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Automatica 43 (2007) 639–646
automatica
www.elsevier.com/locate/automatica

Brief paper

Input-to-state stability of switched systems and switching adaptive control☆

L. Vu*, D. Chatterjee, D. Liberzon

Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

Received 4 August 2005; received in revised form 15 June 2006; accepted 9 October 2006
Available online 25 January 2007

Abstract

In this paper we prove that a switched nonlinear system has several useful input-to-state stable (ISS)-type properties under average dwell-time switching signals if each constituent dynamical system is ISS. This extends available results for switched linear systems. We apply our result to stabilization of uncertain nonlinear systems via switching supervisory control, and show that the plant states can be kept bounded in the presence of bounded disturbances when the candidate controllers provide ISS properties with respect to the estimation errors. Detailed illustrative examples are included.
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Keywords: Switched systems; Nonlinear systems; Supervisory control; Adaptive control; Input-to-state stability

1. Introduction

Switched systems arise in situations where there are several dynamical subsystems and a switching signal specifying the active subsystem at each instant of time. In general, a switched system does not inherit properties of the individual subsystems; a well-known example is that switching among globally exponentially stable subsystems could lead to instability (see, e.g., Liberzon, 2003). Morse has shown that for *dwell-time switching signals*, a switched linear system is exponentially stable if the individual subsystems are exponentially stable (Morse, 1996). This result was later extended to *average dwell-time switching signals* and to switched linear systems with inputs and switched nonlinear systems without inputs (Hespanha & Morse, 1999b). For switched nonlinear systems with inputs and dwell-time switching signals, a switched system is *input-to-state stable* (ISS) if the individual subsystems are ISS (Xie, Wen, & Li, 2001); see also (Liberzon, 1999, Section 5). If the individual subsystems are *integral input-to-state stable* (iISS), the switched system remains iISS with state-dependent dwell-time switching signals (De Persis, De Santis, & Morse, 2003).

This paper extends the results in Hespanha and Morse (1999b) to switched nonlinear systems with inputs. When the individual subsystems of a switched system are ISS and their ISS–Lyapunov functions satisfy a suitable condition (which was also used in Hespanha & Morse, 1999b), we show that for switching signals with sufficiently large average dwell-time, the switched system has *ISS*, *exponentially weighted-ISS*, and *exponentially weighted-iISS* properties. Similar to the linear case in Hespanha and Morse (1999b), these exponentially weighted properties provide quantitative descriptions of external stability. Unlike the ISS result in Xie et al. (2001) which relies on dwell-time switching, our result only requires average dwell-time switching, which is a less stringent requirement. Compared to state-dependent dwell-time switching employed in De Persis et al. (2003) which requires the knowledge of the state, average dwell-time switching can be achieved using simple hysteresis-based switching logics (Hespanha, Liberzon, & Morse, 2000; Hespanha & Morse, 1999b).

We apply our results on switched systems to the problem of stabilizing uncertain nonlinear systems in the presence of disturbances via *switching supervisory control* (Hespanha,

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Bernard Brogliato under the direction of Editor H.K. Khalil. This work was supported by NSF ECS-0134115 CAR and DARPA/AFOSR MURI F49620-02-1-0325 grants.
* Corresponding author.
E-mail addresses: linhvu@uiuc.edu (L. Vu), dchatter@control.csl.uiuc.edu (D. Chatterjee), liberzon@uiuc.edu (D. Liberzon).

0005-1098/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2006.10.007

Liberzon, & Morse, 2003a,b; Morse, 1996, 1997). In switching supervisory control, a *supervisor* orchestrates switching among a parameterized family of *candidate controllers* by appropriately filtering the estimation errors coming out of the *multi-estimator*. This control scheme with the *scale-independent hysteresis switching logic* has been applied successfully to linear systems in the presence of modeling uncertainty and disturbances (Hespanha et al., 2001), and a recent research direction is to extend the result to nonlinear plants. For nonlinear plants with the same switching logic, it has been shown that if there are no disturbances, then switching stops in finite time and the states converge to zero (Hespanha, Liberzon, & Morse, 2002; Hespanha & Morse, 1999a). However, in the presence of disturbances, switching is not guaranteed to stop and the states can diverge. We show that using switching supervisory control with the scale-independent hysteresis switching logic, the states of an uncertain nonlinear plant can be kept bounded for arbitrary initial conditions and bounded disturbances when the controllers provide the ISS property with respect to the estimation errors.

2. Preliminaries

Consider a family of systems

$$\dot{x} = f_p(x, v), \quad p \in \mathscr{P}, \tag{1}$$

where the state $x \in \mathbb{R}^n$, the input $v \in \mathbb{R}^\ell$, and $\mathscr{P}$ is an index set. For each $p \in \mathscr{P}$, f_p is locally Lipschitz and $f_p(0,0) = 0$. A *switched system* generated by the family of systems (1) and a *switching signal* σ is

$$\dot{x} = f_\sigma(x, v), \tag{2}$$

where $\sigma : [0,\infty) \to \mathscr{P}$ is a piecewise constant function, continuous from the right, specifying at every time the index of the active system. The input $v \in \mathscr{V}$, the set of measurable functions $v : [0,\infty) \to \mathbb{R}^\ell$. Assume that there are no jumps in the state at the switching instants, and that a finite number of switches occur on every bounded time interval. Denote by $x(t)$ the trajectory of the system (2) at time $t \geqslant 0$, starting at x_0 at $t = 0$.

The switched system (2) is ISS (Sontag & Wang, 1995) if $\exists\, \beta \in \mathscr{KL}$ and $\alpha, \gamma \in \mathscr{K}_\infty$, such that[1] $\forall v \in \mathscr{V}, x_0 \in \mathbb{R}^n$ we have

$$\alpha(|x(t)|) \leqslant \beta(|x_0|, t) + \gamma\left(\|v\|_{[0,t)}\right), \quad \forall t \geqslant 0, \tag{3}$$

where $|\cdot|$ the Euclidean norm, and $\|\cdot\|_{\mathscr{I}}$ is the supremum norm of a signal over an interval $\mathscr{I} \subseteq [0,\infty)$.

