} - \frac{2P_{2,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\overline{P}^{2}_{2,\text{AND}}\right)^2}\\
&\quad-\frac{P_{2,\text{AND}}}{\left(1-\overline{P}^{2}_{2,\text{AND}}\right)} + \frac{P_{2,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\overline{P}^{2}_{2,\text{AND}}\right)},\\
\text{III} &= \frac{2\overline{P}^{(o)}_{1,AND}P^{(e)}_{1,AND}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\right)^2} - \frac{2\overline{P}^{(o)}_{1,\text{AND}}P^{(e)}_{1,\text{AND}}\overline{P}^{2}_{1,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\overline{P}^{2}_{2,\text{AND}}\right)^2},\\
\text{IV} &= \frac{2\overline{P}_{2,\text{AND}}P_{2,\text{AND}}}{\left(1-\overline{P}^{2}_{2,\text{AND}}\right)^2} - \frac{2\overline{P}^{(o)}_{1,\text{AND}}\overline{P}_{2,\text{AND}}P_{2,\text{AND}}}{\left(1-\overline{P}^{(o)}_{1,\text{AND}}\overline{P}^{(e)}_{1,\text{AND}}\overline{P}^{2}_{2,\text{AND}}\right)^2}.
\end{aligned} \tag{A.6}$$

B. Proof of Equation (47)

Combining (45) and (46) gives

$$\begin{aligned}
P^{(e)}_{1,\text{AF}} &= \iint_{\Theta}\frac{1}{\sigma^2_{p1}\sigma^2_{p2}}\exp\left(-\frac{x}{\sigma^2_{p1}}-\frac{y}{\sigma^2_{p2}}\right)dx\,dy,\\
\Theta &= \{(x,y)\mid x+y>\tau\},
\end{aligned} \tag{B.1}$$

where $\tau = \max(\lambda - 2N_0/Es, 0)$. From (B.1), it is easy to obtain

$$\begin{aligned}
P^{(e)}_{1,\text{AF}} &= \int_0^{\tau}\left[\frac{1}{\sigma^2_{p1}}\exp\left(-\frac{x}{\sigma^2_{p1}}\right)\int_{\tau-x}^{+\infty}\frac{1}{\sigma^2_{p2}}\exp\left(-\frac{y}{\sigma^2_{p2}}\right)dy\right]dx\\
&\quad+\int_{\tau}^{+\infty}\left[\frac{1}{\sigma^2_{p1}}\exp\left(-\frac{x}{\sigma^2_{p1}}\right)\int_0^{+\infty}\frac{1}{\sigma^2_{p2}}\exp\left(-\frac{y}{\sigma^2_{p2}}\right)dy\right]dx.
\end{aligned} \tag{B.2}$$

Performing the integration in (B.2) yields

$$P_{1,\mathrm{AF}}^{(e)} = \exp\left(-\frac{\tau}{\sigma_{p2}^2}\right)\int_0^{\tau}\frac{1}{\sigma_{p1}^2}\exp\left(-\frac{\sigma_{p2}^2-\sigma_{p1}^2}{\sigma_{p1}^2\sigma_{p2}^2}x\right)dx+\exp\left(-\frac{\tau}{\sigma_{p1}^2}\right). \tag{B.3}$$

If $\sigma_{p1}^2=\sigma_{p2}^2$, we can get

$$P_{1,\mathrm{AF}}^{(e)}=\left(1+\frac{\tau}{\sigma_{p1}^2}\right)\exp\left(-\frac{\tau}{\sigma_{p1}^2}\right). \tag{B.4}$$

Else if $\sigma_{p1}^2\neq\sigma_{p2}^2$, $P_{1,\mathrm{AF}}^{(e)}$ is given by

$$P_{1,\mathrm{AF}}^{(e)}=\frac{\sigma_{p2}^2}{\sigma_{p2}^2-\sigma_{p1}^2}\exp\left(-\frac{\tau}{\sigma_{p2}^2}\right)\left[1-\exp\left(-\frac{\sigma_{p2}^2-\sigma_{p1}^2}{\sigma_{p1}^2\sigma_{p2}^2}\tau\right)\right]+\exp\left(-\frac{\tau}{\sigma_{p1}^2}\right) \tag{B.5}$$

which can be further simplified to

$$P_{1,\mathrm{AF}}^{(e)}=\frac{\sigma_{p1}^2}{\sigma_{p1}^2-\sigma_{p2}^2}\exp\left(-\frac{\tau}{\sigma_{p1}^2}\right)+\frac{\sigma_{p2}^2}{\sigma_{p2}^2-\sigma_{p1}^2}\exp\left(-\frac{\tau}{\sigma_{p2}^2}\right). \tag{B.6}$$

