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Random Sign Repetition Time-Hopping UWB with Multiuser Detection

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A modified time-hopping (TH) ultra-wideband (UWB), called the random sign repetition TH-UWB, is considered to improve the performance of the minimum mean square error (MMSE) multiuser detector. We show that the increase of dimension or the number of repetitions is important to improve the performance of the MMSE detector in the random sign repetition TH-UWB for either coded or uncoded signals.

Keywords and phrases: ultra-wideband, multiuser detection, performance analysis.

1. INTRODUCTION

Recently, ultra-wideband (UWB) technology, which uses a very short pulse for wireless digital communications, has been extensively investigated due to its significance that enables to transmit data sequences at a very high rate. Although the propagation range of UWB signals is short (about ten meters [1]), its impact on wireless home networks can be quite significant. Apart from the support of a high data rate, UWB can also provide multiple access so that multiple transmitters can be active simultaneously [2] and construct wireless networks [3].

A conventional UWB uses time-hopping (TH) sequences for multiple access and pulse-position modulation (PPM) for signaling [2]. There are other variations of UWB. In [4], an UWB based on direct sequence spreading has been considered and compared to the conventional TH-UWB. Due to the cochannel interference from other UWB transmitters, the performance is generally limited by the cochannel interference. In [5, 6], a characterization of the cochannel interference and performance analysis for the conventional TH-UWB are discussed. Since UWB systems suffer from the cochannel interference as code-division multiple-access (CDMA) systems, it would be possible to apply some interference suppression methods in CDMA to UWB. In [7], the multiuser detection for UWB that employs CDMA signaling is discussed. In [8], the multiuser detection for the conventional TH-UWB is investigated.

In this paper, we consider the multiuser detection in UWB. Using the multiuser formulation, we are able to characterize UWB signals in a multiuser environment. From this, a modification of TH-PPM signaling has been made for improving the performance of multiuser detection. The modified TH-PPM signaling is called the random sign repetition TH-PPM signaling. Through some analysis and simulation results, we can observe that the random sign repetition TH-UWB can provide better performance than the conventional TH-UWB. In addition, we find the increase of the number of repetitions is more effective to improve the performance when the multiuser detector is employed in the random sign repetition TH-UWB.

The rest of the paper is organized as follows. In Section 2, a background of the conventional TH-UWB is presented. For the multiuser detection, a multiuser formulation is also derived in Section 2. The random sign repetition TH-UWB is introduced in Section 3 with a performance analysis. In Section 4, simulation results are presented. We conclude the paper with some remarks in Section 5.

2. BACKGROUND: TH-UWB AND DETECTION

In this section, we briefly review the UWB system in [2, 9] and derive statistical properties for detection.

2.1. TH-UWB signal model and characterization of interference

In TH-UWB, the signature waveform of the kth active transmitter is written as

$$\psi_k(t) = \sum_{i=0}^{N-1} g\left(t - iT_f - c_{k,i}T_c\right), \tag{1}$$

where N is the number of repetitions, T_f is the frame interval, $c_{k,i}$ is the TH sequence, and T_c is the duration of addressable time delay bin. Here, we assume that the TH sequence $c_{k,i} \in \{0, 1, \ldots, N_h - 1\}$, where N_h is a positive integer, is different for each transmitter. Furthermore, we assume that $c_{k,i}$, $i = 0, 1, \ldots, N - 1$, is independently and identically distributed (i.i.d.) random sequence. Throughout this paper, N_h is called the TH factor. The monocycle $g(t)$ is assumed to have finite time support with the pulse width T_g such that

$$|g(t)| = \begin{cases} B(t) > 0 & \text{if } (N_h - 1)T_c + \Delta, \\ & \quad \leq t < (N_h - 1)T_c + \Delta + T_g < T_f, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where $B(t)$ is a positive function, and an energy $\int g^2(t)dt = 1/N$ for normalization. In addition, we assume that $T_g < \Delta$ to have an orthogonal binary PPM signaling. Equation (2) implies that

$$\begin{aligned} \int_{iT_f}^{(i+1)T_f} g^2(t - iT_f)dt &= \int_{iT_f}^{(i+1)T_f} g^2(t - iT_f - jT_c)dt \\ &= \int_{iT_f}^{(i+1)T_f} g^2(t - iT_f - jT_c - \Delta)dt \\ &= \frac{1}{N}, \quad j = 0, 1, 2, \ldots, N_h - 1. \end{aligned} \tag{3}$$

Using the signature waveform, the kth transmitter can transmit the signal as follows:

$$s_k(t) = \sum_l A_k \psi_k(t - lNT_f - a_{k,l}\Delta), \tag{4}$$

where A_k and $a_{k,l} \in \{0, 1\}$ are the amplitude and the bit sequence of the kth transmitter, respectively, and Δ is the modulation index for PPM. In (4), $T_s = NT_f$ becomes the symbol (bit) interval and we can transmit one bit per T_s second. We assume that $T_f \geq N_h T_c$ to avoid the intersymbol interference (ISI) which is caused by overlapping between the two adjacent signal waveforms, for example, $\psi_k(t - lT_s)$ and $\psi_k(t - (l \pm 1)T_s)$. In the conventional TH-UWB, the two parameters N_h and N will be properly determined to optimize the performance.

Suppose that all there are K active transmitters and all the transmissions from the active transmitters are synchronized. Thus, we only consider one symbol interval, especially for the 0th bit ($l = 0$). In addition, for convenience, let $a_{k,l} = a_k$, $k = 1, 2, \ldots, K$. Then, the received signal is written as

$$r(t) = \sum_{k=1}^{K} A_k \psi_k(t - a_k\Delta) + n(t), \quad 0 \leq t \leq T_s, \tag{5}$$

where $a_k \in \{0, 1\}$ are binary symbols. In general, the received signal is distorted by multipaths [2]. In the paper, however, we assume that the received signal is not distorted for the sake of simplicity in analysis as in (5). At the qth receiver, the correlator output to the signal from the kth transmitter $\psi_k(t)$ is given by

$$\int_0^{T_s} \psi_k(t)v_q(t)dt = \sum_{i=0}^{N-1} \int_{iT_f}^{(i+1)T_f} g(t - iT_f - c_{k,i}T_c)v_q(t)dt, \tag{6}$$

