13th Summer Workshop on Interval Methods

July 19-21, 2022

Book of Abstracts

Steffen Schön[1]

Andreas Rauh[2]

[1]Institute of Geodesy
Leibniz Universität Hannover

[2]Distributed Control in Interconnected Systems
Carl von Ossietzky Universität Oldenburg

Scope & Topics

SWIM aims at gathering researchers working on/with interval methods and their applications. The goal is to review the state-of-the-art in this field. Contributions can be for example in the domain of

- Verification and Validation
- Robust and Nonlinear Control Systems
- State Estimation
- Interval Observer Design
- Parameter Identification
- Fault Detection and Diagnosis, Fault Tolerant Systems
- Stability, Reachability, Observability
- Reliable Software Design
- Robotics
- Mathematics
- Verified Solution of Algebraic and Dynamic System Models
- Verified Numerics and Scientific Computing
- Linear Algebra
- and any other applications of interval methods, verified numerics, and other related set-membership techniques (e.g.: affine arithmetics, polytopes, etc.)

Previous Editions of SWIM

The SWIM workshop series was initiated by the French MEA working group on Set Computation and Interval Techniques of the French research group on Automatic Control GDR MACS, where the MEA group especially aimed at promoting interval analysis techniques and applications to a broader community of researchers. Since 2008, SWIM has become an annual keystone event for researchers dealing with various aspects of interval and set-membership methods.

Previous editions of SWIM were held in:

- Paris, France in 2019
- Rostock, Germany in 2018
- Manchester, UK in 2017
- Lyon, France in 2016
- Prague, Czech Republic in 2015
- Uppsala, Sweden in 2014
- Brest, France in 2013
- Oldenburg, Germany in 2012
- Bourges, France in 2011
- Nantes, France in 2010
- Lausanne, France in 2009
- Montpellier, France in 2008

Monday, July 18th 2022			
Time Slot	**Presenter**	**Title**	**Moderation**
From 18:00		Welcome Reception at the Leibnizhaus	

Tuesday, July 19th 2022			
Time Slot	**Presenter**	**Title**	**Moderation**
9:00 - 9:15		Welcome Address	
9:15 - 10:15	Nathalie Revol	Plenary Lecture Testing interval arithmetic libraries	Ekaterina Auer
10:15 - 10:45		Coffee Break	
		Session 1: Mathematical Tools and Software	
10:45 - 11:15	Simon Rohou	The Codac library	Pieter Collins
11:15 - 11:45	Luc Jaulin	A new type of intervals for solving problems involving partially defined functions	
11:45 - 12:15	Damien Massé	Differential Inclusion using Matrix Exponential	
12:15 - 13:45		Lunch at "Stadtmauer - Genuss am Fluss"	
		Session 2: Fault Tolerant Systems and Identification	
13:45 - 14:15	Carlos Eduardo Valero	Fault Detection in Networked Control Systems. A Robust Approach	Simon Rohou
14:15 - 14:45	Marit Lahme	Online Identification of the Open-Circuit Voltage of Lithium-Ion Batteries with the Use of Interval Methods	
14:45 - 15:15	Marco de Angelis	Rigorous bounds on the failure probability with the SIVIA algorithm	
15:15 - 15:45		Coffee Break	
16:00 - 18:00		Guided Tour Kestnermuseum, exhibition "Good Demon Bes"	
18:00		Dinner at "Meiers Lebenslust"	

Wednesday, July 20th 2022			
Time Slot	**Presenter**	**Title**	**Moderation**
9:00 - 10:00	Michael Beer	Plenary Lecture Efficient engineering analysis with imprecise probabilities	Steffen Schön
10:00 - 10:30		Coffee Break	
		Session 3: Methods with Result Verification 1	
10:30 - 11:00	Julien Alexandre dit Sandretto	A constraint programming approach for polytopic simulation of ordinary differential equations - a collision detection application	Andreas Rauh
11:00 - 11:30	Luc Jaulin	Symmetries for Interval Analysis	
11:30 - 12:00	Julien DAMERS	Lie symmetries applied to guaranteed integration: application to mobile robotics localisation	
12:00 - 12:30	Martin Fränzle	Affine Encodings for Optimal Monitoring of Temporal Properties under Uncertain Observation	
12:30 - 14:00		Lunch at "bona'me"	

	Session 4: Methods with Result Verification 2		
14:00 - 14:30	Pieter Collins	Higher-Order Methods for Differential Inclusions	Nathalie Revol
14:30 - 15:00	Jonathan Wunderlich	Computer-assisted Existence Proofs for Navier-Stokes Equations on an Unbounded Strip with Obstacle	
15:00 - 15:30	Ekaterina Auer	Uses of Methods with Result Verification in the Context of MIMO Systems	
15:30 - 16:00	Suman Maiti	Eigenvalues enclosures of skew symmetric/Hermitian matrices having bounded uncertainty	
16:00 - 16:30	Coffee Break		
	Session 5: Interval Methods for Localization 1		
16:30 - 17:00	Maxime ZAGAR	Robust 3D target localization using UAVs with state uncertainty	Julien Alexandre dit Sandretto
17:00 - 17:30	Maria Luiza Costa Vianna	A Geometric Approach to the Coverage Measure of the Area Explored by a Robot	
17:30 - 18:00	Quentin Brateau	Sea route monitoring by weather buoys using interval analysis	

Thursday, July 21st 2022			
Time Slot	**Presenter**	**Title**	**Moderation**
	Session 6: Interval Methods for Localization 2		
9:00 - 9:30	Jingyao Su	How to determine uncertainty interval: Practice in GNSS and LiDAR localisation	Damien Massé
9:30 - 10:00	Steffen Schön	An investigation of interval and set-based uncertainty representation for GNSS navigation	
10:00 - 10:30	Aaronkumar Ehambram	Interval-based Global Localization in Building Maps	
10:30 - 11:00	Coffee Break		
	Session 7: Robust Control and State Estimation		
11:00 - 11:30	Antoine Hugo	High-gain interval observer for continuous-discrete time systems : Application to a quadcopter	Luc Jaulin
11:30 - 12:00	Oussama Benzinane	Stabilizing Controller Design Using an Iterative LMIs Approach for Quadrotors	
12:00 - 12:30	Andreas Rauh	Validated Model Predictive Control based on Exponential Enclosures	
12:30	End		

Testing interval arithmetic libraries

Nathalie Revol[1], Luis Benet Fernández[2], Luca Ferranti[3] and Sergei Zhilin[4]

[1] INRIA - LIP UMR 5668, ENS Lyon, University Lyon 1, Inria, CNRS - France
`Nathalie.Revol@inria.fr` (presenting author)
[2] Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México
`benet@icf.unam.mx`
[3] University of Vaasa, Vaasa, Finland
`luca.ferranti@uwasa.fi`
[4] CSort LLC, Barnaul, Russia
`szhilin@gmail.com`

Keywords: Interval Arithmetic, Library, Test, Unit Test

Testing interval arithmetic libraries: why

Interval arithmetic is used to get guarantees on numerical results. Indeed, it provides an anclosure of the sought result. However, the user must trust the library implementing interval arithmetic that is employed to solve the given problem. What guarantees that this library is correct? Formal proof is a desirable approach, and in particular it is available within the CoqInterval library [1]. Another, complementary, approach consists in testing the library: it covers aspects that are usually not covered by formal proof, such as the specifics of the language, compiler (with the notable exception of CompCert [2]) or hardware.

Unit tests

In what follows, only unit tests will be considered, and tests that are complete applications will not be discussed, see [3] for a first step in this direction. Unit tests target only one function of the library at a time; typically, they consist of a list of test cases, that is, of input values along with the expected output values: one checks whether the function returns the expected output values for each input arguments. If this is the case, the function passes the test.

Testing interval arithmetic libraries: how

The goal of this talk is to discuss the different aspects that unit tests should cover, and how to devise corresponding test cases, with a specific focus on compliance with the IEEE 1788-2015 standard for interval arithmetic [4]. The ultimate goal would be to create a collection of test cases for each function required or recommended by this standard, and to share them. We emphasize that this collection should be easy to use for libraries written in different programming languages, such as MPFI [5] written in C, libieee1788 [6] written in C++, JInterval [7] written in Java, Intlab [8] available in MatLab, Octave/interval [9] written in Octave or JuliaIntervals/IntervalArithmetic.jl [10] written in Julia. We will survey two approaches in this direction, namely JInterval [11] and ITF-1788 [12], and discuss their limitations. An even more desirable goal would be to design a generator of test cases, we will discuss this point as well.

References

[1] E. Martin-Dorel and G. Melquiond, *Proving tight bounds on univariate expressions with elementary functions in Coq*, Journal of Automated Reasoning 57(3), pp. 187–217 (2016).
`https://coqinterval.gitlabpages.inria.fr`

[2] X. Leroy et al., *CompCert-a formally verified optimizing compiler*, in ERTS 2016: Embedded Real Time Software and Systems, 8th European Congress.
`https://compcert.org`

[3] X. Tang, Z. Ferguson, T. Schneider, D. Zorin, S. Kamil, D. Panozzo, *A Cross-Platform Benchmark for Interval Computation Libraries*, arXiv 2021. `https://arxiv.org/abs/2110.06215`

[**4**] IEEE: Institute of Electrical and Electronic Engineers, *1788-2015 - IEEE Standard for Interval Arithmetic.*

[**5**] N. Revol and F. Rouillier, *Motivations for an Arbitrary Precision Interval Arithmetic and the MPFI Library*, Reliable Computing **11**(4), pp. 275–290 (2005).
`https://gitlab.inria.fr/mpfi/mpfi`

[**6**] M. Nehmeier, *libieeep1788: A C++ Implementation of the IEEE interval standard P1788*, in 2014 IEEE Conference on Norbert Wiener in the 21st Century (21CW), pp. 1–6.
`https://github.com/nehmeier/libieeep1788`

[**7**] D.Y. Nadezhin and S.I. Zhilin, *JInterval Library: Principles, Development, and Perspectives*, Reliable Computing **19**(3), pp. 229–247 (2013).
`https://github.com/jinterval/jinterval/`

[**8**] S.M. Rump, *INTLAB - INTerval LABoratory*, in Developments in Reliable Computing, Tibor Csendes (ed), pp. 77–104. Kluwer Academic Publishers (1999).
`https://www.tuhh.de/ti3/rump/intlab/`

[**9**] O. Heimlich, *Interval arithmetic in GNU Octave*, in SWIM 2016, 9th Summer Workshop on Interval Methods, Lyon, France.
`https://octave.sourceforge.io/interval/index.html`

[**10**] D.P. Sanders and L. Benet Fernández, *JuliaIntervals/-IntervalArithmetic.jl: v0.20.5*, Zenodo, DOI 10.5281/zenodo.6337817
`https://github.com/JuliaIntervals/ValidatedNumerics.jl`

[**11**] *P1788 Test Launcher (based on JInterval Library).*
`https://github.com/jinterval/jinterval/tree/master/p1788-launcher-java`

[**12**] M. Kiesner, M. Nehmeier, and J. Wolff von Gudenberg, *ITF1788: An Interval Testframework for IEEE 1788*, Report no 495, Dpt Computer Science, University of Würzburg (2015).
`https://github.com/oheim/ITF1788`

The Codac library

Simon Rohou[1], Benoît Desrochers[1]

[1] ENSTA Bretagne, Lab-STICC, UMR CNRS 6285, Brest, France
simon.rohou@ensta-bretagne.fr

Keywords: interval analysis, constraint programming, robotics

Introduction

Codac (Catalog Of Domains And Contractors) is a C++/Python library providing interval tools for constraint programming over reals, trajectories and sets. In the field of robotics, complex problems such as non-linear state estimation, parameter estimation, delays, or SLAM can be solved in a very few steps by using constraint programming. Even though the Codac library is not meant to target only robotic problems, the design of its interface has been largely influenced by the needs of the above class of applications. Codac provides solutions to deal with these problems, that are usually hardly solvable by conventional methods such as particle approaches or Kalman filters.

A framework of domains and contractors

Codac extends the tools proposed in the IBEX library to a wider class of problems. A *catalog* of domains such as intervals $[x]$, boxes $[\mathbf{x}]$, tubes $[x](t)$ (intervals of trajectories), thicksets $[\mathbb{X}]$ (intervals of sets) is available in Codac. These sets are contractible by *contractor* operators that aim at narrowing their bounds in a reliable way, according to several constraints defining a problem. The provided contractors are associated with publications from the literature and allow to deal with, for instance, non-linear constraints $\mathbf{f}(\mathbf{x}) = \mathbf{0}$, inequalities, geometric constraints (distance, polar equation, circles), continuous differential

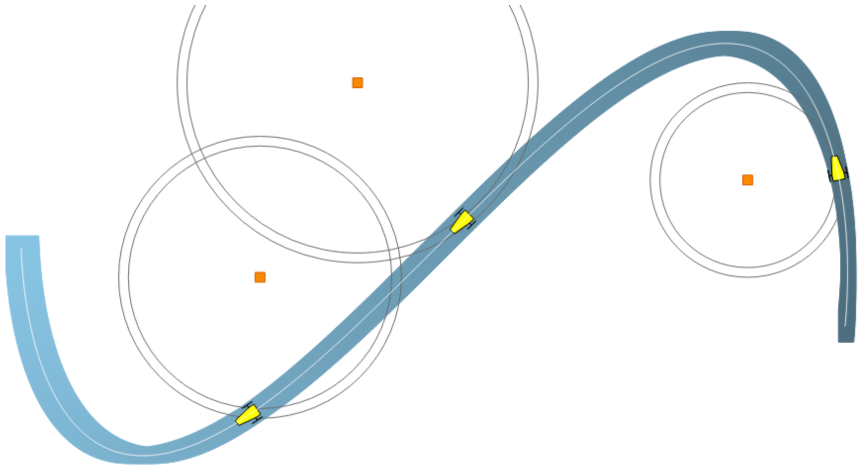


Figure 1: Guaranteed computation of a tube enclosing the feasible trajectories of a robot measuring bounded distances from three landmarks, without prior knowledge about its initial position.

equations: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$, time uncertainties: $\mathbf{y} = \mathbf{x}(t)$ with $t \in [t]$, delays: $x(t) = y(t - \tau)$ [2], *etc.*

Domains and contractors can be combined in a *Contractor Network* that will manage the propagation of the contractions and ease the implementation of the related interval solver. In a few steps, we first: (1) define the initial domains (boxes, tubes) of our variables (vectors, trajectories) ; (2) take contractors from a catalog of already existing operators, provided in the library ; (3) add the contractors and domains to a Contractor Network ; (4) let the Contractor Network solve the problem ; (5) obtain a reliable set of feasible variables. The presentation will provide a simple application of Codac on a robotic problem.

References

[1] S. Rohou, B. Desrochers and Others, *The Codac library, constraint-programming for robotics*, `http://codac.io`, 2022.

[2] R. Voges, Bounded-Error Visual-LiDAR Odometry on Mobile Robots Under Consideration of Spatiotemporal Uncertainties, *PhD Thesis*, 2020.

A new type of intervals for solving problems involving partially defined functions

Pierre Filiol, Théotime Bollengier, Luc Jaulin,
Jean-Christophe Le Lann

Lab-STICC, ENSTA Bretagne
Brest, France

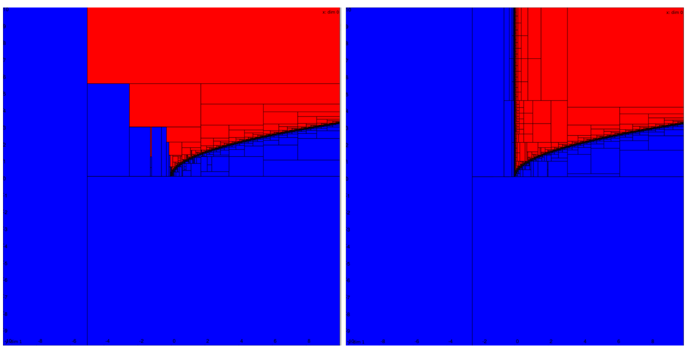
Keywords: Intervals, Contractors, Not a Number

Introduction

If we want to characterize an inner and an outer approximation of

$$\mathbb{S} = \{(x, y) \,|\, y - \sqrt{2x - x} \geq 0\}$$

a classical set inversion algorithm [1], yields the left figure, where as we would like to obtain the right figure



The outer contractor works well, but the inner contractor is overcontracting. Note that, the multi-occurence of x in the expression $\sqrt{2x - x}$, allows the inner contractor to show its weakness. This type of problems occurs several times in our real applications when dealing with functions such as log, $\sqrt{\cdot}$ that are not defined everywhere. We want to identify the reasons of the problem and find a way to fix it.

