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Research Article

Threshold-Based Relay Selection for Detect-and-Forward Relaying in Cooperative Wireless Networks

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This paper studies two-hop cooperative demodulate-and-forward relaying using multiple relays in wireless networks. A threshold based relay selection scheme is considered, in which the reliable relays are determined by comparing source-relay SNR to a threshold, and one of the reliable relays is selected by the destination based on relay-destination SNR. The exact bit error rate of this scheme is derived, and a simple threshold function is proposed. It is shown that the network achieves full diversity order $(N + 1)$ under the proposed threshold, where N is the number of relays in the network. Unlike some other full diversity achieving protocols in the literature, the requirement that the instantaneous/average SNRs of the source-relay links be known at the destination is eliminated using the appropriate SNR threshold.

1. Introduction

1.1. Background. Cooperative relaying can induce spatial diversity in wireless networks without the need for multiple antennas on a single terminal. Various decode-and-forward protocols have been proposed based on selective relaying, distributed space-time coding, and relay selection and have been shown to achieve full diversity [1–5]. Recently, detection aspects of cooperative relaying have been analyzed [5–10]. These works study the detect-and-forward (or demodulate-and-forward) cooperative relaying protocols, in which the relaying does not rely on any error correction or detection codes. Such protocols are particularly attractive for systems that do not use error detection/correction codes due to tight energy constraints. One possible application is sensor networks, which typically function under extremely limited battery-supplied energy. Most coding schemes can consume significant energy, and thus their use reduces sensor and network lifetime if each relay decodes the data. Moreover, the messages transmitted in sensor networks are usually very short while coding is usually efficient only for long messages.

Another relaying scheme that does not rely on any error correction or detection is the amplify-and-forward protocol, in which the relay amplifies and forwards the received waveforms to the destination. The main disadvantage of this scheme is noise amplification, which cannot be avoided due to the physical presence of the thermal noise at the relay receiver. The focus of this paper is on detect-and-forward relaying.

1.2. Related Work. The detect-and-forward protocol has the well-known disadvantage of error propagation. Unlike in ideal decode-and-forward relaying, in detect-and-forward relaying the relays can forward erroneous information, and with a conventional combining scheme such as Maximal Ratio Combining (MRC), these errors *propagate* to the destination, causing end-to-end (e2e) detection errors. Existing techniques for mitigating error propagation can be classified into two groups. The first of these comprises selective and adaptive relaying techniques, which include link adaptive relaying (LAR) [6] and threshold digital relaying (TDR) [11–13]. Both techniques use link SNRs to evaluate the reliability of the data received by the relay. In TDR a relay forwards the received data only when its received SNR is above a threshold value. In LAR the relay transmits with a fraction α of its maximum transmit power, where α depends on the source-relay and relay-destination SNRs. In [6], a function for calculating α is provided, and the resulting scheme is shown to achieve full diversity if the relays are capable of adjusting their transmit powers continuously. However, the proposed function cannot provide diversity if reduced to two power levels, that is, on/off power adaptation. TDR can also be viewed as on/off power adaptation, and it is shown in [13] that it can achieve full diversity in the single relay case. In [5], a relay selection scheme similar to ours is studied. In this paper an approximate expression for bit error probability is derived as a function of relay threshold assuming that MRC is performed at the destination. It is observed that the performance of threshold based relay selection is sensitive to the value of threshold.

The second approach to mitigate error propagation is to develop better combining schemes for the destination. These schemes take the possibility of error propagation into account and require the relays to send their source-relay link SNRs (average or instantaneous) to the destination. In [8], Wang et al. assume that the destination knows the instantaneous source-relay SNR and derive a linear combining technique, called Cooperative MRC (C-MRC), that approximates the Maximum Likelihood (ML) receiver. This receiver achieves full diversity at the expense of increased signaling to convey the first hop SNR information to the destination. In [7], the authors propose a piecewise linear receiver approximating the ML detector that requires knowledge of the average SNRs of the first hop. Conveying the average link SNRs is less costly than conveying the instantaneous SNR. However, this protocol cannot achieve full diversity for more than one relay. As will be shown in this paper, the protocol we consider requires minimal information on the first hop and still achieves full diversity.

We note that relay selection protocols are much more bandwidth efficient than protocols that require the relays to transmit in multiple orthogonal time slots and also have the potential to achieve full diversity [3, 4]. Recent work in the literature on relay selection covers relay selection for a single hop [10, 14, 15] as well as the selection of relays jointly for multiple hops [16, 17]. Different relay selection criteria have been proposed including those based on instantaneous SNR [3, 18, 19], average SNR [20, 21], and distance [22].

A relay selection protocol related to ours has been proposed in [10]. In this protocol, the relay selection is performed based on the equivalent e2e bit error rate (BER) of each relay channel. This protocol can be viewed as a selection version of C-MRC of [8]. As in C-MRC, it requires the destination to obtain the channel coefficients of the first and second hops or their product in the case of a simpler scheme, to make relay selection. However, in our protocol, the specific source-relay channel information is not required in order to perform relay selection.

1.3. Contributions of the Paper. In this paper, we consider the Threshold based Relay Selection Cooperation (TRSC) protocol, which generalizes threshold digital relaying to multiple relays. The specific contributions of the present paper are as follows.

(i) We derive the e2e BER of the TRSC protocol with N relays in a closed form given a common threshold value γ_t at the relays, assuming that all the relays have identical average source-relay and relay-destination SNRs.

(ii) We derive a simple threshold function with which TRSC can achieve full diversity $N + 1$ in an N relay network. We show that this function scales as $N \log \text{SNR}$.

(iii) We find the optimal threshold value that minimizes the e2e BER through computer simulation and observe that it shares common properties with the proposed suboptimal threshold function.