Definition 2.1. The switched system (2) is $e^{\lambda t}$*-weighted input-to-state stable (*$e^{\lambda t}$*-weighted ISS)* for some $\lambda > 0$ if $\exists\, \alpha_1, \alpha_2, \gamma \in \mathscr{K}_\infty$, such that $\forall v \in \mathscr{V}, x_0 \in \mathbb{R}^n$ we have

$$e^{\lambda t}\alpha_1(|x(t)|) \leqslant \alpha_2(|x_0|) + \sup_{s \in [0,t)} \{e^{\lambda s}\gamma(|v(s)|)\}, \quad \forall t \geqslant 0. \tag{4}$$

[1] See, e.g., Khalil (2002, p. 144) for the definitions of class $\mathscr{KL}$ and $\mathscr{K}_\infty$ functions.

The switched system (2) is $e^{\lambda t}$*-weighted integral input-to-state stable (*$e^{\lambda t}$*-weighted iISS)* for some $\lambda > 0$ if $\exists\, \alpha_1, \alpha_2, \gamma \in \mathscr{K}_\infty$, such that $\forall v \in \mathscr{V}, x_0 \in \mathbb{R}^n$ we have

$$e^{\lambda t}\alpha_1(|x(t)|) \leqslant \alpha_2(|x_0|) + \int_0^t e^{\lambda \tau}\gamma(|v(\tau)|)\, d\tau, \quad \forall t \geqslant 0. \tag{5}$$

The $e^{\lambda t}$-weighted ISS and $e^{\lambda t}$-weighted iISS properties generalize ISS and iISS properties[2] in the spirit of exponentially weighted induced norms considered in Hespanha and Morse (1999b). While the ISS property characterizes stability in general, the $e^{\lambda t}$-weighted ISS and $e^{\lambda t}$-weighted iISS properties characterize stability with a "stability margin" λ (similar to stability margin of linear systems), which is useful in quantitative analysis (e.g., in supervisory control as we will see later).

3. Input-to-state properties of switched systems

Recall that a switching signal σ has an *average dwell-time* τ_a if there are numbers $N_\circ, \tau_a > 0$ such that

$$N_\sigma(T, t) \leqslant N_\circ + \frac{T - t}{\tau_a}, \quad \forall T \geqslant t \geqslant 0, \tag{6}$$

where $N_\sigma(T, t)$ is the number of switches in the interval $[t, T)$ (Hespanha & Morse, 1999b) (see, e.g., Liberzon, 2003, p. 58 for more discussions). The following theorem is an extension of the results in Hespanha and Morse (1999b) to switched nonlinear systems with inputs.

Theorem 3.1. *Consider the switched system* (2). *Suppose that there exist continuously differentiable functions* $V_p : \mathbb{R}^n \to [0,\infty)$, $p \in \mathscr{P}$, *class* $\mathscr{K}_\infty$ *functions* $\bar{\alpha}_1, \bar{\alpha}_2, \bar{\gamma}$, *and numbers* $\lambda_\circ > 0$, $\mu \geqslant 1$ *such that* $\forall \xi \in \mathbb{R}^n, \eta \in \mathbb{R}^\ell$, *and* $\forall p, q \in \mathscr{P}$, *we have*

$$\bar{\alpha}_1(|\xi|) \leqslant V_p(\xi) \leqslant \bar{\alpha}_2(|\xi|), \tag{7}$$

$$\frac{\partial V_p}{\partial \xi} f_p(\xi, \eta) \leqslant -\lambda_\circ V_p(\xi) + \bar{\gamma}(|\eta|), \tag{8}$$

$$V_p(\xi) \leqslant \mu V_q(\xi). \tag{9}$$

Let σ *be a switching signal having average dwell-time* τ_a.

(i) *If* $\tau_a > \ln \mu/\lambda_\circ$, *then the switched system* (2) *is ISS.*
(ii) *If* $\tau_a > \ln \mu/(\lambda_\circ - \lambda)$ *for some* $\lambda \in (0, \lambda_\circ)$, *then the switched system* (2) *is* $e^{\lambda t}$*-weighted ISS.*
(iii) *If*

$$\tau_a \geqslant \ln \mu/\lambda_\circ - \lambda \tag{10}$$

for some $\lambda \in (0, \lambda_\circ)$, *then the switched system* (2) *is* $e^{\lambda t}$*-weighted iISS.*

Proof. For notational brevity, define $G_a^b(\lambda) := \int_a^b e^{\lambda s}\bar{\gamma}(|v(s)|)\, ds$. Let $T > 0$ be an arbitrary time. Denote by $\tau_1, \ldots, \tau_{N_\sigma(T,0)}$ the switching times on the interval $(0, T)$ (by convention,

[2] See Sontag (1998) for the original definition of iISS.

$\tau_0 := 0, \tau_{N_\sigma(T,0)+1} := T$). Consider the piecewise continuously differentiable function

$$W(s) := \mathrm{e}^{\lambda_\circ s} V_{\sigma(s)}(x(s)). \tag{11}$$

On each interval $[\tau_i, \tau_{i+1})$, the switching signal is constant. From (11) and (8), $\dot{W}(s) \leqslant \mathrm{e}^{\lambda_\circ s}\overline{\gamma}(|v(s)|), \forall s \in [\tau_i, \tau_{i+1})$. Integrating both sides of the foregoing inequality from τ_i to τ_{i+1}^- and using (9) we arrive at

$$W(\tau_{i+1}) \leqslant \mu W(\tau_{i+1}^-) \leqslant \mu(W(\tau_i) + G_{\tau_i}^{\tau_{i+1}}(\lambda_\circ)). \tag{12}$$

Iterating (12) from $i = 0$ to $i = N_\sigma(T, 0)$, we get

$$W(T^-) \leqslant \mu^{N_\sigma(T,0)} \left(W(0) + \sum_{k=0}^{N_\sigma(T,0)} \mu^{-k} G_{\tau_k}^{\tau_{k+1}}(\lambda_\circ) \right). \tag{13}$$

From (10), for every $\delta \in [0, \lambda_\circ - \lambda - \ln \mu/\tau_a]$, we have $\tau_a \geqslant \ln \mu/(\lambda_\circ - \lambda - \delta)$, and by virtue of (6), we get

$$N_\sigma(T, s) \leqslant N_\circ + (\lambda_\circ - \lambda - \delta)(T - s)/\ln \mu, \quad \forall s \in [0, T].$$