New type of interval

Consider the extended set of reals $\mathring{\mathbb{R}} = \mathbb{R} \cup \iota$ where ι stands for *Not A Number* [2]. Operations on real numbers can be extended to $\mathring{\mathbb{R}}$ as follows:

$$\begin{array}{ll} f(x) = \iota & \text{if } x \notin \mathrm{dom}(f) \\ f(\iota) = \iota & \\ \iota \diamond x = \iota & \end{array}$$

where $f : \mathbb{R} \to \mathbb{R}$, $x \in \mathbb{R}$ and $\diamond$ is a binary operator. The set $\mathring{\mathbb{R}}$ can be equipped with a partial order relation derived from rules:

$$\begin{array}{ll} \iota \leq \iota & \\ a \in \mathbb{R}, b \in \mathbb{R} & \text{then} \quad a \leq_{\mathring{\mathbb{R}}} b \text{ iff } a \leq_{\mathbb{R}} b \end{array}$$

and intervals can be derived from these relations. Examples of intervals of $\mathring{\mathbb{R}}$ are $[2, 5], [2, 5] \cup \{\iota\}, \{\iota\}, \emptyset$. In the extended paper, we show that this new type of intervals allows us to solve inequalities where functions are not defined everywhere.

References

[1] L. Jaulin, M. Kieffer, O. Didrit, E. Walter, *Applied Interval Analysis*, Springer-Verlag, 2022.

[2] Institute of Electrical and Electronics Engineers A.N.S.I. *A standard for binary floating-point arithmetic.* ANSI/IEEE Std. 754-1985, New York, 1985

Differential Inclusion using Matrix Exponential

Damien Massé[1]

[1] University of Brest, Lab-STICC, Robex team
20 avenue Le Gorgeu, 29200 Brest, France
damien.masse@univ-brest.fr

Keywords: Matrix Exponentiation, Differential Inclusion, Guaranteed Integration.

Introduction

On $\mathbb{R}^n$, we consider the differential inclusion problem defined as:

$$\dot{x} = f(x, u) \quad \text{where } u \in [u] \tag{1}$$

where f is differentiable and u can take any value in a box $[u]$ at any time. From a set of initial states (at $t = 0$), our goal is to get an overapproximation of the possible states at time t.

Contribution

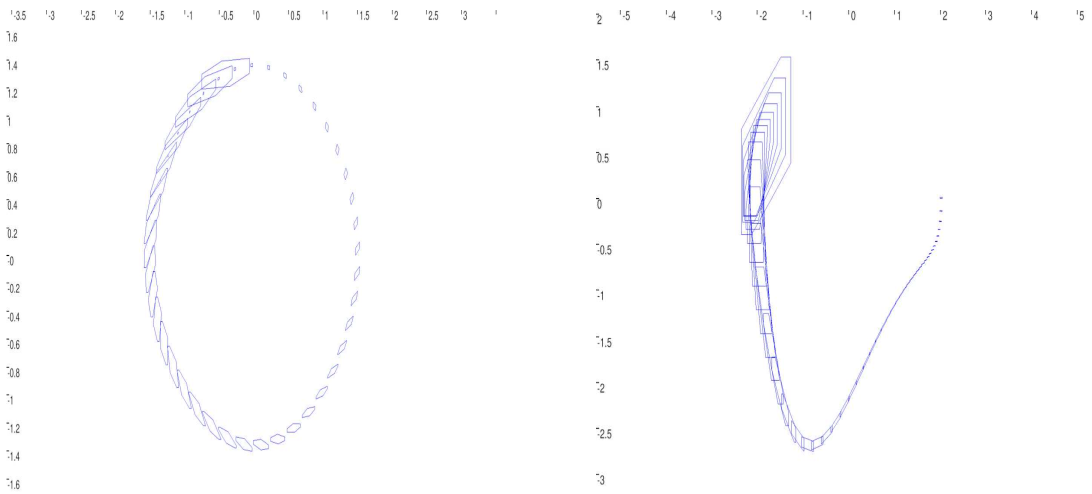
On $[0, t]$, we can express f as:

$$f(x, u) = C + A(x - x_m) + \phi(x, t) \tag{2}$$

where C is an vector of intervals, A is a matrix of intervals and $\phi(x, t) \in [\Phi]$ where Φ is a zero-centered box.

In this case, if $x(0) = x_0$, the solution of the differential equation (for a given ϕ) is:

$$x(t) = x_m + e^{tA}(x_0 - x_m) + \int_0^t e^{(t-\tau)A} d\tau C + \int_0^t e^{(t-\tau)A} \phi(x(\tau), \tau) d\tau \tag{3}$$



(A) Pendulum with perturbation (B) Van der Pol oscillator

Figure 1: Solutions of two differential inclusion. (A) A pendulum with uncertainties. (B) A Van der Pol oscillator. We represent sets as intersections of parallepipeds.

Following previous works on exponentiation of interval matrices[1], we compute precise and safe overapproximations of e^{tA} and $\int_0^t e^{(t-\tau)A} d\tau$ using Taylor developments as well as scaling and squaring techniques.

We show that bounding $\int_0^t e^{(t-\tau)A}\phi(x(\tau),\tau)d\tau$ can be done by bounding $I(A,t) = \int_0^t |e^{\tau A}| d\tau$ (which $|V|$ being the component-wise absolute value of V). This is done by computing $[K]$ such that $e^{\tau A} \in \mathrm{Id} + \tau[K]$ and bounding $I(A,t)$ from the components of $[K]$.

Fig 1 graphically shows the evolution of the solutions for a few classical examples. We compared our approach with CAPD [2] on a Van der Pol oscillator with a small perturbation:

$$(\dot{x}; \dot{y}) = (y + [-10^{-4}, 10^{-4}]; (1 - x^2) * y - x + [-10^{-4}, 10^{-4}])$$

Fig 2 gives the enclosing boxes for $t = 1$, for CAPD and our approach. The precision depends heavily on the number of time steps, but these results indicate the interest of our approach.

Initial state	Our approach	CAPD (CW method)
(2;0)	$[1.507982, 1.508306]$ $\times[-0.780351, -0.780088]$	$[1.508005, 1.508283]$ $\times[-0.780311, -0.780126]$
(2;3)	$[2.300337, 2.300655]$ $\times[-0.479899, -0.479744]$	$[2.300371, 2.300625]$ $\times[-0.479863, -0.479778]$

Figure 2: Comparaison of our approach and CAPD on a simple example.

Acknowledgement

Thanks to Luc Jaulin, Simon Rohou and Thomas Le Mezo for their suggestions.

References

[1] A. Goldsztejn, and A. Neumaier. On the Exponentiation of Interval Matrices. 2009. `https://hal.archives-ouvertes.fr/hal-00411330v1`

[2] T. Kapela, M. Mrozek, D. Wilczak, and P. Zgliczyński. CAPD::DynSys: A flexible C++ toolbox for rigorous numerical analysis of dynamical systems. *Communications in Nonlinear Science and Numerical Simulation*, Volume 101, 2021, 105578, ISSN 1007-5704, `https://doi.org/10.1016/j.cnsns.2020.105578`.

Fault Detection in Networked Control Systems. A Robust Approach

Carlos E. Valero[1], radoslav Paulen [1]

[1] Slovak University of Technology in Bratislava,
Radlinského 9, 812 37 Bratislava, Slovakia
{carlos.valero,radoslav.paulen}@stuba.sk

Keywords: Set-membership state estimation, fault detection, zonotopes,

Introduction

Networked Control Systems (NCSs) are spatially distributed systems in which the controller and/or other elements are connected through a network. They have been used in a wide variety of applications, due to the lower cost of implementation and the growing trend in the Internet of Things (IoT). Aperiodic measurements have been proven useful to decrease the traffic in the network, mechanisms such as self/event-triggered are the most used. On the other hand, the problem of fault detection and isolation (FDI) is still an issue in this type of structure. Many techniques have been applied for FDI. In general, these are classified into active or passive techniques. In this work, we propose a framework to zonotope state estimation in an NCS subject to the event-triggered mechanism for robust FDI. The framework takes measurements from the past to improve the reduction of the feasible set, ergo, FDI. The framework is tested over a well-known FDI method over a double spring-mass system. The results show that the framework improves FDI by 20% compared to the traditional method without the framework.

General Structure

The principle of FDI using zonotope is simple. After performing the reachability step on a standard zonotope state estimation, the output measurement is taken to construct an output set that intersects with the reachable set to reduce the final feasible set. However, if this intersection is empty, it will imply that a fault or an attack has occurred. The framework adds virtual output sets in this intersection.

Acknowledgement

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References

[**1**] Daniel Silvestre, Paulo Rosa, Joao P Hespanha, and Carlos Silvestre, Self-triggered and event-triggered set-valued observers, *Information Sciences*, 426:61–86, 2018.

[**2**] Ingimundarson, A., Bravo, J. M., Puig, V., Alamo, T., and Guerra, P., Robust fault detection using zonotope-based set-membership consistency test, *International journal of adaptive control and signal processing* 23(4), 311-330, 2009.

[**3**] Xiaohua Ge, Qing-Long Han, Xian-Ming Zhang, Lei Ding, and Fuwen Yang., Distributed event-triggered estimation over sensor networks: A survey, *IEEE transactions on cybernetics* 50(3), 1306–1320, 2019.

Online Identification of the Open-Circuit Voltage of Lithium-Ion Batteries with the Use of Interval Methods

Marit Lahme and Andreas Rauh

Carl von Ossietzky Universität Oldenburg,
Distributed Control in Interconnected Systems
D-26111 Oldenburg, Germany
{marit.lahme, andreas.rauh}@uol.de

Keywords: Online identification, Interval methods, Lithium-ion batteries

Introduction

The charging/discharging dynamics of Lithium-ion batteries can be approximated by using equivalent circuit models. According to [1]–[3], these models consist of a finite number of RC sub-networks as well as series resistances and a state of charge (SOC) dependent voltage source (open-circuit voltage $v_{\mathrm{OC}}(t)$) as shown in Fig. 1.

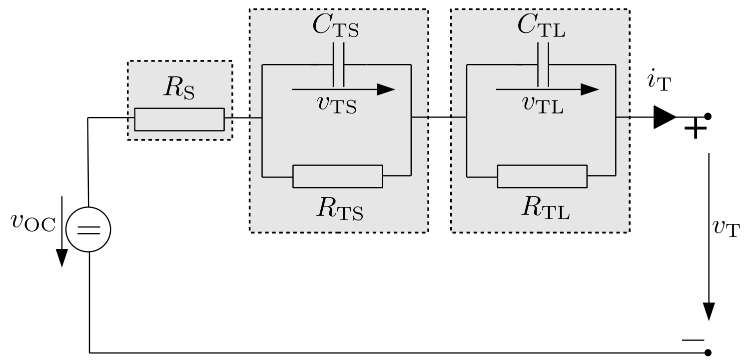

Figure 1: Equivalent circuit model of a Lithium-ion battery (cf. [1]).

In this presentation, two RC sub-networks representing processes with short (T_{TS}) and large (T_{TL}) time constants are considered, which result from polarization effects and concentration losses as described in [1], [2].

The SOC $\sigma(t)$ and the voltages across the RC sub-networks $v_{\mathrm{TS}}(t)$ and $v_{\mathrm{TL}}(t)$ are chosen as the state variables. With the state vector

$$\mathbf{x}(t) = \begin{bmatrix} \sigma(t) & v_{\mathrm{TS}}(t) & v_{\mathrm{TL}}(t) \end{bmatrix}^T \quad , \tag{1}$$

the quasi-linear, continuous-time battery model is obtained as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\left(\sigma(t)\right) \cdot \mathbf{x}(t) + \mathbf{b}\left(\sigma(t)\right) \cdot u(t) \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-1}{C_{\mathrm{TS}}(\sigma(t)) \cdot R_{\mathrm{TS}}(\sigma(t))} & 0 \\ 0 & 0 & \frac{-1}{C_{\mathrm{TL}}(\sigma(t)) \cdot R_{\mathrm{TL}}(\sigma(t))} \end{bmatrix} \cdot \mathbf{x}(t) + \begin{bmatrix} \frac{-1}{C_{\mathrm{Bat}}} \\ \frac{1}{C_{\mathrm{TS}}(\sigma(t))} \\ \frac{1}{C_{\mathrm{TL}}(\sigma(t))} \end{bmatrix} \cdot i_{\mathrm{T}}(t) \end{aligned} \tag{2}$$

with the terminal current $i_{\mathrm{T}}(t)$ as the system input.
By applying Kirchhoff's voltage law, the terminal voltage is obtained as

$$v_{\mathrm{T}}(t) = v_{\mathrm{OC}}(\sigma(t)) - v_{\mathrm{TS}}(t) - v_{\mathrm{TL}}(t) - i_{\mathrm{T}}(t) \cdot R_{\mathrm{S}}\left(\sigma(t)\right) \quad , \tag{3}$$

where $v_{\mathrm{OC}}(\sigma(t))$ and $R_{\mathrm{S}}\left(\sigma(t)\right)$ are represented by nonlinear expressions of the SOC. For more details, see [1], [2].

In classical state estimation approaches, the parameters are identified experimentally (cf. [1]–[3]). But they are subject to aging- and temperature-induced variations, which are shown in [2]. The aging of battery cells leads to a loss of the total capacity, an increasing Ohmic cell resistance and changes in the charging/discharging efficiency as well as changes of the time constants. Additionally, there are other influence factors such as the cell temperature. The first-mentioned variations can be estimated with the help of an augmented state vector, but this approach does not allow for estimating nonlinear dependencies of the circuit elements on the SOC or other influence factors.
For the identification of the nonlinear dependency of the open-circuit voltage on the SOC with underlying aging- and temperature-induced

variations, it is assumed that the parameters are known and not yet affected by aging. The aging- and temperature-induced variations can be mapped onto the open-circuit voltage.
This presentation proposes a two-stage identification for nonlinear dependencies with the dependency of the open-circuit voltage on the SOC as an example. The state variables of the dynamic system are estimated in the first stage with an interval observer. In the second stage, the a-priori knowledge is corrected using the estimated state variables.

In this presentation the notations $\underline{\mathbf{M}}$ and $\overline{\mathbf{M}}$ for a matrix $\mathbf{M}$ denote the element-wise lower and upper bounds.

With the bounding system $\mathbf{x} \in [\underline{\mathbf{x}} \; ; \; \overline{\mathbf{x}}]$ and $\hat{\mathbf{x}} \in \left[\underline{\hat{\mathbf{x}}} \; ; \; \overline{\hat{\mathbf{x}}}\right]$, $\mathbf{x}$ is given as $\mathbf{x} \in \left[\underline{\hat{\mathbf{x}}} \; ; \; \overline{\hat{\mathbf{x}}}\right]$ and the interval observer is obtained according to ([3], [4])

$$\underline{\mathbf{A}}_{\mathrm{O}}\underline{\hat{\mathbf{x}}} + \underline{\mathbf{B}}\mathbf{u} + \mathbf{H}\underline{\mathbf{y}}_{\mathrm{m}} \leq \dot{\hat{\mathbf{x}}} \leq \overline{\mathbf{A}}_{\mathrm{O}}\overline{\hat{\mathbf{x}}} + \overline{\mathbf{B}}\mathbf{u} + \mathbf{H}\overline{\mathbf{y}}_{\mathrm{m}} \tag{4}$$

with the observer system matrices

$$\underline{\mathbf{A}}_{\mathrm{O}} = \underline{\mathbf{A}} - \mathbf{HC} \quad \text{and} \quad \overline{\mathbf{A}}_{\mathrm{O}} = \overline{\mathbf{A}} - \mathbf{HC} \tag{5}$$

and the uncertain measurements

$$[\mathbf{y}_{\mathrm{m}}] := \left[\underline{\mathbf{y}}_{\mathrm{m}} \; ; \; \overline{\mathbf{y}}_{\mathrm{m}}\right] = \mathbf{y}_{\mathrm{m}} + [-\Delta\mathbf{y}_{\mathrm{m}} \; ; \; \Delta\mathbf{y}_{\mathrm{m}}] \quad . \tag{6}$$

Here, the system matrix $\mathbf{A}\left(\sigma(t)\right)$ has the following sign pattern

$$\mathbf{A}\left(\sigma(t)\right) = \begin{bmatrix} \leq 0 & \geq 0 & \geq 0 \\ \geq 0 & \leq 0 & \geq 0 \\ \geq 0 & \geq 0 & \leq 0 \end{bmatrix} \in \left[\; \underline{\mathbf{A}} \;\; ; \;\; \overline{\mathbf{A}} \;\right] \quad . \tag{7}$$

The output equation is given as

$$\mathbf{y}(t) = \tilde{v}_{\mathrm{T}}(t) = \left[\tilde{v}_{\mathrm{OC}}(t) - v_{\mathrm{TS}}(t) - v_{\mathrm{TL}}(t) - i_{\mathrm{T}}(t) \cdot R_{\mathrm{S}}(t)\right] \tag{8}$$

with the associated quasi-linear representation

$$\begin{aligned}\mathbf{y}^*(t) &= \mathbf{y}(t) - \mathbf{D}\left(\sigma(t)\right) \cdot i_{\mathrm{T}}(t) = \mathbf{C}\left(\sigma(t)\right) \cdot \mathbf{x}(t) \\ &= \begin{bmatrix} \eta_{\mathrm{OC}}\left(\sigma(t)\right) & -1 & -1 \end{bmatrix} \cdot \mathbf{x}(t) \in [\mathbf{y}_{\mathrm{m}}] \ \ ;\end{aligned} \tag{9}$$

$\tilde{v}_{\mathrm{OC}}(t)$ is obtained by subtracting the constant, state independent terms from the expression for the open-circuit voltage $v_{\mathrm{OC}}(t)$ to turn this expression into a quasi-linear form, see [1].

