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Research Article

Performance Analysis for Linearly Precoded LTE Downlink Multiuser MIMO

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The average channel capacity and the SINR distribution for multiuser multiple input multiple output (MIMO) systems in combination with the base station based packet scheduler are analyzed in this paper. The packet scheduler is used to exploit the available multiuser diversity in all the three physical domains (i.e., space, time and frequency). The analysis model is based on the generalized 3GPP LTE downlink transmission for which two spatial division multiplexing (SDM) multiuser MIMO schemes are investigated: single user (SU) and multiuser (MU) MIMO schemes. The main contribution of this paper is the establishment of a mathematical model for the SINR distribution and the average channel capacity for multiuser SDM MIMO systems with frequency domain packet scheduler, which provides a theoretical reference for the future version of the LTE standard and a useful source of information for the practical implementation of the LTE systems.

1. Introduction

In 3GPP long term evolution (LTE) (also known as evolved-UMTS terrestrial radio access (E-UTRA)), multiple-input multiple-output (MIMO) and orthogonal frequency division multiple access (OFDMA) have been selected for downlink transmission [1]. Both Spatial Division Multiplexing (SDM) and frequency domain packet scheduling (FDPS) have been proposed. SDM simply divides the data stream into multiple independent sub-streams, which are subsequently transmitted by different antennas simultaneously. It is used to improve the spectral efficiency of the system. FDPS allows the packet scheduler at the base station (BS) to exploit the available multiuser diversity in both time and frequency domain. In [2], it is shown that the MIMO schemes with combined SDM and FDPS can further enhance the system performance.

This paper investigates the average channel capacity of the multiuser SDM MIMO schemes with FDPS for the generalized 3GPP LTE MIMO-OFDMA based downlink transmission. Both open loop and closed loop MIMO (open loop and closed loop MIMO correspond to the MIMO systems without and with channel state information at the transmitter, resp. [1]) are considered as possible solutions in 3GPP LTE. However, the closed loop solution provides both diversity and array gains, and hence a superior performance. Due to its simplicity and robust performance, the use of linear precoding has been widely studied as a closed loop scheme [2, 3]. In this paper, we refer to the open loop MIMO as the SDM MIMO without precoding, and the closed loop MIMO as the linearly precoded SDM MIMO.

Most of the existing work on linear precoding focuses on the design of the transmitter precoding matrix, for example, [3, 4]. In [5, 6], the interaction between packet scheduling and array antenna techniques is studied based on a system level simulation model. The interactions between multiuser diversity and spatial diversity is investigated analytically in [7], with the focus on space time block coding. In a more recent paper [8], system performance for open loop MIMO systems with zero forcing receiver was analyzed. To the authors knowledge, theoretical analysis of linearly precoded multiuser SDM MIMO systems combined with FDPS has not been studied so far. In this paper, we conduct a theoretical analysis for signal to interference plus noise ratio (SINR) distribution and the average channel capacity in multiuser MIMO systems with SDM-FDPS. The packet scheduler is able to exploit the available multiuser diversity in time, frequency and spatial domains. Although our study is conducted for the generalized 3GPP LTE-type downlink packet data transmission [1], the analysis method is generally applicable to other packet switched systems.

In the remainder of this paper, we present the multiuser SDM MIMO system model in Section 2, where the FDPS algorithm is also discussed. Sections 3 and 4 describe the SINR distribution for open loop and closed loop MIMO schemes, respectively. The average channel capacity of the investigated systems are given in Section 5. The analytical and numerical results are provided and discussed in Section 6. Finally, the conclusions are drawn in Section 7.

2. System Model

In this section, we describe the system model of multiuser SDM MIMO schemes for 3GPP LTE downlink transmission with packet scheduling. The basic scheduling unit in LTE is the physical resource block (PRB), which consists of a number of consecutive OFDM sub-carriers reserved during the transmission of a fixed number of OFDM symbols. One PRB of 12 contiguous subcarriers can be configured for localized transmission in a sub-frame (in the localized FDMA transmission scheme, each user's data is transmitted by consecutive subcarriers, while for the distributed FDMA transmission scheme, the user's data is transmitted by distributed subcarriers [1].) With the localized transmission scheme, two SDM schemes are now under investigation [1], that is, single user (SU) MIMO and multi-user (MU) MIMO schemes. They differ in terms of the freedom allowed to the scheduler in the spatial domain [1]. With SU-MIMO scheme, only one single user can be scheduled per PRB; whereas with MU-MIMO scheme, multiple users can be scheduled per PRB, one user for each substream per PRB.

The frequency domain (FD) scheduling algorithm considered in this work is the FD proportional fair (PF) [9] packet scheduling algorithm, which is being investigated under LTE. With the FD PF scheduling algorithm, the scheduler selects users at the kth time slot according to $k^* = \arg\max_{k\in\{1,2,\dots,K\}}\{\mathrm{SINR}_{l,k}/\overline{\mathrm{SINR}_{l,k}}\}$, where $\overline{\mathrm{SINR}_{l,k}}$ is the average received SINR for user k at the lth time slot over a sliding window of T_{win} time slots. When the average received SINR for different users are different, which is the usual case of the system, the distribution of the average received SINR has to be calculated based on the distribution for the instantaneous received SINR. Since the average SINR is obtained by averaging the instantaneous received SINRs in a predefined time interval, with the knowledge of the distribution of the instantaneous received SINR, the distribution of the average SINR can be calculated based on the characteristic function [10]. In this paper, for simplicity, we only consider the case that all users in the system have equal received SINR based on a simplifing assumption similar to those made in [11]. In our future work, we will extend it to the case that all users have different average received SINR.

The simplifying assumptions are fading statistics for all users are independent identically distributed, users move with same speed and have the same access ability, T_{win} is sufficiently large so that the average received user data rates are stationary, and the SINRs for all users are within a dynamic range of the system, where a throughput increase is proportional to an increase of SINR, which is usually a reasonable assumption. When all users have equal average received SINR, the scheduler at the BS just selects the users with the best effective SINRs (the unified effective SINR is defined as the equivalent single stream SINR which offers the same instantaneous (Shannon) capacity as a MIMO scheme with multiple streams [12]. Let γ_q, $q \in \{1,2,\dots\}$, be the SINR of the qth substream, and γ_u be the unified effective SINR, then $\log_2(1+\gamma_u) = \sum_q \log_2(1+\gamma_q)$, so $\gamma_u = \prod_q(1+\gamma_i) - 1$. The distribution of γ_u can be derived given the distribution of γ_q. The purpose of introducing unified SINR is to facilitate the SINR comparison between SU MIMO and MU MIMO schemes.) This assumption becomes valid when all users have roughly the same channel condition, so that the received average throughput for all users are approximately the same.

The system considered here has n_t transmit antennas at the base station (BS) and n_r receive antennas for the MS in SU-MIMO case, and a single receive antenna for each MS in MU-MIMO case. In the latter case, we assumer n_r MSs group together to form a virtual MIMO between BS and the group of MSs. We define $M = \min(n_t, n_r)$ and $N = \max(n_t, n_r)$. The number of users simultaneously served on each PRB for the MU-MIMO scheme is usually limited by the number of transmitter antennas n_t. The scheduler in BS select at most n_t users per PRB from the K active users in the cell for data transmission. Denote by ζ_k the set of users scheduled on the kth PRB and $|\zeta_{\mathbf{k}}| = n_t$. The received signal vector at the nth PRB can then be modeled as

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{x}_n + \mathbf{n}_n, \tag{1}$$

where $\mathbf{n}_n \in \mathbb{C}^{n_r\times 1}$ is a circularly symmetric complex Gaussian noise vector with a zero mean and covariance matrix $N_0\mathbf{I} \in \mathbb{R}^{n_r\times n_r}$, that is, $\mathbf{n}_n \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$. $\mathbf{H}_n \in \mathbb{C}^{n_r\times n_t}$ is the channel matrix between the BS and the MSs at the nth PRB and $\mathbf{x}_n = [x_{n,1} \cdots x_{n,n_t}]^T$ is the transmitted signal vector at the nth PRB, and the $x_{n,\mu}$ is the data symbol transmitted from the μth MS, $\mu \in \zeta_n$.

With linear precoding, the received signal vector for the scheduled group of MSs can be obtained by

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{B}_n\mathbf{x}_n + \mathbf{n}_n, \tag{2}$$

where $\mathbf{B}_n \in \mathbb{C}^{n_t\times n_t}$ is the precoding matrix.

For the MU-MIMO SDM scheme with linear precoding, we use the transmit antenna array (TxAA) technique [13] which is also known as the closed loop transmit diversity (CLTD) [14] in the terminology of 3GPP. The TxAA technique is to use channel state information (CSI) to perform eigenmode transmission. For the TxAA scheme, the antenna weight vector is selected to maximize the SNR at the MS. Furthermore, we assume that the selected users can be cooperated for receiving and investigate the scenarios where the downlink cooperative MIMO is possible. Practical situations where such assumption could apply: (1) users are close, such as they are within the range of WLAN, Bluetooth, and so forth, (2) for eNB to Relay communications where the relays play the role of users; Relays could be assumed to be deployed as a kind of meshed sub-network and therefore able to cooperate in receiving over the downlink MIMO channel. In both cases, one could foresee the need in connection with hot-spots—specific areas where capacity needs to be relieved by multiplexing transmissions in the downlink.

