

# Summary of TLA<sup>+</sup>

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# Module-Level Constructs

MODULE  $M$

Begins the module or submodule named  $M$ .

EXTENDS  $M_1, \dots, M_n$

Incorporates the declarations, definitions, assumptions, and theorems from the modules named  $M_1, \dots, M_n$  into the current module.

CONSTANTS  $C_1, \dots, C_n$ <sup>(1)</sup>

Declares the  $C_j$  to be constant parameters (rigid variables). Each  $C_j$  is either an identifier or has the form  $C(-, \dots, -)$ , the latter form indicating that  $C$  is an operator with the indicated number of arguments.

VARIABLES  $x_1, \dots, x_n$ <sup>(1)</sup>

Declares the  $x_j$  to be variables (parameters that are flexible variables).

ASSUME  $P$

Asserts  $P$  as an assumption.

$F(x_1, \dots, x_n) \triangleq exp$

Defines  $F$  to be the operator such that  $F(e_1, \dots, e_n)$  equals  $exp$  with each identifier  $x_k$  replaced by  $e_k$ . (For  $n = 0$ , it is written  $F \triangleq exp$ .)

$f[x \in S] \triangleq exp$ <sup>(2)</sup>

Defines  $f$  to be the function with domain  $S$  such that  $f[x] = exp$  for all  $x$  in  $S$ . (The symbol  $f$  may occur in  $exp$ , allowing a recursive definition.)

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(1) The terminal s in the keyword is optional.

(2)  $x \in S$  may be replaced by a comma-separated list of items  $v \in S$ , where  $v$  is either a comma-separated list or a tuple of identifiers.

INSTANCE  $M$  WITH  $p_1 \leftarrow e_1, \dots, p_m \leftarrow e_m$

For each defined operator  $F$  of module  $M$ , this defines  $F$  to be the operator whose definition is obtained from the definition of  $F$  in  $M$  by replacing each declared constant or variable  $p_j$  of  $M$  with  $e_j$ . (If  $m = 0$ , the WITH is omitted.)

$N(x_1, \dots, x_n) \triangleq \text{INSTANCE } M \text{ WITH } p_1 \leftarrow e_1, \dots, p_m \leftarrow e_m$

For each defined operator  $F$  of module  $M$ , this defines  $N(d_1, \dots, d_n)!F$  to be the operator whose definition is obtained from the definition of  $F$  by replacing each declared constant or variable  $p_j$  of  $M$  with  $e_j$ , and then replacing each identifier  $x_k$  with  $d_k$ . (If  $m = 0$ , the WITH is omitted.)

THEOREM  $P$

Asserts that  $P$  can be proved from the definitions and assumptions of the current module.



LOCAL  $def$

Makes the definition(s) of  $def$  (which may be a definition or an INSTANCE statement) local to the current module, thereby not obtained when extending or instantiating the module.



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Ends the current module or submodule.

# The Constant Operators

## Logic

$\wedge \vee \neg \Rightarrow \equiv$   
TRUE FALSE BOOLEAN [the set {TRUE, FALSE}]  
 $\forall x \in S : p$  <sup>(1)</sup>  $\exists x \in S : p$  <sup>(1)</sup>  
CHOOSE  $x \in S : p$  [An  $x$  in  $S$  satisfying  $p$ ]

## Sets

$= \neq \in \notin \cup \cap \subseteq \setminus$  [set difference]  
 $\{e_1, \dots, e_n\}$  [Set consisting of elements  $e_i$ ]  
 $\{x \in S : p\}$  <sup>(2)</sup> [Set of elements  $x$  in  $S$  satisfying  $p$ ]  
 $\{e : x \in S\}$  <sup>(1)</sup> [Set of elements  $e$  such that  $x$  in  $S$ ]  
SUBSET  $S$  [Set of subsets of  $S$ ]  
UNION  $S$  [Union of all elements of  $S$ ]

## Functions

$f[e]$  [Function application]  
DOMAIN  $f$  [Domain of function  $f$ ]  
 $[x \in S \mapsto e]$  <sup>(1)</sup> [Function  $f$  such that  $f[x] = e$  for  $x \in S$ ]  
 $[S \rightarrow T]$  [Set of functions  $f$  with  $f[x] \in T$  for  $x \in S$ ]  
 $[f \text{ EXCEPT } ![e_1] = e_2]$  <sup>(3)</sup> [Function  $\hat{f}$  equal to  $f$  except  $\hat{f}[e_1] = e_2$ ]