Based on the design of a robust interval observer shown in [3], the observer matrix $\mathbf{H}$ is hereby assigned as

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 & 0 \end{bmatrix}^T \ \ , \ \ h_1 > 0 \ . \tag{10}$$

With the help of interval methods, the nonlinear dependency of the open-circuit voltage on the SOC is identified as shown in Fig. 2.

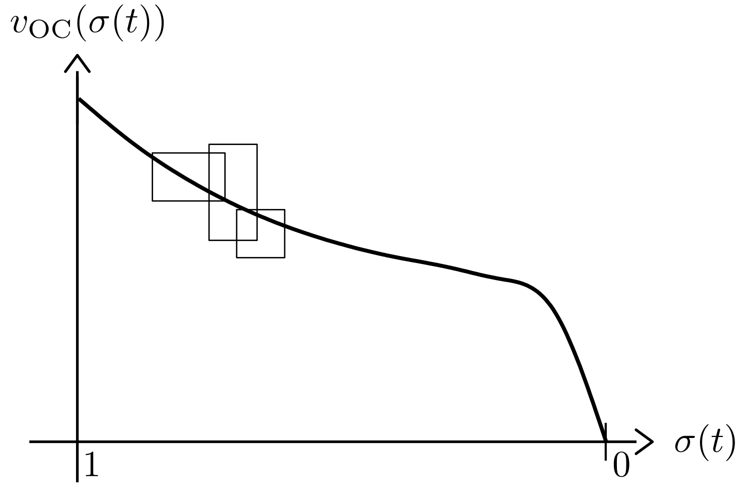

Figure 2: Identification of nonlinear dependencies using interval methods.

References

[1] A. Rauh, T. Chevet, T. N. Dinh, J. Marzat and T. Raïssi, Robust Iterative Learning Observers Based on a Combination of

Stochastic Estimation Schemes and Ellipsoidal Calculus, *Proc. of the 25th International Conference on Information Fusion*, Linköping, SE, 2022, accepted.

[**2**] O. Erdinc, B. Vural and M. Uzunoglu, A Dynamic Lithium-Ion Battery Model Considering the Effects of Temperature and Capacity Fading, *Proc. of International Conference on Clean Electrical Power*, 383–386, 2009.

[**3**] E. Hildebrandt, J. Kersten, A. Rauh and H. Aschemann, Robust Interval Observer Design for Fractional-Order Models with Applications to State Estimation of Batteries, *IFAC-PapersOnLine* (vol. 53) 2, 3683-3688, 2020.

[**4**] T. Raïssi and D. Efimov, Some Recent Results on the Design and Implementation of Interval Observers for Uncertain Systems, *Automatisierungstechnik* (vol. 66) 3, 213-224, 2018.

Rigorous bounds on the failure probability with the SIVIA algorithm

Marco de Angelis[1], Ander Gray [1]

[1] University of Liverpool, Institute for Risk and Uncertainty
Peach Street, L69 3GQ Liverpool, United Kingdom
{marco.de-angelis,ander.gray}@liverpool.ac.uk

Keywords: Bounding, Rigorous probability, SIVIA algorithm, Reliability analysis

Summary

A common verification problem in reliability analysis is establishing the correctness of Monte Carlo methods. When the system is complex and the failure event is rare, reducing the variance of the Monte Carlo estimator can be a challenge. To combat this verification problem, we present an adaptation of the SIVIA algorithm (Set Inversion Via Interval Analysis) that computes rigorous bounds on the failure probability of rare events. With this method, the nonlinearity of the system and the magnitude of the failure event no longer constitute a limitation. The method is rigorous i.e. inclusive and outside-in, so the more computational effort is invested the tighter the bounds. Because the engineering and the probability problem are separated, the method opens exciting avenues towards computing rigorous imprecise failure probability.

Set-inversion reliability

SIVIA (Set Inversion Via Interval Analysis) is a popular algorithm for constraint back propagation [1,2]. In this section, we show how SIVIA can be deployed to output a lower subtiling of the failure domain, i.e. the domain where the failure measure is integrated. Rigorous integration of the lower subtiling of the failure domain leads to a lower bound on the failure probability. SIVIA also outputs an outer subtiling of the boundary of the failure domain, which failure measure corresponds to the imprecision in the failure probability. The size of this outer subtiling determines the accuracy of the calculation.

Rigorous integration of probability

Rigorous integration takes place knowing the probability distribution exactly and evaluating the probability measure in each subbox of the subtiling by means of the so called H-volume [3]. The total failure probability is the sum of the measures of each sub-box.

Reproducibility

The code and algorithms used in this document are available at `https://github.com/marcodeangelis/set-inversion-reliability`. Interval computations were run using *intervals* `https://github.com/marcodeangelis/intervals` a code library for interval computing in Python.

Acknowledgement

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References

[1] Jaulin, L., M. Kieffer, O. Didrit, and E. Walter, Interval analysis, *Applied Interval Analysis: With Examples in Parameter and State Estimation, Robust Control and Robotics*, pp. 11–43. 2001.

[2] Jaulin, L. and E. Walter Set inversion via interval analysis for nonlinear bounded-error estimation. 1993.

[3] Schweizer, B. and A. Sklar *Probabilistic metric spaces.* Courier Corporation. 2011.

Efficient engineering analysis with imprecise probabilities

Michael Beer[1,5,6], Matteo Broggi[1], Matthias G.R. Faes[2], Marcos A. Valdebenito[3], and Pengfei Wei[4]

[1] Institute for Risk and Reliability. Leibniz University Hannover, Germany.
[2] Chair for Reliability Engineering. TU Dortmund, Germany.
[3] Faculty of Engineering and Sciences. Universidad Adolfo Ibáñez, Chile.
[4] School of Power and Energy. Northwestern Polytechnical University, China.
[5] Institute for Risk and Uncertainty. University of Liverpool, UK.
[6] International Joint Research Center for Engineering Reliability and Stochastic Mechanics (ERSM) & International Joint Research Center for Resilient Infrastructure (ICRI). Tongji University, China.

Abstract

An efficient analysis of our engineering structures and systems is a key requirement for a most suitable design at the required level of reliability. This requirement, however, is challenging engineers to come up with innovative solutions that can cope with the increasing complexity of our structures and their behaviour and with the uncertainties involved. Imprecise probabilities have shown useful conceptual features to facilitate a modelling at a reasonable level of detail and capturing the remaining epistemic uncertainty in a set-valued manner. This approach allows for an optimal balance between model detailedness and imprecision of results to still derive useful decisions. However, it is also associated with some extensive numerical cost when applied in a crude way. This presentation will highlight selected solutions for efficient numerical analysis with imprecise probabilities, specifically for reliability analysis, to attack high-dimensional and nonlinear problems. After an introductory overview on conceptual pathways for solution one intrusive and three non-intrusive specific developments will be discussed. These solutions include operator norm theory to solve first passage problems by linear algebra, intervening variables to moderate nonlinearities for linearized approximate solutions, and the utilization

of high dimensional model representation of the failure probability for non-intrusive efficient sampling. Engineering examples are presented to demonstrate the capabilities of the approaches and concepts.

References

[**1**] Beer, M. (2020), *Fuzzy Probability Theory*, revised and updated chapter, In: Meyers, R. (ed.), Encyclopedia of Complexity and Systems Science, Springer, Berlin, Heidelberg. (Invited Chapter), pages 1–25, $https://doi.org/10.1007/978-3-642-27737-5_237-2$

[**2**] A. Beer, M.; Ferson, S.; Kreinovich, V. (2013), *Imprecise Probabilities in Engineering Analyses*, Mechanical Systems and Signal Processing 37, 4–29.

[**3**] Beer, M.; Zhang, Y.; Quek, S.T.; Phoon, K.K. (2013), *Reliability analysis with scarce information: Comparing alternative approaches in a geotechnical engineering context*, Structural Safety 41, 1–10.

[**4**] Behrendt, M.; de Angelis, M.; Comerford, L.A.; Zhang, Y.J.; Beer, M. (2022), *IProjecting interval uncertainty through the discrete Fourier transform: an application to time signals with poor precision Processing*, Mechanical Systems and Signal Processing, 172, Article 108920.

[**5**] Dang, C.; Wei, P.F.; Faes, M.G.R.; Beer, M. , *Bayesian probabilistic propagation of hybrid uncertainties: Estimation of response expectation function, its variable importance and bounds, Computers and Structures*, (in Press).

[**6**] Dang, C.; Wei, P.F.; Faes, M.G.R.; Valdebenito, M.A.; Beer, M. (2022), *Interval uncertainty propagation by a parallel Bayesian global optimization method*, Applied Mathematical Modelling, 108, 220–235.

[**7**] De Angelis, M.; Patelli, E.; Beer, M. (2015), *Advanced Line Sampling for Efficient Robust Reliability Analysis*, Structural Safety 52, 170–182.

[**8**] Faes, M.G.R.; Broggi, M.; Chen, G., Phoon, K.K.; Beer M. (2022), *Distribution-free P-box processes based on translation theory: definition and simulation*, Probabilistic Engineering Mechanics, 69, Article 103287.

[**9**] Faes, M.G.R., Daub, M.; Marelli, S.; Patelli, E.; Beer, M. (2021), *Engineering analysis with probability boxes: a review on computational methods*, Structural Safety, 93, Article 102092.

[**10**] Faes, M.; Sadeghi, J.; Broggi, M.; De Angelis, M.; Patelli, E.; Beer, M.; Moens, D. (2019), *On the robust estimation of small failure probabilities for strong non-linear models*, ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering 5, 041007.

[**11**] Faes, M.; Valdebenito, M.A.; Moens, D.; Beer, M. (2020), *Bounding the First Excursion Probability of Linear Structures Subjected to Imprecise Stochastic Loading*, Computers and Structures 239, 106320.

[**12**] Faes, M.; Valdebenito, M.A.; Moens, D.; Beer, M. (2021), *Operator norm theory as an efficient tool to propagate hybrid uncertainties and calculate imprecise probabilities*, Mechanical Systems and Signal Processing 152, 107482.

[**13**] Faes, M.; Valdebenito, M.A., Yuan, X.K.; Wei, P.F.; Beer, M. (2021), *Efficient imprecise reliability analysis using the Augmented Space Integral*, Reliability Engineering and System Safety, 210, Article 107477.

[**14**] Faes, M.G.R.; Valdebenito, M.A.; Yuan, X.K.; Wei, P.F.; Beer, M. (2021) , *Augmented Reliability Analysis for Estimating Imprecise First Excursion Probabilities in Stochastic*

Linear Dynamics, Advances in Engineering Software, 155, Article 102993

[**15**] Möller, B.; Graf, W.; Beer, M. (2000), *Fuzzy structural analysis using α-level optimization*, Computational Mechanics 26, 547–565.

[**16**] Song, J.W.; Valdebenito, M.; Wei, P.F.; Beer, M.; Lu, Z.Z. (2020), *Non-intrusive imprecise stochastic simulation by line sampling*, Structural Safety 84, 101936.

[**17**] Song, J.W.; Wei, P.F.; Valdebenito, M.; Beer, M. (2020), *Adaptive reliability analysis for rare events evaluation with global imprecise line sampling*, Computer Methods in Applied Mechanics and Engineering 372, 113344.

[**18**] Song, J.W.; Wei, P.F.; Valdebenito, M.; Beer, M. (2020), *Active Learning Line Sampling for Rare Event Analysis*, Mechanical Systems and Signal Processing 147, Article 107113

[**19**] Song, J.W.; Wei, P.F.; Valdebenito, M.; Bi, S.F.; Broggi, M.; Beer, M.; Lei, Z.X. (2019) , *): Generalization of non-intrusive imprecise stochastic simulation for mixed uncertain variables*, Mechanical Systems and Signal Processing, 134, 06316.

[**20**] Valdebenito, M.A.; Beer, M.; Jensen, H.A.; Chen, J.B.; Wei, P.F. (2020) , *Fuzzy Failure Probability Estimation Applying Intervening Variables*, Structural Safety 83, 101909.

[**21**] Valdebenito, M.A.; Pérez, C.A.; Jensen, H.A.; Beer, M. (2016), *Approximate fuzzy analysis of linear structural systems applying intervening variables*, Computers & Structures 162, 116–129.

[**22**] Wang, C.; Zhang, H.; Beer, M. (2018), *Computing tight bounds of structural reliability under imprecise probabilistic information*, Computers and Structures 208, 92–104.

[**23**] Wei, P.F.; Hong, F.Q.; Phoon, K.K.; Beer, M. (2021), *Bounds optimization of model response moments: a twin-engine Bayesian active learning method*, Computational Mechanics, 67, 1273–1292.

[**24**] Wei, P.F.; Liu, F.C.; Valdebenito, M.; Beer, M. (2021), *Bayesian Probabilistic Propagation of Imprecise Probabilities with Large Epistemic Uncertainty*, Mechanical Systems and Signal Processing 149, 107219.

[**25**] Wei, P.F.; Song, J.W.; Bi, S.F.; Broggi, M.; Beer, M.; Lu, Z.Z.; Yue, Z.F. (2019), *Non-intrusive stochastic analysis with parameterized imprecise probability models: I. Performance estimation*, Mechanical Systems and Signal Processing 124, 349–368.

[**26**] Wei, P.F.; Song, J.W.; Bi, S.F.; Broggi, M.; Beer, M.; Lu, Z.Z.; Yue, Z.F. (2019), *Non-intrusive stochastic analysis with parameterized imprecise probability models: II. Reliability and rare events analysis*, Mechanical Systems and Signal Processing 126, 227–247.

[**27**] Yang, L.C.; Bi, S.F.; Faes, M.G.R.; Broggi, M.; Beer, M. (2022), *Bayesian inversion for imprecise probabilistic models using a novel entropy-based uncertainty quantification metric*, Mechanical Systems and Signal Processing, 162, Article 107954.

[**28**] Zhu, W.Q.; Chen, N.; Liu, J.; Beer, M. (2021), *A probability-box-based method for propagation of multiple types of epistemic uncertainties and its application on composite structural-acoustic system*, Mechanical Systems and Signal Processing 149, 107184.

[**29**] Zhang, H.; Dai, H.; Beer, M.; Wang, W. (2013), *Structural reliability analysis on the basis of small samples: an interval quasi-Monte Carlo method*, Mechanical Systems and Signal Processing, 37(1–2), 137–151.