(iv) We propose two strategies to apply the TRSC protocol in asymmetric networks, in which relays have different average SNRs. Through numerical examples we analyze the performance of these strategies and show that the TRSC protocol is applicable to asymmetric networks as well.

The rest of this paper is organized as follows. In Section 2, we describe the system model and the TRSC protocol. In Section 3, we derive the e2e BER of the protocol, and in Section 4 we show that the protocol achieves full diversity using a threshold function we propose. We present some numerical results in Section 5 and conclude in Section 6 with a summary of our results.

2. System Model and Protocol

A network as shown in Figure 1 is considered in which a source node S communicates with a destination node D with the assistance of N relays denoted by $R_1, R_2, \ldots, R_N$. All links experience independent Rayleigh fading. For each link we assume quasistatic fading, in which the fading is constant over a two-stage transmission interval but then can change at the next interval. We assume a general modulation scheme for which the bit error probability can be expressed as $P_b(\gamma) = b\,\text{erfc}(\sqrt{a\gamma})$, where a and b are positive and where γ is the received SNR. Some of our derivations are even more general; they are given in terms of P_b and $\overline{P}_b$ and can be evaluated for any modulation scheme.

The SNRs of the $S-D$, $S-R_i$, and R_i-D links are denoted by γ_{sd}, $\gamma_{sr,i}$, and $\gamma_{rd,i}$, respectively. To simplify the analysis, we assume that all the relays have the same average SNRs to the source and to the destination. The variation in SNR is due to the Rayleigh channel gain and the noise and transmit powers do not vary. As $\overline{\gamma}_{sr,i} = \overline{\gamma}_{sr}$ and $\overline{\gamma}_{rd,i} = \overline{\gamma}_{rd}$ for $i = 1, 2, \ldots, N$, the link SNRs are characterized by $\overline{\gamma}_{sd}$, $\overline{\gamma}_{sr}$, and $\overline{\gamma}_{rd}$. The *Threshold based Relay Selection Cooperation (TRSC)* protocol has two phases. In the first phase the source transmits while all the relays and the destination listen. Then each relay R_i decides independently whether its detection is reliable by comparing its received SNR $\gamma_{sr,i}$ to a threshold value. Those

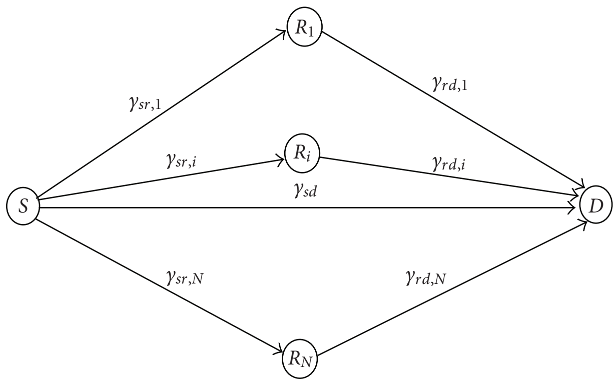

FIGURE 1: The network model.

relays whose received SNRs are larger than the threshold are called *reliable relays*.

One of the reliable relays is selected by the destination for possible retransmission. Each reliable relay informs the destination by sending a short message. The destination can estimate the values of $\gamma_{rd,i}$ for all the reliable relays from these messages. The destination replies with another message conveying which relay is selected for retransmission. Let us denote the number of reliable relays by N_r and reindex the reliable relays to simplify the notation. The destination, then, makes a decision based on the SNRs of the reliable relays and the source to the destination, that is, γ_{sd} and $\gamma_{rd,1},\ldots,\gamma_{rd,N_r}$. Among N_r+1 branches D selects the one with the largest SNR. If the branch from the source is selected, the relays do not transmit and the source transmits the next data. Otherwise, the selected reliable relay transmits and D performs detection based on the selected branch only.

Alternatively, the relay selection can be performed by all nodes in a distributed manner using a timer at each relay as in [4]. Comparing with relay selection by the destination, this distributed selection scheme does not require the relay to forward the single bit information with regard to the $S-R_i$ channel condition, however it requires each relay to obtain the channel gain (i.e., $\gamma_{rd,i}$), which can be assured through feedback from the destination.

In TRSC the information passed from the relay to the destination regarding the first hop is limited to whether the relay is a reliable relay or not, which can be represented by a single bit. For comparison we study the performance of cooperative relaying schemes in which the destination also knows either the instantaneous $S-R_i$ SNRs $\gamma_{sr,i}$ for all links or their average $\overline{\gamma}_{sr}$.

3. End-to-end BER of the TRSC

In this section, we derive the e2e BER of the system described in Section 2. Since all the relays are assumed to be identical in terms of average SNRs to the relay and the destination, the optimal value of their thresholds must be the same. Hence, we derive the e2e BER of the system for a given common threshold γ_t for all relays. Then the e2e BER is given by

$$\mathrm{BER}_{\mathrm{e2e}} = \sum_{i=0}^{N} \mathbb{P}(N_r = i)\mathbb{P}(\mathcal{E}_{\mathrm{e2e}} \mid N_r = i), \tag{1}$$

where $\mathbb{P}\{\ \}$ denotes the probability of an event and

$$\mathbb{P}\{N_r = i\} = \binom{N}{i}\left(e^{-\gamma_t/\overline{\gamma}_{sr}}\right)^i\left(1-e^{-\gamma_t/\overline{\gamma}_{sr}}\right)^{N-i}, \tag{2}$$

as the received SNRs have exponential probability distributions. For $N_r = 0$, the destination detects based on the direct link only and, thus, $\mathbb{P}(\mathcal{E}_{\mathrm{e2e}}|N_r = 0) = \overline{P}_b(\overline{\gamma}_{sd})$. For $N_r \geq 1$, let $\mathcal{A}_s$ denote the event that the destination selects the signal received from the source, and let $\mathcal{A}_{r,k}$ denote the event that the destination selects the signal from the kth reliable relay ($k \in \{1,\ldots,N_r\}$), respectively:

$$\mathcal{A}_s = \left\{\gamma_{sd} > \gamma_{rd,j},\ \forall j \in \{1,\ldots,N_r\}\right\},$$
$$\mathcal{A}_{r,k} = \left\{\gamma_{rd,k} > \gamma_{sd},\ \gamma_{rd,k} > \gamma_{rd,j},\ \ \forall j \in \{1,\ldots,N_r\}, j \neq k\right\}. \tag{3}$$