With a linear minimum mean square error (MMSE) receiver, also known as a Wiener filter, the optimum precoding matrix under the sum power constraint can be generally expressed as $\mathbf{B}_n = \mathbf{U}_n\sqrt{\mathbf{\Sigma}_n}\mathbf{V}_n$ [15]. Here $\mathbf{U}_n$ is an $n_t \times n_t$ eigenvector matrix with columns corresponding to the n_t largest eigenvalues of the matrix $\mathbf{H}_n\mathbf{H}_n^H$, where $\mathbf{H}_n^H$ is the Hermitian transpose of the channel matrix $\mathbf{H}_n$. For Schur-Concave objective functions, $\mathbf{V}_n \in \mathbb{C}^{n_t\times n_t}$ is an unitary matrix, and $\mathbf{\Sigma}_n$ is a diagonal matrix with the ηth diagonal entry $\mathbf{\Sigma}_n(\eta,\eta)$ representing the power allocated to the ηth established data sub-stream, $\eta \in \{1, 2, \ldots, n_t\}$.

3. SINR Distribution for Open Loop Spatial Multiplexing MIMO

For an open loop single user MIMO-OFDM system with n_t transmit antennas and n_r receive antennas, assuming the channel is uncorrelated flat Rayleigh fading channel at each subcarrier (this is a valid assumption since the OFDM technique transforms the broadband frequency selective channel into many narrow band subchannels, each of which can be treated as a flat Rayleigh fading channel.) the received signal vector at the receive antennas for the nth subcarrier can be expressed as (1).

With a ZF receiver, the SINR on the kth sub-stream has a Chi-squared probability density distribution (PDF) [16]

$$f_{\Gamma_k}(\gamma) = \frac{n_t\sigma_k^2 e^{-n_t\gamma\sigma_k^2/\gamma_0}}{\gamma_0(n_r - n_t)!}\left(\frac{n_t\gamma\sigma_k^2}{\gamma_0}\right)^{(n_r-n_t)}, \quad (3)$$

where $\gamma_0 = E_s/N_0$, E_s is the average transmit symbol energy per antenna and N_0 is the power spectral density of the additive white Gaussian noise and Γ_k represents the instantaneous SINR on the kth spatial sub-stream, σ_k^2 is the kth diagonal entry of $\mathbf{R}_t^{-1}$ where $\mathbf{R}_t$ is the transmit covariance matrix (in the rest of this paper, we denote by an upper case letter a random variable and by the corresponding lower case letter its realization.) Equation (3) is for the flat Rayleigh fading channel with uncorrelated receive antennas and with transmit correlation. For uncorrelated transmit antennas, $\mathbf{R}_t$ becomes an identity matrix, therefore, $\sigma_k^2 = 1$ in (3). For a dual stream spatial multiplexing MIMO scheme with a 2 $\times$ 2 antenna configuration, combining the two sub-stream SINRs of each PRB into an unified SINR with the same total (Shannon) capacity, the unified effective SINR $\Gamma_u = \prod_{i=1}^{2}(1+\Gamma_i) - 1$, the cumulative distribution function (CDF) for the post scheduling effective SINR can then be expressed as

$$\begin{aligned} F_{\Gamma_u}(\gamma) &= \Pr((\Gamma_1 + 1)(\Gamma_2 + 1) - 1 \le \gamma) \\ &= \int_0^\infty \Pr\left(\Gamma_2 \le \frac{\gamma - x}{x+1} \mid \Gamma_1 = x\right) f_{\Gamma_1}(x)dx. \end{aligned} \quad (4)$$

Under the assumption of the independence of the dual sub-streams, (4) becomes

$$F_{\Gamma_u}(\gamma) = \int_0^\infty f_{\Gamma_1}(x) F_{\Gamma_2}\left(\frac{\gamma - x}{x+1}\right)dx, \quad (5)$$

where $F_{\Gamma_k}(\gamma)$ is the CDF of the received SINR for the kth sub-stream and $F_{\Gamma_k}(\gamma) = \int_0^\gamma f_{\Gamma_k}(x)dx = (1 - e^{-n_t\gamma/\gamma_0})$ for the case of $n_t = n_r = 2$. Consequently, the CDF of the unified effective SINR can be represented by [12] $F_{\Gamma_u}(\gamma) = P_r(\Gamma_u \le \gamma) = \int_0^\gamma (2/\gamma_0)e^{-2x/\gamma_0}(1 - e^{-2(\gamma-x)/\gamma_0(1+x)})dx$. It was shown in [17] that in SDM with a ZF receiver, the MIMO channel can be decomposed into a set of parallel channels. Therefore, the received sub-stream SINRs are independent, which means that the assumption for (5) is valid.

For localized downlink transmission with SU-MIMO SDM scheme [1] and FDPF algorithm under the simplifying assumptions as mentioned in Section 2, the probability that the SINR of a scheduled user is below a certain threshold, that is, the CDF of the post scheduling SINR per PRB can be computed as

$$\begin{aligned} F_{\Gamma_u}^{OS}(\gamma) &= \Pr\left(\Gamma_u^1 \le \gamma, \Gamma_u^2 \le \gamma, \ldots, \Gamma_u^{K_T} \le \gamma\right) \\ &= \prod_{i=1}^{K_T} \Pr\left(\Gamma_u^i \le \gamma\right) = \left[F_{\Gamma_u}(\gamma)\right]^{K_T}, \end{aligned} \quad (6)$$

where Γ_u^i, $i \in \{1, 2, \ldots, K_T\}$, is the effective SINR for the ith user and K_T is the number of active users in the cell or the so-called user diversity order (UDO). Equation (6) is for the distribution of the best user, that is, the largest SINR selected from the K_T users.

The PDF of the post scheduling SINR, that is, the SINR after scheduling, per PRB can be obtained by differentiating its corresponding CDF as

$$\begin{aligned} f_{\Gamma_u}^{OS}(\gamma) &= \frac{d}{d\gamma}F_{\Gamma_u}^{OS}(\gamma) \\ &= K_T\left[\int_0^\gamma \frac{2}{\gamma_0}e^{-2x/\gamma_0}\left(1 - e^{-2(\gamma-x)/\gamma_0(1+x)}\right)dx\right]^{K_T-1} \\ &\quad \times \int_0^\gamma \left[\frac{4}{\gamma_0^2(1+x)}\exp\left(-\frac{2(\gamma + x^2)}{\gamma_0(1+x)}\right)\right]dx. \end{aligned} \quad (7)$$

For a MU-MIMO SDM scheme, multiuser diversity can also be exploited in the spatial domain, which effectively increases the UDO. This is due to the fact that for localized transmission under an MU-MIMO scheme in LTE, we can schedule multiple users per PRB, that is, one user per sub-stream. With a ZF receiver and K_T active users over an uncorrelated flat Rayleigh fading channel, the CDF of post scheduling SINR for each sub-stream is

$$F_{\Gamma_k}^{Ms}(\gamma) = \left(\int_0^{\gamma} \frac{n_t e^{-n_t\alpha/\gamma_0}}{\gamma_0(n_r - n_t)!}\left(\frac{n_t\alpha}{\gamma_0}\right)^{(n_r-n_t)} d\alpha\right)^{K_T}. \quad (8)$$

In the case of $n_r = n_t$, the above equation can be written in a closed form as $F_{\Gamma_k}^{Ms}(\gamma) = (1 - e^{-n_t\gamma/\gamma_0})^{K_T}$.