Since $N_\sigma(T, 0) - k - 1 \leqslant N_\sigma(T, \tau_{k+1})$, it follows that

$$\mu^{N_\sigma(T,0)-k} \leqslant \mu^{1+N_\circ} \mathrm{e}^{(\lambda_\circ-\lambda-\delta)(T-\tau_{k+1})} \tag{14}$$

for all $k = 0, \dots, N_\sigma(T, 0)$. Since $\lambda + \delta < \lambda_\circ$, we have

$$G_{\tau_k}^{\tau_{k+1}}(\lambda_\circ) \leqslant \mathrm{e}^{(\lambda_\circ-\lambda-\delta)\tau_{k+1}} G_{\tau_k}^{\tau_{k+1}}(\lambda + \delta). \tag{15}$$

From (13)–(15), we then arrive at

$$\overline{\alpha}_1(|x(T)|) \leqslant c\mathrm{e}^{-(\lambda+\delta)\mathrm{T}}(\overline{\alpha}_2(|x_0|) + G_0^{\mathrm{T}}(\lambda + \delta)), \tag{16}$$

$$c := \mu^{1+N_\circ}, \tag{17}$$

by virtue of (11), (7) and since x is continuous. Letting $\delta = 0$ in (16), we obtain (5) with $\alpha_1 := \overline{\alpha}_1$, $\alpha_2 := c\overline{\alpha}_2$, $\gamma := c\overline{\gamma}$. From the definition of $G_a^b(\lambda)$, we have

$$G_0^{\mathrm{T}}(\lambda + \delta) \leqslant \frac{c_1}{c} \mathrm{e}^{(\lambda+\delta-\bar{\lambda})\mathrm{T}} \sup_{\tau\in[0,T)} \{\mathrm{e}^{\bar{\lambda}\tau}\overline{\gamma}(|v(\tau)|)\} \tag{18}$$

for every $\bar{\lambda} \in [0, \lambda + \delta)$ where $c_1 := c/(\lambda + \delta - \bar{\lambda})$. From (18) and (16), we obtain

$$\overline{\alpha}_1(|x(T)|) \leqslant c\mathrm{e}^{-(\lambda+\delta)\mathrm{T}}\overline{\alpha}_2(|x_0|) + c_1\mathrm{e}^{-\bar{\lambda}\mathrm{T}} \sup_{\tau\in[0,T)} \{\mathrm{e}^{\bar{\lambda}\tau}\overline{\gamma}(|v(\tau)|)\}, \quad \forall T \geqslant 0. \tag{19}$$

Picking some $\delta \in (0, \lambda_\circ - \lambda - \ln \mu/\tau_a)$, and letting $\bar{\lambda} = \lambda$ in (19), we have property (4) with $\alpha_1 := \overline{\alpha}_1$, $\alpha_2 := c\overline{\alpha}_2$, and $\gamma := c_1\overline{\gamma}$. If we let $\bar{\lambda} = 0$, $\delta = 0$ in (19), we have property (3) with $\alpha := \overline{\alpha}_1$, $\beta(r, s) := c\mathrm{e}^{-\lambda s}\overline{\alpha}_2(r)$, and $\gamma := c\overline{\gamma}/\lambda$ by the fact that $\sup_{\tau\in[0,T)} \overline{\gamma}(|v(\tau)|) \leqslant \overline{\gamma}\left(\|v\|_{[0,T)}\right)$. □

An immediate consequence of Theorem 3.1 is that for a switched system satisfying (7)–(9), if the input v is bounded, then the state x is bounded for average dwell-time switching signals with average dwell-time τ_a satisfying (10).[3] If the input goes to zero, the average dwell-time switching implies global asymptotic stability of the switched system.

Remark 1. If each individual subsystem in the family (1) is ISS, then for every $p \in \mathscr{P}$ there exist functions $\overline{\alpha}_{1,p}, \overline{\alpha}_{2,p}, \overline{\gamma}_p \in \mathscr{K}_\infty$, numbers $\lambda_{\circ,p} > 0$, and ISS–Lyapunov functions V_p, satisfying $\overline{\alpha}_{1,p}(|\xi|) \leqslant V_p(\xi) \leqslant \overline{\alpha}_{2,p}(|\xi|)$ and $(\partial V_p/\partial\xi) f_p(\xi) \leqslant -\lambda_{\circ,p}V_p(\xi) + \overline{\gamma}_p(|\eta|)$ $\forall \xi \in \mathbb{R}^n, \eta \in \mathbb{R}^\ell$; see Praly and Wang (1996), and Sontag and Wang (1995). If the set $\mathscr{P}$ is finite or if the set $\mathscr{P}$ is compact and suitable continuity assumptions on $\left\{\overline{\alpha}_{1,p}, \overline{\alpha}_{2,p}, \overline{\gamma}_p\right\}_{p\in\mathscr{P}}$ and $\left\{\lambda_{\circ,p}\right\}_{p\in\mathscr{P}}$ with respect to p hold, then (7) and (8) follow. The set of possible ISS–Lyapunov functions is restricted by (9). This inequality does not hold, for example, if V_p is quadratic for one value of p and quartic for another. If $\mu = 1$, (9) implies that $V = V_p$, $p \in \mathscr{P}$, is a common ISS–Lyapunov function for the family of systems (1), and the switched system is ISS for *arbitrary switching* (also called *uniformly input-to-state stable* Mancilla-Aguilar & Garcia, 2000).

4. Application to switching supervisory control of nonlinear systems

We quickly review here the switching supervisory control framework; for details, see, e.g., Liberzon (2003, Chapter 6) and references therein. Suppose that a plant $\mathbb{P}$ belongs to a family of plants parameterized by a parameter $p \in \mathscr{P}$, for some known index set $\mathscr{P}$ of m elements, and denote by $p^\star \in \mathscr{P}$ the true value of the unknown parameter:

$$\dot{x} = f(x, u, p^\star, d), \quad y = h(x, p^*),$$

where x, y, u, d are the state, output, input and disturbance, respectively. These parameterized plants are common in the context of adaptive control; see, e.g., Hespanha et al. (2003b) for more discussions and some practical systems. A family of *candidate controllers*

$$\dot{x}_{\mathbb{C}} = g_q(x_{\mathbb{C}}, y, u), \quad u_q = r_q(x_{\mathbb{C}}, y), \quad q \in \mathscr{P}, \tag{20}$$

are designed such that the qth controller stabilizes the plant with index q (see, e.g., Freidovich & Khalil, 2005) for an example of using a particular type of controllers—sliding-mode controllers). Controller selection is carried out by a high-level *supervisor*, which comprises three subsystems:

(i) The first subsystem is a *multi-estimator*:

$$\dot{x}_{\mathbb{E}} = F(x_{\mathbb{E}}, y, u), \quad y_p = h_p(x_{\mathbb{E}}), \quad p \in \mathscr{P}. \tag{21}$$

Let $e_p = y_p - y$, $p \in \mathscr{P}$ be the estimation errors. The multi-estimator has the following property.