Then, the e2e BER conditioned on the number of reliable relays is equal to

$$\begin{aligned}\mathbb{P}\{\mathcal{E}_{\mathrm{e2e}} \mid N_r = i\} &= \mathbb{P}\{\mathcal{E}_{\mathrm{e2e}} \mid \mathcal{A}_s, N_r = i\}\mathbb{P}\{\mathcal{A}_s \mid N_r = i\} \\ &\quad + \sum_{k=1}^{i} \mathbb{P}\{\mathcal{E}_{\mathrm{e2e}} \mid \mathcal{A}_{r,k}, N_r = i\}\mathbb{P}\{\mathcal{A}_{r,k} \mid N_r = i\}.\end{aligned} \tag{4}$$

Since all relays are assumed to be identical in their average SNRs to the source and the destination, the terms included in $\mathcal{A}_{r,k}$ are the same for all k, and the index k can be dropped. When the destination selects the source signal, its bit error rate depends only on the source-destination link. However, if the destination selects reliable relay j, it will have a bit error if either the $S-R_j$ link or the R_j-D link has a bit error:

$$\begin{aligned}\mathbb{P}\{\mathcal{E}_{\mathrm{e2e}} \mid \mathcal{A}_s, N_r = i\} &= \mathbb{P}\{\mathcal{E}_{sd} \mid \mathcal{A}_s, N_r = i\}, \\ \mathbb{P}\{\mathcal{E}_{\mathrm{e2e}} \mid \mathcal{A}_r, N_r = i\} &= \mathbb{P}\{\mathcal{E}_{rd} \mid \mathcal{A}_r, N_r = i\}\left(1-\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\}\right) \\ &\quad + \left(1-\mathbb{P}\{\mathcal{E}_{rd} \mid \mathcal{A}_r, N_r = i\}\right)\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\}.\end{aligned} \tag{5}$$

The probability of bit error at a reliable relay is given by [11]

$$\begin{aligned}\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\} &= b\Bigg[\mathrm{erfc}\left(\sqrt{a\gamma_t}\right) \\ &\quad - e^{\gamma_t/\overline{\gamma}_{sr}}\sqrt{\frac{a\overline{\gamma}_{sr}}{1+a\overline{\gamma}_{sr}}}\,\mathrm{erfc}\left(\sqrt{\gamma_t\left(a+\frac{1}{\overline{\gamma}_{sr}}\right)}\right)\Bigg].\end{aligned} \tag{6}$$

Substituting (5) into (4), we obtain the e2e BER conditioned on N_r as

$$\begin{aligned}\mathbb{P}(\mathcal{E}_{\text{e2e}} \mid N_r = i) &= \mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\} \\ &\quad + i(\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\} \\ &\qquad + \mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\}(1 - 2\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\}) \\ &\qquad + \mathbb{P}\{\mathcal{A}_r \mid N_r = i\}\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\}).\end{aligned} \tag{7}$$

The probability that a particular reliable relay is selected by the destination is equal to

$$\mathbb{P}\{\mathcal{A}_r \mid N_r = i\} = \frac{1}{i}\left(1 - \sum_{j=0}^{i}\binom{i}{j}(-1)^j \frac{1}{1 + j\left(\overline{\gamma}_{sd}/\overline{\gamma}_{rd}\right)}\right). \tag{8}$$

The terms $\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\}$ and $\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\}$ are given by

$$\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\} = \sum_{j=0}^{i}\left\{\binom{i}{j}(-1)^j \frac{\overline{\gamma}_{rd}}{j\overline{\gamma}_{sd} + \overline{\gamma}_{rd}} \overline{P}_b\left(\frac{\overline{\gamma}_{sd}\overline{\gamma}_{rd}}{j\overline{\gamma}_{sd} + \overline{\gamma}_{rd}}\right)\right\}, \tag{9}$$

$$\begin{aligned}\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\} &= \sum_{j=0}^{i-1}\left\{\binom{i-1}{j}(-1)^j \right. \\ &\quad \times \left[\frac{1}{j+1}\overline{P}_b\left(\frac{\overline{\gamma}_{rd}}{j+1}\right)\right. \\ &\quad \left.\left. - \frac{\overline{\gamma}_{sd}}{\overline{\gamma}_{sd}(j+1) + \overline{\gamma}_{rd}}\overline{P}_b\left(\frac{\overline{\gamma}_{sd}\overline{\gamma}_{rd}}{\overline{\gamma}_{sd}(j+1) + \overline{\gamma}_{rd}}\right)\right]\right\}.\end{aligned} \tag{10}$$

See Appendix A for the derivations of (8)–(10). By substituting (8)–(10) into (7) and then substituting (4), (7) into (1), we obtain an exact expression for the e2e BER of the threshold based relay selection protocol described in Section 2.

4. Diversity Order of TRSC

In this section, we consider a modulation scheme with $P_b(\gamma) = b\,\text{erfc}(\sqrt{a\gamma})$, where γ is the received SNR. Based on the insight from the e2e BER minimizing threshold derived in [13], for a network with N relays we propose to use a threshold function in the form of $\log(c_1 \text{SNR}^{N/a})$, where c_1 is a positive constant. Next, we show that TRSC can achieve full diversity with the proposed threshold function.