where $v_q(t) = \psi_q(t) - \psi_q(t - \Delta)$ is the function to correlate with the received signal for the PPM detection. When the kth signature waveform without delay is presented, the ith partial output of the qth receiver correlator is written as

$$\begin{aligned} u_{k,q;i} &= \int_{iT_f}^{(i+1)T_f} g(t - iT_f - c_{k,i}T_c)v_q(t)dt \\ &= \begin{cases} \dfrac{1}{N} & \text{if } c_{k,i} = c_{q,i}; \\ 0, & \text{if } c_{k,i} \neq c_{q,i}, \end{cases} \end{aligned} \tag{7}$$

because we assume the orthogonal binary PPM signaling ($T_g < \Delta$). We can also find the ith partial output of the qth receiver correlator with delay Δ as

$$\begin{aligned} \overline{u}_{k,q;i} &= \int_{iT_f}^{(i+1)T_f} g(t - iT_f - c_{k,i}T_c - \Delta)v_q(t)dt \\ &= \begin{cases} -\dfrac{1}{N} & \text{if } c_{k,i} = c_{q,i}; \\ 0, & \text{if } c_{k,i} \neq c_{q,i} \end{cases} \\ &= -u_{k,q;i}. \end{aligned} \tag{8}$$

From (7) and (8), the qth output of the matched filter with $v_q(t)$ is given by

$$\begin{aligned} r_{q,i} &= \int_{iT_f}^{(i+1)T_f} r(t)v_q(t)dt \\ &= \int_{iT_f}^{(i+1)T_f} \left(\sum_{k=1}^{K} A_k \psi_k(t - a_k\Delta) + n(t) \right) v_q(t)dt \\ &= \sum_{k=1}^{K} A_k u_{k,q;i} b_k + n_{q;i}, \quad i = 0, 1, \ldots, N - 1, \end{aligned} \tag{9}$$

where $b_k = 1 - 2a_k$ and $n_{q;i} = \int_{iT_f}^{(i+1)T_f} n(t)v_q(t)dt$. From (9), the signal vector can be written as

$$\begin{aligned} \mathbf{r}_q &= \begin{bmatrix} r_{q;0} & r_{q;1} & \cdots & r_{q;N-1} \end{bmatrix}^T \\ &= \mathbf{U}_q \mathbf{A} \mathbf{b} + \mathbf{n}_q, \end{aligned} \tag{10}$$

where

$$\begin{aligned}\mathbf{A} &= \operatorname{diag}(A_1, A_2, \ldots, A_K),\\ \mathbf{n}_q &= \begin{bmatrix} n_{q;0} & n_{q;1} & \cdots & n_{q;N-1}\end{bmatrix}^T,\\ \mathbf{b} &= \begin{bmatrix} b_1 & b_2 & \cdots & b_K\end{bmatrix}^T,\\ \mathbf{U}_q &= \begin{bmatrix} u_{1,q;0} & u_{2,q;0} & \cdots & u_{K,q;0}\\ u_{1,q;1} & u_{2,q;1} & \cdots & u_{K,q;1}\\ \vdots & \vdots & \ddots & \vdots\\ u_{1,q;N-1} & u_{2,q;N-1} & \cdots & u_{K,q;N-1}\end{bmatrix}.\end{aligned} \tag{11}$$

Note that the qth column vector of $\mathbf{U}_q$ is $[(1/N)\ (1/N)\ \cdots\ (1/N)]^T$, which is denoted by $\mathbf{u}_q$, from (7) and (8). In addition, from (3) and the orthogonality of $\psi_k(t)$ and $\psi_k(t-\Delta)$, we can show that

$$\begin{aligned}E[n_{q;i}^2] &= \frac{N_0}{2}\int_{iT_f}^{(i+1)T_f} v_q^2(t)\,dt\\ &= N_0\int_{iT_f}^{(i+1)T_f} g^2(t-iT_f)\,dt\\ &= \frac{N_0}{N}.\end{aligned} \tag{12}$$

Then, it follows that

$$E[\mathbf{n}_q\mathbf{n}_q^T] = \frac{N_0}{N}\mathbf{I}. \tag{13}$$

Equation (10) will play a key role to devise multiuser detectors for TH-UWB signals and allows us to determine the two parameters N and N_h for better performance.

For multipath fading channels, there exists the interpath interference (IPI). The IPI can be considered as the transmitted signals from other transmitters. In this case, the matrix $\mathbf{U}_q$ has more than K columns. In addition, due to asynchronous interarrival time of multipath signals, the correlation coefficient would be differently obtained from (7) and (8). As the extension to multipath fading channels involves UWB channel models, it can be considered as a future research topic.

2.2. Single-user detector from analysis of interference

Since we assume that the TH sequences $c_{k,i}$'s are randomly generated and are i.i.d., we have

$$\Pr(c_{k,i} = c_{q,i}) = \frac{1}{N_h}, \quad \text{for } k \neq q. \tag{14}$$

Then, from (7), we can show that

$$u_{k,q;i} = \begin{cases}\dfrac{1}{N}, & \text{w.p. } \dfrac{1}{N_h};\\ 0, & \text{w.p. } 1-\dfrac{1}{N_h},\end{cases} \quad \text{for } k \neq q, \tag{15}$$

where w.p. stands for "with probability." We define the interference-plus-noise vector as

$$\mathbf{v}_q = \sum_{k\neq q} A_k\mathbf{u}_{k,q}b_k + \mathbf{n}_q, \tag{16}$$

where $\mathbf{u}_{k,q}$ stands for the kth column vector of $\mathbf{U}_q$. Then, for i.i.d. b_k's, the interference-plus-noise vector has the covariance matrix as

$$\left[E[\mathbf{v}_q\mathbf{v}_q^T]\right]_{n,n'} = \begin{cases}\dfrac{1}{N^2N_h}\displaystyle\sum_{k\neq q}A_k^2 + \dfrac{N_0}{N} & \text{if } n = n',\\ \dfrac{1}{N^2N_h^2}\displaystyle\sum_{k\neq q}A_k^2 & \text{otherwise.}\end{cases} \tag{17}$$