[3] It has come to the authors' attention that the ISS property of switched nonlinear systems under average dwell-time switching (but not the $\mathrm{e}^{\lambda t}$-weighted ISS and $\mathrm{e}^{\lambda t}$-weighted iISS properties) has been independently reported without proof in Feng and Zhang (2005).

Assumption 4.1. The disturbance d is bounded by $\bar{d}$. There exists a constant $c_0 > 0$ (c_0 depends on $\bar{d}$ and may depend on the initial states $x(0)$ and $x_{\mathbb{E}}(0)$) such that $|e_{p^\star}(t)| \leqslant c_0 \ \forall t \geqslant 0$.

There is a family of *injected systems*, where the injected system indexed by $q \in \mathscr{P}$ comprises the multi-estimator and the corresponding controller:

$$\dot{x}_{\mathbb{CE}} = \begin{bmatrix} g_q(x_{\mathbb{C}}, y, r_q(x_{\mathbb{C}}, y)) \\ F(x_{\mathbb{E}}, y, r_q(x_{\mathbb{C}}, y)) \end{bmatrix} =: f_q(x_{\mathbb{CE}}, e_q)$$

by virtue of $y = h_q(x_{\mathbb{E}}) - e_q \ \forall q \in \mathscr{P}$, where $x_{\mathbb{CE}} := [x_{\mathbb{C}}^{\mathrm{T}} \ x_{\mathbb{E}}^{\mathrm{T}}]^{\mathrm{T}}$ is the state of the injected system; $x_{\mathbb{CE}} \in \mathbb{R}^n$; $e_q \in \mathbb{R}^\ell$. The *switched injected system* is generated by the above family of injected systems and some switching signal σ defined in (iii) below.[4]

(ii) The second subsystem is the *monitoring signal generator* generating the *monitoring signals* μ_p, $p \in \mathscr{P}$:

$$\dot{z}_p = -\lambda z_p + \bar{\gamma}(|e_p|), \quad z_p(0) = 0,$$
$$\mu_p(t) = \varepsilon + z_p(t), \tag{22}$$

for some $\varepsilon > 0$, $\lambda \in (0, \lambda_\circ)$, where $\lambda_\circ$, $\bar{\gamma}$ are in (8).

(iii) The third subsystem is a *switching logic*. We use the *scale-independent hysteresis switching logic*:

$$\sigma(t) := \begin{cases} \arg\min_{q\in\mathscr{P}} \mu_q(t) & \text{if } \exists q \in \mathscr{P} \text{ such that} \\ & (1+h)\mu_q(t) \leqslant \mu_{\sigma(t^-)}(t), \\ \sigma(t^-) & \text{else,} \end{cases} \tag{23}$$

where $h > 0$ is a design parameter such that

$$\frac{\ln(1+h)}{\lambda m} > \frac{\ln \mu}{\lambda_\circ - \lambda}. \tag{24}$$

Note that the above hysteresis switching logic is scale-independent—the switching signal σ is unaltered when we multiply all the monitoring signals by a positive scalar. Let $\bar{\mu}_p(t) := \mathrm{e}^{\lambda t}\mu_p(t),\ t \geqslant 0,\ p \in \mathscr{P}$, be the scaled version of μ_p. From (22), for each $p \in \mathscr{P}$, we have

$$\bar{\mu}_p(t) = \varepsilon \mathrm{e}^{\lambda t} + \int_0^t \mathrm{e}^{\lambda s}\bar{\gamma}(|e_p(s)|)\,\mathrm{d}s, \quad t \geqslant 0, \tag{25}$$

which indicates that $\bar{\mu}_p$ is continuous and monotonically nondecreasing. Lemma 4.2 below provides a property of the switching signals generated by the switching logic (23) (cf. Hespanha et al., 2000, Theorem 1); the proof is along the lines of Hespanha et al. (2000) (with more careful counting) and is omitted due to space limitation.

[4] By switched injected system we mean that there are no jumps in $x_{\mathbb{CE}}$ at switching times. When implementing (20), at each switching instant τ_i, we can ensure that $x_{\mathbb{C}}(\tau_i^-) = x_{\mathbb{C}}(\tau_i)$, and thus $x_{\mathbb{C}}$ is continuous; $x_{\mathbb{E}}$ is continuous in view of (21).

Lemma 4.2. *For every $q \in \mathscr{P}$ and $t \geqslant t_0 \geqslant 0$, we have*

$$N_\sigma(t, t_0) \leqslant m + \frac{m}{\ln(1+h)} \ln\left(\frac{\bar{\mu}_q(t)}{\min_{p\in\mathscr{P}}\bar{\mu}_p(t_0)}\right), \tag{26}$$

$$\sum_{k=0}^{N_\sigma(t,t_0)} (\bar{\mu}_{\sigma(\tau_k)}(\tau_{k+1}) - \bar{\mu}_{\sigma(\tau_k)}(\tau_k)) \leqslant m((1+h)\bar{\mu}_q(t) - \min_{p\in\mathscr{P}}\bar{\mu}_p(t_0)), \tag{27}$$

where $\tau_1, \tau_2, \ldots, \tau_{N_\sigma(t,t_0)}$ are the discontinuities of σ on (t_0, t) and $\tau_{N_\sigma(t,t_0)+1} := t$, $\tau_0 := t_0$.