## Records

$e.h$  [The  $h$ -field of record  $e$ ]  
 $[h_1 \mapsto e_1, \dots, h_n \mapsto e_n]$  [The record whose  $h_i$  field is  $e_i$ ]  
 $[h_1 : S_1, \dots, h_n : S_n]$  [Set of all records with  $h_i$  field in  $S_i$ ]  
 $[r \text{ EXCEPT } !.h = e]$  <sup>(3)</sup> [Record  $\hat{r}$  equal to  $r$  except  $\hat{r}.h = e$ ]

## Tuples

$e[i]$  [The  $i^{\text{th}}$  component of tuple  $e$ ]  
 $\langle e_1, \dots, e_n \rangle$  [The  $n$ -tuple whose  $i^{\text{th}}$  component is  $e_i$ ]  
 $S_1 \times \dots \times S_n$  [The set of all  $n$ -tuples with  $i^{\text{th}}$  component in  $S_i$ ]

- (1)  $x \in S$  may be replaced by a comma-separated list of items  $v \in S$ , where  $v$  is either a comma-separated list or a tuple of identifiers.
- (2)  $x$  may be an identifier or tuple of identifiers.
- (3)  $![e_1]$  or  $!.h$  may be replaced by a comma separated list of items  $!a_1 \dots a_n$ , where each  $a_i$  is  $[e_i]$  or  $.h_i$ .

# Miscellaneous Constructs

IF $p$ THEN $e_1$ ELSE $e_2$	[ $e_1$ if $p$ true, else $e_2$ ]
CASE $p_1 \rightarrow e_1 \square \dots \square p_n \rightarrow e_n$	[Some $e_i$ such that $p_i$ true]
CASE $p_1 \rightarrow e_1 \square \dots \square p_n \rightarrow e_n \square$ OTHER $\rightarrow e$	[Some $e_i$ such that $p_i$ true, or $e$ if all $p_i$ are false]
LET $d_1 \triangleq e_1 \dots d_n \triangleq e_n$ IN $e$	[ $e$ in the context of the definitions]
$\wedge p_1$ [the conjunction $p_1 \wedge \dots \wedge p_n$ ]	$\vee p_1$ [the disjunction $p_1 \vee \dots \vee p_n$ ]
$\vdots$	$\vdots$
$\wedge p_n$	$\vee p_n$

# Action Operators

$e'$	[The value of $e$ in the final state of a step]
$[A]_e$	$[A \vee (e' = e)]$
$\langle A \rangle_e$	$[A \wedge (e' \neq e)]$
ENABLED $A$	[An $A$ step is possible]
UNCHANGED $e$	$[e' = e]$
$A \cdot B$	[Composition of actions]

# Temporal Operators

$\square F$	[ $F$ is always true]
$\diamond F$	[ $F$ is eventually true]
$\text{WF}_e(A)$	[Weak fairness for action $A$ ]
$\text{SF}_e(A)$	[Strong fairness for action $A$ ]
$F \rightsquigarrow G$	[ $F$ leads to $G$ ]

# User-Definable Operator Symbols

## Infix Operators

$+^{(1)}$	$-^{(1)}$	$*^{(1)}$	$/^{(2)}$	$\circ^{(3)}$	$++$
$\div^{(1)}$	$\%^{(1)}$	$\cdot^{(1,4)}$	$\dots^{(1)}$	$\dots$	$--$
$\oplus^{(5)}$	$\ominus^{(5)}$	$\otimes$	$\oslash$	$\odot$	$**$
$\langle^{(1)}$	$\rangle^{(1)}$	$\leq^{(1)}$	$\geq^{(1)}$	$\sqcap$	$//$
$\lrcorner$	$\rceil$	$\lrcorner$	$\rceil$	$\sqcup$	$\hat{\hat{}}$
$\ll$	$\gg$	$\langle :$	$: \rangle^{(6)}$	$\&$	$\&\&$
$\sqcap$	$\sqcup$	$\sqcap^{(5)}$	$\sqcup$	$ $	$\% \%$
$\subset$	$\supset$		$\supset$	$\star$	$@ @^{(6)}$
$\top$	$\perp$	$\perp$	$\perp$	$\bullet$	$\#\#\$
$\sim$	$\approx$	$\approx$	$\Re$	$\$$	$\$\$\$
$\bigcirc$	$::=$	$\times$	$\doteq$	$??$	$!!$
$\propto$	$\}$	$\notin$			

## Postfix Operators <sup>(7)</sup>

$\hat{+}$     $\hat{*}$     $\hat{\#}$

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- (1) Defined by the *Naturals*, *Integers*, and *Reals* modules.
  - (2) Defined by the *Reals* module.
  - (3) Defined by the *Sequences* module.
  - (4)  $x \hat{y}$  is printed as  $x^y$ .
  - (5) Defined by the *Bags* module.
  - (6) Defined by the *TLC* module.
  - (7)  $e \hat{+}$  is printed as  $e^+$ , and similarly for  $\hat{*}$  and  $\hat{\#}$ .