Note that the statistical properties in (17) are available without knowing the signature waveforms explicitly. Using a Gaussian approximation, we can derive a single-user detector. Suppose that $\mathbf{v}_q$ is a Gaussian vector with mean zero and covariance matrix $\mathbf{R}_{\mathbf{v}_q} = E[\mathbf{v}_q\mathbf{v}_q^T]$. From the received signal which is given by

$$\mathbf{r}_q = A_q\mathbf{u}_qb_q + \mathbf{v}_q, \tag{18}$$

the optimal single-user maximum-likelihood (ML) detector can be found as

$$\begin{aligned}b_{q,\text{sml}} &= \arg\min_{b\in\{-1,+1\}}(\mathbf{r}_q - \mathbf{u}_qA_qb)^T\mathbf{R}_{\mathbf{v}_q}^{-1}(\mathbf{r}_q - \mathbf{u}_qA_qb)\\ &= \operatorname{sign}\left(\mathbf{u}_q^T\mathbf{R}_{\mathbf{v}_q}^{-1}\mathbf{r}_q\right).\end{aligned} \tag{19}$$

Note that $\mathbf{u}_q^T\mathbf{R}_{\mathbf{v}_q}^{-1}$ can be obtained from (17) in advance. From (17), we can show that

$$\mathbf{R}_{\mathbf{v}_q} = \frac{\sum_{k\neq q}A_k^2}{N^2N_h^2}\mathbf{1}\mathbf{1}^T + \left(\frac{\left(\sum_{k\neq q}A_k^2\right)(N_h-1)}{N^2N_h^2} + \frac{N_0}{N}\right)\mathbf{I}, \tag{20}$$

where $\mathbf{1}$ is a vector whose elements are all 1's. It follows that

$$\mathbf{R}_{\mathbf{v}_q}^{-1}\mathbf{u}_q = \alpha\mathbf{u}_q, \tag{21}$$

where α is a positive constant. Therefore, the detector in (19) can be shown as

$$b_{q,\text{sml}} = \operatorname{sign}\left(\mathbf{u}_q^T\mathbf{R}_{\mathbf{v}_q}^{-1}\mathbf{r}_q\right) = \operatorname{sign}\left(\mathbf{u}_q^T\mathbf{r}_q\right). \tag{22}$$

This implies that the detector in (19) is identical to the single-user correlator detector and indicates that statistical properties in (17) and Gaussian approximation cannot help to improve the detection performance. This is the same as in CDMA. Note that in CDMA, the optimal detector is the correlator detector when the receiver only has statistical properties of the cochannel interference (generated by random sequence) [10]. However, the statistical properties in (10) can help to determine some parameters for improving the performance.

2.3. Multiuser detectors

There are various multiuser detectors [11]. We can apply them to TH-UWB based on (10). The multiuser ML detector is optimal and is given by

$$\mathbf{b}_{q,\text{ml}} = \arg \min_{\mathbf{b}\in\{-1,+1\}^K} \left\|\mathbf{r}_q - \mathbf{U}_q\mathbf{A}\mathbf{b}\right\|^2. \tag{23}$$

However, the complexity grows exponentially with K. The MMSE detector can be considered as a computationally efficient alternative. Using the orthogonality principle [12], the MMSE receiver can be obtained as

$$\begin{aligned}\mathbf{M}_q &= \arg\min_{\mathbf{M}} E\left[\left\|\mathbf{M}\mathbf{r}_q - \mathbf{A}\mathbf{b}\right\|^2\right] \\ &= \mathbf{A}\mathbf{A}^T\mathbf{U}_q^T\left(E\left[\mathbf{r}_q\mathbf{r}_q^T\right]\right)^{-1},\end{aligned} \tag{24}$$

where $E[\mathbf{r}_q\mathbf{r}_q^T]$ is the covariance matrix of $\mathbf{r}_q$ and is given by

$$E\left[\mathbf{r}_q\mathbf{r}_q^T\right] = \mathbf{U}_q\mathbf{A}\mathbf{A}^T\mathbf{U}_q^T + \frac{N_0}{N}\mathbf{I}. \tag{25}$$

Then, the MMSE estimate of $\mathbf{b}$ at the qth receiver is given by

$$\mathbf{b}_{q,\text{MMSE}} = \mathbf{A}^{-1}\mathbf{M}_q\mathbf{r}_q. \tag{26}$$

As shown in (23) and (26), the matrix $\mathbf{U}_q$ is similar to that of signature vectors in CDMA systems [11]. Hence, this determines the interference and, thereby, the performance. Therefore, it is important to understand the properties of $\mathbf{U}_q$ for improving the performance.

The MMSE detector in (24) can be implemented by adaptive algorithms as in CDMA systems [11, 13]. In addition, it would be possible to extend for multipath fading channels using the rake structure (see [14] for CDMA systems). However, since it is beyond the scope of the paper, we do not pursue it for further generalization.

3. RANDOM SIGN REPETITION TH-PPM SIGNALING

3.1. Random sign repetition

The performance of multiuser detection depends on the matrix $\mathbf{U}_q$. In order to have a good performance, it is necessary that $\mathbf{U}_q$ be full rank. In general, for an underloaded case, that is, $K \le N$, the rank of $\mathbf{U}_q$ needs to be K. Unfortunately, since the elements of $\mathbf{U}_q$ are 1's and 0's, $\mathbf{U}_q$ can be quite easily rank deficient. For example, suppose that $K = 3$, $N = 4$, and $N_\text{h} = 2$. At the first receiver (i.e., $q = 1$), $\mathbf{U}_q = \mathbf{U}_1$ can be given by

$$\mathbf{U}_1 = \frac{1}{N}\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 1 & 0 & 1\\ 1 & 0 & 1\end{bmatrix}. \tag{27}$$

Note that the probability that an element of the second- or third-column vector of $\mathbf{U}_1$ is $1/N$, that is, $\Pr(u_{k,1,i} = 1/N)$, $k = 2, 3$, is $1/N_\text{h} = 1/2$. In this case, the rank of $\mathbf{U}_1$ is 2 and the resulting detection performance is poor whether a single-user detector or a multiuser detector is used. Especially, when $b_2 = b_3 = \bar{b}$, we can show that $\mathbf{u}_1 b_1 = \mathbf{u}_2 b_2 + \mathbf{u}_3 b_3 = (\mathbf{u}_2 + \mathbf{u}_3)\bar{b}$. Clearly, the new signature vector $\mathbf{u}_2 + \mathbf{u}_3$ is identical to the signature vector for the desired signal, $\mathbf{u}_1$. Hence, with the received signal vector $\mathbf{r}_1$, it is impossible to detect b_1 due to $(\mathbf{u}_2 + \mathbf{u}_3)\bar{b}$ even if there is no noise. To avoid this difficulty, the coefficient $u_{k,q;i}$ needs to be more random.