The e2e BER is given in (1). For the first term we have

$$\mathbb{P}\{N_r = i\} = \binom{N}{i}\left(e^{-\gamma_t/\overline{\gamma}_{sr}}\right)^i\left(1 - e^{-\gamma_t/\overline{\gamma}_{sr}}\right)^{N-i}. \tag{11}$$

Let us denote the asymptotic equivalence of two positive functions f and g as $f \sim g$. The functions f and g are called asymptotically equivalent functions if $\lim_{x\to\infty}(f(x)/g(x)) = 1$. If $\limsup_{x\to\infty}(f(x)/g(x)) < \infty$, we say that f is asymptotically less than or equal to g and denote it as $f = O(g)$.

With the proposed threshold as SNR $\to \infty$ we have

$$\begin{aligned}e^{-\gamma_t/\overline{\gamma}_{sr}} &= e^{-\log(c_1\text{SNR}^{N/a})/(\lambda_{sr}\text{SNR})} \sim 1, \\ \left(1 - e^{-\gamma_t/\overline{\gamma}_{sr}}\right) &= 1 - e^{-\log(c_1\text{SNR}^{N/a})/(\lambda_{sr}\text{SNR})} \sim \frac{\log\left(c_1\text{SNR}^{N/a}\right)}{\lambda_{sr}\text{SNR}}.\end{aligned} \tag{12}$$

We note that throughout this paper all the logarithms are in the natural base.

Thus, $\mathbb{P}\{N_r = i\}$ is of order

$$\mathbb{P}\{N_r = i\} = O\left(\frac{\log\left(\text{SNR}^{N/a}\right)^{N-i}}{\text{SNR}^{N-i}}\right). \tag{13}$$

Next, we study how fast the term $\mathbb{P}\{\mathcal{E}_{\text{e2e}} \mid N_r = i\}$ (given in (7)) decays with increasing SNR.

Lemma 1 (Asymptotic behavior of $\mathbb{P}\{\mathcal{E}_{\text{e2e}} \mid N_r = i\}$). *With the proposed threshold $\gamma_t = \log(c_1 SNR^{N/a})$, we have $\mathbb{P}\{\mathcal{E}_{e2e} \mid N_r = i\} = O(1/SNR^{i+1})$.*

See Appendix B for the proof.

Combining the result of Lemma 1 with (13), we observe that in (1) the term with index i, that is, $\mathbb{P}\{N_r = i\}\mathbb{P}\{\mathcal{E}_{\text{e2e}} \mid N_r = i\}$ decreases as $O(\log(\text{SNR}^{N/a})^{N-i}/\text{SNR}^{N+1})$. The order of the sum of these $N+1$ terms is determined by the term that has the slowest decay, which is the term with index $i = 0$. Hence,

$$\mathbb{P}\{\mathcal{E}_{\text{e2e}}\} = O\left(\frac{\log\left(\text{SNR}^{N/a}\right)^N}{\text{SNR}^{N+1}}\right). \tag{14}$$

We observe that while the $N+1$ order diversity achieved by conventional diversity combining schemes will decrease as $1/\text{SNR}^{N+1}$, the cooperative diversity achieved by the TRSC protocol has a decay of $O(\log(\text{SNR}^{N/a})^N/\text{SNR}^{N+1})$. However, at large SNR the log term becomes insignificant and the diversity order, which is defined in [23], is equal to

$$d = -\lim_{\text{SNR}\to\infty}\frac{\log(\mathbb{P}\{\mathcal{E}_{\text{e2e}}\})}{\log(\text{SNR})} = N + 1. \tag{15}$$

We summarize the results in this section as the following theorem.

Theorem 1. *Assume a general modulation scheme with bit error rate $P_b(\gamma) = b\,\text{erfc}(\sqrt{a\gamma})$ given receive SNR γ. TRSC can achieve diversity order of $N+1$ in an N-relay network if the threshold function at each relay is in the form of*

$$\gamma_t = \log\left(c_1 SNR^{N/a}\right), \tag{16}$$

where c_1 is a positive constant.

5. Results

In this section we compare the e2e BER of TRSC to two other Relay Selection Cooperation (RSC) protocols that are described below. We also present simulation results for asymmetric networks in which average SNRs of different relays are not identical.

5.1. Benchmark Protocols

5.1.1. RSC-Inst. In the first protocol, *RSC-inst*, the relay is selected based on the equivalent instantaneous BER of branches. The equivalent BER of relay k is given by

$$P_k^{\text{inst}}(\gamma_{sr,k},\gamma_{rd,k}) = P_b(\gamma_{sr,k})(1-P_b(\gamma_{rd,k})) + P_b(\gamma_{rd,k})(1-P_b(\gamma_{sr,k})), \quad k=1,\ldots,N, \tag{17}$$

and $P_0^{\text{inst}} = P_b(\gamma_{sd})$. The destination selects the branch with the minimum equivalent BER. Note that in this protocol, the specific $S-R_i$ and R_i-D channel information is required at the destination in order to perform selection. This protocol is very similar to C-MRC with relay selection introduced in [9]. The only difference is that the scheme in [9] combines the direct signal with one of the relay signals, whereas RSC-inst selects either the destination or one of the relays.