The PDF for the post scheduling sub-stream SINR can be derived as

$$f_{\Gamma_k}^{Ms}(\gamma) = \frac{n_t}{\gamma_0} e^{-n_t\gamma/\gamma_0} K_T \left(1 - e^{-n_t\gamma/\gamma_0}\right)^{(K_T-1)}. \quad (9)$$

For a dual stream MU-MIMO scheme with 2 antennas at both the transmitter and the receiver, the CDF for the post scheduling effective SINR per PRB can then be expressed as

$$F_{\Gamma_u}^{OM}(\gamma) = \int_0^{\gamma} \frac{n_t}{\gamma_0} e^{-n_t x/\gamma_0} K_T \left(1 - e^{-n_t x/\gamma_0}\right)^{(K_T-1)} \times \left(1 - e^{-n_t((\gamma-x)/(x+1))/\gamma_0}\right)^{K_T} dx. \quad (10)$$

4. SINR Distribution for Linearly Precoded SDM MIMO Schemes

In the previous section, the analysis of the SINR distribution was addressed for open loop multiuser MIMO-OFDMA schemes with packet scheduling. Now let us look at the linearly precded MIMO schemes which is also termed as closed loop MIMO scheme. The system model for a linearly precoded MIMO-OFDMA scheme using the linear MMSE receiver is described in Section 2. The received signal at the kth MS, $k \in \boldsymbol{\zeta}_\iota$, for the nth subcarrier after the linear MMSE equalizer is given by (2) for both the SU and MU MIMO schemes. The received SINR at the jth spatial sub-stream can be related to its mean square error (MSE) as [15] (here for simplicity, we omit the subcarrier index n),

$$\Gamma_j = \lambda_j' p_j = \lambda_j \rho_j, \quad j \in \{1, 2, \ldots, n_t\}, \quad (11)$$

where λ_j is the jth non-zero largest eigenvalue of the matrix $\mathbf{H}_i\mathbf{H}_i^H$, p_j is the power allocated to the jth established sub-stream of the ith MS and $\rho_j = p_j/N_0$, where N_0 is the noise variance It is well known that for Rayleigh MIMO fading channels, the complex matrix $\mathbf{H}_i\mathbf{H}_i^H$ is a complex central Wishart matrix [18].

The joint density function of the ordered eigenvalues of $\mathbf{H}_i\mathbf{H}_i^H$ can be expressed as [18]

$$f_{\mathbf{\Lambda}}(\lambda_1, \ldots, \lambda_\kappa) = \prod_{i=1}^{\kappa} \frac{\lambda_i^{\vartheta-\kappa}}{(\kappa - i)!(\vartheta - i)!} \prod_{i<j}^{\kappa-1} \left(\lambda_i - \lambda_j\right)^2 \cdot \exp\left(-\sum_{i=1}^{\kappa} \lambda_i\right), \quad (12)$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_\kappa$ and $\vartheta = \max(n_t, n_r)$, $\kappa = \min(n_t, n_r)$. For unordered eigenvalues, the joint density function can be obtained by $f_{\mathbf{\Lambda}}(\lambda_1, \ldots, \lambda_\kappa)/\kappa!$.

4.1. Linearly Precoded SDM SU-MIMO Schemes. For localized downlink transmission with linearly precoded SU-MIMO system with 2 antennas at both the transmitter and the receiver side, applying the FDPF scheduling algorithm the probability that the SINR of a scheduled user is below a certain threshold, that is, the CDF of the post scheduling SINR per PRB is, as shown in Appendix A, given by

$$F_{\Gamma_u}^{CS}(\gamma) = \left[\int_0^{\gamma} dv \frac{1}{(\rho_1\rho_2)^3} \exp\left(-\frac{v}{\rho_1}\right) \varphi(\gamma, v)\right]^{K_T}, \quad (13)$$

where K_T is the number of active users in the cell and

$$\begin{aligned} &\varphi(\gamma, v) \\ &= \rho_2{}^3 v^2 \left[1 - \exp\left(-\frac{\gamma - v}{\rho_2(v+1)}\right)\right] \\ &\quad - 2\rho_1\rho_2{}^3 v \cdot \left[1 - \exp\left(-\frac{\gamma - v}{\rho_2(v+1)}\right)\left(1 + \frac{\gamma - v}{\rho_2(v+1)}\right)\right] \\ &\quad + 2\rho_1{}^2\rho_2{}^3 - \rho_1{}^2\rho_2{}^3 \\ &\quad \times \exp\left(-\frac{\gamma - v}{\rho_2(v+1)}\right) \cdot \left(\left(\frac{\gamma - v}{\rho_2(v+1)}\right)^2 + \frac{2(\gamma - v)}{\rho_2(v+1)} + 2\right). \end{aligned} \quad (14)$$

By differentiating the distribution function expressed by (13), the PDF of the effective post scheduling SINR for the linearly precoded SDM SU-MIMO scheme can be derived as

$$\begin{aligned} f_{\Gamma_u}^{CS}(\gamma) = K_T &\int_0^{\gamma} \frac{1}{(\rho_1\rho_2)^3(1+v)} \exp\left(-\frac{v}{\rho_1} - \frac{\gamma - v}{\rho_2(1+v)}\right) \\ &\cdot \left(\rho_2 v - \frac{\gamma - v}{1+v}\rho_1\right)^2 dv \\ &\cdot \left(\int_0^{\gamma} \frac{1}{(\rho_1\rho_2)^3} \exp\left(-\frac{v}{\rho_1}\right) \varphi(\gamma, v) dv\right)^{K_T-1}. \end{aligned} \quad (15)$$

4.2. Linearly Precoded SDM MU-MIMO Schemes. For MU-MIMO, the distribution of instantaneous SINR for each sub-stream of each scheduled user should be computed first in order to get the distribution of the unified effective SINR for the scheduled users per PRB. This requires the derivation of the marginal PDF of each eigenvalue. The marginal density function of the ςth ordered eigenvalue can be obtained by [19]

$$f_{\Lambda_\varsigma}(\lambda_\varsigma) = \int_{\lambda_\varsigma}^{\infty} d\lambda_{\varsigma-1} \cdots \int_{\lambda_2}^{\infty} d\lambda_1 \int_0^{\lambda_\varsigma} d\lambda_{\varsigma+1} \cdots \int_0^{\lambda_{\kappa-1}} d\lambda_\kappa f_{\mathbf{\Lambda}}(\lambda_1, \ldots, \lambda_\kappa), \quad (16)$$

where $f_{\mathbf{\Lambda}}(\lambda_1, \ldots, \lambda_\kappa)$ is given by (12). Complex expressions of the distribution of the largest and the smallest eigenvalues can be found in [20, 21], but not for the other eigenvalues. In [22], the marginal PDF of eigenvalues is approximated as

$$f_{\Lambda_i}(\lambda_i) \simeq \frac{1}{[\beta(i)-1]!}\frac{\lambda_i^{\beta(i)-1}}{\widetilde{\lambda}_i^{\beta(i)}}\exp\left(-\frac{\lambda_i}{\widetilde{\lambda}_i}\right), \tag{17}$$

where $\beta(i) = (n_t - i + 1)(n_r - i + 1)$ and $\widetilde{\lambda}_i = (1/\beta(i))\overline{\lambda}_i = (1/\beta(i))\int_0^\infty \lambda_i f_\Lambda(\lambda_i)d\lambda_i$. It was verified by simulations in [22] that despite its simple form, (17) provides an accurate estimation of eigenvalues distribution of the complex central Wishart matrix $\mathbf{HH}^H$ for Rayleigh MIMO fading channel.

Based on (11) and (17), the density function of the instantaneous SINR of the ith sub-stream can be expressed as

$$\begin{aligned} f_{\Gamma_i}(\gamma) &= \frac{1}{\rho_i} f_{\Lambda_i}\left(\frac{\gamma}{\rho_i}\right) \\ &\simeq \frac{1}{\rho_i}\frac{1}{[\beta(i)-1]!}\frac{(\gamma/\rho_i)^{\beta(i)-1}}{\widetilde{\lambda}_i^{\beta(i)}}\exp\left(-\frac{\gamma}{(\rho_i\widetilde{\lambda}_i)}\right). \end{aligned} \tag{18}$$

The outage probability, which is defined as the probability of the SINR going below the targeted SINR within a specified time period, is a statistical measure of the system. From the definition, the outage probability is simply the CDF of the SINR evaluated at the targeted SINR. The outage probability can be obtained by

$$\begin{aligned} \Pr(\Gamma_i \le \gamma) &= \int_{-\infty}^{\gamma} f_{\Gamma_i}(\alpha)d\alpha = \Pr\left(\lambda_i \le \frac{\gamma}{\rho_i}\right) \\ &\simeq 1 - \sum_{j=0}^{\beta(i)-1}\frac{\left(\gamma/\left(\rho_i\widetilde{\lambda}_i\right)\right)^j}{j!}\exp\left(-\frac{\gamma}{(\rho_i\widetilde{\lambda}_i)}\right). \end{aligned} \tag{19}$$

With the MU-MIMO SDM scheme and the FDPF packet scheduling algorithm, the distribution function of the instantaneous SINR for the ith sub-stream of each subcarrier can be obtained as

$$\begin{aligned} F_{\Gamma_i}^{CM}(\gamma) &= \Pr(\Gamma_1 \le \gamma, \dots, \Gamma_{K_T} \le \gamma) \\ &\simeq \left[1 - \sum_{j=0}^{\beta(i)-1}\frac{\left(\gamma/\left(\rho_i\widetilde{\lambda}_i\right)\right)^j}{j!}\exp\left(-\frac{\gamma}{(\rho_i\widetilde{\lambda}_i)}\right)\right]^{K_T}. \end{aligned} \tag{20}$$