Letting $p = p^\star$ in (25), in view of Assumption 4.1, we get

$$\bar{\mu}_{p^\star}(t) \leqslant \kappa \mathrm{e}^{\lambda t}, \quad \kappa := \varepsilon + \bar{\gamma}(c_0)/\lambda. \tag{28}$$

Since $\min_{p\in\mathscr{P}} \bar{\mu}_p(t_0) \geqslant \varepsilon \mathrm{e}^{\lambda t_0} \ \forall t_0 \geqslant 0$, (26) with $q = p^\star$ and (28) yield $N_\sigma(t, t_0) \leqslant N_\circ + (t - t_0)/\tau_a$, where $N_\circ := m + m\ln(\kappa/\varepsilon)/\ln(1+h)$, and $\tau_a := \ln(1+h)/(\lambda m)$. With $q = p$ in (27), from (25) and (28), we arrive at

$$\int_0^t \mathrm{e}^{\lambda s}\bar{\gamma}(|e_{\sigma(s)}(s)|)\,\mathrm{d}s + \varepsilon \mathrm{e}^{\lambda t} - \varepsilon = \sum_{k=0}^{N_\sigma(t,t_0)} (\bar{\mu}_{\sigma(\tau_k)}(\tau_{k+1}) - \bar{\mu}_{\sigma(\tau_k)}(\tau_k)) \leqslant m(1+h)\kappa \mathrm{e}^{\lambda t}. \tag{29}$$

We have the following result on switching supervisory control of nonlinear plants with disturbances.

Theorem 4.3. *Suppose that*

(i) *the state x of the process $\mathbb{P}$ is bounded when the input u, output y and disturbance d are bounded,*
(ii) *the multi-estimator is designed such that Assumption* 4.1 *holds,*
(iii) *the candidate controllers are designed such that the hypotheses of Theorem* 3.1 *hold for the switched injected system.*

Then under the supervisor with scale-independent hysteresis switching logic, all continuous states of the closed-loop system are bounded for arbitrary initial conditions and bounded disturbances.

Proof. From hypothesis (iii) and the condition on average dwell-time (24), it follows from Theorem 3.1 that the state of switched injected system $x_{\mathbb{CE}}$ has the $\mathrm{e}^{\lambda t}$-weighted iISS property:

$$\mathrm{e}^{\lambda t}\bar{\alpha}_1(|x_{\mathbb{CE}}(t)|) \leqslant c\bar{\alpha}_2(|x_{\mathbb{CE}}(0)|) + c\int_0^t \mathrm{e}^{\lambda s}\bar{\gamma}(|e_\sigma(s)|)\,\mathrm{d}s,$$

where $\bar{\alpha}_1, \bar{\alpha}_2$, and $\bar{\gamma}$ are $\mathscr{K}_\infty$ functions as in (7). The above inequality and (29) yield

$$|x_{\mathbb{CE}}(t)| \leqslant \bar{\alpha}_1^{-1}(c\bar{\alpha}_2(|x_{\mathbb{CE}}(0)|) + cm(1+h)\kappa) =: c_2 \quad \forall t \geqslant 0. \tag{30}$$

We have $\forall q \in \mathscr{P}$, $\forall t \geqslant 0$, $|y_q(t)| = |h_q(x_{\mathbb{E}}(t))| \leqslant \sup_{p\in\mathscr{P},|\xi|\leqslant c_2} \{|h_p(\xi)|\} =: c_3$. Since $y = y_{p^\star} - e_{p^\star}$ and $|e_{p^\star}(t)| \leqslant c_0\ \forall t \geqslant 0$ (by Assumption 4.1), it follows that $|y(t)| \leqslant c_3 + c_0 =: c_4\ \forall t \geqslant 0$. Also $e_q = y_q - y$, and therefore $|e_q(t)| \leqslant c_0 + 2c_3 =: c_5\ \forall q \in \mathscr{P}$, $\forall t \geqslant 0$. Further, we have $\forall t \geqslant 0$, $|u(t)| \leqslant \sup_{q\in\mathscr{P},|\xi|\leqslant c_2,|\eta|\leqslant c_4}\{|r_q(\xi,\eta)|\} =: c_6$. Since d, u and y are bounded, the state x is bounded in view of hypothesis (i). Finally, every monitoring signal μ_q, $q \in \mathscr{P}$, is bounded since $|e_q|$ is bounded $\forall\ q \in \mathscr{P}$. □

Remark 2. Hypothesis (i) of Theorem 4.3 holds, for example, when the plant is input-output-to-state stable (see Sontag & Wang, 1997 for the definition). Hypothesis (ii) requires that at least one estimator provides a bounded estimation error in the presence of disturbances. This is more or less a standard assumption in multi-estimator design; a similar assumption was used in Hespanha and Morse (1999a) for plants without disturbances. Hypothesis (iii) stipulates that the injected systems are ISS (which was also an assumption in Hespanha & Morse, 1999a); the design of ISS injected systems is nontrivial, and is a topic of ongoing research (cf. Liberzon, Sontag, & Wang, 2002). All three hypotheses can be completely characterized via detectability and stabilizability of the plant for linear systems Morse, 1996, but characterizing the nonlinear plants for which these hypotheses hold is still an open question. However, there are certain nonlinear systems for which these conditions hold (see Example 1 below).

Remark 3. If the disturbance d is vanishing and in Assumption 4.1 we replace the constant bound c_0 with a time-varying bound $c_0(t) \to 0$ as $t \to \infty$, and further, if the plant is IOSS, then we can have $|x(t)| \to 0$ as $t \to \infty$ if we use a nonnegative decaying $\varepsilon(t)$ in the monitoring signal generator such that $\varepsilon(t) \to 0$ and $\overline{\gamma}(c_0(t))/\varepsilon(t) < \infty$ as $t \to \infty$ (which means ε should decay more slowly than $\overline{\gamma}(c_0)$). If this is the case, $\kappa \to 0$ in (28) and the chatter bound $N_\circ < \infty$. Then the iISS property of the switched injected system together with (29) yields $|x_{\mathbb{CE}}(t)| \leqslant \alpha_1^{-1}(\mathrm{e}^{-\lambda t}\alpha_2(|x_{\mathbb{CE}}(0)|) + cm(1+h)\kappa(t)) \to 0$ as $t \to \infty$; thus, c_2 in (30) becomes a time-varying $c_2(t) \to 0$ as $t \to \infty$. It then follows that $c_3(t), c_4(t), c_5(t), c_6(t) \to 0$ as $t \to \infty$ where $c_3(t), c_4(t), c_5(t), c_6(t)$ are the time-varying bounds in places of c_3, c_4, c_5, c_6 in the proof of the theorem. Since $|u(t)| \to 0$, $|y(t)| \to 0$ and the plant is IOSS, the state norm $|x(t)| \to 0$ as $t \to \infty$.