5.1.2. RSC-Avr. The second protocol we compare to is *RSC-avr* in which the destination has no knowledge of $\gamma_{sr,k}$ values and the relay selection is based on $\overline{\gamma}_{sr,k}$, γ_{sd}, and $\gamma_{rd,k}$ values. Then, the equivalent BER of relay k is given by

$$P_k^{\text{avr}}\left(\overline{\gamma}_{sr,k},\gamma_{rd,k}\right) = \overline{P}_b\left(\overline{\gamma}_{sr,k}\right)(1-P_b(\gamma_{rd,k})) + P_b(\gamma_{rd,k})\left(1-\overline{P}_b\left(\overline{\gamma}_{sr,k}\right)\right), \quad k=1,\ldots,N, \tag{18}$$

and $P_0^{\text{avr}} = P_b(\gamma_{sd})$. The destination selects the link with the lowest P_k^{avr}. While RSC-inst is the selection version of the C-MRC of [8], RSC-avg can be viewed as the selection version of the maximum likelihood receiver of [7].

5.2. Numerical Results. For numerical results, we first consider a symmetric network scenario, in which all average link SNRs are the same ($\overline{\gamma}_{sr} = \overline{\gamma}_{rd} = \overline{\gamma}_{sd} = \overline{\gamma}$). Binary phase shift keying (BPSK) modulation is used by all the nodes, that is, $(b,a) = (0.5,1)$. Figures 2 and 3 show the e2e BER of different protocols as a function of $\overline{\gamma}$ for $N=1$ and $N=2$ relays, respectively. In each figure, there are two curves for TRSC: *optimal TRSC* and *suboptimal TRSC*. The threshold values for *optimal TRSC* are determined from the numerical minimization of the analytical e2e BER expression obtained in Section 3. For suboptimal TRSC the threshold values are calculated according to the threshold function we propose (given in (16)) with $c_1 = 1$, that is, $\gamma_t = \log(\overline{\gamma}^N)$.

For $N=1$, TRSC and RSC-avr perform similarly, while RSC-inst performs slightly better than these two protocols and all protocols achieve full diversity gain as observed from the slopes of the BER curves. However, as the number of relays is increased to $N=2$, RSC-avr cannot deliver full diversity. In fact, by analyzing RSC-avr for different N values, we observe that the diversity order of RSC-avr is limited to 2. The TRSC with the suboptimal threshold achieves full diversity for both N values as evident from the slope of the BER curves, in accordance with our claims in Section 4.

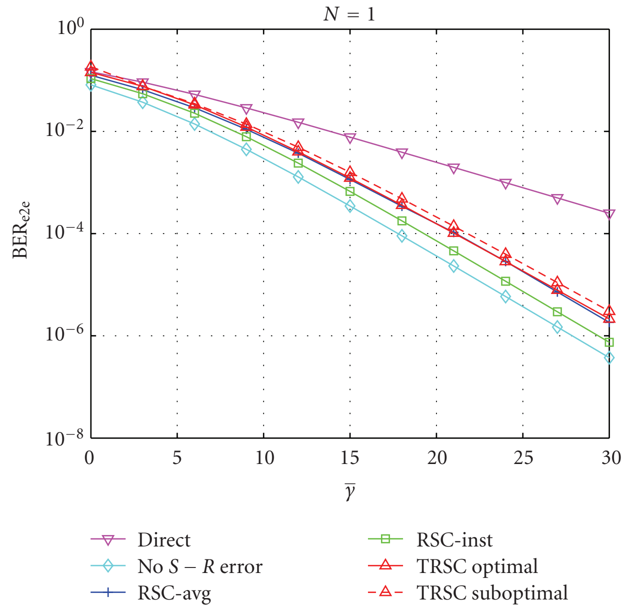

FIGURE 2: The e2e BER for all relaying protocols for $N = 1$ relay. The BER of direct transmission and the BER in the absence of errors in the $S-R_i$ links are also shown as reference curves.

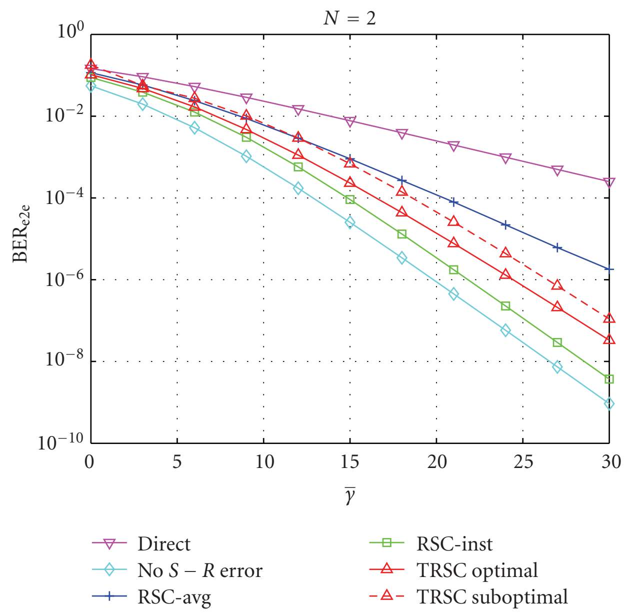

FIGURE 3: The e2e BER for all relaying protocols for $N = 2$ relays. The BER of direct transmission and the BER in the absence of errors in the $S-R_i$ links are also shown as reference curves.

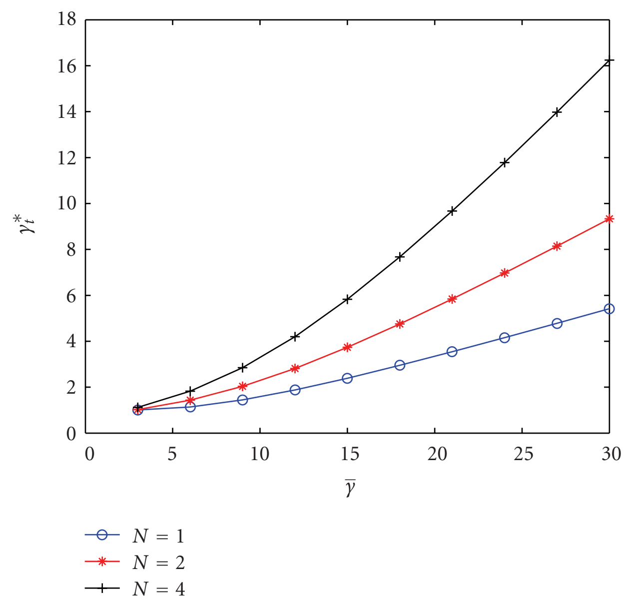

FIGURE 4: Threshold values that minimize e2e BER of TRSC in symmetric networks with different numbers N of relays.