As in CDMA, if $u_{k,q;i}$ can be $-1/N$, the rank deficiency of $\mathbf{U}_1$ can occur less frequently. For example, if one element of the previous $\mathbf{U}_1$, for example, $(1, 2)$th element, has been changed to $-1/N$ as

$$\mathbf{U}_1 = \frac{1}{N}\begin{bmatrix}1 & -1 & 0\\ 1 & 1 & 0\\ 1 & 0 & 1\\ 1 & 0 & 1\end{bmatrix}, \tag{28}$$

the new $\mathbf{U}_1$ becomes full rank.

As shown above, to avoid the rank deficiency of $\mathbf{U}_q$, we can consider the random sign repetition which allows $u_{k,q;i}$ to have one of $\{-1/N, 0, 1/N\}$ randomly. To this end, we need to modify the signaling method of TH-UWB. Suppose that there are two signature waveforms $\psi_k(t;0)$ and $\psi_k(t;1)$ for data bit 0 and 1, respectively, as follows:

$$\begin{aligned}\psi_k(t;0) &= \sum_{i=0}^{N-1} g\left(t - iT_\text{f} - c_{k,i}T_\text{c} - \beta_{k,i}\Delta\right),\\ \psi_k(t;1) &= \sum_{i=0}^{N-1} g\left(t - iT_\text{f} - c_{k,i}T_\text{c} - \left(1-\beta_{k,i}\right)\Delta\right),\end{aligned} \tag{29}$$

where $\beta_{k,i} \in \{0, 1\}$, $i = 0, 1, \ldots, N-1$, is a random binary sequence which is independent of $c_{k,i}$. Then, the transmitted signal given the data bit sequence $a_{k,l}$ is written as

$$s_k(t) = \sum_l A_k\psi_k\left(t - lNT_\text{f}; a_{k,l}\right). \tag{30}$$

Note that in the conventional TH-UWB, $\beta_{k,i} = 0$ for all k and i. From (29), we can show that

$$\begin{aligned}u_{k,q;i} &= \int_{iT_\text{f}}^{(i+1)T_\text{f}} g\left(t - iT_\text{f} - c_{k,i}T_\text{c} - \beta_{k,i}\Delta\right)v_q(t)\,dt\\ &= \begin{cases}\dfrac{1}{N} & \text{if } c_{k,i} = c_{q,i},\ \beta_{k,i} = 0,\\ -\dfrac{1}{N} & \text{if } c_{k,i} = c_{q,i},\ \beta_{k,i} = 1,\\ 0 & \text{if } c_{k,i} \ne c_{q,i}.\end{cases}\end{aligned} \tag{31}$$

Then, when we assume that $\Pr(\beta_{k,i} = 1) = \Pr(\beta_{k,i} = 0) = 1/2$, $u_{k,q;i}$, for $k \ne q$, has the following statistical property:

$$u_{k,q;i} = \begin{cases}\dfrac{1}{N}, & \text{w.p. } \dfrac{1}{2N_\text{h}};\\ -\dfrac{1}{N}, & \text{w.p. } \dfrac{1}{2N_\text{h}};\\ 0, & \text{w.p. } 1 - \dfrac{1}{N_\text{h}}.\end{cases} \tag{32}$$

According to (32), the matrix $\mathbf{U}_q$ can be rank deficient with less probability. It can certainly improve the performance of multiuser detection. It is noteworthy that the modified PPM in (29) is used for random sign repetition without any antipodal signaling such as binary phase-shift keying (BPSK). Using the following antipodal signaling,[1] we can have the same statistical property in (32):

$$\begin{aligned}\psi_k(t;0) &= \sum_{i=0}^{N-1} g\left(t - iT_\mathrm{f} - c_{k,i}T_\mathrm{c}\right),\\ \psi_k(t;1) &= \sum_{i=0}^{N-1} \left(1 - 2\beta_{k,i}\right)g\left(t - iT_\mathrm{f} - c_{k,i}T_\mathrm{c}\right).\end{aligned} \tag{33}$$

3.2. *Performance and the impact of the number of repetitions, TH factor, and channel coding*

As mentioned earlier, the performance depends on the number of repetitions N, and the TH factor N_h. According to (10), N decides the dimension and N_h decides the interference density. Hence, the performance of the multiuser detection can be improved by increasing both N and N_h. However, there is a constraint. Since $T_\mathrm{f} \geq N_\mathrm{h}T_\mathrm{c}$ and $T_\mathrm{s} = NT_\mathrm{f}$, with a fixed symbol interval T_s, we have

$$T_\mathrm{s} = NT_\mathrm{f} = NN_\mathrm{h}T_\mathrm{c}, \tag{34}$$

where T_c is generally determined by the duration of the monocycle and is fixed. This implies that

$$NN_\mathrm{h} \leq \frac{T_\mathrm{s}}{T_\mathrm{c}} = \text{const.} \tag{35}$$

With the constraint in (35), we can consider the impact of N and N_h on the performance.