Example 1. Consider a scalar nonlinear plant

$$\dot{y} = y^2 + p^\star u + d, \tag{31}$$

where $p^\star$ is an unknown constant belonging to a finite index set $\mathscr{P}$ of m elements, $\mathscr{P} := \{p_1, \ldots, p_m\}$, and d is a disturbance. Our objective is to keep the state bounded in the presence of a bounded disturbance. The unknown parameter enters as the input gain, which makes the problem challenging to solve in the framework of conventional adaptive control when the sign of $p^\star$ is unknown.

The multi-estimator and the candidate controllers are

$$\begin{aligned}\dot{y}_p &= -(y_p - y) - (y_p - y)^3 + pu + y^2, \\ u_p &= \tfrac{1}{p}(-y - y^2 - y^3),\end{aligned} \qquad p \in \mathscr{P}.$$

The injected system for the controller with an index q is

$$\dot{y}_p = -(y_p - y) - (y_p - y)^3 + \frac{p}{q}(-y - y^2 - y^3) + y^2, \quad p \in \mathscr{P}. \tag{32}$$

Consider the candidate ISS–Lyapunov functions $V_q(x_{\mathbb{CE}}) := a_1 y_q^4 + b_1 y_q^2 + \sum_{p\neq q, p\in\mathscr{P}} a_0 y_p^4 + b_0 y_p^2$, $q \in \mathscr{P}$, where $x_{\mathbb{CE}} := [y_{p_1}, \ldots, y_{p_m}]^{\mathrm{T}}$ is the state of the injected system, for some $a_1, b_1, a_0, b_0 > 0$ to be determined. One can pick $\mu := \max\{a_1/a_0, a_0/a_1, b_1/b_0, b_0/b_1\}$. The derivative of V_q along the qth injected system is $\dot{V}_q = 4a_1 y_q^3 \dot{y}_q + 2b_1 y_q \dot{y}_q + \sum_{p\neq q, p\in\mathscr{P}} 4a_0 y_p^3 \dot{y}_p + 2b_0 y_p \dot{y}_p$. Substituting (32) into the foregoing $\dot{V}_q$, after some expansions and simplifications, we arrive at

$$\begin{aligned}\dot{V}_q \leqslant &-a_1 y_q^6 - 4a_1 y_q^4 - 2b_1 y_q^2 \\ &+ \sum_{\substack{p\neq q \\ p\in\mathscr{P}}} (-a_0 y_p^6 - 4a_0 y_p^4 - 2b_0 y_p^2 \\ &+ (4a_0 y_p^3 + 2b_0 y_p)\kappa_{pq} g(y)),\end{aligned} \tag{33}$$

where $\kappa_{pq} := (1 - p/q)$ and $g(y) := y + y^2 + y^3$. Define $\kappa_{\max} := \max\{|\kappa_{pq}| : p, q \in \mathscr{P}\}$. Using completions of the squares with $-a_0 y_p^6 - b_0 y_p^2 + (4a_0 y_p^3 + 2b_0 y_p^2)\kappa_{pq} g(y)$ and using the triangle inequality with $|g(y)|^2$ in (33), after some computations, we obtain

$$\begin{aligned}\dot{V}_q \leqslant &-V_q - (a_1 - 256(4a_0 + b_0)m\kappa_{\max}^2)y_q^6 \\ &- (b_1 - 16(4a_0 + b_0)m\kappa_{\max}^2)y_q^2 \\ &+ (4a_0 + b_0)m\kappa_{\max}^2(16e_q^2 + 256e_q^6).\end{aligned}$$

If b_0, a_0 are chosen such that $(4a_0 + b_0)m\kappa_{\max}^2 \leqslant 1$, $a_1 \geqslant 256$, $b_1 \geqslant 16$, we then get $\dot{V}_q \leqslant -V_q + \overline{\gamma}(|e_q|)$, where $\overline{\gamma}(r) := 16r^2 + 256r^6$ is a class $\mathscr{K}_\infty$ function. The foregoing inequality shows that for each fixed controller with index q, the corresponding injected system is ISS with respect to the output error e_q. By Theorem 4.3, all the continuous states are bounded for arbitrary initial conditions and bounded disturbances under the supervisor with scale-independent hysteresis switching logic for a large enough h satisfying (24).

For $\mathscr{P} = \{-2, -1, 1, 2\}$, $p^\star = 1$, numerical values are $m = 4$, $\kappa_{\max} = 3$, $a_0 = 6.5 \times 10^{-3}$, $b_0 = 0.5 \times 10^{-3}$, $a_1 = 256$, $b_1 = 16$, $\mu = 3.94 \times 10^4$. Choose $\varepsilon = 10^{-6}$, $\lambda = 2 \times 10^{-4}$. The hysteresis constant $h = 0.02$ satisfies the condition on average dwell-time (24). Simulation results in MATLAB® with disturbance uniformly distributed between -5 and 5 (solid lines) and exponentially decaying disturbance (dotted lines), and $x_0 = 0.1$, $x_{\mathbb{E}}(0) = 0$ are plotted in Fig. 1. The simulation shows that the state is bounded for the bounded disturbance and is decaying for the vanishing disturbance (indeed, the bound in the simulation is much smaller than those given by the analysis).

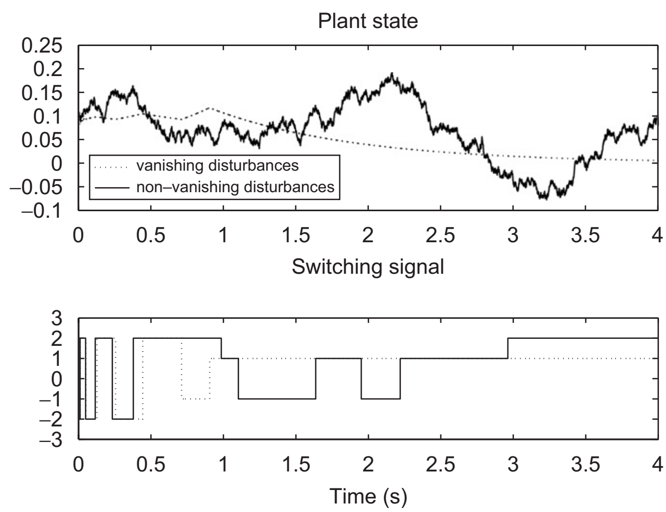

Fig. 1. Example 1.