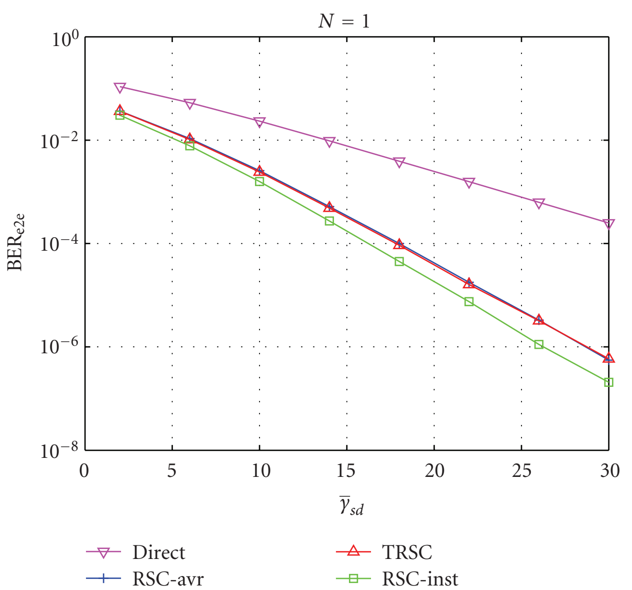

FIGURE 5: The e2e BER for all relaying protocols for $N = 1$ relay under random topologies.

By comparing the TRSC curves with the optimal and the suboptimal threshold we observe that there is approximately 0.5 dB and 2 dB loss in SNR for $N = 1$ and $N = 2$, respectively. We note that the suboptimal threshold we propose achieves full diversity for any positive constant c_1. However, this threshold function is not necessarily optimal even if the c_1 value is selected carefully.

In order to examine the behavior of the optimal threshold, in Figure 4, we show the threshold values used by TRSC to minimize e2e BER through numerical optimization. It is seen that the optimal threshold increases with increasing

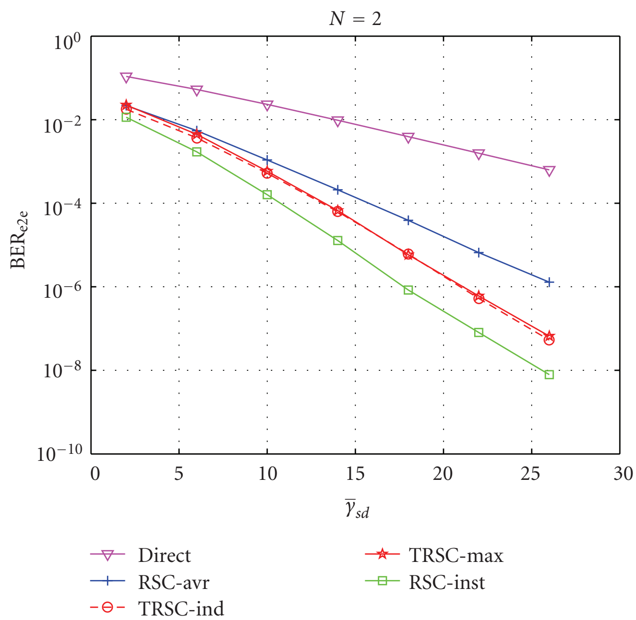

FIGURE 6: The e2e BER for all relaying protocols for $N = 2$ relays under random topologies.

number of relays. As a function of average SNR, the optimal threshold increases logarithmically. We note that the suboptimal threshold we have proposed in this paper also shares these two properties. However, finding optimal thresholds analytically remains a challenging problem for future work.

The motivation of the analysis in this paper was to obtain insight to the threshold selection problem. To simplify the analysis we have assumed that the average source-relay and relay-destination SNRs are common for all relays. In practice, all links are expected to have nonidentical average SNRs and it is desirable to understand the performance of TRSC under different scenarios. For the multiple relay case, finding the optimal threshold for each relay following the same approach as in Section 3 seems intractable. We propose two strategies to determine the threshold values in this case. In the first strategy, which we call *TRSC-ind*, each relay calculates the optimal threshold (as in TRSC optimal described above) assuming that the remaining $N - 1$ relays have the same $\bar{\gamma}_{SR}$ and $\bar{\gamma}_{RD}$ values as itself. Note that we also assume that each relay knows the number of relays N. Different relays employ TRSC based on their individual threshold values. In the second strategy, which is called *TRSC-max*, after calculating individual relay thresholds as in TRSC-ind, the largest of N thresholds is set as the common threshold of all relays. We assume a separate mechanism to convey the value of the maximum threshold among all relays.

Next, we present simulation results on the average performance of TRSC over random SNR values. We assume that the relay positions are selected randomly on the line connecting S and D. The average SNR values are calculated based on distance assuming a pathloss exponent of $\alpha = 3$. The results are averaged over many random relay positions. First, as a reference, in Figure 5 we plot the performance of optimal TRSC for the single relay case. It is seen that the conclusions are similar to those for the symmetric network shown in Figure 2. In Figure 6 we plot the average performance of these two strategies for $N = 2$ relays. The two strategies perform very closely and the gain over RSC-avr is preserved.

We conclude that TRSC offers a good tradeoff between performance and signaling overhead since it performs comparable to RSC-inst with no instantaneous $S - R$ SNR knowledge at the destination.