A constraint programming approach for polytopic simulation of ordinary differential equations - a collision detection application

Julien Alexandre dit Sandretto[1], Alexandre Chapoutot[1],Christophe Garion[2], Xavier Thirioux[2] and Ghiles Ziat[2]

[1] ENSTA Paris, Institut Polytechnique de Paris,
828 boulevard des Maréchaux, France
{alexandre,chapoutot}@ensta.fr
[2] ISAE-SUPAERO, Université de Toulouse, France
{garion,thirioux,ziat}@isae-supaero.fr

Keywords: Constraint programming, Abstract domains, ODEs

Introduction

Starting from a set of possible initial points, the solution of an ODE can be represented by a *reachable tube* describing the evolution of the system from this initial set. *Abstract domains* can be used to enclose the tube: *boxes* (cartesian product of intervals), *zonotopes*, *ellipsoids*, and nonconvex sets such as *Taylor models* (see [2] for a review of these abstract domains). The more accurate an abstract domain is, *i.e.* the smallest the difference between the hull of the abstraction and the abstracted set is, the more accurate the enclosure of the reachable tube will be. Polytope enclosure is a promising approach as it is more precise than the interval or zonotope abstract domains, but suffers from the expensiveness of its geometrical computation. Considering a polytope as *an intersection of zonotopes* and therefore benefiting from affine arithmetic is a possible solution to overcome the limitations (e.g. zonotope bundles [3], i.e., a set of zonotopes is used and therefore the intersection is not computed; or the intersection is computed when necessary [1]).

Reachable tubes as abstract trees

Considering a tube, a disjunction of predicates, $\mathcal{T} = (t_1 \wedge e_1) \vee (t_2 \wedge e_2) \cdots \vee (t_n \wedge e_n)$ where each e_i represents the set of values of solution functions within time frame t_i, it is to be understood as the following property: *the solution is either in set e_1 during the time frame t_1, or in set e_2 during the time frame t_2, etc.* Considering initial values given as a polytope $\mathcal{P}$, we decompose $\mathcal{P}$ as the intersection of s zonotopes $\mathcal{Z}_i$. The reachable tube of the corresponding ODE is therefore described by the conjunction of s tubes $\mathcal{T}^1 \wedge \cdots \wedge \mathcal{T}^i \wedge \cdots \wedge \mathcal{T}^s$, each tube $\mathcal{T}^i$ being obtained by the zonotopic simulation with initial value taken in $\mathcal{Z}_i$. This conjunction of disjunctions can be efficiently solved with constraint programming and polytopes as abstract domains. For obstacle avoidance or collision detection, a predicate (or several ones) of the form "and not in" is added to the previous formula.

Acknowledgement

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References

[**1**] Julien Alexandre dit Sandretto and Jian Wan, *Reachability analysis of nonlinear ODEs using polytopic based validated Runge-Kutta*, Reachability Problems, Springer, 2018.

[**2**] Matthias Althoff, Goran Frehse, and Antoine Girard, Set Propagation Techniques for Reachability Analysis, *Annual Review of Control, Robotics, and Autonomous Systems*, 4(1), 2021.

[**3**] Matthias Althoff and Bruce H. Krogh, Zonotope bundles for the efficient computation of reachable sets, IEEE conference on decision and control and European control conference, 2011

Symmetries for Interval Analysis

Luc Jaulin

Lab-STICC, ENSTA Bretagne
Brest, France
{lucjaulin}@gmail.com

Keywords: Interval analysis, Symmetries, Contractors

Introduction

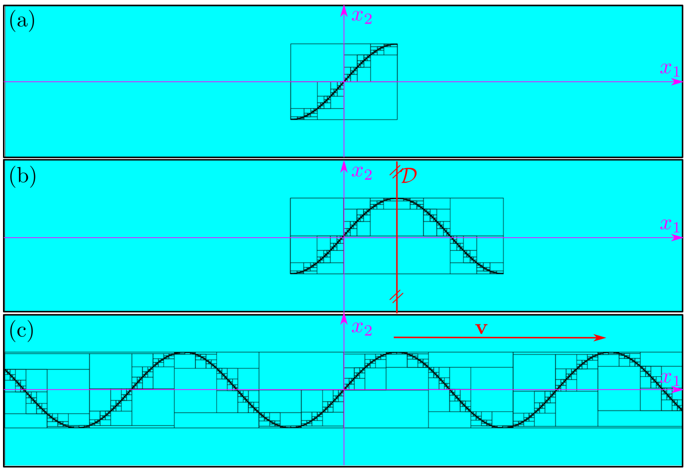
Interval analysis relies on a catalog of basic constraints such as

$$\begin{array}{cl} (i) & x_1 + x_2 = x_3 \\ (ii) & x_1 \cdot x_2 = x_3 \\ (iii) & x_2 = x_1^2 \\ (iv) & x_2 = \sin(x_1) \\ (v) & \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_3 & x_4 \\ x_5 & x_6 \end{pmatrix} \cdot \begin{pmatrix} x_7 \\ x_8 \end{pmatrix} \\ (vi) & \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_3 \cdot \cos(x_4) \\ x_3 \cdot \sin(x_4) \end{pmatrix} \end{array} \qquad (1)$$

For each of these constraints, we have to build minimal contractors. To solve a problem defined by nonlinear constraints [1], an interval solver decomposes it into constraints that are inside the catalog. Then, it calls the associated contractors until no more contractions can be observed.

Minimal contractors

Denote by $\mathbb{IR}^n$ the set of boxes of $\mathbb{R}^n$. A *minimal contractor* $\mathcal{C}^*$ for a constraint can be defined as an operator from $\mathbb{IR}^n$ to $\mathbb{IR}^n$ such that $\mathcal{C}^*([\mathbf{x}])$ corresponds to the smallest box which can be obtained by a contraction of $[\mathbf{x}]$ without removing a single point of the constraint. Now, many constraints such as those in (1) can be generated by applying specific symmetries (translation, hyperoctahedral, scaling, ...) [2] to a monotonic constraint. This will allow us to build efficient optimal contractors for a large class of constraints. The principle is illustrated by the figure below for the constraint (iv) where (a) represents the generator, (b) the action of the axial symmetry $\mathcal{D}$ and (c) the action of the translation symmetry $\mathbf{v}$.



In the presentation we will consider much complex constraints related to localization problems.

References

[**1**] S. Rohou, L. Jaulin, L. Mihaylova, F. Le Bars and S. Veres, *Reliable robot localization*, ISTE group, 2019.

[**2**] B. Desrochers and L. Jaulin, A Minimal Contractor for the Polar Equation; Application to Robot Localization, *Engineering Applications of Artificial Intelligence*, 2016.

Lie symmetries applied to guaranteed integration: application to mobile robotics localisation

Julien Damers[1], Luc Jaulin [1] and Simon Rohou[1]

[1] Lab-STICC, ENSTA Bretagne
2 rue François Verny, 29200 Brest, France
julien.damers@ensta-bretagne.org

Keywords: Guaranteed Integration, Lie Symmetries, Localisation, Robotics

Introduction

When dealing with mobile robots, localisation is one of the main problems one will be required to solve. One may use complex and expensive devices to localise the vehicle. However, with the development of swarms of robots, the need for cheaper units requires finding alternatives to these expensive devices. Here we will use the dynamical model of the system studied.

Guaranteed integration using Lie symmetries

In [1] and [2], we presented a method to perform guaranteed integration for large initial conditions. We recall the principle quickly here. Provided we have a reference trajectory ($\phi_{\mathbf{x}_0^1}(t)$, black arrow in Fig. 1) computed for an initial condition $\mathbf{x}_0^1$ (the bottom left black car), it is possible, using Lie symmetries of the system (if they are known) to find the solution (top right green car) for another initial condition (bottom right green car).

Combining interval analysis tools to solve the localisation problem

Consider a robot with a known dynamical model and an unknown initial state. We want to localise it, using range-only measurements to beacons for which we know the exact location as it progresses throughout the mission. Applying both our integration method and contractors related to range measurements on tubes [3] used to enclose the robot trajectory, it is possible to estimate the initial condition of the robot with backward constraint propagation. Then, propagating the constraints in a forward manner, one can estimate the trajectory of the robot. This is illustrated in a video which can be found on youtube[1].

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References

[1] J. Damers, L. Jaulin and S. Rohou, Lie symmetries applied to interval integration, *Automatica* (accepted), 2022.

[2] J. Damers, L. Jaulin and S. Rohou, Guaranteed interval integration for large initial boxes *SWIM* 2019.

[3] S. Rohou et al., *Reliable Robot Localization: A Constraint-Programming Approach Over Dynamical System*, John Wiley & Sons, London, 2019.

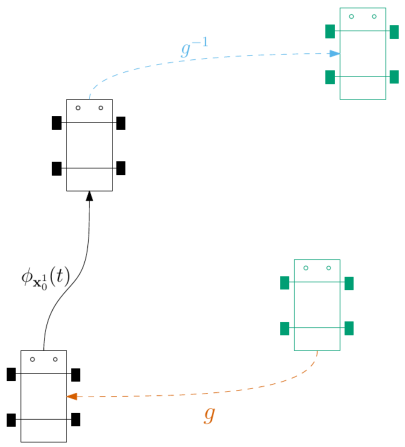

Figure 1: Guaranteed integration principle using Lie symmetries

[1] State estimation using Lie symmetries by Julien DAMERS

Affine Encodings for Optimal Monitoring of Temporal Properties under Uncertain Observation

Martin Fränzle[1]

Carl von Ossietzky Universität Oldenburg
Dpt. of CS, Foundations and Applications of Systems of Cyber-Physical Systems
26111 Oldenburg, Germany

martin.fraenzle@uol.de

Keywords: Temporal logic; Online monitoring; Uncertain information; Partial observation; Affine encodings

Introduction

State estimation in a dynamic system subject to uncertain state observation, where uncertainties can be both aleatoric due to noisy measurements and epistemic due to partial observation, is a classical problem. Optimal state estimation algorithms [6] can provide as precise as possible verdicts on state conditions, answering questions like whether the current position of a drone violates a geo-fencing condition. Many interesting safety properties of interacting cyber-physical agents are, however, more complex than state conditions, calling for specification as a durational property in an adequate temporal logic [5, 4]. This provokes the quest for optimal, in the sense of as precise as possible, monitoring algorithms evaluating properties expressed in temporal logic based on noisy and incomplete, i.e. uncertain sensory information.

[1]joint work with Bernd Finkbeiner (CISPA Helmholtz Center for Information Security; Stuhlsatzenhaus 5, 66123 Saarbrücken, Germany), Florian Kohn (CISPA Helmholtz Center for Information Security), and Paul Kröger (Carl von Ossietzky Universität Oldenburg, Foundations and Applications of Systems of Cyber-Physical Systems,)

Results

In this talk, we will demonstrate that contrary to common belief, optimal monitoring under uncertainty cannot be achieved by first applying optimal state estimation and then evaluating the temporal logic property in question upon this sequence of as precise as possible state estimates. Based on the indicative example of Signal Temporal Logic (STL) [4], a linear-time temporal logic specifically designed for classifying the time-dependent signals originating from continuous-state or hybrid-state dynamical systems according to formal specifications, we demonstrate that more precise statements can be computed based on affine-arithmetic encodings of STL semantics. For this, we first define the pertinent notion of precision, namely that verdicts provided by a monitor ought be sound (yield 'true' or 'false' only if *all* ground-truth trajectories consistent with the uncertain measurements satisfy, or violate, resp., the property of interest) and informative (monitoring yields 'inconclusive' only if some ground-truth trajectories consistent with the uncertain measurements satisfy and other consistent ones violate the property of interest).

In a setting where measurements are subject to both an interval-bounded per-sample error and an unknown, yet fixed offset, sequential execution of optimal state estimation and STL evaluation yields a sound, yet not an informative monitoring algorithm. That means that this combination sometimes fails to provide conclusive verdicts though these would be adequate. For the model-free as well as for the linear model-based case of dynamic system monitoring, we then provide precise, i.e. sound and informative, evaluation algorithms based on affine arithmetic [2] and SAT modulo theory solving over linear arithmetic [7, 1]. We prove preciseness of these algorithms in the cases of interval-bounded measurement noise and, when a linear system model is provided, partial observation.

For full constructions and proofs, we refer the reader to [3].

References

[1] A. Cimatti, A. Griggio, B. Schaafsma, and R. Sebastiani. The MathSAT5 SMT Solver. In N. Piterman and S. Smolka, editors, *TACAS 2013*, volume 7795 of *Lecture Notes in Computer Science*. Springer, 2013.

[2] L. H. de Figueiredo and J. Stolfi. Affine arithmetic: Concepts and applications. *Numerical Algorithms*, 37(1-4):147–158, 2004.

[3] B. Finkbeiner, M. Fränzle, F. Kohn, and P. Kröger. A truly robust signal temporal logic: Monitoring safety properties of interacting cyber-physical systems under uncertain observation. *Algorithms*, 15(4):126, 2022.

[4] O. Maler and D. Nickovic. Monitoring temporal properties of continuous signals. In Y. Lakhnech and S. Yovine, editors, *FORMATS and FTRTFT 2004*, volume 3253 of *Lecture Notes in Computer Science*. Springer, 2004.

[5] Z. Manna and A. Pnueli. *The temporal logic of reactive and concurrent systems - specification*. Springer, 1992.

[6] P. S. Maybeck. *Stochastic models, estimation, and control*, volume 141 of *Mathematics in Science and Engineering*. 1979.

[7] S. A. Wolfman and D. S. Weld. The lpsat engine & its application to resource planning. In *IJCAI'99 - Volume 1*, page 310–316. Morgan Kaufmann Publishers Inc., 1999.

Higher-Order Methods for Differential Inclusions

Pieter Collins[1], Luca Geretti[2], Tiziano Villa[2] and Sanja Živanović Gonzalez[3]

[1] Department of Data Science and Knowledge Engineering, Maastricht University, Postbus 616, 6200MD Maastricht, The Netherlands
pieter.collins@maastrichtuniversity.nl
[2] Dipartimento di Informatica, Università di Verona, Strada le Grazie 15, 37134 Verona, Italy
{luca.geretti,tiziano.villa}@univr.it
[3] Department of Mathematics, Barry University, Miami Shores, Florida, USA
sanja51@gmail.com

Keywords: Differential inclusions, rigorous numerics, Taylor models

Introduction

Differential inclusions are nondeterministic continuous-times systems with set-valued uncertainties $\dot{x}(t) \in F(x(t))$. A solution is an absolutely-continuous function satisfying the equation almost-everywhere. They arise from noisy systems $\dot{x}(t) = f(x(t), v(t))$ with $v(t) \in V$. We give a method for computing solution sets for systems with affine noise:

$$\dot{x}(t) = f(x(t)) + \textstyle\sum_{i=1}^{m} g_i(x(t))\, v_i(t);\ \ v_i(t) \in [-V_i, +V_i];\ \ x(t_0) = x_0.$$

Method

To over-approximate the set of solutions, we consider an auxiliary system of ordinary differential equations

$$\dot{y}(t) = f(y(t), w(t; a)), \quad y(0) = x_0,\ w(\cdot; a) \in W,\ a \in A$$

where A is a finite-dimensional compact set of parameters, and analytically determine a uniform error bound on the difference between solutions of the original system and its auxiliary counterpart.

We take constants $\Lambda \geq \lambda(Df(\cdot))$,, the logarithmic norm, and

$$\|f(z(t))\| \leq K, \quad \|g_i(z(t))\| \leq K_i \quad K' = \textstyle\sum_{i=1}^{m} V_i K_i,$$
$$\|Df(z(t))\| \leq L, \; \|Dg_i(z(t))\| \leq L_i, \quad L' = \textstyle\sum_{i=1}^{m} V_i L_i.$$

Then if f and g are C^1 and the w_i are measurable functions such that $\int_0^h v_i(\tau) - w_i(\tau)\, d\tau = 0$, the single-time-step error is

$$|x(h) - y(h)| \leq h^2\big((K + K')L'/3 + 2K'(L + L')\,(e^{\Lambda h} - 1)/\Lambda h\big) \quad (1)$$

By using two-parameter piecewise-constant or affine functions for each component $w_j(\cdot)$, we can obtain an error with terms of order $O(h^2)$ and $O(h^3)$, and full order of $O(h^3)$ is obtained in the cases of additive inputs $\dot{x} = f(x) + v$ and the one-input case.

The auxiliary system is solved using rigorous integration using polynomial models. The method has been implemented in the tool Ariadne, and performed competitively on the ARCH benchmarks [3].

Acknowledgement

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References

[1] S. Živanović Gonzalez and P. Collins, Computing Reachable Sets of Differential Inclusions *Coordination Control of Distributed Systems*, Springer, 2015.