To see the performance dependency on N and N_h, the signal-to-interference-plus-noise ratio (SINR) can be used. Firstly, we consider the conventional TH-UWB with the single-user correlator detector. From (10) and (15), we can show that

$$\begin{aligned}\gamma_q &= \frac{A_q^2\|\mathbf{u}_q\|^2}{E\left[\left|\mathbf{u}_q^T\mathbf{v}_q\right|^2\right]}\\ &= \frac{A_q^2}{N_0 + \sum_{k\neq q} A_k^2\left(1/NN_\mathrm{h} + (N-1)/NN_\mathrm{h}^2\right)}.\end{aligned} \tag{36}$$

From (35), let $\overline{N} = NN_\mathrm{h}$. Then, we have

$$\gamma_q = \frac{A_q^2}{N_0 + (1/\overline{N})\sum_{k\neq q} A_k^2\left(1 + (N-1)/N_\mathrm{h}\right)}. \tag{37}$$

When $\overline{N}$ is fixed, we can readily see that the SINR γ_q increases with N_h (note that N decreases with N_h since $\overline{N} = NN_\mathrm{h}$). This shows that the TH factor N_h should be large for better performance.

In the random sign repetition TH-UWB, from (32), the covariance matrix of $\mathbf{v}_q$ becomes

$$E\left[\mathbf{v}_q\mathbf{v}_q^T\right] = \left(\frac{1}{N^2N_\mathrm{h}}\sum_{k\neq q} A_k^2 + \frac{N_0}{N}\right)\mathbf{I}. \tag{38}$$

From this, the SINR is given by

$$\overline{\gamma}_q = \frac{A_q^2}{N_0 + (1/\overline{N})\sum_{k\neq q} A_k^2}. \tag{39}$$

This implies that the SINR is independent of the values of (N, N_h) as long as $\overline{N} = NN_\mathrm{h}$ is fixed. In addition, we can show that

$$\overline{\gamma}_q \geq \gamma_q. \tag{40}$$

That is, the random sign repetition TH-UWB can perform better than the conventional TH-UWB when the single-user correlator detector is used.

In general, the average SINR of the multiuser detector with respect to random TH sequences is difficult to obtain. Fortunately, however, there are some approaches we can use from CDMA systems including a large system analysis [15]. We can adopt the approach in [15] to understand the performance of the multiuser MMSE detector.

From (24), (26), and [15], we can show that the SINR of the multiuser MMSE detector is given by

$$\gamma_{\mathrm{MMSE},1} = P_q\mathbf{u}_q^T\left(\widetilde{\mathbf{U}}_q\widetilde{\mathbf{D}}_q\widetilde{\mathbf{U}}_q^T + \frac{N_0}{N}\mathbf{I}\right)^{-1}\mathbf{u}_q, \tag{41}$$

where $P_q = A_q^2$, $\widetilde{\mathbf{U}}_q$ is the submatrix of $\mathbf{U}_q$ obtained by deleting the qth column vector, and $\widetilde{\mathbf{D}}_q$ is the diagonal matrix that is given by

$$\widetilde{\mathbf{D}}_q = \operatorname{diag}\left(P_1, \ldots, P_{q-1}, P_{q+1}, \ldots, P_K\right). \tag{42}$$

For convenience, let $q = 1$. Using the eigendecomposition, we have

$$\widetilde{\mathbf{U}}_1\widetilde{\mathbf{D}}\widetilde{\mathbf{U}}_1^T = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^T, \tag{43}$$

where $\mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_N]$ and $\mathbf{\Lambda} = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$. Here, λ_l stands for the lth (smallest) eigenvalue of $\widetilde{\mathbf{U}}_1\widetilde{\mathbf{D}}\widetilde{\mathbf{U}}_1^T$ and $\mathbf{e}_l$ is the corresponding eigenvector. We assume that the random sign repetition TH-UWB is used. Then, the entries of $\widetilde{\mathbf{U}}_q$ are i.i.d. Furthermore, from (32), the column vector of $\widetilde{\mathbf{U}}_q$ can be normalized as follows:

$$\mathbf{u}_{q,l} = P\left[u_{q,l;1} \;\; u_{q,l;2} \;\; \cdots \;\; u_{q,l;N}\right]^T, \tag{44}$$

where $\mathbf{u}_{q,l}$ stands for the lth column vector of $\widetilde{\mathbf{U}}_q$, $P = 1/NN_\mathrm{h}$, and $u_{q,l;n}$, $n = 1, 2, \ldots, N$, are i.i.d. random variables with

[1]This approach is proposed by one of the reviewers.

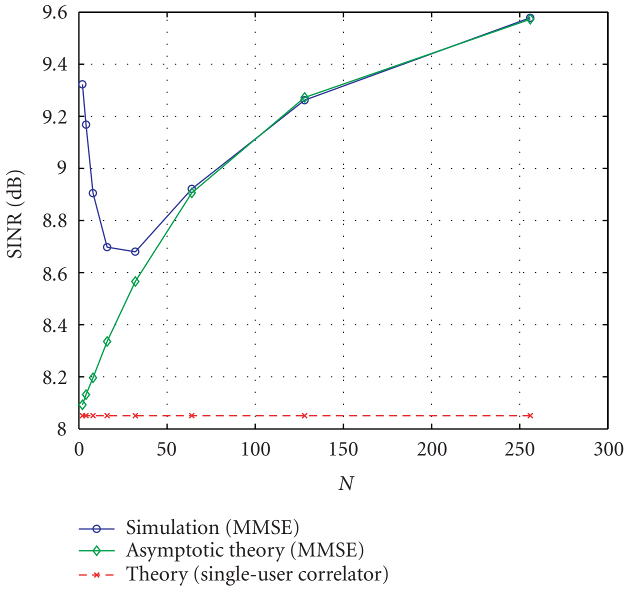


FIGURE 1: SINR performance for different pairs of (N, N_h) when $\overline{N} = NN_h = 512$. MMSE detector $K = 30$, SNR=10 dB, $N \times N_h = 512$.