# Precedence Ranges of Operators

The relative precedence of two operators is unspecified if their ranges overlap.  
Left-associative operators are indicated by (a).



## Prefix Operators

$\neg$	4-4	$\square$	4-15	UNION	8-8
ENABLED	4-15	$\diamond$	4-15	DOMAIN	9-9
UNCHANGED	4-15	SUBSET	8-8	-	12-12

## Infix Operators

$\Rightarrow$	1-1	$\leq$	5-5	$<:$	7-7	$\ominus$	11-11 (a)
$\Rightarrow$	2-2	$\ll$	5-5	$\setminus$	8-8	-	11-11 (a)
$\equiv$	2-2	$\succ$	5-5	$\cap$	8-8 (a)	--	11-11 (a)
$\sim$	2-2	$\succ$	5-5	$\cup$	8-8 (a)	$\&$	13-13 (a)
$\wedge$	3-3 (a)	$\propto$	5-5	$\dots$	9-9	$\&\&$	13-13 (a)
$\vee$	3-3 (a)	$\sim$	5-5	$\dots$	9-9	$\odot$	13-13 (a)
$\neq$	5-5	$\approx$	5-5	!!	9-13	$\oslash$	13-13
$\dagger$	5-5	$\sqcap$	5-5	##	9-13 (a)	$\otimes$	13-13 (a)
::=	5-5	$\sqsupset$	5-5	\$	9-13 (a)	*	13-13 (a)
: =	5-5	$\sqsubset$	5-5	\$\$	9-13 (a)	**	13-13 (a)
$<$	5-5	$\sqsupseteq$	5-5	??	9-13 (a)	/	13-13
$=$	5-5	$\subset$	5-5	$\square$	9-13 (a)	//	13-13
$\perp$	5-5	$\supset$	5-5	$\sqcup$	9-13 (a)	$\bigcirc$	13-13 (a)
$>$	5-5	$\supseteq$	5-5	$\boxplus$	9-13 (a)	$\bullet$	13-13 (a)
$\approx$	5-5	$\supseteq$	5-5	$\wr$	9-14	$\div$	13-13
$\asymp$	5-5	$\cup$	5-5	$\oplus$	10-10 (a)	$\circ$	13-13 (a)
$\cong$	5-5	$\cup$	5-5	+	10-10 (a)	*	13-13 (a)
$\doteq$	5-5	$\top$	5-5	++	10-10 (a)	$\wedge$	14-14
$\geq$	5-5	$\perp$	5-5	%	10-11	$\wedge\wedge$	14-14
$\gg$	5-5	$\cdot^{(1)}$	5-14 (a)	%%	10-11 (a)	$\cdot^{(2)}$	17-17 (a)
$\in$	5-5	@@	6-6 (a)		10-11 (a)		
$\notin$	5-5	:>	7-7		10-11 (a)		

## Postfix Operators

$\sim+$	15-15	$\sim*$	15-15	$\sim\#$	15-15	'	15-15
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(1) Action composition ( $\setminus\text{cdot}$ ).

(2) Record field (period).

# Operators Defined in Standard Modules.

## Modules *Naturals, Integers, Reals*

+	− <sup>(1)</sup>	*	/ <sup>(2)</sup>	^ <sup>(3)</sup>	..	<i>Nat</i>	<i>Real</i> <sup>(2)</sup>
÷	%	≤	≥	<	>	<i>Int</i> <sup>(4)</sup>	<i>Infinity</i> <sup>(2)</sup>

(1) Only infix − is defined in *Naturals*.

(2) Defined only in *Reals* module.

(3) Exponentiation.

(4) Not defined in *Naturals* module.



## Module *Sequences*

◦	<i>Head</i>	<i>SelectSeq</i>	<i>SubSeq</i>
<i>Append</i>	<i>Len</i>	<i>Seq</i>	<i>Tail</i>

## Module *FiniteSets*

*IsFiniteSet*    *Cardinality*

## Module *Bags*

⊕	<i>BagIn</i>	<i>CopiesIn</i>	<i>SubBag</i>
⊖	<i>BagOfAll</i>	<i>EmptyBag</i>	
⊆	<i>BagToSet</i>	<i>IsABag</i>	
<i>BagCardinality</i>	<i>BagUnion</i>	<i>SetToBag</i>	

## Module *RealTime*

*RTBound*    *RTnow*    *now* (declared to be a variable)