[2] S. Živanović Gonzalez, P. Collins, L. Geretti, D. Bresolin and T. Villa, Higher Order Method for Differential Inclusions, arXiv:2001.11330, 2020

[3] L. Geretti et al., ARCH-COMP21 Category Report: Continuous and Hybrid Systems with Nonlinear Dynamics. *EPiC Series in Computing* 80, 32-54, 2021.

Computer-assisted Existence Proofs for Navier-Stokes Equations on an Unbounded Strip with Obstacle

Jonathan Wunderlich

Karlsruhe Institute of Technology, Department of Mathematics
Englerstraße 2, 76131 Karlsruhe, Germany
Jonathan.Wunderlich@kit.edu

Keywords: Computer-assisted proof, Navier-Stokes, existence, enclosure

Introduction

The incompressible stationary 2D Navier-Stokes equations

$$\left.\begin{aligned} -\Delta v + Re\,[(v\cdot\nabla)v + \nabla q] &= f \\ \operatorname{div} v &= 0 \end{aligned}\right\} \text{ in } \Omega$$
$$v = 0 \qquad \text{on } \partial\Omega$$

are considered on an unbounded strip domain $\Omega \subseteq \mathbb{R}^2$ perturbed by a compact obstacle D, i.e., $\Omega = \mathbb{R}\times(0,1)\setminus D$. Here, Re denotes the Reynolds number and f models external forces acting on the fluid.

With U denoting the Poiseuille flow and P its associated pressure we are interested in solutions of the form $v = U + \bar{u}$ where $\bar{u}(x,y)\to 0$ as $|x|\to\infty$ and $q = P + p$.

Since such functions $\bar{u}$ do not satisfy the Dirichlet boundary conditions anymore we perform a second transformation using a solenoidal vector field V with

$$V = 0 \text{ on } \partial\Omega\setminus\partial D, \quad V = U \text{ on } \partial\Omega\cap\partial D \text{ and } V(x,y)\to 0\ (|x|\to\infty)$$

which finally leads to the transformed Navier-Stokes equations

$$\left.\begin{aligned} -\Delta u + Re\,[(u\cdot\nabla)u + (u\cdot\nabla)\Gamma + (\Gamma\cdot\nabla)u + \nabla p] &= g \\ \operatorname{div} u &= 0 \end{aligned}\right\}\ \text{in } \Omega$$
$$u = 0 \qquad \text{on } \partial\Omega$$

with $\Gamma := U - V$ and the right-hand side $g := f - \Delta V - Re(\Gamma\cdot\nabla)\Gamma$.

Modeling the divergence free condition in the solution space $H(\Omega) := \{u \in H_0^1(\Omega,\mathbb{R}^2) \colon\ \operatorname{div} u = 0\}$ we can eliminate the pressure from the first equation which leads to the following weak formulation:

Find velocity $u \in H(\Omega)$ such that

$$\int_\Omega \left(\nabla u\cdot\nabla\varphi + Re\,[(u\cdot\nabla)u + (u\cdot\nabla)\Gamma + (\Gamma\cdot\nabla)u]\cdot\varphi\right)\,d(x,y)$$
$$= \int_\Omega g\cdot\varphi\,d(x,y) \quad (\varphi\in H(\Omega)).$$

Main results

Applying computer-assisted techniques to this problem, we are able to prove existence of a (non-trivial) solution $u^* \in H(\Omega)$ to the weak formulation (with $f \equiv 0$) for different Reynolds numbers and several domains Ω.

Starting from an approximate solution (computed with divergence-free finite elements), we determine a bound for its defect, and a norm bound for the inverse of the linearization at the approximate solution. For the latter, bounds for the essential spectrum and for eigenvalues play a crucial role, especially for the eigenvalues “close to” zero.

With these data at hand, we can use a fixed-point argument to obtain the existence of a solution “nearby“ the approximate one as well as an error bound (in the Sobolev space $H(\Omega)$).

Finally, if our computer-assisted proof provides the existence of a solution u^* to the weak formulation for the velocity we additionally prove existence of a corresponding pressure p^* such that the pair (u^*, p^*) is a weak solution to the transformed Navier-Stokes equations.

Uses of Methods with Result Verification in the Context of MIMO Systems

Ekaterina Auer[1] and Andreas Ahrens[1]

[1] Department of Electrical Engineering
University of Applied Sciences Wismar, D-23966 Wismar, Germany
ekaterina.auer@hs-wismar.de

Keywords: MIMO, interval analysis, optimization, BER, SVD, GMD

Technical simulations that take into account the underlying uncertainties have become indispensable in modern engineering. One possibility to deal with bounded uncertainty, for example, in parameters, is to employ methods with result verification such as interval analysis. In this way, it is possible not only to propagate such uncertainty through systems in a forward, deterministic manner, but also to verify that the result obtained using a computer definitely contains the true result of a simulation. In this contribution, we propose to use interval analysis to increase the reliability and to account for uncertainty in the context of the multiple-input multiple-output (MIMO) systems.

Placing multiple antennas at the transmitter and receiver sides has been shown to improve both the capacity and the integrity of a communication link. To model a frequency flat MIMO link consisting of n_{T} transmitting and n_{R} receiving antennas, a linear stochastic model [1]

$$\vec{y} = H \cdot \vec{a} + \vec{n}, \quad \vec{y}, \vec{n} \in \mathbb{C}^{n_{\mathrm{R}}},\ \vec{a} \in \mathbb{C}^{n_{\mathrm{T}}},\ H \in \mathbb{C}^{n_{\mathrm{R}} \times n_{\mathrm{T}}}\ , \tag{1}$$

is widely employed. Here, $\vec{y}$ is the received data vector, $\vec{a}$ is the transmitted signal vector, $\vec{n}$ is the vector of the additive white Gaussian noise at the receiver side with the zero mean and the variance σ^2 in both real and imaginary parts, and H is the channel matrix. In practice, working with this model can be roughly divided into the steps shown in Figure 1.

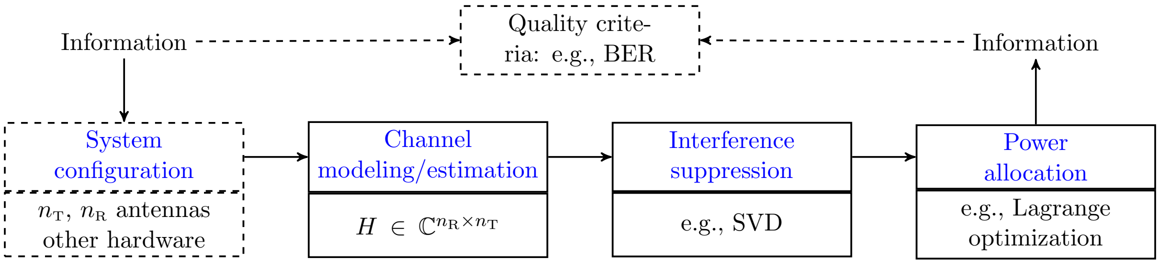

Figure 1: Modeling, simulation and optimization of a MIMO link

This entire process is influenced by several uncertain factors. For example, the matrix H is not known exactly (e.g., due to quantization errors or time variations of the channel). This can lead to faulty channel separation and subsequent failure to eliminate inter-antenna interference. Additionally, measurement errors or unknown noise power might complicate the optimization step.

In this contribution, we focus on uses of interval analysis at the stages of interference suppression and resource allocation. We show how correlated and uncorrelated MIMO systems can be optimized wrt. power if singular value decomposition (SVD) is employed at the stage of interference suppression. As an outlook, we compare the SVD to the so-called geometric mean decomposition (GMD), which can also be used for channel separation, from the point of view of the achievable bit error ratio (BER) and the influence of numerical errors and uncertainty.

References

[**1**] G. Foschini, Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas, *Bell Labs Technical Journal* 1(2), 41–59, 1996.

[**2**] E. Auer and A. Ahrens, Guaranteed Minimization of the Bit Error Ratio for MIMO Systems: A Mathematical Viewpoint, *ASME Journal of Risk and Uncertainty Part B* 7(2), 020910, 2021.

Eigenvalues enclosures of skew symmetric/Hermitian matrices having bounded uncertainty

Suman Maiti, Snehashish Chakraverty

National institute of Technology Rourkela,
Rourkela 769008, Odisha, India.
{518ma1009,chakravertys}@nitrkl.ac.in

Keywords: Skew symmetric/Hermitian interval matrices, Eigenvalues bounds

Introduction

We have concentrated on binding the eigenvalues of skew symmetric/Hermitian interval matrices. It has been noticed that significantly less attention has been given to this kind of matrices. The benefits and drawbacks of handling these problems are discussed here.

Basic properties

The skew symmetric interval matrix $\boldsymbol{A}^{ss}$ is defined as

$$\boldsymbol{A}^{ss} = \{A \in \boldsymbol{A} | A = -A^T\}, \tag{1}$$

where $\boldsymbol{A}$ is a real interval matrix. We know that eigenvalues of a skew symmetric matrix are either zero or purely imaginary. Thus our focus is on the real eigenvalues of the interval matrix $i\boldsymbol{A}^{ss}$.

There is no way to compute exact eigenvalue bounds for the complex interval matrix. Therefore finding outer bounds of eigenvalues of the complex interval matrix $i\boldsymbol{A}^{ss}$ is a good option.

The exact interval eigenvalue $\boldsymbol{\Lambda}$ of $\boldsymbol{A}^{ss}$ is described as

$$\boldsymbol{\Lambda} = \{ib : Az = ibz, A \in \boldsymbol{A}^{ss}, z \neq 0\}. \tag{2}$$

Main results

One bound [4] of the eigenvalues of $i\boldsymbol{A}^{ss}$ can be found by Gershgorin bound as following,

$$-\Sigma_{k \neq j} |\boldsymbol{a}_{jk}| \leq \lambda \leq \Sigma_{k \neq j} |\boldsymbol{a}_{jk}|, \tag{3}$$

for $j = 1, \ldots, n$ and $\boldsymbol{A}^{ss} = (\boldsymbol{a}_{jk})$, where n is the order of the matrix.

Another bound [1] for complex interval matrices can be found by the following equation,

$$\underline{\lambda}_n \begin{pmatrix} \boldsymbol{O} & \boldsymbol{A}^{ssT} \\ \boldsymbol{A}^{ss} & \boldsymbol{O} \end{pmatrix}^s \leq \lambda \leq \overline{\lambda}_1 \begin{pmatrix} \boldsymbol{O} & \boldsymbol{A}^{ssT} \\ \boldsymbol{A}^{ss} & \boldsymbol{O} \end{pmatrix}^s . \tag{4}$$

Also, there are iterative approaches for the eigenvalue enclosure of complex interval matrices [2, 3].

Purely imaginary skew Hermitian interval matrices:

Let $i\boldsymbol{B}^s$ be a skew Hermitian interval matrix. Then $\boldsymbol{B}^s$ is a real symmetric matrix. We can bound the eigenvalues for symmetric interval matrices, and consequently, we will get eigenvalue bounds for $i\boldsymbol{B}^s$.

The eigenvalues of purely imaginary skew Hermitian interval matrices will be zero or purely imaginary. The main distinction with skew symmetric matrices is that eigenvalues do not occur in conjugate pairs.

Complex skew Hermitian interval matrices:

We can obtain enclosures for eigenvalue clusters of complex skew Hermitian interval matrices by different methods developed for complex interval matrices. However, we need to find the tighter enclosure of the eigenvalue clusters for the complex skew Hermitian interval matrices.

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References

[1] Hladík, M, Bounds on eigenvalues of real and complex interval matrices, *Applied Mathematics and Computation*, 219(10): 5584–5591, 2013.

[2] Matcovschi, M H and Pastravanu, O, Novel estimations for the eigenvalue bounds of complex interval matrices, *Applied Mathematics and Computation*, 234: 645–666, 2014.

[3] Roy, F and Gupta, D K, Sufficient regularity conditions for complex interval matrices and approximations of eigenvalues sets, *Applied Mathematics and Computation*, 317: 193–209, 2018.

[4] Maiti, S and Chakraverty, S, Gershgorin disk theorem in complex interval matrices, *Proceedings of the Estonian Academy of Sciences*, 71(1): 65–76, 2022.

Robust 3D target localization using UAVs with state uncertainty

M. Zagar[1], M. Kieffer[1], H. Piet-Lahanier[2], and L. Meyer[2]

[1] Université Paris-Saclay - CNRS - CentraleSupélec - L2S, F-91192 Gif-sur-Yvette
[2] ONERA, F-91120 Palaiseau

Keywords: bounded errors, cooperative estimation, 3D localization, UAV state uncertainty

Introduction

This work considers the estimation of the 3D location of a target observed by several cooperating Unmanned Aerial Vehicles (UAVs). The measurement errors in the body frame of each UAV are assumed to be bounded. Moreover, the proposed approach, accounts for the UAV state uncertainty, assumed bounded, and for the presence of outliers.

Robust estimation

N UAVs observe a target located at $\boldsymbol{x}^{\mathrm{t}}$. A measurement in the frame of UAV i, expressed in spherical coordinates, is assumed to satisfy

$$\mathbf{y}_i = \mathbf{h}\left(\mathbf{x}_i^{\mathrm{u}}, \boldsymbol{x}^{\mathrm{t}}\right) + \mathbf{w}, \tag{1}$$

where $\mathbf{x}_i^{\mathrm{u}}$ is the state of the UAV only known to belong to the set $\in \mathbb{X}_i^{\mathrm{u}}$ and $\mathbf{w} \in [\mathbf{w}]$ is the bounded measurement noise. Consider a set of indexes $\mathcal{N} \subset \{1, \dots, N\}$. The estimate of all target positions in the search domain $\mathbb{X}_0$, consistent the measurements $\mathbf{y}_i$, $i \in \mathcal{N}$, the measurement model (1), the noise bounds, and the UAV state uncertainty is

$$\mathbb{X}_{\mathcal{N}}^{\mathrm{t}} = \left\{\boldsymbol{x} \in \mathbb{X}_0 \mid \exists \mathbf{x}_i \in \mathbb{X}_i^{\mathrm{u}},\ \mathbf{h}\left(\mathbf{x}_i, \boldsymbol{x}\right) \in \left[\mathbf{y}_i\right], i \in \mathcal{N}\right\}. \tag{2}$$

Combining the Thick-SIVIA algorithm [1] and the q-relaxed intersection [2], we propose the Robust Thick SIVIA (RTSIVIA) algorithm to determine the largest sets $\mathcal{N}$ such that the set $\mathbb{X}^{\mathrm{t}}_{\mathcal{N}}$ is not empty. Starting with $q = 0$, the corresponding set estimate is evaluated as

1. Contract $\mathbb{X}_0$ using i-th measurement only to get $\mathbb{X}_i$, $i \in \{1, \ldots, N\}$.

2. Use Thick SIVIA to evaluate the set

$$\mathbb{X}^{\mathrm{t}}_q = \bigcap_{i \in \{1,\ldots N\}}^{q} \{\boldsymbol{x} \in \mathbb{X}_i \mid \exists \mathbf{x}_i \in \mathbb{X}^{\mathrm{u}}_i,\ \mathbf{h}(\mathbf{x}_i, \boldsymbol{x}) \in \mathbf{y}_i - [\mathbf{w}]\} \quad (3)$$

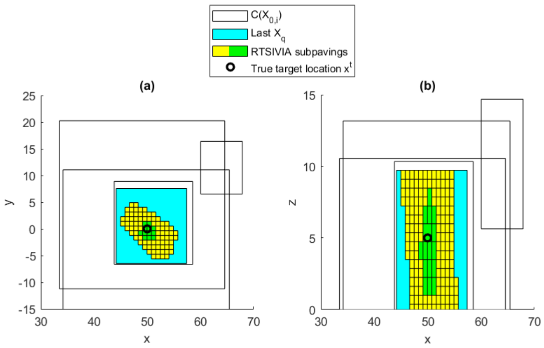
   of all target locations consistent with at least $N - q$ measurements.

3. If $\mathbb{X}^{\mathrm{t}}_q = \emptyset$, more outliers have to be tolerated: $q = q + 1$; Go to 2.

Results

Consider 4 UAVs getting target distance and elevation measurements (one produces outliers). UAV's position and attitude uncertainties are ± 1m and $\pm 2^{\mathrm{o}}$; Measurement noise is with ± 1m and $\pm 0.5^{\mathrm{o}}$ uncertainty.

