Appliquées from the University of Louvain, Belgium, in 1977 and the Ph.D. degree in mechanical engineering from Columbia University, New York, in 1981. In 1983, he joined the Aerospace Engineering Department of the University of Michigan, where he is currently a professor, with a joint appointment in the Department of Electrical Engineering and Computer Science. His research and teaching interests are in the areas of dynamic systems, control, guidance, and navigation. He is a Fellow of the IEEE.

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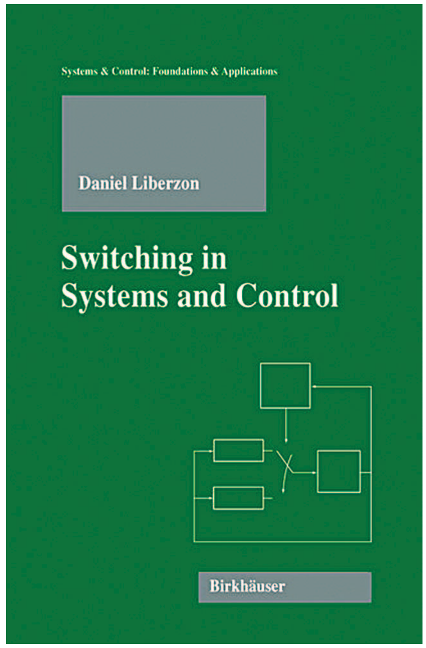
***Switching in Systems and Control*** by Daniel Liberzon, Birkhauser, 2003, ISBN: 8-176-42988, US$69.96. *Reviewed by João P. Hespanha.*




## Hybrid Versus Switched Systems

Hybrid systems are loosely defined as dynamical systems whose state has two components, one of which evolves in a continuous set such as $\mathbb{R}^n$ (typically satisfying a differential or difference equation) while the other evolves in a discrete set such as $\mathbb{N}$ according to some transition-logic-based rule. Perhaps the simplest model of a hybrid system is given by

$$\dot{x}(t) = f_{\sigma(t)}(x(t)), \quad x \in \mathbb{R}^n, \tag{1}$$

$$\sigma(t) = \lim_{\tau \uparrow t} \phi(x(\tau), \sigma(\tau)), \ \sigma \in \mathbb{N}, \tag{2}$$

where $x$ and $\sigma$ denote the continuous and discrete components of the state, respectively. The solution to such a system is depicted in Figure 1.

Hybrid systems attract researchers from two communities with distinct backgrounds and interests: computer scientists who need to model variables (such as time) that do not fit into the realm of discrete mathematics as well as control theorists who want to explore the discrete nature of certain variables to simplify the analysis and design of nonlinear systems. While this book will especially appeal to the latter, it can also provide the former with new perspectives on hybrid systems.

The terminology *switching systems* implicitly assumes a hybrid system model in which the discrete dynamics are "simple." This simplification is extreme when all possible evolutions of the discrete component of the state are admissible solutions, as in chapter 2. However, constraints on the discrete evolution can also be imposed, as in chapter 3. In the context of (1) and (2), an example of the former would be to completely ignore the dynamics imposed by (2) and accept any $\sigma(t)$ as an admissible solution, whereas an example of the latter would be to ignore the details of (2) but allow only signals $\sigma(t)$ with intervals between consecutive discontinuities larger than a given positive constant.

The terminology *switching systems* also hints toward a focus on the behavior of the continuous state. Consistent with this terminology, this book is mostly concerned with asymptotic properties of the continuous state, such as boundedness and convergence. With this viewpoint, Liberzon can avoid several unpleasant technicalities that arise in defining solutions to (1)–(2) by considering systems described by (2), with the understanding that the time-varying index $\sigma(t)$, now called the *switching signal*, satisfies prespecified constraints. In the context of hybrid systems, $\sigma(t)$ is generated by discrete dynamics such as (2). This pragmatic approach avoids burdensome notation usually associated with more complex models of hybrid systems, while still retaining most of what is needed for stability analysis.

## The Book

The book is divided into three parts covering modeling, stability analysis, and control design. Chapter 1 addresses switching system modeling. The

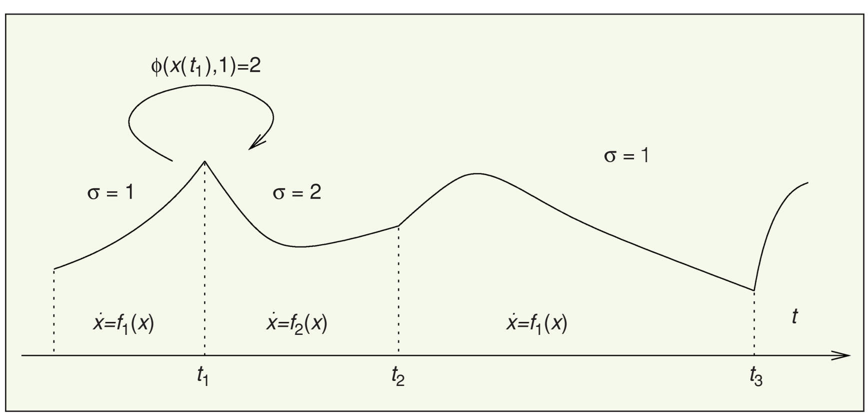



**Figure 1.** *Trajectory of a hybrid system. The switching signal $\sigma(t)$ takes on integer values that change at discrete-time instances.*

exposition is informal, avoiding a broad definition of switching systems. Instead, the book presents several classes of systems that are to be investigated. This chapter also discusses issues related to the existence of solutions. The chapter briefly reviews existence and uniqueness results for ordinary differential equations (ODEs) and then discusses issues that are specific to hybrid/switching systems, such as Zeno and sliding modes.