Using the K_Tth order statistics [23], the PDF of the instantaneous SINR of the ith sub-stream of each subcarrier with linearly precoded MU-MIMO scheme using FDPF packet scheduling algorithm can then be obtained as

$$\begin{aligned} f_{\Gamma_i}^{CM}(\gamma) \simeq{}& \frac{K_T(\gamma/\rho_i)^{\beta(i)-1}}{\rho_i[\beta(i)-1]!\widetilde{\lambda}_i^{\beta(i)}} \\ &\times \exp\left(-\frac{\gamma}{(\rho_i\widetilde{\lambda}_i)}\right) \\ &\times \left[1 - \sum_{j=0}^{\beta(i)-1}\frac{\left(\gamma/\left(\rho_i\widetilde{\lambda}_i\right)\right)^j}{j!}\exp\left(-\frac{\gamma}{(\rho_i\widetilde{\lambda}_i)}\right)\right]^{K_T-1}. \end{aligned} \tag{21}$$

Note that for a dual sub-stream linearly precoded SDM MU MIMO scheme with a FDPF packet scheduling algorithm, the distribution of instantaneous SINRs for the two sub-streams within a PRB are independent. The reason is that the investigated precoding scheme separates the channel into parallel subchannels, each sub-stream occupies one subchannel. In the case of 2 antennas at both the transmitter and the receiver side, the CDF of the unified effective instantaneous SINR of the two sub-streams can be obtained by substituting (20) and (21) into (5) and limiting the integral region

$$\begin{aligned} F_{\Gamma_u}^{CM}(\gamma) \simeq{}& \int_0^\gamma dx \frac{K_T}{\rho_1\widetilde{\lambda}_1}\frac{\left(x/\left(\rho_1\widetilde{\lambda}_1\right)\right)^{(\beta(i)-1)}}{(\beta(i)-1)!}e^{(-x/(\rho_1\widetilde{\lambda}_1))} \\ &\cdot \left[1 - \sum_{j=0}^{\beta(i)-1}\frac{\left(x/\left(\rho_1\widetilde{\lambda}_1\right)\right)^j}{j!}e^{(-x/(\rho_1\widetilde{\lambda}_1))}\right]^{K_T-1} \\ &\cdot \left[1 - \sum_{j=0}^{\beta(2)-1}\frac{\left((\gamma-x)/((x+1)\rho_2\widetilde{\lambda}_2)\right)^j}{j!}\right. \\ &\left. \times\ e^{(-(\gamma-x)/(x+1)\rho_2\widetilde{\lambda}_2)}\right]^{K_T}. \end{aligned} \tag{22}$$

The corresponding PDF can be derived by differentiating (5) with respect to γ.

5. The Average Channel Capacity

The average channel capacity [24] or the so-called Shannon (ergodic) capacity [25] per PRB can be obtained by

$$C = \int_0^\infty \log_2(1+\gamma) f_\Gamma(\gamma)d\gamma. \tag{23}$$

Here, $f_\Gamma(\gamma)$ is the PDF of the effective SINR, which can be obtained by differentiating the CDF of the SINR for the corresponding SDM schemes. With the investigated linear receivers, which decompose the MIMO channel into independent channels, the total capacity for the multiple input sub-stream MIMO systems is equal to the sum of the capacities for each sub-stream, that is,

$$C_{\text{total}} = \sum_i \int_0^\infty \log_2(1+\gamma) f_{\Gamma_i}(\gamma)d\gamma. \tag{24}$$

5.1. Average Channel Capacity for SDM MIMO without Precoding. The average channel capacity for SDM SU-MIMO without precoding can be obtained as

$$\begin{aligned} C_{\mathrm{SU}}^{\mathrm{O}} &= \int_0^\infty d\gamma \log_2(1+\gamma) K_T \frac{4e^{4/\gamma_0}}{\gamma_0^2}\left[\gamma_0^2(1+\gamma)\right]^{-1/4} \\ &\times \exp\left(-\frac{4}{\gamma_0}\sqrt{1+\gamma}\right)\sqrt{\frac{\pi}{2}}\gamma_0 \sum_{n=0}^{\infty}\frac{(1/2-n)^{2n}}{2^{n/2}\left((4/\gamma_0)\sqrt{1+\gamma}\right)^n} \\ &\times \left[1-e^{-2\gamma/\gamma_0}-2e^{4/\gamma_0}\int_{\gamma_0}^{\gamma_0(1+\gamma)} e^{-2/\gamma_0^2 u-2(1+\gamma)u^{-1}}du\right]^{K_T-1}. \end{aligned} \tag{25}$$

The derivation of (25) is given in Appendix B. The average channel capacity of SDM MU MIMO without precoding is the sum of the average channel capacity for each substream. Substituting the PDF for the post scheduling substream SINR (9) into (24) yields

$$\begin{aligned} C_{\mathrm{MU}}^{\mathrm{O}} &= \sum_i \int_0^\infty \log_2(1+x) f_{\Gamma_i}(x)dx \\ &= \frac{n_t K_T}{\gamma_0}\sum_i \int_0^\infty \log_2(1+x)e^{-n_t x/\gamma_0}\left(1-e^{-n_t x/\gamma_0}\right)^{K_T-1}dx \\ &= \frac{n_t K_T}{\gamma_0 \ln 2}\sum_i \sum_{j=0}^{K_T-1}(-1)^j \binom{K_T-1}{j}\frac{e^{-a_j}E_i(a_j)}{a_j}, \end{aligned} \tag{26}$$

where $a_j = -(j+1)n_t/\gamma_0$, and $E_i(\cdot)$ is the exponential integral function defined as [26, pages 875–877]

$$E_i(x) = \int_{-\infty}^{x}\frac{e^t}{t}dt = \ln(-x) + \sum_{m=1}^{\infty}\frac{x^m}{m\cdot m!}, \quad x<0. \tag{27}$$

The derivation of (26) is given in Appendix C.

5.2. Average Channel Capacity for SDM MIMO with Precoding. For a linearly precoded SDM SU MIMO scheme without FDPS, substituting (18) into (24), we obtain

$$\begin{aligned} C &= \sum_i \int_0^\infty \log_2(1+\gamma)\frac{1}{\rho_i} f_{\Lambda_i}\left(\frac{\gamma}{\rho_i}\right)d\gamma \\ &\simeq \sum_i \int_0^\infty \log_2(1+\gamma)\frac{1}{\rho_i}\frac{1}{[\beta(i)-1]!}\frac{(\gamma/\rho_i)^{\beta(i)-1}}{\tilde{\lambda}_i^{\beta(i)}} \\ &\quad\cdot \exp\left(-\frac{\gamma}{\left(\rho_i\tilde{\lambda}_i\right)}\right)d\gamma. \end{aligned} \tag{28}$$

For a linearly precoded multiuser SDM SU-MIMO scheme with FDPS, the probability density function of the effective SINR can be obtained by (15). Substituting (15) into (23), the post scheduling average channel capacity of a linearly precoded SDM SU-MIMO scheme can be derived as

$$\begin{aligned} C_{\mathrm{SU}}^{\mathrm{C}} &= \int_0^\infty d\gamma \log_2(1+\gamma)K_T \\ &\times \left(\int_0^\gamma \frac{1}{(\rho_1\rho_2)^3}\exp\left(-\frac{\nu}{\rho_1}\right)\varphi(\gamma,\nu)d\nu\right)^{K_T-1} \\ &\cdot \int_0^\gamma d\nu \frac{1}{(\rho_1\rho_2)^3(1+\nu)} \\ &\cdot \exp\left(-\frac{\nu}{\rho_1}-\frac{\gamma-\nu}{\rho_2(1+\nu)}\right)\left(\rho_2\nu-\frac{\gamma-\nu}{1+\nu}\rho_1\right)^2. \end{aligned} \tag{29}$$

Substituting (21) into (24), the average channel capacity of the linearly precoded multiuser SDM MU-MIMO scheme can be derived as

$$\begin{aligned} C_{\mathrm{MU}}^{\mathrm{C}} &\simeq \sum_{i=1}^{2}\int_0^\infty d\gamma \log_2(1+\gamma)\frac{K_T}{\rho_i\tilde{\lambda}_i}\frac{\left(\gamma/\left(\rho_i\tilde{\lambda}_i\right)\right)^{(\beta(i)-1)}}{(\beta(i)-1)!}\exp\left(-\frac{\gamma}{\rho_i\tilde{\lambda}_i}\right)\cdot\left[1-\sum_{j=0}^{\beta(i)-1}\frac{\left(\gamma/\left(\rho_i\tilde{\lambda}_i\right)\right)^j}{j!}\exp\left(-\frac{\gamma}{\rho_i\tilde{\lambda}_i}\right)\right]^{K_T-1} \\ &= \underbrace{\frac{K_T}{\left(\rho_1\tilde{\lambda}_1\right)^{\beta(i)}(\beta(i)-1)!}\int_0^\infty \log_2(1+\gamma)\underbrace{\gamma^{\beta(i)-1}e^{-\gamma/\rho_1\tilde{\lambda}_1}\left[1-\sum_{j=0}^{\beta(i)-1}\frac{\gamma^j}{j!\left(\rho_1\tilde{\lambda}_1\right)^j}e^{-\gamma/\rho_1\tilde{\lambda}_1}\right]^{K_T-1}}_{\Psi(\gamma)}d\gamma}_{\Omega} \\ &+ \underbrace{\frac{K_T}{\rho_2\tilde{\lambda}_2}\int_0^\infty \log_2(1+\gamma)e^{-\gamma/\rho_2\tilde{\lambda}_2}\left(1-e^{-\gamma/\rho_2\tilde{\lambda}_2}\right)^{K_T-1}d\gamma.}_{\Phi} \end{aligned} \tag{30}$$