The figure shows the results of RTSIVIA: white boxes are the $\mathbb{X}_i$s; the first non-empty $\mathbb{X}^{\mathrm{t}}_q$ is contained in the cyan box; (a) and (b) are the projection on X-Y and X-Z planes of the subpaving approximating $\mathbb{X}^{\mathrm{t}}_q$



References

[1] B. Desrochers and L. Jaulin, *Thick set inversion*, Artificial Intelligence, 249:1-18, 2017

[2] L. Jaulin, *Robust set-membership state estimation; application to underwater robotics*, Automatica, 45(1), 202–206, 2009

A Geometric Approach to the Coverage Measure of the Area Explored by a Robot

Maria Luiza Costa Vianna[1,2], Eric Goubault [1], Luc Jaulin[2] and Sylvie Putot [1]

[1] LIX, École Polytechnique, CNRS & Institut Polytechnique de Paris, 91128 Palaiseau, France
{costavianna, goubault, putot}@lix.polytechnique.fr
[2] Lab-STICC - ENSTA Bretagne, Brest, France
luc.jaulin@ensta-bretagne.fr

Keywords: Robotics, interval analysis, exploration, topology

Introduction

Area covering missions are a common task for autonomous robots, where the robot must cover with its embedded sensors or tools a whole area of interest. Estimating the explored area is essential for determining if path-planning algorithms lead to complete coverage. Some applications might also require the robot to revisit an area of interest, in this case, to verify the mission's completion, one has to be capable of determining how many times each part of the space has been in the robot's range of detection. This information can also be exploited in localization, being directly related to the notion of loop closure, a key concept in Simultaneous Localization and Mapping (SLAM) algorithms [4].

In this paper, we propose a solution capable of determining how many times the robot has been sensing each point in the space. Then, using a set-membership approach, we define the explored area as a set of points in the space that were sensed at least once. We use a novel approach based on topological properties of the environment that has been scanned. More precisely, we demonstrate that the computation of certain winding numbers enables to estimate the explored area while also determining the "coverage measure" of each point, i.e. how many times each point in the space was explored by the robot during its mission.

The approach that we propose is adapted to safety-critical applications, where a guaranteed estimation of the robot's explored area is necessary. For this purpose, we use interval analysis to compute guaranteed approximations of the winding number, as briefly outlined in next Section.

Winding number guaranteed computation

In practice, we never have exact robot localization data, so we need to estimate winding numbers around "imprecise" points, that are abstracted using interval analysis. We propose a new method for algorithmically computing the winding number $[\eta]([f], b)$ of the envelope $[f]$ of a continuous function $f : [0, 1] \to \mathbb{R}^2$ with respect to a box $b \in \mathbb{IR}^2$.

The computation of the winding number using interval analysis is not new, e.g [3]. One of the contributions of this work is that we deal with what we call here uncertain boxes, estimating a guaranteed interval for the winding number value. A box is uncertain if $\exists t \in [0, 1]$ s.t. $b \cap [f](t) \neq \emptyset$: only in this case the winding number is not uniquely determined.

Main results

We propose a new approach for estimating the area explored by a mobile robot. The use of interval analysis makes the approach adapted to deal with uncertainties in the robot's estimated trajectory, making it suitable for safety-critical applications. In comparison to previous works, e.g. [1] and [2], our method estimates how many times each part of the space has been sensed, this is a direct result of the relation established between the winding number and the exploration in the plane. We demonstrated the efficiency of the proposed method using data acquired during a real experience.

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References

[1] B. Desrochers and L. Jaulin, Computing a guaranteed approximation of the zone explored by a robot. IEEE Transactions on Automatic Control, 62(1):425–430, 2017.

[2] V. Drevelle, L. Jaulin, and B. Zerr., Guaranteed Characterization of the Explored Space of a Mobile Robot by using Subpavings. In NOLCOS 2013, Toulouse, France, September 2013.

[3] P. Franek and S. Ratschan, Effective topological degree computation based on interval arithmetic, 2012.

[4] S. Rohou, P. Franek, C. Aubry, and L. Jaulin, Proving the existence of loops in robot trajectories, 2017.

Sea route monitoring by weather buoys using interval analysis

Quentin Brateau[1], Luc Jaulin[1]

[1]ENSTA Bretagne, UMR 6285, Lab-STICC,
2 rue François Verny, 29806 Brest CEDEX 09, France
quentin.brateau@ensta-bretagne.org

Keywords: Detection, Surface Vessels, State estimation, Wake, Weather Buoys, Set inversion, Interval Analysis

Introduction

The maritime environment is complex and difficult to monitor. It is quite easy for a boat to navigate furtively if it is not visible from the shore. For instance, it is possible to practice illegal fishing in the vastness of the ocean, even if today innovative methods are developed to counter these practices [1]. Ocean monitoring then requires the implementation of tools to reliably detect surface vessels that evolve in the marine space. This can be used to detect enemy ships sailing in unauthorized areas, but also to know the ship's position and manage maritime traffic.

Basic properties

The movement of the boats creates a wake that betrays their presence. A mathematical model shows that wake's angle is constant regardless of the boat and is $\alpha = \arcsin\left(\frac{1}{3}\right) \approx 19.47°$ [2], [3]. Recent studies have established a more accurate model that takes into account the decrease of the wake angle with increasing Froude number [4].

There are all over the world weather buoys at sea to monitor wind, currents, temperature, and water height [1]. Disturbances in water

[1] https://www.ndbc.noaa.gov/

height induced by sailing surface vessels can then be detected on the weather buoys which interfere with the measurements. By combining data from a network of buoys, the states of the boats can be estimated (position and velocity).

Main results

This work is not focused on methods of disturbances detection on weather buoys but assumes that the ship's wake is detectable within a time interval. These buoys are placed around a maritime route to enclose the ship's state using an accurate wake model and set inversion algorithms [5]. It is possible to retrieve the number of boats as well as to see their trajectories with enough buoys. The presented solution does not rely on combinatorial complexity due to the number of sensors but rather on efficient methods to characterize the boat's state.

References

[1] H. Weimerskirch, J. Collet, A. Corbeau, *et al.*, "Ocean sentinel albatrosses locate illegal vessels and provide the first estimate of the extent of nondeclared fishing," *Proceedings of the National Academy of Sciences*, vol. 117, no. 6, pp. 3006–3014, 2020.

[2] W. Thomson, "On ship waves," *Proceedings of the Institution of Mechanical Engineers*, vol. 38, no. 1, pp. 409–434, 1887.

[3] J. Stoker, *Water Waves: The Mathematical Theory with Applications* (Wiley Classics Library). Wiley, 1992.

[4] M. Rabaud and F. Moisy, "Ship wakes: Kelvin or mach angle?" *Physical Review Letters*, vol. 110, no. 21, May 2013.

[5] L. Jaulin and E. Walter, "Set inversion via interval analysis for nonlinear bounded-error estimation," *Automatica*, vol. 29, no. 4, pp. 1053–1064, 1993.

How to determine uncertainty interval: Practice in GNSS and LiDAR localization

Jingyao Su[1] Yuehan Jiang[2] Steffen Schön[1] and Bernardo Wagner[2]

[1] Leibniz University Hannover, Institute of Geodesy
Schneiderberg 50, 30167 Hannover, Germany
{suj,schoen}@ife.uni-hannover.de
[2] Leibniz University Hannover, Institute for Systems Engineering, Real Time Systems Group (RTS)
Appelstraße 9A, 30167 Hannover, Germany
{jiang, wagner}@rts.uni-hannover.de

Keywords: GNSS positioning, LiDAR localization, uncertainty interval, sensitivity analysis, error bounding

Introduction

Intervals can be seen as a natural way to bound observation uncertainty in navigation systems such as Global Navigation Satellite System (GNSS), Inertial Measurement Units (IMU) or optical sensors like LiDAR, since they are in principle free of any assumption about probability distributions and can thus describe adequately remaining systematic effects [1]. Transferring the uncertainty from the observation domain to the state domain, such as the position and pose, the uncertainty is represented as a set-value, e.g., polytope [2,3], zonotope [1,3], and interval box [4,8]. This is applicable in navigation integrity monitoring as an alternative approach for error bounding [5], in contrast to the conventional stochastic handling.

The critical issue is how to determine the observation uncertainty interval properly. Some researchers constructed intervals based on the confidence level, which is unavoidably associated with probabilistic distributions. In this contribution, we report methods that we have applied in the context of GNSS range-based positioning and LiDAR localization and show examples of uncertainty intervals due to different error sources in GNSS signal propagation and LiDAR measuring process.

Methodology

GNSS: The measurement result after correcting recognized systematic effects is still only an estimate for the measurand value because of the uncertainty arising from random effects and imperfect correction of the results for systematic effects [6]. Thus, the overall uncertainty of the pseudorange observation p_r^k has contributions from all influence factors d_i of the applied correction models and can be determined as:

$$[\underline{p_r^k}, \overline{p_r^k}] = \left| \frac{\partial p_r^k(\mathbf{d})}{\partial \mathbf{d}} \right| \cdot [\underline{\mathbf{d}}, \overline{\mathbf{d}}]$$

The absolute values of the partial derivatives are used to linearly transfer the interval-described uncertainty of the vector of influences parameters $[\underline{\mathbf{d}}, \overline{\mathbf{d}}]$ into an uncertainty interval of the pseudorange. This method is used for modeling residual tropospheric errors and residual ionospheric errors that are two major components of the pseudorange uncertainty.

The upper bounds of multipath error on pseudorange measurements is typically represented by a multipath error envelope, dependent on the signal modulation and extra path delay. In this situation, we can construct interval values in a straight-forward way.

LiDAR: The error sources of the 3D LiDAR measurements on a moving vehicle consist of imperfect measuring of the laser beam [7] and the ego-motion of the sensor base. An interval error model is proposed for 3D LiDAR to account for systematic errors like range offset, beam divergence and beam footprint [8]. To acquire a coherent point cloud relative to a fixed coordinate frame, the general technique is to utilize the rotation estimation from the IMU with linear interpolation [9], assuming continuous change in the angular velocity which yields an interpolation error to the measurements. Thus, the interpolation error is considered as an additional contribution to the interval:

$$[\alpha_t] = [\alpha_{t-1}] + [-\delta\omega_t,\, \delta\omega_t] \cdot \delta t$$

Where $[\alpha_t]$ represents the rotation angle with interval interpolation error in either vertical or horizontal direction at current timestamp t. δt is the time difference between two neighboring laser measurements. $\delta\omega_t$ is the angular error rate per unit time. It can be represented by the first-order coning error [10].

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References

[1] S. Schön and H. Kutterer, Uncertainty in GPS Networks due to Remaining Systematic Errors: The Interval Approach, *Journal of Geodesy* 80(3):150-162, 2006.

[2] H. Dbouk and S. Schön, Reliable bounding zones and inconsistency measures for GPS positioning using geometrical constraints, *Acta Cybernetica* vol.24, no.3, pp.573–591, 2020.

[3] J. Su and S. Schön, Deterministic approaches for bounding GNSS uncertainty: A comparative analysis, *2022 10th ESA Workshop on Satellite Navigation Technologies and European Workshop on GNSS Signals and Signal Processing (NAVITEC)*, 2022.

[4] V. Drevelle and P. Bonnifait, High integrity GNSS location zone characterization using interval analysis, *Proceedings of the*

22nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2009) pp.2178-2187, 2009.

[**5**] J. Su and S. Schön, Improved observation interval bounding for multi-GNSS integrity monitoring in urban navigation, *Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021)* pp.4141-4156, 2021.

[**6**] JCGM, *Evaluation of measurement data - Guide to the expression of uncertainty in measurement (GUM 1995 with mino rcorrections)*, JCGM 100:2008.

[**7**] P. Skrzypczynski, How to Recognize and Remove Qualitative Errors in Time-of-Flight Laser Range Measurements, *2008 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2008)* pp.2958-2963, 2008.

[**8**] R. Voges and B. Wagner, Interval-Based Visual-LiDAR Sensor Fusion, *IEEE Robotics and Automation Letters* vol.6, no.2, pp.1304-1311, 2021.

[**9**] J. Zhang and S. Singh, Low-drift and real-time lidar odometry and mapping, *Auton Robot* vol.41, no.2, pp.401-416, 2017.

[**10**] C. Jekeli, Inertial Navigation Systems with Geodetic Applications, *De Gruyter*, 2001.

An investigation of interval and set-based uncertainty representation for GNSS navigation

Jingyao Su[1] and Steffen Schön[1]

[1] Leibniz University Hannover, Institute of Geodesy
Schneiderberg 50, 30167 Hannover, Germany
{suj,schoen}@ife.uni-hannover.de

Keywords: GNSS positioning, uncertainty interval, zonotope, polytope, error bounding

Introduction

Uncertainty modeling and bounding are of vital importance for high-integrity GNSS applications. All contributing observation and system errors should be adequately assessed to ensure safety operations of navigation. Classical approaches are mostly developed in a stochastic manner with probabilistic assumptions. However, the exact error distribution is often unknown, and remaining systematics may persist, so that a purely stochastic modeling of all error sources will not be adequate, and alternative uncertainty bounding and propagation should be studied. Intervals and sets, i.e., zonotope and polytopes, can be seen as natural ways to represent unknown-but-bounded uncertainty. They are not linked with any probabilistic assumptions, therefore, are deterministic [1,3]. Subsequently, a linear uncertainty propagation is applied instead of the quadratic variance propagation.

In this contribution, we report the interval and set-based uncertainty methods that we have applied in the context of GNSS range-based positioning and discuss its feasibility in future integrity applications.

Acknowledgement

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References

[**1**] S. Schön, C. Brenner, H. Alkhatib, et al., Integrity and collaboration in dynamic sensor networks, *Sensors* 18(7): 2400, 2018.

[**2**] S. Schön and H. Kutterer, Uncertainty in GPS Networks due to Remaining Systematic Errors: The Interval Approach, *Journal of Geodesy* 80(3):150-162, 2006.

[**3**] J. Su and S. Schön, Deterministic approaches for bounding GNSS uncertainty: A comparative analysis, *2022 10th ESA Workshop on Satellite Navigation Technologies and European Workshop on GNSS Signals and Signal Processing (NAVITEC)*, 2022.

[**4**] JCGM, *Evaluation of measurement data - Guide to the expression of uncertainty in measurement (GUM 1995 with mino rcorrections)*, JCGM 100:2008.