4.1. Boundedness under weaker hypotheses

As noted in Remark 1, the existence of μ as in (9) for all ξ restricts the set of possible ISS–Lyapunov functions. We now assume that we only have μ such that the inequality (9) holds in some annulus $\Omega := \{\xi \in \mathbb{R}^n : r_1 \leqslant |\xi| \leqslant r_2\}$, for some numbers $r_2 > r_1 \geqslant 0$.

Consider the switched injected system described in the previous section. Suppose $\mu \geqslant 1$ such that $V_p(\xi) \leqslant \mu V_q(\xi)$, $\forall r_1 \leqslant |\xi| \leqslant r_2\ \forall p, q \in \mathscr{P}$. We can set $x_{\mathbb{CE}}(0) = 0$. Let $\hat{t}_1 := \inf\{t \geqslant 0 : |x_{\mathbb{CE}}(t)| > r_1\}$. If $\hat{t}_1 = \infty$, then $|x_{\mathbb{CE}}(t)| \leqslant r_1\ \forall t \geqslant 0$. Otherwise, let $\hat{t}_2 := \inf\{t \geqslant \hat{t}_1 : |x_{\mathbb{CE}}(t)| > r_2\}$ and $\check{t}_1 := \inf\{t \geqslant \hat{t}_1 : |x_{\mathbb{CE}}(t)| < r_1\}$ and $\bar{t} := \min\{\hat{t}_2, \check{t}_1\}$. Since $r_1 \leqslant |x_{\mathbb{CE}}(t)| \leqslant r_2\ \forall t \in [\hat{t}_1, \bar{t})$, it follows from (30) that $\forall t \in [\hat{t}_1, \bar{t})$, we have

$$|x_{\mathbb{CE}}(t)| \leqslant \overline{\alpha}_1^{-1}(c\overline{\alpha}_2(r_1) + cm(1+h)\kappa) =: c_2 \tag{34}$$

with c and κ being as in (17) and (28).

Let $\bar{x}_0$ and $\bar{d}$ be the bounds on the plant initial state and disturbance, respectively. Then the bound c_0 in Assumption 4.1 depends on $\bar{x}_0$ and $\bar{d}$ only. Suppose that $\bar{x}_0$ and $\bar{d}$ are sufficiently small such that $c_2 \leqslant r_2$. This inequality together with (34) and the definition of $\hat{t}_2$ imply that we must have $\hat{t}_2 = \infty$. If $\check{t}_1 = \infty$, then $\bar{t} = \infty$ and hence, $|x_{\mathbb{CE}}(t)| \leqslant c_2\ \forall t \geqslant 0$. If $\check{t}_1 < \infty$, then $\bar{t} = \check{t}_1$, and let $\hat{t}_3 := \inf\{t \geqslant \check{t}_1 : |x_{\mathbb{CE}}(t)| > r_1\}$. If $\hat{t}_3 = \infty$, then $|x_{\mathbb{CE}}(t)| \leqslant r_1 \leqslant c_2\ \forall t \geqslant \check{t}_1$, and hence, $|x_{\mathbb{CE}}(t)| \leqslant c_2\ \forall t \geqslant 0$; otherwise, repeat the current argument with $\hat{t}_3$ playing the role of $\hat{t}_1$. We can then conclude that $|x_{\mathbb{CE}}(t)| \leqslant c_2\ \forall t \geqslant 0$. From the boundedness of $x_{\mathbb{CE}}$, we can prove that all continuous states are bounded using similar arguments as in the proof of Theorem 4.3. We then have the following result.

Theorem 4.4. *Suppose that*

(i) *the state x of the process $\mathbb{P}$ is bounded when the input u, output y and disturbance d are bounded*;
(ii) *the multi-estimator is designed such that Assumption* 4.1 *holds*;
(iii) *the candidate controllers are designed such that hypotheses* (7) *and* (8) *of Theorem* 3.1 *hold for the switched injected system for some family of ISS–Lyapunov functions* $\{V_p\}_{p\in\mathscr{P}}$;
(iv) *there exist positive numbers r_1, r_2, μ, such that $V_q(\xi) \leqslant \mu V_q(\xi)\ \forall r_1 \leqslant |\xi| \leqslant r_2$ and positive numbers $\bar{x}_0$, $\bar{d}$ such that $c_2 \leqslant r_2$ for some $\varepsilon > 0, h > 0, 0 < \lambda < \lambda_\circ$ where c_2 is as in* (34).

Then under the supervisor with the scale-independent hysteresis switching logic, with hysteresis constant h, all continuous states of the closed-loop system are bounded for bounded disturbances $|d(t)| \leqslant \bar{d}$, $t \geqslant 0$ whenever the initial plant state $|x(0)| \leqslant \bar{x}_0$.

Example 2. Consider the scalar nonlinear plant in Example 1, and the following simpler multi-estimator and candidate controllers:

$$\begin{aligned} \dot{y}_p &= -(y_p - y) + y^2 + pu, \\ u_p &= -\tfrac{1}{p}(y + y^2), \end{aligned} \qquad p \in \mathscr{P}.$$

The injected system with the controller indexed by q is

$$\dot{y}_p = -(y_p - y) + y^2 + \frac{p}{q}(-y - y^2), \quad p \in \mathscr{P}.$$

Using the candidate ISS–Lyapunov function $V_q := b_1 y_q^4 + b_2 y_q^2 + a\sum_{p\neq q,\, p\in\mathscr{P}} y_p^2$, it can be checked that for each fixed controller indexed by $q \in \mathscr{P}$, the injected system is ISS:

$$\begin{aligned} \dot{V}_q &= -4b_1 y_q^4 - 2b_2 y_q^2 + 2a \sum_{p\neq q,\, p\in\mathscr{P}} y_p(-y_p + \kappa_{pq} y + \kappa_{pq} y^2) \\ &\leqslant -\lambda_\circ V_q + \overline{\gamma}(|e_q|), \end{aligned}$$

with $y^2 := |y_q - e_q|^2 \leqslant 2(y_q^2 + e_q^2)$ and $y^4 \leqslant 8(y_q^4 + e_q^4)$ for some $0 < \lambda_\circ < 2$, $a_1, a_2 > 0$, such that $a_1 + a_2 = 2 - \lambda_\circ$, where $\kappa_{pq} := (1 - p/q)$, $\kappa_{\max} := \max_{p,q\in\mathscr{P}}\{|\kappa_{pq}|\}$, $b_3 := a(m-1)\kappa_{\max}^2$, $\overline{\gamma}(r) := b_3(2r^2/a_1 + 8r^4/a_2)$, and b_1, b_2, a such that $b_3 < \min\{(4-\lambda_\circ)b_1 a_2/8, (2-\lambda_\circ)b_2 a_1/2\}$.