The second part of the book is devoted to stability analysis. Chapter 2 overviews results concerning the well-studied but exceedingly difficult question of determining whether a switching system such as (1) is stable for *every* switching signal $\sigma(t)$. Lyapunov-based necessary and sufficient conditions for global uniform asymptotic stability (GUAS) are provided. The main challenge in applying these conditions is, of course, finding appropriate Lyapunov functions. For switching systems, this problem is especially severe because, even for linear (or linearized) systems, restricting the search to quadratic Lyapunov functions may lead to conservative results, as shown by the counterexample in 2.1.5. Chapter 2 also provides several algebraic conditions that are sufficient for GUAS. These conditions are generally simpler to verify but can be quite conservative. This is the case, for example, in the elegant but nongeneric Lie-algebraic conditions in Section 2.2.

Chapter 3 addresses the stability of systems for which the switching signal satisfies prespecified constraints. These constraints can be formulated in terms of how often switching is allowed (Section 3.2) or where in the state space switching can occur (Section 3.3). The main tool used to establish these results relies on multiple Lyapunov functions, popularized by DeCarlo, Branicky, and coauthors. This chapter also addresses the design question of when it is possible to stabilize a switching system by appropriate selection of the switching signal.

The third part of the book is devoted to control problems where switching is introduced by design. Chapter 4 considers the stabilization of nonlinear systems that are open-loop controllable to the origin but for which there is no continuous feedback controller that makes the origin asymptotically stable. Brockett's nonholonomic integrator, as well as many underactuated vehicles, falls in this class. The use of a hybrid controller provides a way out of this difficulty, as illustrated in this chapter. Essentially, a smooth stabilizing controller is used away from a zero-measure singular surface, and switching is used to "kick" the state of the closed-loop system away from this surface. This technique, which has general applicability, is illustrated for swing-up and stabilization of an inverted pendulum.

Chapter 5 addresses switching systems that arise from sensor and actuation limitations. The use of bang-bang controls in the presence of actuator saturation is a well-known example of such switching systems. Quantization of sensor measurements and control signals also results in switching systems. This topic is covered thoroughly in Section 5.3, which is not surprising since Liberzon made significant contributions to this area, starting with his Ph.D. work. Section 5.2 also provides a brief discussion of linear system stabilization using static output feedback.

The last chapter of the book focuses on supervisory control, which can be viewed as an alternative to adaptive control, where continuous tuning is replaced by controller switching. However, the basic objective of stabilizing a process with large modeling uncertainty remains the same. Stability analysis of supervisory control closed loops is fundamentally different from that of adaptive control in that supervisory control makes full use of switching-system stability results. In fact, concepts such as (average) dwell-time switching and related results, now standard in the switching systems literature, first appeared as (somewhat obscure) definitions and lemmas in early supervisory control papers. Chapter 6 provides what is perhaps one of the most tutorial treatments on supervisory control published so far, covering both the linear and nonlinear cases.

References to related work are mostly absent from the body of the text; they appear at the end of the book in a separate section, which also contains historical remarks. This arrangement avoids distractions from the presentation of the material. The book also contains an appendix on stability theory, the use of which permeates the book, and another appendix on Lie algebras, which is needed only for Section 2.2.

## Who Can Use This Book?

The book reads somewhere between a graduate textbook and a research monograph. I can envision two ways in which the book can be used in teaching. First, the book can serve as the primary textbook of a graduate course on switching systems that covers the introductory material in Part 1, most of the stability material in Part 2, and a subset of the control applications in Part 3. Alternatively, this book can support the second half of a course on hybrid systems, which would start with an overview of hybrid systems modeling and reachability analysis (not covered in this book) and then proceed to the topic of switching systems, covering only subsets of Parts 2 and 3. A course along the lines of the first option is taught at the University of Illinois at Urbana-Champagne by Liberzon [1], whereas a course following the second approach is taught, for example, at the University of California, Santa Barbara [2]. In both cases, students are likely to appreciate the clear writing style, concise proofs, and unpretentious notation of the book.

As a teaching aid, the book's main weakness is the lack of exercises that illustrate the applicability of the concepts and results to practical problems. Although the book contains several exercises, these problems largely complement the theoretical derivations but do not focus on practical applications of switched systems. The third part of the book covers application areas for hybrid systems, but one can imagine impatient students being eager to explore practical applications of switching systems earlier in the course. An instructor might want to seek permission to complement the problems in the book with examples and exercises taken from the Web sites listed below.

## Conclusions

In summary, this book provides a fairly complete picture of the stability results available in the literature for switching systems. The book's strength is its unified, coherent coverage of the topics. Liberzon is successful in presenting this material in a form that is pleasant to read and yet mathematically accurate. The book does not attempt to cover the general topic of hybrid systems, which I believe contributes to its success. However, questions such as controllability and observability of switching systems are conspicuously absent. Perhaps in a second edition.

## References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