Following the same procedure as shown in Section 5.1 for SDM MU-MIMO without precoding, we have

$$\Phi = \frac{1}{\ln 2}\sum_{j=0}^{K_T-1}(-1)^j\binom{K_T-1}{j}\frac{e^{-b_j}E_i(b_j)}{b_j}, \tag{31}$$

where $b_j = -(j+1)/\rho_2\lambda_2$. With binomial expansion, we have

$$\Psi(\gamma) = \gamma^{\beta(i)-1}e^{-\gamma/\rho_1\tilde{\lambda}_1}\left[1-\sum_{j=0}^{\beta(i)-1}\frac{\gamma^j}{j!\left(\rho_1\tilde{\lambda}_1\right)^j}e^{-\gamma/\rho_1\tilde{\lambda}_1}\right]^{K_T-1}$$

$$= \gamma^{\beta(i)-1} \sum_{n=0}^{K_T-1} (-1)^n \frac{(K_T-1)!}{(K_T-1-n)!n!} \times \left(\sum_{j=0}^{\beta(i)-1} \frac{\gamma^j}{j!\left(\rho_1\tilde{\lambda}_1\right)^j} \right)^n e^{-(n+1)\gamma/\rho_1\tilde{\lambda}_1}$$

$$= \gamma^{\beta(i)-1} \sum_{n=0}^{K_T-1} c_n \left(\sum_{j=0}^{\beta(i)-1} \frac{\gamma^j}{j!\left(\rho_1\tilde{\lambda}_1\right)^j} \right)^n e^{-(n+1)\gamma/\rho_1\tilde{\lambda}_1}, \tag{32}$$

where

$$c_n = (-1)^n \frac{(K_T-1)!}{(K_T-1-n)!n!} = (-1)^n \binom{K_T-1}{n}. \tag{33}$$

According to (30), $\Omega = \int_0^\infty \log_2(1+\gamma)\Psi(\gamma)d\gamma$. For a large number of transmit and receiver antennas (assume $\eta = n_t = n_r$ and the ordered eigenvalues of $\mathbf{HH}^H$ as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_\eta$), that is, $\beta(i)$ is sufficiently large, the average channel capacity for SDM MU-MIMO with precoding can be approximated by a closed form

$$C_{\text{MU}}^C \approx \sum_{i=1}^{\eta-1} \frac{K_T}{(\rho_i\lambda_i)^{\beta(i)-1}(\beta(i)-1)!} \frac{1}{\ln 2} \times \sum_{n=0}^{K_T-1} (-1)^n \frac{(K_T-1)!}{(K_T-1-n)!n!} \mathcal{I}_{\beta(i)}\left(\frac{1}{\rho_i\lambda_i}\right) + \frac{K_T}{\rho_\eta\tilde{\lambda}_\eta} \frac{1}{\ln 2} \sum_{j=0}^{K_T-1} (-1)^j \binom{K_T-1}{j} \frac{e^{-d_j}E_i\left(d_j\right)}{d_j}, \tag{34}$$

where the function $\mathcal{I}(\cdot)$ is defined as [27] $\mathcal{I}_i(\mu) = \int_0^\infty \ln(1+x)x^{i-1}e^{-\mu x}dx = (i-1)!e^\mu \sum_{k=1}^{i} \Gamma(-i+k,\mu)/\mu^k$, where $\mu > 0$, $i = 1, 2, \ldots$ and $\Gamma(\cdot,\cdot)$ is the complementary incomplete gamma function defined as [27] $\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1}e^{-t}dt$, and $d_j = -(j+1)/\rho_\eta\lambda_\eta$. The derivation of (34) is given in Appendix D.

6. Analytical and Numerical Results

We consider the case with 2 antennas at the transmitter and 2 receiver antennas at the MS for SU-MIMO case and single antenna at the MS for MU-MIMO case. For MU-MIMO case, two MSs are grouped together to form a virtual MIMO between the MSs and the BS. We first give the results for open loop SU/MU SDM MIMO schemes of LTE downlink transmission.

Figure 1 shows a single stream SINR and the effective SINR distribution per PRB for MIMO schemes with and without FDPS. When FDPS is not used, the scheduler randomly selects users for transmission. The number of active users available for scheduling in the cell is 20. It can be seen that without packet scheduling, MU-MIMO can exploit available multiuser diversity gain, therefore has better stream SINR distribution than SU-MIMO. For SDM SU-MIMO at 50% percentile of effective SINR, approximately 10 dB gain can be obtained by using FDPS. More gain can be achieved by using MU-MIMO scheme with packet scheduling. This is due to the fact that the multiuser diversity is further exploited in SDM MU-MIMO schemes.

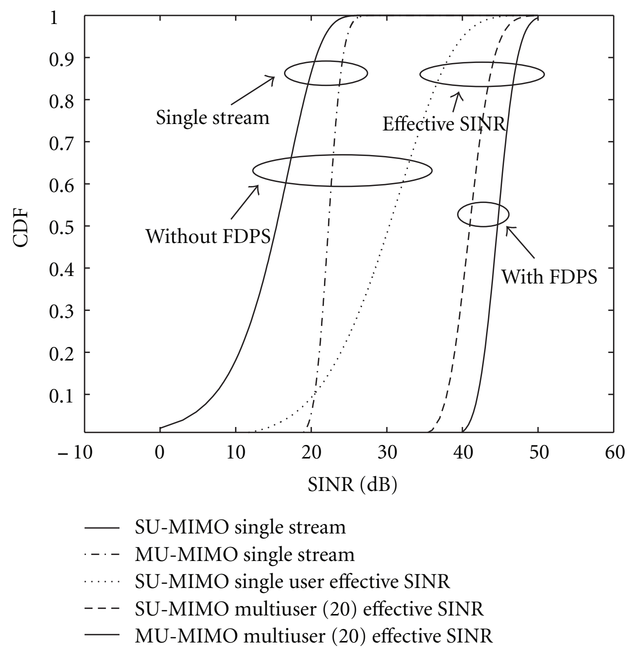

Figure 1: SINR distribution for SDM multiuser SU and MU-MIMO schemes with 20 active users in the cell.

Figure 2 shows the effective SINR distribution per PRB for linearly precoded SDM MIMO scheme. The precoding scheme which we used is from [15] as mentioned in Section 2. The number of active users, that is, the user diversity order, is 10. These plots are obtained under the assumption of evenly allocated transmit power at the two transmitter antennas, and a transmitted signal to noise ratio (SNR), defined as the total transmitted power of the two sub-streams divided by the variance of the complex Gaussian noise, is equal to 20 dB. Both the simulation results and analytical results are shown in this figure. In the simulation, the system bandwidth is set to 900 kHz with a subcarrier spacing of 15 kHz. Hence there are 60 occupied subcarriers for full band transmission. We further assume these 60 subcarriers are arranged in 5 consecutive PRBs per sub-frame, so that each PRB contains 12 subcarriers. At each Monte-Carlo run, 100 sub-frames are used for data transmission. The simulation results are averaged over 100 Monte-Carlo runs. One can see from Figure 2 that the simulation results are in close agreement with the analytical results. It can also be seen that for SU-MIMO scheme, the multiuser diversity gain at the 10th percentile of the post scheduled SINR per PRB is about 11 dB with 10 users, while an MU-MIMO scheme with SDM-FDPS can achieve an additional 2 dB gain compared with a SDM-FDPS SU-MIMO scheme. This implies that the MU-MIMO scheme has more freedom or selection diversity than the SU-MIMO in the spatial domain.

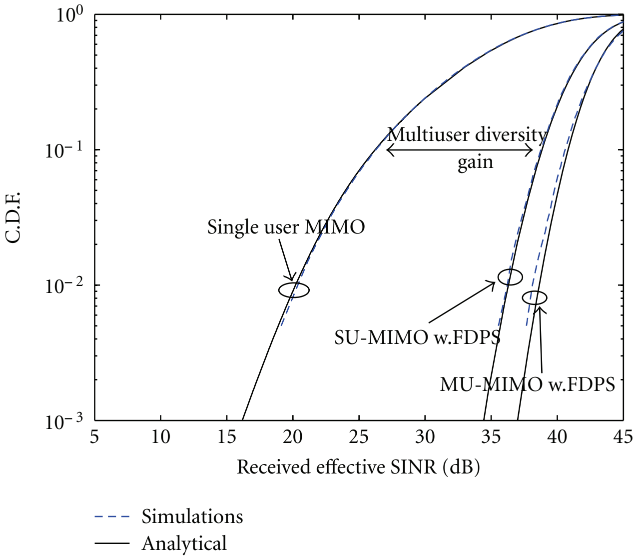

Figure 2: Analytical and simulation results of SINR distribution for linearly precoded SU and MU-MIMO schemes with 10 active users in the cell. In the figure, “w.FDPS” represents “with FDPS”.