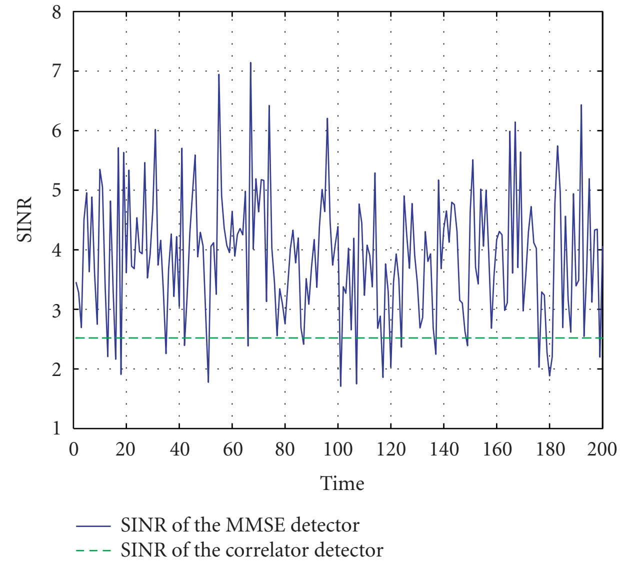


FIGURE 2: Time-varying SINR when long TH sequences are employed ($N = 8$, $N_h = 8$, $K = 20$, and SNR $(= P_k/N_0) = 10$ dB, where $P_1 = P_2 = \cdots = P_K$). $N = 4$, $N_h = 16$, $K = 20$, SNR=10 dB.

zero mean and unit variance. Note that $E[\|\mathbf{u}_{q,l}\|^2] = P$. From [15], the asymptotic SINR when N is large is given by

$$\begin{aligned}\gamma_{\text{MMSE}} &= P_1\|\mathbf{u}_1\|^2 \int_0^\infty \frac{1}{\lambda + N_0/N} dG(\lambda) \\ &= \frac{P_1}{N}\int_0^\infty \frac{1}{\lambda + N_0/N} dG(\lambda),\end{aligned} \tag{45}$$

where $\|\mathbf{u}_1\|^2 = 1/N$ and $G(\lambda)$ is the empirical distribution of the eigenvalue of $\tilde{\mathbf{U}}_1\tilde{\mathbf{D}}\tilde{\mathbf{U}}_1^T$. For the sake of simplicity, let $A_k = 1$ for all k. After some manipulations based on [15], we have

$$\gamma_{\text{MMSE}} \simeq \frac{P_1}{N_0 + ((K-1)/N)(P_1P/(P1 + P\gamma_{\text{MMSE}}))}, \tag{46}$$

where $P_1 = 1$. The asymptotic SINR in (46) can provide some insights into the impact of N and N_h on the performance of the multiuser MMSE detector.

In Figure 1, the simulation results for the SINR are presented with different values of (N, N_h) when $K = 30$ and $\overline{N} = NN_h$ is fixed and set to 512. The signal-to-noise ratio (SNR) P_k/N_0 is set to as 10 dB for all k. As shown in Figure 1, the asymptotic SINR according to (46) increases with N or decreases with N_h. This implies that we need to increase N when the multiuser detector is used (note that this is contrary to the case of the single-user correlator detector in the conventional TH-UWB in which N_h needs to be large for better performance). However, the actual simulation results are different from (46) when N is small. The SINR from simulation results decreases with N until $N = 32$ and then increases with N. This shows that the asymptotic SINR in (46) is only valid when N is sufficiently large and urges the need of performance analysis for the case of small N.

When a channel code is employed, the performance of the multiuser MMSE detector can depend on the type of TH sequence. As in CDMA, we can consider two different types of TH sequences. One is a short TH sequence which repeats every symbol interval; this is the case in (1). The other is a long TH sequence. In this case, TH sequence is different for every symbol interval. The main advantage of a short TH sequence is that adaptive techniques can be used for the multiuser detection [11]. The adaptive multiuser detector can suppress the interfering signals from other transmitters without knowing their TH sequences [13]. On the other hand, when a long TH sequence is employed, it is hard to implement adaptive multiuser detectors. Hence, the TH sequences of all the active transmitters should be known, which is difficult in a realistic environment. Although the use of long TH sequence makes the implementation of adaptive multiuser detectors difficult, it can provide a better performance with channel coding. Since the SINR changes from symbol to symbol when long TH sequences are used, a diversity gain can be induced and exploited by channel coding. This implies that the performance (in terms of coded bit error rate (BER)) for long TH sequences can outperform that for short TH sequences. In Figure 2, an illustration of time-varying SINR is presented when $N = 8$, $N_h = 8$, $K = 20$, and the SNR P_k/N_0 is 10 dB for all k.

4. SIMULATION RESULTS

To see the impact of random sign repetition on the performance, simulations are carried out with $N = 8$, $N_h = 4$, and $K = 5$. The system loading K can be normalized as K/NN_h. From this, we can see that the system for simulations is substantially underloaded as the normalized loading is 5/32. The (uncoded) BER results are shown in Figure 3. Note that the

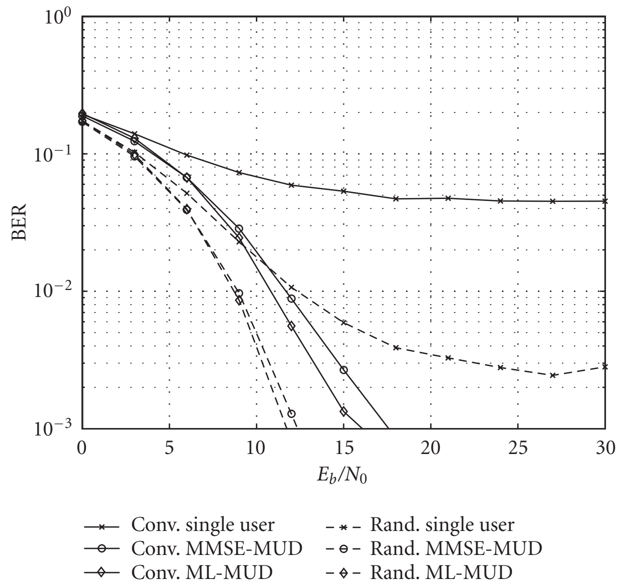


Figure 3: BER performance in terms of E_b/N_0 and conventional TH-UWB (solid lines: conventional TH-PPM; dashed lines: random sign repetition TH-PPM).