6. Conclusions and Discussion

In this paper, we have analyzed a threshold based relay selection protocol for two hop, multirelay cooperative communication. This protocol requires minimal information at the destination about the SNRs of the source-relay links. We have proposed a threshold function that increases logarithmically with the link SNRs and linearly with the number of relays. We have shown that, with a threshold of this form, threshold based relay selection protocol achieves full diversity.

We have presented performance results for the threshold based relay selection with the proposed threshold function and optimal threshold values determined through numerical optimization. We have compared the BER of threshold based relay selection to similar protocols found in the literature. Although our mathematical analysis has assumed simplified network scenarios, through simulations we have verified the applicability of threshold based selection relaying for general scenarios.

Appendices

A. Derivation of (8), (9), and (10)

A.1. Derivation of (8). The probability of $\mathcal{A}_s$ can be expressed as

$$\begin{aligned}\mathbb{P}\{\mathcal{A}_s \mid N_r = i\} &= \mathbb{P}\{\gamma_{sd} > \gamma_{rd,1}, \ldots, \gamma_{rd,i}\} \\ &= \int_0^\infty p_{\gamma_{sd}}(\gamma_{sd}) \int_0^{\gamma_{sd}} p_{\gamma_{rd,1}}(\gamma_{rd,1}) \cdots \\ &\quad \times \int_0^{\gamma_{sd}} p_{\gamma_{rd,i}}(\gamma_{rd,i}) \mathrm{d}\gamma_{rd,i} \cdots \mathrm{d}\gamma_{rd,1} \mathrm{d}\gamma_{sd} \\ &= \int_0^\infty \frac{1}{\overline{\gamma}_{sd}} e^{\gamma_{sd}/\overline{\gamma}_{sd}} \left(1 - e^{-\overline{\gamma}_{sd}/\overline{\gamma}_{rd}}\right)^i \mathrm{d}\gamma_{sd}.\end{aligned} \tag{A.1}$$

Using the binomial expansion for $(1 - e^{-\gamma_{sd}/\overline{\gamma}_{rd}})^i$ we obtain

$$\mathbb{P}\{\mathcal{A}_s \mid N_r = i\} = \sum_{j=0}^{i} \binom{i}{j} (-1)^j \frac{1}{1 + j\left(\overline{\gamma}_{sd}/\overline{\gamma}_{rd}\right)}. \tag{A.2}$$

Since the probability of being selected by the destinations is the same for all potential relays and independent of index k, we denote it by $\mathbb{P}\{\mathcal{A}_r\}$ and calculate it as $\mathbb{P}\{\mathcal{A}_r\} = (1/i)(1 - \mathbb{P}\{\mathcal{A}_s\})$. Hence,

$$\mathbb{P}\{\mathcal{A}_{r,k} \mid N_r = i\} = \frac{1}{i}\left(1 - \sum_{j=0}^{i} \binom{i}{j} (-1)^j \frac{1}{1 + j\left(\overline{\gamma}_{sd}/\overline{\gamma}_{rd}\right)}\right). \tag{A.3}$$

A.2. Derivation of (9). The term $\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\}$ is equal to the following integral:

$$\begin{aligned}&\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\} \\ &\quad = \int_{\mathcal{A}_s} P_b(\gamma_{sd}) p_{\gamma_{rd,1}}(\gamma_{rd,1}) \cdots \\ &\qquad \times p_{\gamma_{rd,i}}(\gamma_{rd,i}) p_{\gamma_{sd}}(\gamma_{sd}) \mathrm{d}\gamma_{rd,1} \cdots \mathrm{d}\gamma_{rd,i} \mathrm{d}\gamma_{sd} \\ &\quad = \int_0^\infty P_b(\gamma_{sd}) \left(1 - e^{-\gamma_{sd}/\overline{\gamma}_{rd}}\right)^i \frac{1}{\overline{\gamma}_{sd}} e^{-\gamma_{sd}/\overline{\gamma}_{sd}} \mathrm{d}\gamma_{sd}.\end{aligned} \tag{A.4}$$

Again, using the binomial expansion for $(1 - e^{-\gamma_{sd}/\overline{\gamma}_{rd}})^i$ we obtain

$$\begin{aligned}&\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\} \\ &\quad = \sum_{j=0}^{i} \left\{ \binom{i}{j} (-1)^j \frac{\overline{\gamma}_{rd}}{j\overline{\gamma}_{sd} + \overline{\gamma}_{rd}} \overline{P}_b\left(\frac{\overline{\gamma}_{sd}\overline{\gamma}_{rd}}{j\overline{\gamma}_{sd} + \overline{\gamma}_{rd}}\right) \right\}.\end{aligned} \tag{A.5}$$

A.3. Derivation of (10). Similarly, the error probability given that a particular relay R_k is selected is equal to

$$\begin{aligned}&\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_{r,k} \mid N_r = i\} \\ &\quad = \int_{\mathcal{A}_{r,k}} P_b(\gamma_{rd,k}) p_{\gamma_{rd,1}}(\gamma_{rd,1}) \cdots p_{\gamma_{rd,i}}(\gamma_{rd,i}) \\ &\qquad \times p_{\gamma_{sd}}(\gamma_{sd}) \mathrm{d}\gamma_{rd,1} \ldots \mathrm{d}\gamma_{rd,i} \mathrm{d}\gamma_{sd} \\ &\quad = \int_0^\infty P_b(\gamma_{rd,k}) \left(1 - e^{-\gamma_{rd,k}/\overline{\gamma}_{rd}}\right)^{i-1} \\ &\qquad \times \left(1 - e^{-\gamma_{rd,k}/\overline{\gamma}_{sd}}\right) \frac{1}{\overline{\gamma}_{rd}} e^{-\gamma_{rd,k}/\overline{\gamma}_{rd}} \mathrm{d}\gamma_{rd,k} \\ &\quad = \sum_{j=0}^{i-1} \left\{ \binom{i-1}{j} (-1)^j \left[\frac{1}{j+1} \overline{P}_b\left(\frac{\overline{\gamma}_{rd}}{j+1}\right) \right.\right. \\ &\qquad - \frac{\overline{\gamma}_{sd}}{\overline{\gamma}_{sd}(j+1) + \overline{\gamma}_{rd}} \\ &\qquad \left.\left. \times \overline{P}_b\left(\frac{\overline{\gamma}_{sd}\overline{\gamma}_{rd}}{\overline{\gamma}_{sd}(j+1) + \overline{\gamma}_{rd}}\right) \right] \right\}.\end{aligned} \tag{A.6}$$