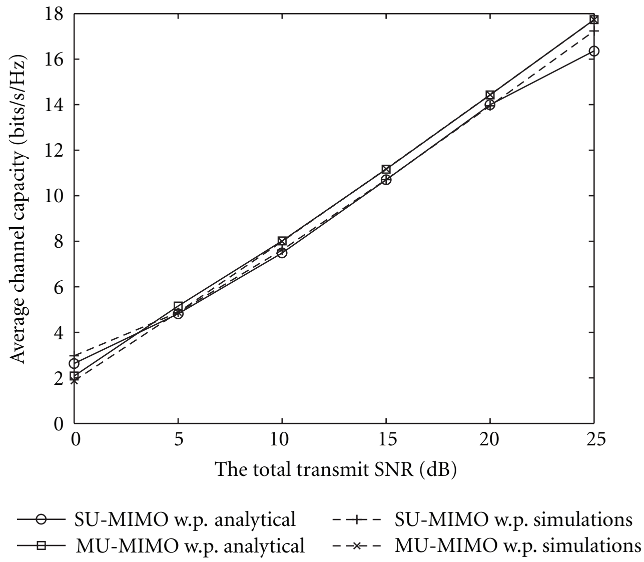

Figure 3: Analytical and simulation results of average channel capacity for SU and MU-MIMO schemes with linear precoding, number of active users is 10. In the figure, “w.p.analytical” represents “with precoding analytical results” and “w.p.simulation” represents “with precoding simulation results”.

The average channel capacity for SU and MU MIMO schemes versus transmitted SNR are shown in Figures 3 and 4. The number of active users in the cell is 10. Figure 3 shows the simulation and the analytical results for the linearly precoded SU and MU-MIMO systems, it can be seen that the simulation results match the analytical results rather well. Figure 4 shows the average channel capacity comparison between open loop MIMO and closed loop MIMO system. Figure 5 shows the average channel capacity for SU and MU MIMO schemes versus the number of active users in the cell. Both the simulation results and the analytical results for the open loop and the linearly precoded MIMO systems are shown. It can be seen that the simulation results almost coincide with the analytical results. Figure 5 indicates that in a cell with 10 active users, the MU-MIMO schemes (no matter with or without precoding) always perform better than the SU-MIMO schemes. Notice that the performance for the closed loop SU-MIMO denoted by w.p. in Figure 5 is slightly worse than the one for the open loop MU-MIMO. This implies that MU-MIMO exploits more multiuser diversity gain than SU-MIMO does. Interestingly, the precoding gain for SU-MIMO is much larger than for MU-MIMO.

Figure 5 shows that the average channel capacity for SU-MIMO schemes with precoding is always higher than the one for the SU-MIMO scheme without precoding regardless of the number of users. However, for the MU-MIMO scheme, the above observation does not hold especially for systems with a large number of active users. As the number of active users increases, the advantages using schemes with precoding gradually vanish. This can be explained by the fact that the multiuser diversity gain has already been exploited by MU-MIMO schemes and the additional diversity gain by using precoding does not contribute too much in this case. Note that we used ZF receiver for the open loop scheme while for the closed loop scheme, the MMSE receiver was employed. One reason why we use ZF receiver instead of MMSE for the open loop scheme is that the SINR distribution for the open loop scheme with MMSE receiver is very difficult to obtain. Another reason is that the ZF receiver can separate the received data sub-streams, while MMSE receiver cannot, the independence property of the received data sub-streams is used for computing the effective SINR as we mentioned earlier.

7. Conclusions

In this paper, we analyzed the multiuser downlink transmission for linearly precoded SDM MIMO schemes in conjunction with a base station packet scheduler. Both SU and MU MIMO with FDPS are investigated. We derived mathematical expressions of SINR distribution for linearly precoded SU-MIMO and MU-MIMO schemes, based upon which the average channel capacities of the corresponding systems are also derived. The theoretical analyses are verified by the simulations results and proven to be accurate. Our investigations reveal that the system using a linearly precoded MU-MIMO scheme has a higher average channel capacity than the one without precoding when the number of active users is small. When the number of users increase, linearly precoded MU-MIMO has comparable performance to MU-MIMO without precoding.

Appendices

A. Derivation of (13)

For a 2×2 linearly precoded spatial multiplexing MIMO system, (12) can be simplified as

$$f_{\Lambda}(\lambda_1, \lambda_2) = (\lambda_1 - \lambda_2)^2 \exp(-(\lambda_1 + \lambda_2)). \qquad (A.1)$$

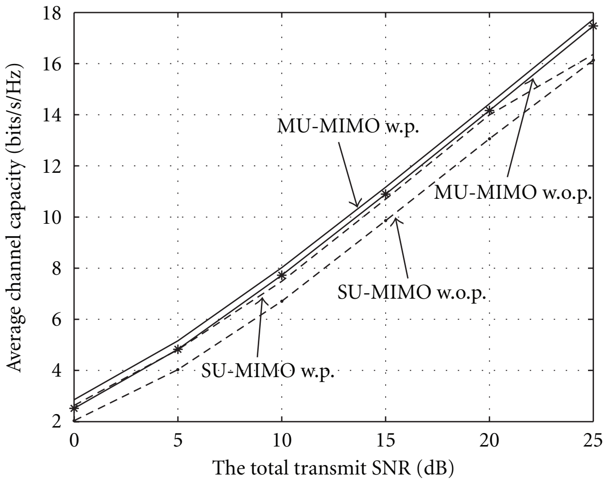

Figure 4: Analytical average channel capacity comparison for SU and MU-MIMO schemes with and without linear precoding, number of active users is 10. In the figure, "w.p" represents "with precoding", "w.o.p." represents "without precoding".

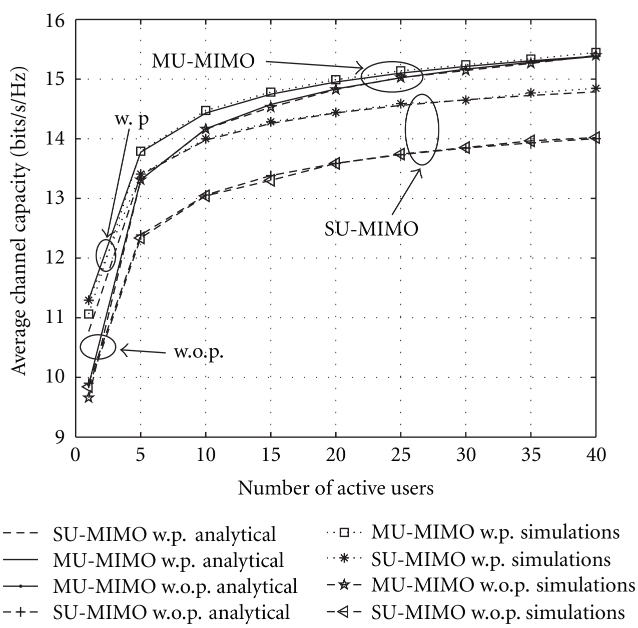

Figure 5: Average channel capacity versus number of active users for SU and MU-MIMO schemes with/without linear precoding, transmit SNR is 20 dB. "w.p" represents "with precoding", "w.o.p." represents "without precoding".