Thus, substituting $\tau=\max(\lambda-2N_0/Es,0)$ into (B.4) and (B.6) yields

$$P_{1,\mathrm{AF}}^{(e)}=\begin{cases}\left(1+\dfrac{1}{\sigma_{p1}^2}\max\left(\dfrac{\lambda-2N_0}{Es},0\right)\right)\\ \quad\times\exp\left(-\dfrac{1}{\sigma_{p1}^2}\max\left(\dfrac{\lambda-2N_0}{Es},0\right)\right); \quad \sigma_{p1}^2=\sigma_{p2}^2\\ \dfrac{\sigma_{p1}^2}{\sigma_{p1}^2-\sigma_{p2}^2}\exp\left(-\dfrac{1}{\sigma_{p1}^2}\max\left(\dfrac{\lambda-2N_0}{Es},0\right)\right)\\ \quad+\dfrac{\sigma_{p2}^2}{\sigma_{p2}^2-\sigma_{p1}^2}\exp\left(-\dfrac{1}{\sigma_{p2}^2}\max\left(\dfrac{\lambda-2N_0}{Es},0\right)\right);\\ \qquad\qquad \sigma_{p1}^2\neq\sigma_{p2}^2.\end{cases} \tag{B.7}$$

and this is (47).

C. Proof of Equation (49)

Rewrite (48) as

$$\alpha=\Pr\left\{|n_1(2k)|^2+|n_2(2k-1)|^2>\lambda\right\}. \tag{C.1}$$

Note that both $n_1(2k)$ and $n_2(2k-1)$ are complex white Gaussian noise with zero mean and double-sided power spectral density N_0. Without loss of generality, let z denote the complex Gaussian noise, which can be expressed as

$$z=z_x+iz_y, \tag{C.2}$$

where $z_x\sim N(0,N_0)$ and $z_y\sim N(0,N_0)$. Obviously, $z_x/\sqrt{N_0}$ and $z_y/\sqrt{N_0}$ follow the standard normal distribution, giving $|z_x/\sqrt{N_0}|^2+|z_y/\sqrt{N_0}|^2\sim\chi^2(2)$, where $\chi^2(2)$ is the chi-square distribution with 2 degrees of freedom. From (C.2), we can find $|z_x/\sqrt{N_0}|^2+|z_y/\sqrt{N_0}|^2=|z/\sqrt{N_0}|^2$, implying $|z/\sqrt{N_0}|^2\sim\chi^2(2)$. Consequently, it is easily shown that $|n_1(2k)/\sqrt{N_0}|^2$ and $|n_2(2k-1)/\sqrt{N_0}|^2$ are distributed as $\chi^2(2)$. Noting that $n_1(2k)$ and $n_2(2k-1)$ are independent from each other, we obtain $|n_1(2k)/\sqrt{N_0}|^2+|n_2(2k-1)/\sqrt{N_0}|^2\sim\chi^2(4)$. Let $X=|n_1(2k)/\sqrt{N_0}|^2+|n_2(2k-1)/\sqrt{N_0}|^2$, and thus its PDF (probability density function), $f_X(x)$, can be given by

$$f_X(x)=\frac{x}{4}\exp\left(-\frac{x}{2}\right)U(x). \tag{C.3}$$

Let $Y=|n_1(2k)|^2+|n_2(2k-1)|^2$, resulting in $Y=N_0X$. Hence, the CDF (cumulative distribution function) of Y, $P_Y(y)$, is calculated as

$$P_Y(y)=\Pr(Y<y)=\Pr\left(\frac{Y}{N_0}<\frac{y}{N_0}\right)=\Pr\left(X<\frac{y}{N_0}\right). \tag{C.4}$$

Combining (C.3) and (C.4) gives

$$P_Y(y)=\int_{-\infty}^{y/N_0}\frac{x}{4}\exp\left(-\frac{x}{2}\right)U(x)dx. \tag{C.5}$$