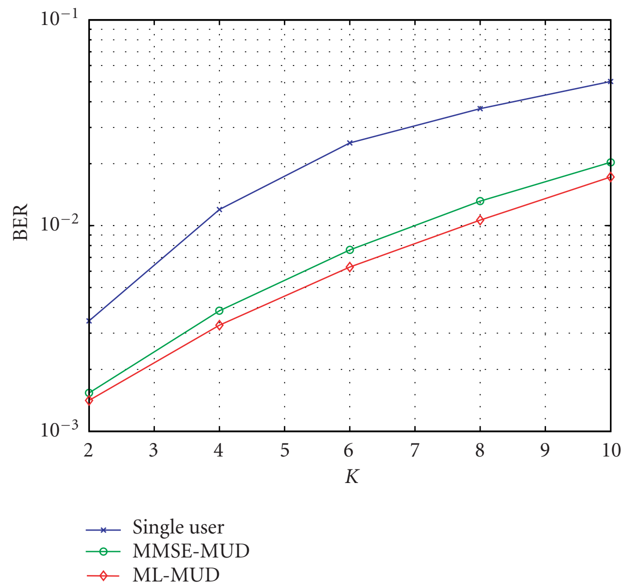


Figure 4: BER performance in terms of K and modified TH-PPM when $E_b/N_0 = 10$ dB.

bit energy $E_b = A_k^2 \int_0^{T_s} \psi_k^2(t)dt$ is normalized to be 1. According to (3), this implies that $A_k = 1$ for all k. Then, from (13), the SNR at the qth receiver (without any interference from other transmitters) is given by

$$\text{SNR} = \frac{A_q^2 ||\mathbf{u}_q||^2}{E[|\mathbf{u}_q^T \mathbf{n}_q|^2]} = \frac{E_b}{N_0} = \frac{1}{N_0}. \tag{47}$$

This shows that the SNR is identical to E_b/N_0. In Figure 3, it is shown that the random sign repetition TH-UWB can provide about 5 dB E_b/N_0 gain at a BER of 10^{-3} compared to the conventional TH-UWB when the multiuser MMSE detector is employed. In addition, we can observe that the multiuser detectors can provide much better performance than the single-user detector, especially at high E_b/N_0. As in CDMA [11], since the performance of the single-user detector is limited by the interference, the BER is saturated (the error floor occurs at a BER of about 3×10^{-3}) at high E_b/N_0, but the multiuser detectors do not encounter the error floor up to a BER of 10^{-4}.

In Figure 4, the BER performance in terms of the system loading (i.e., the number of transmitters K) is presented. We assume that the random sign repetition TH-UWB is used with $N = 8$, $N_h = 4$, and $E_b/N_0 = 10$ dB. In Figure 4, comparing to the single-user correlator detector, the multiuser detectors can accommodate about double users at a BER of 10^{-2}. Note that as K increases, there are more interfering signals. Hence, the performance becomes worse as shown in Figure 4.

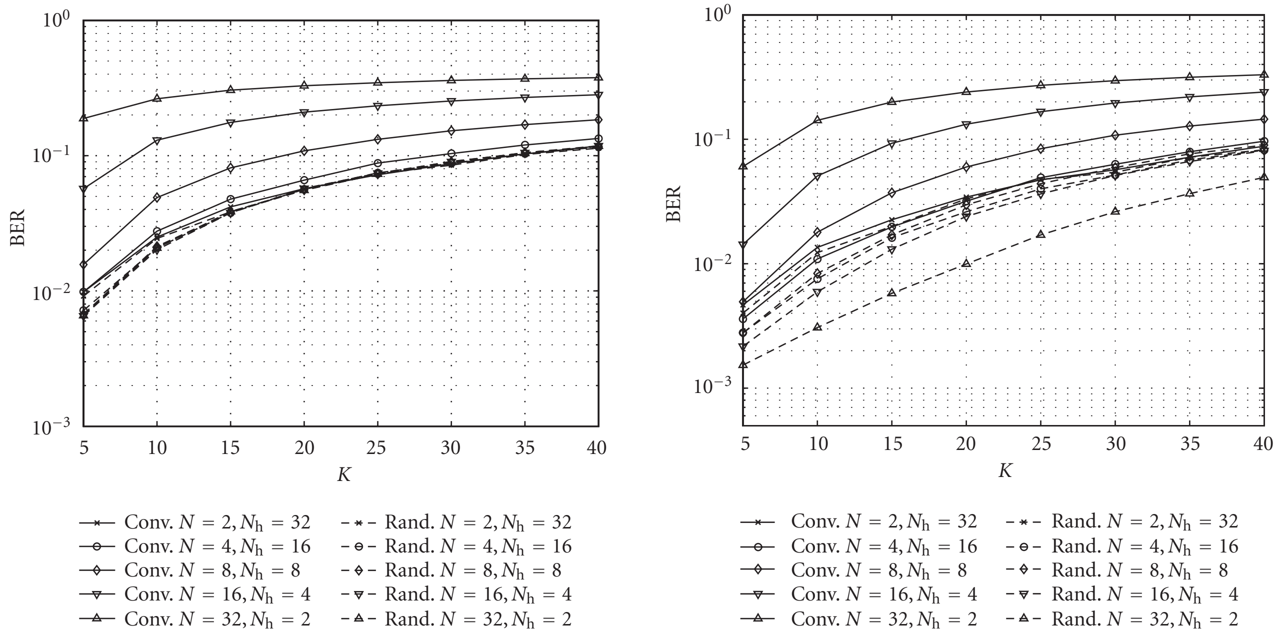
In order to see the impact of (N, N_h) for a fixed $\overline{N} = NN_h$, we consider the following pairs:

$$(N, N_h) \in \{(2,32), (4,16), (8,8), (16,4), (32,2)\} \tag{48}$$

with $\overline{N} = 64$. In Figure 5, the BER simulation results are shown in terms of the loading K with different pairs of (N, N_h). In Figure 5a, as we have seen in Section 3.2, the performance of the single-user correlator detector can be improved as N_h increases in the conventional TH-UWB (see (37)). On the other hand, the performance of the single-user correlator detector is unchanged for different values of (N, N_h) as long as $\overline{N}$ is fixed in the random sign repetition TH-UWB. This observation is predicted with the SINR (see (39)) in Section 3.2.