The joint probability density function of the SINRs of the two (assumed) established sub-streams using the Jacobian transformation [10] is $f_\Gamma(\gamma_1, \gamma_2) = (1/\rho_1\rho_2) f_\Lambda(\lambda_1/\rho_1, \lambda_2/\rho_2)$. Let $x = 1 + \gamma_1$ and $y = 1 + \gamma_2$, then the unified effective SINR is given by $\Gamma_u = xy - 1$, and the distribution function of Γ_u can be expressed as

$$
\begin{aligned}
F_{\Gamma_u}(\gamma) &= \int_{-\infty}^{\infty} dx \int_{-\infty}^{(\gamma+1)/x} dy\, f_\Gamma(x-1, y-1) \\
&= \int_{-\infty}^{\infty} dx \int_{-\infty}^{(\gamma+1)/x} dy \frac{1}{\rho_1\rho_2} f_\Lambda\left(\frac{x-1}{\rho_1}, \frac{y-1}{\rho_2}\right).
\end{aligned} \tag{A.2}
$$

By substituting (A.1) into (A.2), and limiting the integral region, we have

$$
\begin{aligned}
F_{\Gamma_u}(\gamma) &= \int_1^{\gamma+1} dx \int_1^{(\gamma+1)/x} dy \frac{1}{(\rho_1\rho_2)^3} (\rho_2 x - \rho_1 y + \rho_1 - \rho_2)^2 \\
&\quad \cdot \exp\left(-\frac{1}{\rho_1\rho_2}(\rho_2 x + \rho_1 y - \rho_1 - \rho_2)\right) \\
&= \int_0^{\gamma} dv \int_0^{(\gamma-v)/(v+1)} du \frac{1}{(\rho_1\rho_2)^3} (\rho_2 v - \rho_1 u)^2 \\
&\quad \cdot \exp\left(-\frac{1}{\rho_1\rho_2}(\rho_2 v + \rho_1 u)\right) \\
&= \int_0^{\gamma} dv \frac{1}{(\rho_1\rho_2)^3} \exp\left(-\frac{v}{\rho_1}\right) \varphi(\gamma, v),
\end{aligned} \tag{A.3}
$$

where $\varphi(\gamma, v)$ is given by (14). With the FDPF scheduling algorithm, the scheduled user is the one with the largest effective stream SINR among the K_T users, that is,

$$
\begin{aligned}
F_{\Gamma_u}^{CS}(\gamma) &= \Pr(\gamma_1 < \alpha_1, \gamma_2 < \alpha_2, \dots, \gamma_{K_T} < \alpha_{K_T}) \\
&= \left[P_r(\Gamma_u \le \gamma)\right]^{K_T} = \left[F_{\Gamma_u}(\gamma)\right]^{K_T}.
\end{aligned} \tag{A.4}
$$

Substituting (A.3) into (A.4), we obtain (13).

B. Derivation of (25)

By inserting the PDF of the post scheduling effective SINR (7) into (23), the average channel capacity for SDM SU-MIMO without precoding can be obtained as

$$
\begin{aligned}
C_{\text{SU}}^{\text{O}} &= \int_0^{\infty} d\gamma \log_2(1+\gamma) K_T \\
&\quad \times \Bigg[\underbrace{\int_0^{\gamma} \frac{2}{\gamma_0} e^{-2x/\gamma_0} \left(1 - e^{-2(\gamma-x)/\gamma_0(1+x)}\right) dx}_{\Theta}\Bigg]^{K_T-1} \\
&\quad \cdot \underbrace{\int_0^{\infty} \left[\frac{4}{\gamma_0^2(1+x)} \exp\left(-\frac{2(\gamma+x^2)}{\gamma_0(1+x)}\right)\right] dx}_{\Upsilon},
\end{aligned} \tag{B.1}
$$

where

$$
\begin{aligned}
\Theta &= \int_0^{\gamma} \frac{2}{\gamma_0} e^{-2x/\gamma_0} \left(1 - e^{-2(\gamma-x)/\gamma_0(1+x)}\right) dx \\
&= \frac{2}{\gamma_0} \underbrace{\int_0^{\gamma} e^{-2x/\gamma_0} dx}_{\alpha} - \frac{2}{\gamma_0} \underbrace{\int_0^{\gamma} e^{-(2x^2+2\gamma^2)/\gamma_0(1+x)} dx}_{\beta}.
\end{aligned} \tag{B.2}
$$

Therefore, $\Theta = (2/\gamma_0)\alpha - (2/\gamma_0)\beta$, and α can be computed as

$$\alpha = \int_0^{\gamma} e^{-2x/\gamma_0} dx = -\frac{\gamma_0}{2}\int_0^{\gamma} e^{-2x/\gamma_0} d\left(-\frac{2x}{\gamma_0}\right) = -\frac{\gamma_0}{2} e^{-2x/\gamma_0}\Big|_0^{\gamma} = \frac{\gamma_0}{2}\left(1 - e^{-2\gamma/\gamma_0}\right). \tag{B.3}$$

We derive β by u-Substitution. Let $u = \gamma_0(1+x)$, we have $x = u/\gamma_0 - 1$, $dx = du/\gamma_0$ and $x^2 + \gamma^2 = (u/\gamma_0 - 1)^2 + \gamma^2 = u^2/\gamma_0^2 - 2u/\gamma_0 + 1 + \gamma^2$. Therefore,

$$\beta = \int_0^{\gamma} e^{-2(x^2+\gamma^2)/\gamma_0(1+x)} dx = \frac{1}{\gamma_0}\int_{\gamma_0}^{\gamma_0(1+\gamma)} e^{-2u^{-1}(u^2/\gamma_0^2 - 2u/\gamma_0 + 1 + \gamma^2)} du = \frac{e^{4/\gamma_0}}{\gamma_0}\int_{\gamma_0}^{\gamma_0(1+\gamma)} e^{bu + au^{-1}} du, \tag{B.4}$$

where $b = -2/\gamma_0^2$ and $a = -2(1+\gamma)$.

From (B.1), we have

$$\Upsilon = \int_0^{\infty} \frac{4}{\gamma_0^2(1+x)} \exp\left(-\frac{2(\gamma + x^2)}{\gamma_0(1+x)}\right) dx. \tag{B.5}$$

Let $u = \gamma_0(1+x)$, we have $x = u/\gamma_0 - 1$, $dx = du/\gamma_0$ and $x^2 + \gamma = (u/\gamma_0 - 1)^2 + \gamma = u^2/\gamma_0^2 - 2u/\gamma_0 + 1 + \gamma$, (B.5) can be represented as

$$\Upsilon = \int_0^{\infty} \frac{4}{\gamma_0 u} \exp\left(-\frac{2}{u}\left[\frac{u^2}{\gamma_0^2} - \frac{2u}{\gamma_0} + 1 + \gamma\right]\right) dx = \frac{4e^{4/\gamma_0}}{\gamma_0^2}\int_0^{\infty} u^{-1} \exp\left(-\frac{2u}{\gamma_0^2} - \frac{2(1+\gamma)}{u}\right) du. \tag{B.6}$$

According to [28, page 144],

$$\int_0^{\infty} e^{-(px+q/x)} x^{-(a+1/2)} dx = \left(\frac{p}{q}\right)^{(1/2)a} \exp\left(-2\sqrt{pq}\right)\sqrt{\frac{\pi}{p}}\sum_{n=0}^{\infty}\frac{(a-n)^{2n}}{2^{n/2}\left(2\sqrt{pq}\right)^n}. \tag{B.7}$$

Let $a = 1/2$, (B.7) becomes

$$\int_0^{\infty} e^{-(px+q/x)} x^{-1} dx = \left(\frac{p}{q}\right)^{1/4} \exp\left(-2\sqrt{pq}\right)\sqrt{\frac{\pi}{p}}\sum_{n=0}^{\infty}\frac{(1/2-n)^{2n}}{2^{n/2}\left(2\sqrt{pq}\right)^n}. \tag{B.8}$$

Assigning $p = 2/\gamma_0^2$, $q = 2(1+\gamma)$ in the above equation, Υ in (B.6) can be derived as

$$\Upsilon = \frac{4e^{4/\gamma_0}}{\gamma_0^2}\left[\gamma_0^2(1+\gamma)\right]^{-1/4} \times \exp\left(-\frac{4}{\gamma_0}\sqrt{1+\gamma}\right)\sqrt{\frac{\pi}{2}}\gamma_0\sum_{n=0}^{\infty}\frac{(1/2-n)^{2n}}{2^{n/2}\left((4/\gamma_0)\sqrt{1+\gamma}\right)^n}. \tag{B.9}$$

C. Derivation of the Average Channel Capacity for MU MIMO without Precoding

For the SDM MU-MIMO without precoding, the average channel capacity has the form

$$C_{\mathrm{MU}}^{\mathrm{O}} = \sum_i \int_0^{\infty} \log_2(1+x)\frac{n_t}{\gamma_0} e^{-n_t x/\gamma_0} K_T\left(1 - e^{-n_t x/\gamma_0}\right)^{K_T-1} dx = \frac{n_t K_T}{\gamma_0}\sum_i \int_0^{\infty} \log_2(1+x) e^{-n_t x/\gamma_0}\left(1 - e^{-n_t x/\gamma_0}\right)^{K_T-1} dx. \tag{C.1}$$