Interval-based Global Localization in Building Maps

Aaronkumar Ehambram[1], Luc Jaulin[2] and Bernardo Wagner[1]

[1] Leibniz Universität Hannover, Real Time Systems Group (RTS)
Appelstraße 9A, 30167 Hannover, Germany
{ehambram,wagner}@rts.uni-hannover.de
[2]Lab-STCC, ENSTA Bretagne, Brest, France
lucjaulin@gmail.com

Keywords: Interval Analysis, Global Localization, Odometry, Building Maps, Robotics

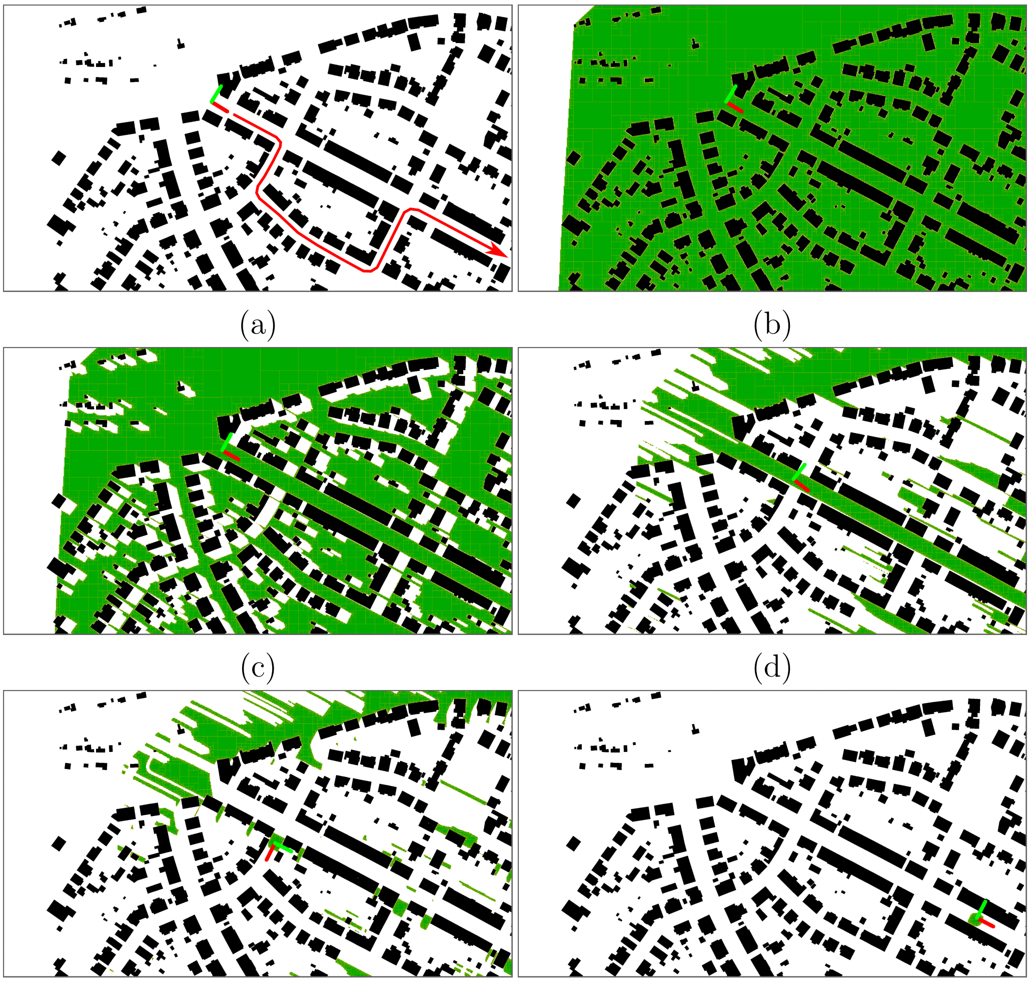
Introduction

Global localization in large maps in the absence of GPS data is one of the key challenges that need to be solved in the context of autonomous driving. In particular urban canyons make global positioning using such systems very challenging. As a result, local sensory needs to be used to localize the robot on a map. Our goal is to localize the robot in an arbitrary building map. A building map may have repetitive symmetrical structures due to which the localization may be ambiguous. A prominent solution to this problem is the Monte Carlo Localization (MCL) [1,2]. However, the major drawback of MCL approaches is that the quality of the solution heavily depends on the number of samples. If the uncertainty is very large, a large number of particles may be required to cover the solution space and therefore can become computationally heavy. Further, due to random sampling of the particles, an unfortunate sequence of samples can cause a wrong convergence of the method.
We introduce a novel global localization method using intervals that copes with ambiguous localization and overcomes the aforementioned

problems. We assume (i) that we know the building map of the environment in which the robot is moving, (ii) that the robot can never be inside a building, and in the scope of this abstract (iii) that the orientation of the robot is known (from a compass for instance). Exploiting the assumptions and by only using the robot's odometry information, our method can narrow down the feasible set of poses of the vehicle along the trajectory without using any exteroceptive sensors such as laser scanners. Desrochers and Jaulin [3] solve a similar problem to ours, but with the major difference, that they use sonar range measurements and restrict the local measurements to always see parts of the map. In contrast to probabilistic approaches as [1,2], our method does not need an association step of local sensor data to the map which is often error-prone. As a result, our method maintains integrity. On the downside, our approach cannot provide as accurate results as classical methods do. Nonetheless, we believe that our approach can be used to effectively reduce the search space for other methods and to find inconsistencies in the used map.

Method

In the 2D case, the pose consists of two translation and one orientation parameter. According to assumption (iii) the orientation angle is known. Hence, the global localization problem simplifies to determining the translation. As the initial position is unknown, according to assumption (i) the robot can be placed everywhere on the map. Assumption (ii) reveals, that the robot cannot be placed within buildings. That means, the position of the robot can be described by a set of positions on the map, that has an empty intersection with the buildings described by polygons. The set of feasible robot positions is represented by subpavings. When the robot moves, the subpavings are updated according to the measured odometry. Those updated subpavings that lie within a building polygon are discarded from the feasible set of positions. Subpavings that intersect but do not fully lie within a building polygon are further bisected and evaluated in a SIVIA approach. Fig. 1 illustrates our method.



(a) (b) (c) (d) (e) (f)

Figure 1: Global Localization on the KITTI 0027 dataset [4].

First results

We experimetally evaluate our method with the KITTI 0027 dataset [4]. Fig. 1a shows the building map of a part of Karlsruhe, Germany. The trajectory is marked red. The ground truth pose of the robot is represented by the green-red coordinate frame. The feasible set of robot positions is visualized in Fig. 1b to 1f in green. Initially, the robot can be everywhere on the map besides within the buildings as shown in Fig. 1b. In Fig. 1c the vehicle has driven forward and the buildings

enable us to carve out infeasible positions of the vehicle. From Fig. 1b to 1f the feasible position set is gradually narrowed down along the trajectory. In Fig. 1e multiple disconnected regions for the position remain. In Fig. 1f the robot is localized unambiguously at the end of the trajectory.
In future work, we plan to extend our method in such a way that we can withdraw assumption (iii) by also estimating the orientation. Therefore, we divide the initial orientation interval into multiple subpavings and apply our method to each of them. Those initial orientation subpavings that lead to empty sets for the feasible set of positions can be discarded and the initial orientation can be narrowed down by exclusion.

Acknowledgement

This work was supported by the German Research Foundation (DFG) as part of the Research Training Group i.c.sens [RTG 2159].

References

[1] M. Hentschel and W. Wagner, Autonomous robot navigation based on OpenStreetMap geodata, *International IEEE Conference on Intelligent Transportation Systems*, pp. 1645–1650, 2010.

[2] P. Ruchti, B. Steder, M. Ruhnke, and W. Burgard, Localization on OpenStreetMap data using a 3D laser scanner, *IEEE International Conference on Robotics and Automation*, pp. 5260-5265, 2015.

[3] B. Desrochers and L. Jaulin, Minkowski Operations of Sets with Application to Robot Localization, *Electronic Proceedings in Theoretical Computer Science*, vol. 247, pp. 34–45, 2017.

[4] A. Geiger, P. Lenz, C. Stiller, and R. Urtasun, Vision meets robotics: The KITTI dataset, *The International Journal of Robotics Research*, vol. 10, no. 11, pp. 1231–1237, 2013.

High-gain interval observer for continuous-discrete time systems : Application to a quadcopter

Antoine Hugo[1], Rihab El Houda Thabet [1], Luc Meyer[2], Sofiane Ahmed Ali[1], Hélène Piet-Lahanier[2] and Felipe Kataoka Ishikawa[1]

[1] Normandie Université, UNIROUEN, ESIGELEC, Laboratoire IRSEEM, 76800 Saint-Etienne-du-Rouvray, France
antoine.hugo@groupe-esigelec.org,
{rihab.hajri, sofiane.ahmedali}@esigelec.fr
felipe.kataoka-ishikawa@etu.u-bordeaux.fr
[2] Université Paris Saclay, ONERA
Chemin de la Vauve aux Granges, Palaiseau, 91120, France
{luc.meyer, helene.piet-lahanier}@onera.fr

Keywords: High-gain observer, interval observer, continuous-discrete time system, discretization, quadcopter

Introduction

State estimation is a key challenge concerning control and fault detection of complex uncertain systems. Interval observer design is a promising way to tackle this issue in the context of bounded uncertainties [1,2]. This work focuses on continuous-discrete interval observer design for a class of partially linear systems subject to sampled data measurements. The proposed approach is an extension of [3]. The observer structure is based upon a discretization of their resulting observer. A sufficient condition on the maximum allowable sampling period is derived in order to ensure the observer stability. The aim is to deal with the implementation issue on a real system. Results will be tested on the Navigation-Guidance-Control loop of a quadcopter.

Main results

The High-Gain Interval Observer (HGIO) design presented in [3] consists in four main steps. The first two are offline computation of the associated gains and parameters. The last two are online, based on the proposed observers, to compute the estimated bounds of the output and the state. In this work, the resulting observer for state bounds estimation has been discretized at a sampling period T_d by a 3$^{\text{rd}}$ order polynomial method. Indeed, this choice allows to work with higher sampling periods than those of the rectangular method.
As a first result, a sufficient implicit condition on the sampling period for the observer stability has been proven based on the non-divergence of the radius dynamics. It consists in checking that a Metzler matrix is Hurwitz. Later, an explicit condition will be given to derive the maximum allowable sampling period. Moreover, as the measurement sampling period has to be lower than T_d, this condition also defines the maximum measurement sampling period.
The implementation of the HGIO for a quadcopter will be initially conducted in Hardware In The Loop (HITL) simulation with a Pixhawk 4. Later it will be tested on an experimental plateform in real conditions through outdoor experiments.

References

[1] D. Efimov and T. Raïssi, Design of Interval Observers for Uncertain Dynamical Systems, *Automation and Remote Control*, Vol.77, No.2, 2016.

[2] T. Chevet, T. N. Dinh, J. Marzat, and T. Raïssi, Interval Estimation for Discrete-Time Linear Parameter-Varying System with Unknown Inputss, *Conference on Decision and Control*, 2021.

[3] R. El Houda Thabet, S. Ahmed Ali and V. Puig, High-gain interval observer for continuous-discrete-time systems using an LMI design approach, *International Journal of Systems Science*, 2022.

Stabilizing Controller Design Using an Iterative LMIs Approach for Quadrotors

Oussama Benzinane, Andreas Rauh

Distributed Control in Interconnected Systems
Carl von Ossietzky Universität Oldenburg
26111 Oldenburg, Germany
o.benzinane@gmail.com, andreas.rauh@uol.de

Keywords: Interval methods, Linear matrix inequalities, Robust control.

Introduction

Linear Matrix Inequalities (LMIs) have recently gained a momentum due to the increasing performance of computing hardware. Many current research activities rely on the advantages of this growth in order to design linear state feedback controllers with provable stability and performance guarantees. As an example, the authors of the paper [1] have established an approach based on an iterative LMI solution to obtain the gain matrices of robust controllers and state observers simultaneously in the presence of bounded parameter uncertainty and stochastic noise.

On the basis of the aforementioned work, this presentation discusses possible paths to follow in order to reduce the dependency effect and the wrapping effect that may turn the control and observer synthesis pessimistic.

Control Loop Structure

Consider the cascaded structure of a quadrotor control as it has been proposed in the paper [2]. The state-dependent model for the inner attitude control loop from this paper has firstly been reformulated in terms of a quasi-linear state-space representation by a suitable factorization of the state equations. Defining bounded intervals for the state vector components that are included in the system matrices, a polytopic realization can be constructed. Hence, an observer-based state feedback control approach can be implemented, after temporal discretization of the continuous-time state equations, with the structure depicted in Fig. 1.

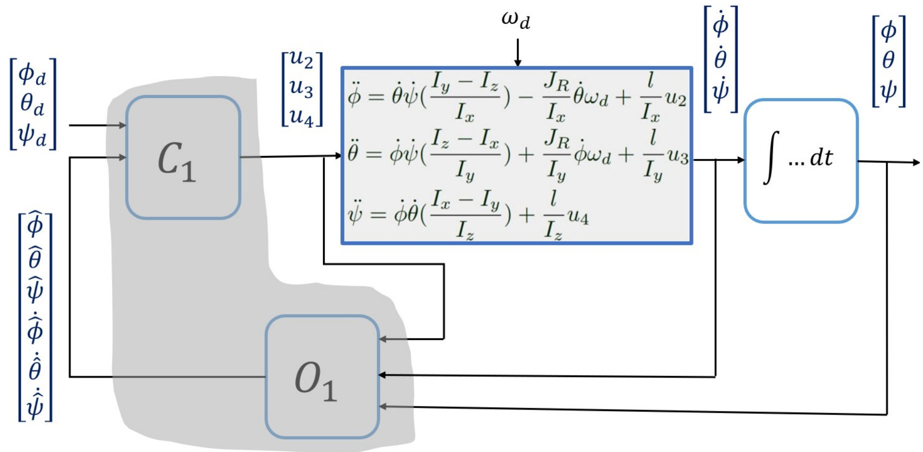


Figure 1: Block diagram of the discrete-time observer-based state feedback control system.

In this graphical representation ϕ, θ, ψ are respectively the roll, pitch, and yaw angles, while ω_d represents disturbances caused by couplings with the outer velocity and position control loops that are not further considered in this contribution; J_R is the rotor inertia while I_x, I_y, I_z are the parameters on the diagonal of the inertia matrix. The blocks C_1 and O_1 are respectively the controller and the state observer. The command signals u_2, u_3, and u_4 are the rolling, pitching, and yawing torque, respectively, which depend on the speed of each rotor.

Preliminary Results and Ongoing Work

Using the proposed approach, it is possible to find controller and observer gains jointly for which stability can be proven despite state and parameter uncertainties with eigenvalue domains strictly included in the interior of the unit circle in the complex z-plane. The response for a desired hovering state is shown in the Fig. 2, where the initial states are fixed at $10°$. The resulting control signals are reasonable in their amplitudes and setting times as shown in Fig. 3.

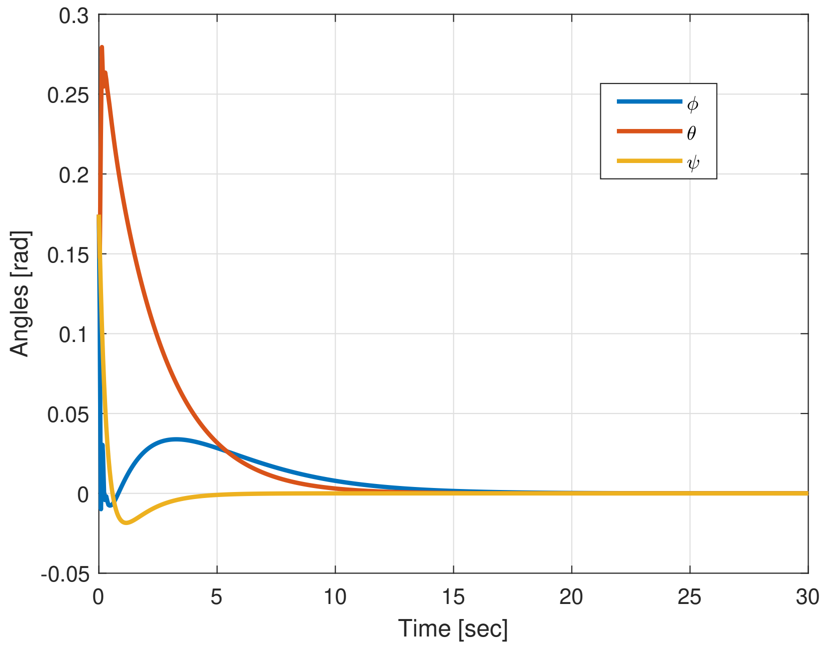


Figure 2: Regulation of the quadrotor's attitude.

So far, the decay rates of the state vector components towards the equilibrium state are restricted by the use of the polytopic uncertainty model in combination with a parameter-independent Lyapunov function approach. In such cases, infeasibility of the LMIs may occur if excessively large distances of the eigenvalues from the boundary of the unit circle are desired.

To overcome this problem, the authors in the cited paper [3] have provided a tool, on the basis of the theorem of Ehlich and Zeller, that provides the possibility to balance between conservatism and the calculation effort. Therefore, ongoing research focuses on combining this method with the algorithm presented in [1] to obtain the gain

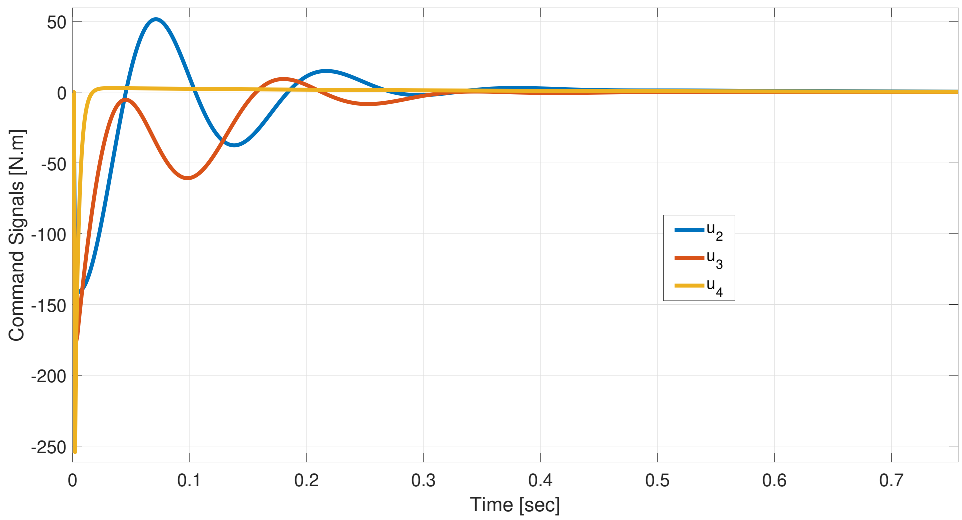


Figure 3: Command Signals applied to the attitude model.

matrices of the controller and the observer with less conservatism and a moderate computational effort.