Performing the integration in (C.5) yields

$$P_Y(y)=\begin{cases}1-\dfrac{y}{2N_0}\exp\left(-\dfrac{y}{2N_0}\right)-\exp\left(-\dfrac{y}{2N_0}\right), & y\geq 0\\ 0, & \text{others}.\end{cases} \tag{C.6}$$

From (C.1), we can obtain

$$\alpha=1-P_Y(\lambda). \tag{C.7}$$

Combining (C.6) and (C.7) gives

$$\alpha=\frac{\lambda}{2N_0}\exp\left(-\frac{\lambda}{2N_0}\right)+\exp\left(-\frac{\lambda}{2N_0}\right). \tag{C.8}$$

Here, we have utilized the positive property of the energy detection threshold λ to obtain (C.8). Besides, (C.8) can be equivalently written as

$$-\alpha e^{-1}=\left(-\frac{\lambda}{2N_0}-1\right)\exp\left(-\frac{\lambda}{2N_0}-1\right). \tag{C.9}$$

By using the Lambert's W function that is used to solve the equation

$$w\exp(w)=x, \tag{C.10}$$

for w as a function of x, the parameter λ can be given by

$$\lambda=-2[W(-\alpha e^{-1})+1]N_0. \tag{C.11}$$

and this is (49).

Acknowledgments

This work was partially supported by the Postgraduate Innovation Programs of Scientific Research of Jiangsu Province (no. CX09B_150Z, CX08B_080Z), the Key Project of Nature Science Funding of Jiangsu Province (no. BK2007729), the National High Technology Research Development Plan (no. 2009AA01Z241), and the Major Development Program of Jiangsu Educational Committee (no. 06KJA51001).

References

[1] J. Mitola III, "Software radios: survey, critical evaluation and future directions," *IEEE Aerospace and Electronic Systems Magazine*, vol. 8, no. 4, pp. 25–36, 1993.

[2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, 2005.

[3] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks*, vol. 50, no. 13, pp. 2127–2159, 2006.

[4] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits in cognitive radio," in *Proceedings of the Allerton Conference on Communication, Control, and Computing*, Monticello, Ill, USA, October 2004.

[5] V.I. Kostylev, "Energy detection of a signal with random amplitude," in *Proceedings of the IEEE International Conference on Communications (ICC 2002)*, vol. 3, pp. 1606–1610, 28 April–2 May 2002.

[6] D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proceedings of the 38th Asilomar Conference on Signals, Systems and Computers*, vol. 1, pp. 772–776, November 2004.

[7] A. Fehske, J. Gaeddert, and J. H. Reed, "A new approach to signal classification using spectral correlation and neural networks," in *Proceedings of the 1st IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN '05)*, pp. 144–150, November 2005.

[8] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proceedings of the 1st IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN '05)*, pp. 131–136, November 2005.

[9] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2415–2425, 2003.

[10] Y. Zou, B. Zheng, and J. Zhu, "Outage analysis of opportunistic cooperation over Rayleigh fading channels," *IEEE Transactions on Wireless Communications*, vol. 8, no. 6, pp. 3077–3085, 2009.

[11] Y. Zou, B. Zheng, and X. Zhu, "A new cooperative diversity scheme for next generation wireless network," in *Proceedings of the 5th IEEE Consumer Communications and Networking Conference (CCNC '08)*, pp. 938–942, Las Vegas, Nev, USA, January 2008.

[12] Y. Zou, B. Zheng, and W.-P. Zhu, "An opportunistic cooperation scheme and its BER analysis," *IEEE Transactions on Wireless Communications*, vol. 8, no. 9, pp. 4492–4497, 2009.

[13] G. Ganesan and Y. G. Li, "Cooperative spectrum sensing in cognitive radio—part I: two user networks," *IEEE Transactions on Wireless Communications*, vol. 6, no. 6, pp. 2204–2213, 2007.

[14] G. Ganesan and Y. G. Li, "Cooperative spectrum sensing in cognitive radio—part II: multiuser networks," *IEEE Transactions on Wireless Communications*, vol. 6, no. 6, pp. 2214–2222, 2007.

[15] H. V. Poor, *An Introduction to Signal Detection and Estimation*, Springer, Berlin, Germany, 1994.