According to the binomial theorem [26, page 25]

$$(1-z)^n = 1 - nz + \frac{n(n-1)}{1\cdot 2}z^2 - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}z^3 + \cdots = \sum_{j=0}^{n}(-1)^j\frac{n!}{(n-j)!j!}z^j, \tag{C.2}$$

we can derive

$$e^{-n_t x/\gamma_0}\left(1 - e^{-n_t x/\gamma_0}\right)^{K_T-1} = \sum_{j=0}^{K_T-1}(-1)^j\binom{K_T-1}{j} e^{-(j+1)n_t x/\gamma_0}, \tag{C.3}$$

where the binomial coefficient is given by

$$\binom{K_T-1}{j} = \frac{(K_T-1)!}{(K_T-j-1)!j!}. \tag{C.4}$$

To solve the integral in (C.1), let us first consider $\int_0^{\infty} \log_2(1+x)e^{a_j x}dx$, where $a_j = -(j+1)n_t/\gamma_0$. Its closed form expression can be derived as

$$\begin{aligned}\int_0^{\infty} \log_2(1+x) e^{a_j x} dx &= \frac{1}{\ln 2}\int_0^{\infty} \ln(1+x) e^{a_j x} dx \\ &= \frac{1}{a_j \ln 2}\int_0^{\infty} \ln(1+x) d(e^{a_j x}) \\ &= \frac{1}{a_j \ln 2}\ln(1+x)e^{a_j x}\Big|_0^{\infty} - \frac{1}{a_j \ln 2}\int_0^{\infty} e^{a_j x} d[\ln(1+x)] \\ &= -\frac{1}{a_j \ln 2}\int_0^{\infty}\frac{e^{a_j x}}{1+x}dx,\end{aligned} \tag{C.5}$$

Equation (C.5) is derived by following the fact that $\lim_{y\to\infty} \ln y/e^{-cy} = 0$ $(c < 0)$, and by assigning $u = \ln(1+x)$, $v = e^{a_j x}$, then performing integral by parts. According to [26, page 337],

$$\int_0^\infty \frac{e^{-\mu x}}{x+\beta} dx = -e^{\beta\mu} E_i(-\mu\beta), \quad \mu > 0. \tag{C.6}$$

Assigning $\beta = 1$, $\mu = -a_j$ in (C.6), the closed form of (C.5) can be obtained as

$$\int_0^\infty \log_2(1+x)e^{a_j x} dx = \frac{e^{-a_j} E_i\left(a_j\right)}{a_j \ln 2}, \tag{C.7}$$

where the exponential integral function $E_i(x)$ is defined in (27). Substituting (C.3) and (C.7) into (C.1), we can derive the average channel capacity for SDM MU-MIMO without precoding

$$\begin{aligned} C_{\text{MU}}^{O} &= \frac{n_t K_T}{\gamma_0} \sum_i \int_0^\infty \log_2(1+x) e^{-n_t x/\gamma_0} \left(1 - e^{-n_t x/\gamma_0}\right)^{K_T - 1} dx \\ &= \frac{n_t K_T}{\gamma_0} \sum_i \int_0^\infty \log_2(1+x) \sum_{j=0}^{K_T-1} (-1)^j \binom{K_T - 1}{j} e^{-(j+1)n_t x/\gamma_0} dx \\ &= \frac{n_t K_T}{\gamma_0} \sum_i \sum_{j=0}^{K_T-1} (-1)^j \binom{K_T - 1}{j} \int_0^\infty \log_2(1+x) e^{-(j+1)n_t x/\gamma_0} dx \\ &= \frac{n_t K_T}{\gamma_0 \ln 2} \sum_i \sum_{j=0}^{K_T-1} (-1)^j \binom{K_T - 1}{j} \frac{e^{-a_j} E_i\left(a_j\right)}{a_j}, \end{aligned} \tag{C.8}$$

where $a_j = -(j+1)n_t/\gamma_0$.

D. Derivation of Channel Capacity for Systems with Large Number of Antennas

For the systems where n_r and/or n_t is large, $\beta(i)$ is sufficiently large. Under such circumstances, we can utilize the series representation of the exponential function

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ &= \sum_{j=0}^{\infty} \frac{x^j}{j!} \approx \sum_{j=0}^{\beta(i)-1} \frac{x^j}{j!}. \end{aligned} \tag{D.1}$$

For a linearly precoded SDM MU-MIMO, the average channel capacity can be expressed as

$$C_{\text{MU}}^{C} \simeq \sum_{i=1}^{\eta} \frac{K_T}{\left(\rho_i \widetilde{\lambda}_i\right)^{\beta(i)} (\beta(i) - 1)!} \underbrace{\int_0^\infty \log_2(1+\gamma) \underbrace{\gamma^{\beta(i)-1} e^{-\gamma/\rho_i \widetilde{\lambda}_i} \left[1 - \sum_{j=0}^{\beta(i)-1} \left(\frac{\gamma^j}{j!\left(\rho_i \widetilde{\lambda}_i\right)^j}\right) e^{-\gamma/\rho_i \widetilde{\lambda}_i}\right]^{K_T - 1}}_{\Delta(\gamma)} d\gamma}_{\Xi}, \tag{D.2}$$

where $\eta = n_t = n_r$ and the ordered eigenvalues of the complex central Wishart matrix $\mathbf{H}\mathbf{H}^H$ is $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_\eta$.

When $i = \eta$, $\beta(\eta) = 1$, following the same procedure as shown in Section 5.1 for SDM MU-MIMO without precoding, we have

$$C_{\text{MU}}^{C}(\beta(\eta)) = \frac{K_T}{\rho_\eta \widetilde{\lambda}_\eta} \frac{1}{\ln 2} \sum_{j=0}^{K_T - 1} (-1)^j \binom{K_T - 1}{j} \frac{e^{-d_j} E_i\left(d_j\right)}{d_j}, \tag{D.3}$$

where $d_j = -(j+1)/\rho_\eta \lambda_\eta$. With binomial expansion, we have

$$\begin{aligned} \Delta(\gamma) &= \gamma^{\beta(i)-1} e^{-\gamma/\rho_i \widetilde{\lambda}_i} \left[1 - \sum_{j=0}^{\beta(i)-1} \frac{\gamma^j}{j!\left(\rho_i \widetilde{\lambda}_i\right)^j} e^{-\gamma/\rho_i \widetilde{\lambda}_i}\right]^{K_T - 1} \\ &= \gamma^{\beta(i)-1} \sum_{n=0}^{K_T - 1} (-1)^n \frac{(K_T - 1)!}{(K_T - 1 - n)!n!} \\ &\quad \times \left(\sum_{j=0}^{\beta(i)-1} \frac{\gamma^j}{j!\left(\rho_i \widetilde{\lambda}_i\right)^j}\right)^n e^{-(n+1)\gamma/\rho_i \widetilde{\lambda}_i} \\ &= \gamma^{\beta(i)-1} \sum_{n=0}^{K_T - 1} c_n \left(\sum_{j=0}^{\beta(i)-1} \frac{\gamma^j}{j!\left(\rho_i \widetilde{\lambda}_i\right)^j}\right)^n e^{-(n+1)\gamma/\rho_i \widetilde{\lambda}_i}, \end{aligned} \tag{D.4}$$

where c_n is given by (33). When $\beta(i)$ is large, (D.4) can be approximated by

$$\begin{aligned} \Delta(\gamma) &= \sum_{n=0}^{K_T - 1} c_n \gamma^{\beta(i)-1} \left(\sum_{j=0}^{\beta(i)-1} \frac{\gamma^j}{j!\left(\rho_1 \widetilde{\lambda}_i\right)^j}\right)^n e^{-(n+1)\gamma/\rho_i \widetilde{\lambda}_i} \\ &= \sum_{n=0}^{K_T - 1} c_n \gamma^{\beta(i)-1} \left(\sum_{j=0}^{\beta(i)-1} \frac{\left[\gamma/\left(\rho_i \widetilde{\lambda}_i\right)\right]^j}{j!}\right)^n e^{-(n+1)\gamma/\rho_i \widetilde{\lambda}_i} \\ &\approx \sum_{n=0}^{K_T - 1} c_n \gamma^{\beta(i)-1} e^{\gamma n/\rho_i \widetilde{\lambda}_i} e^{-(n+1)\gamma/\rho_i \widetilde{\lambda}_i} \\ &= \sum_{n=0}^{K_T - 1} c_n \gamma^{\beta(i)-1} e^{-\gamma/\rho_i \widetilde{\lambda}_i}. \end{aligned} \tag{D.5}$$

Therefore,

$$\begin{aligned}\Xi &= \int_0^{\infty} \log_2(1+\gamma)\Delta(\gamma)d\gamma \\ &= \int_0^{\infty} \log_2(1+\gamma)\sum_{n=0}^{K_T-1} c_n \gamma^{\beta(i)-1} e^{-\gamma/\rho_i\widetilde{\lambda}_i} d\gamma \\ &= \frac{1}{\ln 2}\sum_{n=0}^{K_T-1} c_n \mathcal{I}_{\beta(i)}\left(\frac{1}{\rho_i\widetilde{\lambda}_i}\right) \\ &= \frac{1}{\ln 2}\sum_{n=0}^{K_T-1} (-1)^n \frac{(K_T-1)!}{(K_T-1-n)!n!}\mathcal{I}_{\beta(i)}\left(\frac{1}{\rho_i\widetilde{\lambda}_i}\right).\end{aligned} \tag{D.6}$$

According to (D.2), $C_{\text{MU}}^C \approx \sum_{i=1}^{\eta-1} K_T/(\rho_i\widetilde{\lambda}_i)^{\beta(i)}(\beta(i)-1)!\Xi + C_{\text{MU}}^C(\beta(\eta))$.

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