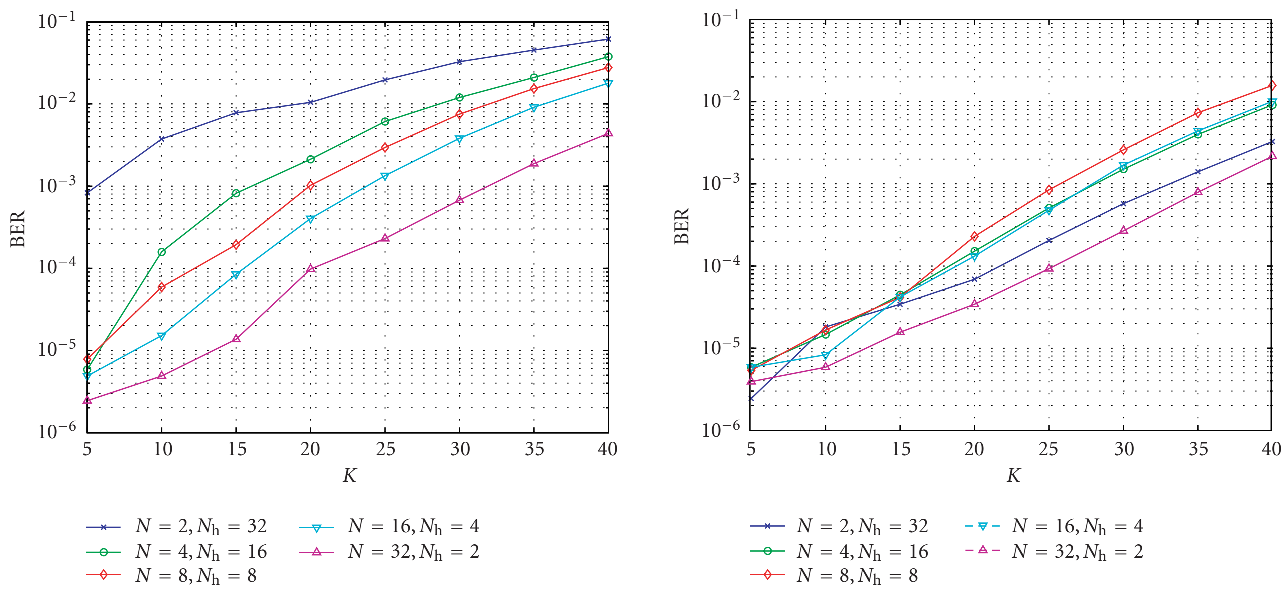
For the multiuser MMSE detector, the BER simulation results are shown in Figure 5b. Note that in the conventional TH-UWB, it is shown that the performance is improved when N_h is large. However, in the random sign repetition TH-UWB, we can see that the performance is improved when N is large as shown in Figure 5b. This shows that the increase of dimension (i.e., increasing N) is more important than the decrease of the interference density (i.e., increasing N_h) to improve the performance when the multiuser detector is employed in the random sign repetition TH-UWB.

Through Figures 3, 4, and 5, we consider uncoded BER performance. To see the impact of channel codes, we consider simulations with the multiuser MMSE detector for coded signals. A half-rate convolutional channel code with generators of (23, 35) in octal and free distance of 7 [16] is used. The results are presented in Figure 6 with different pairs of (N, N_h) when $\overline{N} = 64$ and $E_b/N_0 = 10$ dB. Generally, the coded BER performance can be improved when long TH sequences are used as shown in Figure 6. This has been predicted in Section 3. However, there is an interesting observation in Figure 6. We can see that the best performance can be achieved and there is no significant performance difference between the cases of long TH sequence and short TH sequence for a pair of $(N, N_h) = (32, 2)$. Since the



(a) (b)

FIGURE 5: Uncoded BER performance in terms of K when $E_b/N_0 = 10$ dB: (a) the case of single-user correlator detector, (b) the case of multiuser MMSE detector (solid lines: conventional TH-PPM; dashed lines: random sign repetition TH-PPM).



(a) (b)

FIGURE 6: Coded BER performance of the multiuser MMSE detector in terms of K when $E_b/N_0 = 10$ dB. (A convolutional code of rate 1/2 with generators (23, 35) in octal is used. The Viterbi algorithm is used for decoding.) (a) The modified random sign repetition TH-PPM with short TH sequence, MMSE detector; (b) TH-UWB with the random sign repetition TH-PPM with long TH sequence, MMSE, and different TH sequence for each bit.

multiuser MMSE detector can have good performance when the dimension is sufficiently large and its SINR follows the asymptotic SINR in (46) which is independent of particular realizations of (random) TH sequences, the performance can be maximized and the performance difference between long and short TH sequences can be vanished when N is large.

From this, we can conclude that short TH sequences with large N can be used without performance loss in the multiuser detection. Importantly, as mentioned in Section 3, this makes the implementation of the adaptive MMSE detector easy.

Consequently, we can have a few observations regarding the determination of N and N_h for a fixed $\overline{N} = NN_h$. When the conventional TH-UWB is used with the single-user correlator detector at the receiver, as shown in Figure 3 and (37), N_h should be large for better performance. However, for the multiuser MMSE detector in the random sign repetition TH-UWB, N should be large to improve the performance as discussed in Section 3 and confirmed in Figures 5 and 6. This indicates that when the MMSE multiuser detector is employed, the CDMA signaling is more suitable to improve the performance as the TH-PPM signaling becomes the CDMA signaling when $N_h = 1$ according to (32).

5. CONCLUDING REMARKS

We proposed a modified TH-PPM signaling for improving the performance when the MMSE multiuser detector is used. With the MMSE multiuser detector, the modified TH-PPM can provide about 5 dB E_b/N_0 gain at a BER of 10^{-3} compared to the conventional TH-PPM under a lower system loading. Furthermore, we observed that the increase of dimension or the number of repetitions is more important to improve the performance of the multiuser MMSE detector when the modified TH-PPM signaling is used for either coded or uncoded signals. This is contrary to the case of the conventional TH-PPM signaling, in which the increase of the TH factor is more effective to improve the performance of the single-user correlator detector. Hence, the determination of parameters (e.g., N and N_h) can be different depending on the choices of the detectors and TH-PPM signaling schemes to maximize the performance.

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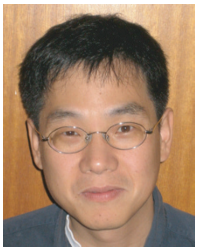

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