## Module *TLC*

*>*    @@    *Print*    *Assert*    *JavaTime*    *Permutations*  
*SortSeq*



# ASCII Representation of Typeset Symbols

$\wedge$	<code>\</code> or <code>\land</code>	$\vee$	<code>\</code> or <code>\lor</code>	$\Rightarrow$	<code>=&gt;</code>
$\neg$	<code>~</code> or <code>\lnot</code> or <code>\neg</code>	$\equiv$	<code>&lt;=&gt;</code> or <code>\equiv</code>	$\triangleq$	<code>==</code>
$\in$	<code>\in</code>	$\notin$	<code>\notin</code>	$\neq$	<code>#</code> or <code>/=</code>
$\langle$	<code>&lt;&lt;</code>	$\rangle$	<code>&gt;&gt;</code>	$\square$	<code>[]</code>
$<$	<code>&lt;</code>	$>$	<code>&gt;</code>	$\diamond$	<code>&lt;&gt;</code>
$\leq$	<code>\leq</code> or <code>=&lt;</code> or <code>&lt;=</code>	$\geq$	<code>\geq</code> or <code>&gt;=</code>	$\rightsquigarrow$	<code>~&gt;</code>
$\ll$	<code>\ll</code>	$\gg$	<code>\gg</code>	$\rightarrowtail$	<code>-+&gt;</code>
$\prec$	<code>\prec</code>	$\succ$	<code>\succ</code>	$\mapsto$	<code> -&gt;</code>
$\preceq$	<code>\preceq</code>	$\succeq$	<code>\succeq</code>	$\div$	<code>\div</code>
$\subset$	<code>\subseteq</code>	$\supset$	<code>\supseteq</code>	$\cdot$	<code>\cdot</code>
$\subsetneq$	<code>\subsetneq</code>	$\supsetneq$	<code>\supsetneq</code>	$\circ$	<code>\circ</code> or <code>\circ</code>
$\sqsubset$	<code>\sqsubset</code>	$\sqsupset$	<code>\sqsupset</code>	$\bullet$	<code>\bullet</code>
$\sqsubseteq$	<code>\sqsubseteq</code>	$\sqsupseteq$	<code>\sqsupseteq</code>	$\star$	<code>\star</code>
$\top$	<code> -</code>	$\perp$	<code>- </code>	$\bigcirc$	<code>\bigcirc</code>
$\vDash$	<code> =</code>	$\vDash$	<code>= </code>	$\sim$	<code>\sim</code>
$\rightarrow$	<code>-&gt;</code>	$\leftarrow$	<code>&lt;-</code>	$\simeq$	<code>\simeq</code>
$\cap$	<code>\cap</code> or <code>\intersect</code>	$\cup$	<code>\cup</code> or <code>\union</code>	$\asymp$	<code>\asymp</code>
$\sqcap$	<code>\sqcap</code>	$\sqcup$	<code>\sqcup</code>	$\approx$	<code>\approx</code>
$\oplus$	<code>(+)</code> or <code>\oplus</code>	$\uplus$	<code>\uplus</code>	$\cong$	<code>\cong</code>
$\ominus$	<code>(-)</code> or <code>\ominus</code>	$\times$	<code>\X</code> or <code>\times</code>	$\doteq$	<code>\doteq</code>
$\odot$	<code>(.)</code> or <code>\odot</code>	$\wr$	<code>\wr</code>	$x^y$	<code>x^y</code> <sup>(2)</sup>
$\otimes$	<code>(\X)</code> or <code>\otimes</code>	$\propto$	<code>\propto</code>	$x^+$	<code>x^+</code> <sup>(2)</sup>
$\oslash$	<code>(/)</code> or <code>\oslash</code>	"s"	<code>"s"</code> <sup>(1)</sup>	$x^*$	<code>x^*</code> <sup>(2)</sup>
$\exists$	<code>\E</code>	$\forall$	<code>\A</code>	$x^\#$	<code>x^\#</code> <sup>(2)</sup>
$\exists$	<code>\EE</code>	$\forall$	<code>\AA</code>	'	,
$]_v$	<code>]_v</code>	$]_v$	<code>&gt;&gt;_v</code>		
$WF_v$	<code>WF_v</code>	$SF_v$	<code>SF_v</code>		

$\overline{\hspace{2cm}}$	<code>-----</code> (3)	$\overline{\hspace{2cm}}$	<code>-----</code> (3)
$\underline{\hspace{2cm}}$	<code>-----</code> (3)	$\underline{\hspace{2cm}}$	<code>=====</code> (3)

- (1)  $s$  is a sequence of characters.  
 (2)  $x$  and  $y$  are any expressions.  
 (3) a sequence of four or more - or = characters.