B. Proof of Lemma 1

We prove this lemma by analyzing the orders of terms in (7) as SNR $\to \infty$.

Part 1. Let us first analyze the asymptotic behavior of $\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\}$ and $\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\}$. In the absence of errors at the reliable relays the bit error probability at the destination would be equal to the performance of $(i+1)$ branch selection combining (SC), where one of the branches has average SNR of $\overline{\gamma}_{sd}$, and the rest have $\overline{\gamma}_{rd}$. The probability of bit error of SC can be expressed as

$$\overline{P}_b^{\mathrm{SC}}\left(i, \overline{\gamma}_{sd}, \overline{\gamma}_{rd}\right) = \mathbb{P}\{\mathcal{E}_{sd}, A_s \mid N_r = i\} + i\mathbb{P}\{\mathcal{E}_{rd}, A_r \mid N_r = i\}. \tag{B.1}$$

Hence, $\mathbb{P}\{\mathcal{E}_{sd}, A_s \mid N_r = i\} \leq \overline{P}_b^{\mathrm{SC}}(i, \overline{\gamma}_{sd}, \overline{\gamma}_{rd})$ and $\mathbb{P}\{\mathcal{E}_{rd}, A_r \mid N_r = i\} \leq \overline{P}_b^{\mathrm{SC}}(i, \overline{\gamma}_{sd}, \overline{\gamma}_{rd})$. Since SC is known to achieve diversity order equal to the number of its branches, we conclude that both $\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\}$ and $\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\}$ decrease at least as fast as $1/\mathrm{SNR}^{i+1}$: $\mathbb{P}\{\mathcal{E}_{sd}, A_s \mid N_r = i\} = O(1/\mathrm{SNR}^{i+1})$ and $\mathbb{P}\{\mathcal{E}_{rd}, A_r \mid N_r = i\} = O(1/\mathrm{SNR}^{i+1})$.

Part 2. Now, let us examine the order of the term $\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\}$ if $\gamma_t = \log(c_1 \mathrm{SNR}^{N/b})$. The analysis closely follows that given in [13] for $N = 1$ relay. In [13] for BPSK and any threshold γ_t it is shown that $\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\} < (1/2\overline{\gamma}_{sr}) \operatorname{erfc}(\sqrt{\gamma_t})$. In the case of $P_b(\gamma) = b \operatorname{erfc}(a\gamma)$, this bound can easily be generalized to

$$\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\} < \frac{1}{\overline{\gamma}_{sr}} b \operatorname{erfc}\left(\sqrt{a\gamma_t}\right). \tag{B.2}$$

Using the well-known bound $\operatorname{erfc}(z) < e^{-z^2}$, we obtain

$$\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\} < \frac{b}{\overline{\gamma}_{sr}} e^{-a\gamma_t}. \tag{B.3}$$

By substituting $\gamma_t = \log(c_1 \mathrm{SNR}^{N/a})$, we conclude that

$$\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\} < \frac{b}{\overline{\gamma}_{sr}} \frac{1}{c_1^a} \frac{1}{\mathrm{SNR}^N} = \frac{b}{c_1^a \lambda_{sr}} \frac{1}{\mathrm{SNR}^{N+1}}. \tag{B.4}$$

Thus $\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\} = O(1/\mathrm{SNR}^{N+1})$.

Part 3. As seen in (8), $\mathbb{P}\{\mathcal{A}_r \mid N_r = i\}$ depends on $\overline{\gamma}_{rd}$ and $\overline{\gamma}_{sd}$ only through their ratio. Hence, this quantity is independent of SNR and $\mathbb{P}\{\mathcal{A}_r \mid N_r = i\} = O(1)$.

Combining Parts 1, 2, and 3, we obtain

$$\begin{aligned}
&\mathbb{P}(\mathcal{E}_{\mathrm{e2e}} \mid N_r = i) \\
&= \underbrace{\mathbb{P}\{\mathcal{E}_{sd}, \mathcal{A}_s \mid N_r = i\}}_{O(1/\mathrm{SNR}^{i+1})} \\
&\quad + i \times \Bigg(\underbrace{\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\}}_{O(1/\mathrm{SNR}^{i+1})} \\
&\qquad + \underbrace{\mathbb{P}\{\mathcal{E}_{rd}, \mathcal{A}_r \mid N_r = i\}}_{O(1/\mathrm{SNR}^{i+1})} \underbrace{\left(1 - 2\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\}\right)}_{O(1)} \\
&\qquad + \underbrace{\mathbb{P}\{\mathcal{A}_r \mid N_r = i\}}_{O(1)} \underbrace{\mathbb{P}\{\mathcal{E}_{sr} \mid \gamma_{sr} > \gamma_t\}}_{O(1/\mathrm{SNR}^{N+1})} \Bigg).
\end{aligned} \tag{B.5}$$

Hence, $\mathbb{P}\{\mathcal{E}_{\mathrm{e2e}} \mid N_r = i\} = O(1/\mathrm{SNR}^{i+1})$.

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