References

[**1**] Robert Dehnert, Michelle Damaszek, Sabine Lerch, Andreas Rauh, and Bernd Tibken, *Robust Feedback Control for Discrete-Time Systems Based on Iterative LMIs with Polytopic Uncertainty Representations Subject to Stochastic Noise*, Frontiers in Control Engineering, 2022. doi: 10.3389/fcteg.2021.786152

[**2**] Holger Voos, *Nonlinear State-Dependent Riccati Equation Control of a Quadrotor UAV*, Proceedings of the 2006 IEEE. International Conference on Control Applications. Munich, Germany, October 4-6, 2006.

[**3**] Sascha A. Warthenpfuhl and Bernd Tibken, *Guaranteed bounds for robust LMI problems with polynomial parameter dependence*, Proceedings of the 17th World Congress. The International Federation of Automatic Control. Seoul, Korea, July 6-11, 2008.

Validated Model Predictive Control based on Exponential Enclosures

Mohamed Fnadi[1] and Andreas Rauh[2]

[1] Laboratoire d'Informatique, Signal et Image de la Côte d'Opale – LISIC UR 4491
Université du Littoral Côte d'Opale, F-62228, France
mohamed.fnadi@univ-littoral.fr
[2] Carl von Ossietzky Universität Oldenburg
Distributed Control in Interconnected Systems D-26111 Oldenburg, Germany
andreas.rauh@uni-oldenburg.de

Keywords: validated simulation, exponential enclosures, model predictive control

Introduction

Guaranteed numerical integration is a fundamental tool to solve initial value problems of ordinary differential equations (IVP-ODEs) with uncertain initial conditions and parameters in a reliable and validated way. Providing guaranteed solution enclosures to these IVP-ODEs is essential for designing and verifying linear and nonlinear feedback controllers, mainly for predictive control approaches. In the literature, many solvers have been developed, such as the DynIbex library, allowing for the computation of enclosures which are guaranteed to contain all possible system states. The DynIbex library is based on Runge-Kutta schemes to obtain tight state enclosures [1]. Nevertheless, it has been shown that — due to the computational complexity of Runge-Kutta methods [2] — fast convergence and high accuracy of the computed enclosures are not always guaranteed for finitely long integration time spans, possibly leading to an excessive duration to get the IVP-ODEs' solutions. To overcome these issues, exponential enclosure techniques for IVP-ODE problems seem to be attractive to remarkably reduce the computing time of validated methods and to approach real-time capability [3,4].

The time aspect is especially crucial, because at each sampling instant, a validated nonlinear model predictive controller (NMPC) needs to compute optimal and guaranteed system inputs along a receding horizon that minimize some interval cost function and ensure compatibility constraints (such as actuator saturations or safety constraints on the state trajectories) [2]. Our motivation is to interface exponential enclosure techniques with the validated NMPC to remarkably speed up the solution.

Guaranteed Nonlinear Model Predictive Control

Consider a dynamic system defined by the following IVP-ODEs :

$$\begin{cases} \dot{\mathbf{x}}_t = \mathbf{f}(t, \mathbf{x}_t, \mathbf{u}, \mathbf{p}) \\ \mathbf{x}_0 \in [\mathbf{x}_0] \subseteq \mathbb{IR}^n \\ \mathbf{u} \in [\mathbf{u}] \subseteq \mathbb{IR}^m \\ \mathbf{p} \in [\mathbf{p}] \subseteq \mathbb{IR}^p, \end{cases} \tag{1}$$

where the state vector is denoted by $\mathbf{x}_t$, the vector of parameters by $\mathbf{p}$, and the control vector by $\mathbf{u}$. The sets $[\mathbf{x}_0] = \big[[x_{10}] \ \ldots \ [x_{n0}]\big]^T$, $[\mathbf{u}] = \big[[u_1] \ \ldots \ [u_m]\big]^T$, and $[\mathbf{p}] = \big[[p_1] \ \ldots \ [p_p]\big]^T$, expressed as interval boxes, are respectively the initial condition of the state vector, the interval-bounded input, and the set of feasible dynamic parameters. The proposed guaranteed NMPC encompasses two stages [2]:

1. **Filtering and branching:** The first step provides a sequence of guaranteed input interval boxes at each time-step k over the prediction horizon N_p, denoted as $[\mathbf{U}]_k = [\mathbf{u}]_k \times [\mathbf{u}]_{k+1} \times \ldots \times [\mathbf{u}]_{k+N_\mathrm{p}-1}$. Branching and filtering procedures allow the computation of safe input intervals along the receding time horizon that satisfy the state constraints (i.e., $\forall j, [x_j] \subseteq [x_{\min,j}, x_{\max,j}]$, where $x_{\min,j}$ and $x_{\max,j}$ are the bounds for the admissible domain for each state variable) and ensure convergence to the reference interval (i.e., $[\mathbf{x}_k] \rightarrow [\mathbf{x}_{\mathrm{r},k}]$).

2. **Interval optimization:** Since safe inputs are computed over a finite time horizon, the optimization algorithm is launched to com-

pute the optimal inputs $[\mathbf{U}]_k^\star$ by minimizing as much as as possible a newly formulated interval objective function to reduce the error between predicted and reference outputs as well as the norm of the input intervals.

Exponential Enclosure Technique

Guaranteed numerical integration methods aim at computing the state enclosure sequences $(t_j, [\mathbf{x}_j])_{j\in\mathbb{N}}$, assuming that the input and parameter boxes $[\mathbf{u}]$ and $[\mathbf{p}]$, respectively, are piecewise constant and known for each validated simulation. Here, the exponential enclosure technique will be applied to approximate the IVP-ODEs' solutions, given in (1). It has been shown that this method improves the accuracy of the computed state enclosures and reduces the required computation time for asymptotically stable systems [3]. The dynamic model (1) can be reformulated by considering that the dynamic parameters are represented by constant intervals, and the input variables are assumed to be included in an augmented state vector, i.e., $\left[\mathbf{x}_t^T \quad \mathbf{u}^T(\mathbf{x}_t)\right]^T$, denoted for brevity again as $\mathbf{x}_t$ with

$$\dot{\mathbf{x}}_t = \mathbf{f}(\mathbf{x}_t). \tag{2}$$

To ensure the (local) asymptotic stability of the system model in the neighborhood of a desired terminal state, we assume — as a prerequisite for the exponential enclosure approach — that a feedback controller is included in a cascaded manner in the control law $\mathbf{u}(\mathbf{x}_t)$ so that the NMPC effectively computes a kind of feedforward control sequence.

To prevent the growth of the diameters of the intervals $(t_j, [\mathbf{x}_j])_{j\in\mathbb{N}}$ for asymptotically stable systems with a minimum computational capacity, the exact solution $\mathbf{x}_t^\star$ can be bracketed into the following exponential state enclosures

$$\mathbf{x}_t^\star \in [\mathbf{x}_e](t) = \exp\Big([\boldsymbol{\Lambda}] \cdot t\Big) \cdot [\mathbf{x}_e](0) \ , \quad [\mathbf{x}_e](0) = [\mathbf{x}_0], \tag{3}$$

where $\boldsymbol{\Lambda}$ represents a yet unknown dynamics matrix. By choosing $[\boldsymbol{\Lambda}] = \mathrm{diag}\{[\lambda_i]\}$, $i = 1, \ldots, n$, as a diagonal matrix, its elements λ_i

need to have negative real parts to describe contracting state enclosures.

Using the exponential state enclosures (3) and a Picard iteration with the iteration index κ, we obtain

$$\begin{aligned} \mathbf{x}_t^\star \in [\mathbf{x}_e]_{(\kappa+1)} &= \exp\Big([\mathbf{\Lambda}]_{(\kappa+1)} \cdot t\Big) \cdot [\mathbf{x}_e](0) \\ &= [\mathbf{x}_e](0) + \int\limits_0^t \mathbf{f}\Big(\exp\Big([\mathbf{\Lambda}]_{(\kappa)} \cdot s\Big) \cdot [\mathbf{x}_e](0)\Big)\,\mathrm{d}s. \end{aligned} \tag{4}$$

The differentiation of (4) with respect to time, belonging to the integration interval $t \in [t]$, leads to

$$\dot{\mathbf{x}}_{[t]}^\star \in [\mathbf{\Lambda}]_{(\kappa+1)} \cdot \exp\Big([\mathbf{\Lambda}]_{(\kappa+1)} \cdot [t]\Big) \cdot [\mathbf{x}_e](0) = \mathbf{f}\Big(\exp\Big([\mathbf{\Lambda}]_{(\kappa)} \cdot [t]\Big) \cdot [\mathbf{x}_e](0)\Big). \tag{5}$$

Assuming a converging iteration with $[\mathbf{\Lambda}]_{(\kappa+1)} \subseteq [\mathbf{\Lambda}]_{(\kappa)}$ and, thus, $[\lambda_i]_{(\kappa+1)} \subseteq [\lambda_i]_{(\kappa)}$, the iteration formula for $[\lambda_i]_{(\kappa+1)}$ can be expressed as

$$[\lambda_i]_{(\kappa+1)} = \frac{f_i\Big(\exp\Big([\mathbf{\Lambda}]_{(\kappa)} \cdot [t]\Big) \cdot [\mathbf{x}_{e,i}](0)\Big)}{\exp\Big([\mathbf{\Lambda}]_{(\kappa)} \cdot [t]\Big).[\mathbf{x}_{e,i}](0)}, \quad i = 1, \ldots, n. \tag{6}$$

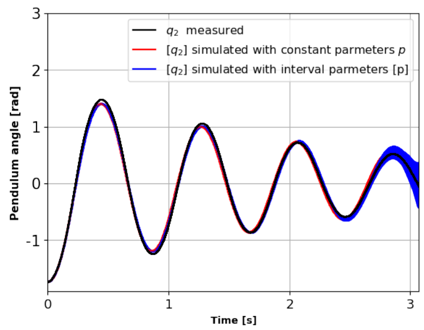
The guaranteed state enclosure at the time instant $t = T = \sup([t])$ is given by

$$\mathbf{x}_t^\star \in [\mathbf{x}_e](t) = \exp\Big([\mathbf{\Lambda}] \cdot T\Big) \cdot [\mathbf{x}_e](0), \tag{7}$$

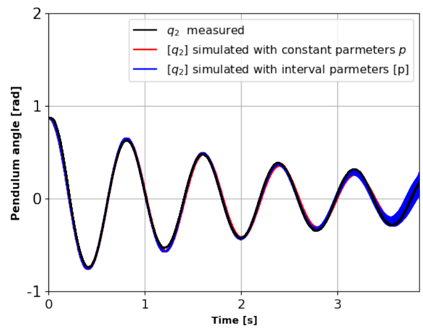
where $[\mathbf{\Lambda}]$ is the final result of the iteration (6).

Preliminary Results using DynIbex

The NMPC strategy is applied to stabilize a nonlinear inverted pendulum with two serial joints, actuated by a DC motor whose angular speed is the input variable. To evaluate the dynamic model of the inverted pendulum, we can solve the IVP-ODEs in a validated way using the DynIbex library. Figs. 1a and 1b show the measured pendulum angle (black lines) with the computed enclosures by DynIbex using point-valued parameters $\mathbf{p}$ (red enclosures) and interval parameters $[\mathbf{p}]$ (blue enclosures). We can notice that the simulated tubes of the pendulum angle are close to the real measured signal. Moreover, we have calculated the coverage ratios between the measurements and the simulated tubes as recapped in Tab. 1. The coverage ratios confirm that the model is identified with high precision when the dynamic parameters are considered as intervals that account for different uncertainties related to the measurements and dynamic modeling. However, the accuracy of the validated simulation should be enhanced because the widths of the computed Rung-Kutta enclosures enlarge with time, and the coverage ratios are not quite satisfactory.



(a) Initial conditions: $[\mathbf{u}_0] = [0\,\mathrm{Nm}, 0\,\mathrm{Nm}]$ and $[\mathbf{x}_0] = \underbrace{[0°, 0°]}_{[x_{10}]} \times \underbrace{[0, 0]}_{[x_{20}]} \times \underbrace{[-98°, -100°]}_{[x_{30}]} \times \underbrace{[0, 0]}_{[x_{40}]}$.



(b) Initial conditions: $[\mathbf{u}_0] = [0.15\,\mathrm{Nm}, 0.18\,\mathrm{Nm}]$ and $[\mathbf{x}_0] = \underbrace{[0°, 0°]}_{[x_{10}]} \times \underbrace{[0, 0]}_{[x_{20}]} \times \underbrace{[46°, 50°]}_{[x_{30}]} \times \underbrace{[0, 0]}_{[x_{40}]}$.

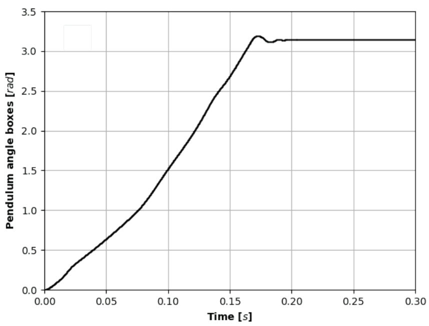
Figure 1: The validation of the dynamic model of a nonlinear inverted pendulum using the DynIbex library. Comparison between the actual and simulated pendulum angles at different initial conditions with point-valued and interval parameters.

Figs. 2a and 2b display the simulation results of the validated NMPC approach. As it can be seen in Fig. 2a, the pendulum arm starts from the downward position, and it is stabilized via the vali-

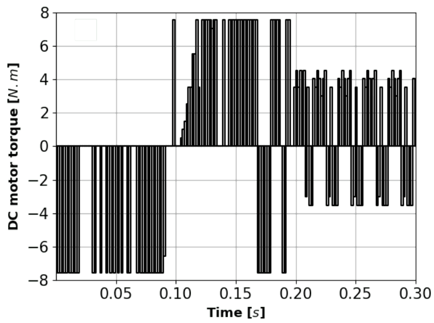
Table 1: Coverage rates between the model and physical reality.

Scenario	(a)	(b)
With interval-valued dynamic parameters $[\mathbf{p}]$	51%	61%
With point-valued dynamic parameters $\mathbf{p}$	38%	34%

dated NMPC in its vertical upright position interval $[\mathbf{x}_\mathrm{r}]$ with a small settling-time (around $t_{\mathrm{r}5\%} \approx 0.18\,\mathrm{s}$). Despite its proven effectiveness in making the system output converge to the desired reference interval, it still has some drawbacks. The main ones are related globally to the computation time, which depends mainly on a large number of bisections of the initial input domain $[\mathbf{u}_k]$ preventing the validation of this approach in real-time. This issue can be reduced by using exponential enclosure techniques in combination with an underlying feedback controller.



(a) Tube for the pendulum angle.



(b) Input Intervals.

Figure 2: Validated NMPC results starting from the downward position.

References

[1] J. Alexandre dit Sandretto and A. Chapoutot. *Validated Explicit and Implicit Runge-Kutta methods*, Reliable Computing, electronic edition, 22, 2016.

[2] M. Fnadi and J. Alexandre dit Sandretto, *Experimental Validation of a Guaranteed Nonlinear Model Predictive Control*,

Algorithms, 14(8), 248, 2021.

[**3**] A. Rauh, R. Westphal, H. Aschemann and E. Auer. *Exponential Enclosure Techniques for Initial Value Problems with Multiple Conjugate Complex Eigenvalues.* In International Symposium on Scientific Computing, Computer Arithmetic, and Validated Numerics, pp. 247–256, Springer, Cham, 2015.

[**4**] A. Rauh, R. Westphal, H. Aschemann. *Verified Simulation of Control Systems with Interval Parameters Using an Exponential State Enclosure Technique.* Proc. of IEEE Intl. Conference on Methods and Models in Automation and Robotics MMAR 2013, Miedzyzdroje, Poland, 2013