The ISS–Lyapunov functions V_q have the property (7), (8); however, there is no global μ as in (9) because V_q is quartic in y_q whilst $V_{p,p\neq q}$ are quadratic in y_q. Nevertheless, we can obtain a stability result using Theorem 4.4.

We can choose $\overline{\alpha}_1(r) := \min\{b_2, a\}r^2 =: \eta_1 r^2$ and $\overline{\alpha}_2(r) := \max\{(b_1 r_2^2 + b_2), a\}r^2 =: \eta_2 r^2$. Then $\mu := \eta_2/\eta_1$. The error dynamics for $p = p^\star$ is $\dot{e}_{p^\star} = -e_{p^\star} - d$ and hence, the bound on $e_{p^\star}$ is $|e_{p^\star}(t)| \leqslant |e_{p^\star}(0)| + \bar{d} \leqslant \bar{x}_0 + \bar{d}$ since $|e_{p^\star}(0)| = |y_{p^\star}(0) - y(0)| \leqslant \bar{x}_0$ by virtue of $y_p(0) = 0\ \forall p \in \mathscr{P}$. Now, $c_2 = (\mu^{1+N_\circ}(\eta_2 r_1^2 + m(1+h)\kappa)/\eta_1)^{1/2} < r_2$ if r_1 and $\bar{d}$ are small enough. Choosing the hysteresis constant h to satisfy the average dwell-time condition, we conclude that all the continuous states $x, x_{\mathbb{CE}}$ are bounded.

For $\mathscr{P} = \{-2, -1, 1, 2\}$, $p^\star = 1$, then $m = 4$, $\kappa_{\max} = 3$. Let $r_2 = 0.1$, $r_1 = 10^{-8}$, $b_1 = 2.96 \times 10^{-9}$, $b_2 = 1.3 \times 10^{-10}$, $a = 8 \times 10^{-12}$, $a_1 = 1.75$, $a_2 = 0.15$, $\lambda_\circ = 0.1$. Then $\mu = 19.95$. Choose

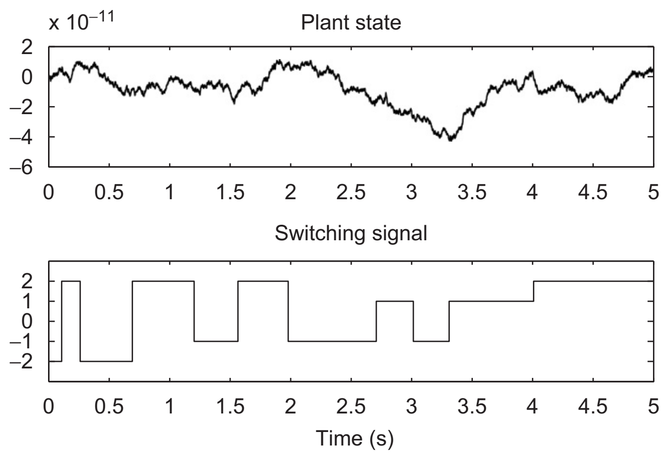

Fig. 2. Example 2.

$h = 0.05$, $\lambda = 0.0003$, $\varepsilon = 3.2914 \times 10^{-21}$. Then $N_\circ = 4.0819$. If $|x(0)| < 10^{-9}$ and $\bar{d} < 10^{-9}$, then all the states are bounded by $c_2 = 0.0836$ for all time. A simulation result with uniformly distributed disturbances between $(-\bar{d}, \bar{d})$ is in Fig. 2.

On the one hand, when ISS–Lyapunov functions satisfying (9) are not available, Theorem 4.4 can provide a way to achieve local boundedness of the plant state. There are more choices of ISS–Lyapunov functions, which can lead to simpler controller and multi-estimator designs, but it may be difficult to find the positive numbers in hypothesis (i) in Theorem 4.4. Also, the hysteresis constant h cannot be chosen arbitrarily small since λ cannot be arbitrarily small (c_2 increases when λ decreases). On the other hand, if we can find ISS–Lyapunov functions satisfying (9), Theorem 4.3 provides a global boundedness result. It also provides the flexibility to choose a small hysteresis constant h, which can be made arbitrarily small by reducing λ (see (24)), and a smaller h possibly leads to a better performance.

5. Conclusions

In this paper, we have shown that under switching signals with large enough average dwell-time, a switched system is ISS, $e^{\lambda t}$-weighted ISS, and $e^{\lambda t}$-weighted iISS, if the individual subsystems are ISS. We applied this result to show that using switching supervisory control with the scale-independent hysteresis switching logic, the states of an uncertain nonlinear plant can be kept bounded for arbitrary initial conditions and bounded disturbances, provided that the injected systems are ISS with respect to the estimation errors and there is a global constant μ as in (9). We relaxed the requirement of a global μ and achieved local boundedness of the plant state in the presence of bounded disturbances.

There are several possible avenues for future research. It is interesting to study the ISS property of switched systems under other classes of slowly switching signals that do not require the existence of a global constant μ in (9). Another consideration is to include measurement noises and unmodeled dynamics, as well as the case of a continuum $\mathscr{P}$. Regarding multi-estimators and controller design for nonlinear plants, this is a challenging problem and has not been solved in general. However, for certain classes of nonlinear systems that possess special structure, it may be possible to obtain interesting results in multi-estimator and controller design.

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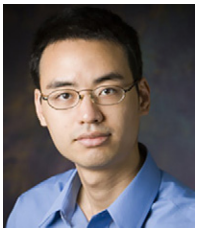

Linh Vu is currently a doctoral candidate in the Department of Electrical and Computer Engineering at the University of Illinois, Urbana-Champaign. He obtained a bachelor degree in Electrical Engineering with the highest distinction from the University of New South Wales, Sydney, Australia in 2002 and a master degree in Electrical Engineering from the University of Illinois, Urbana-Champaign in 2004. His research interests include nonlinear control, switched systems and hybrid dynamical systems, system identification, adaptive control, and broad applications of control.

Debasish Chatterjee was born in 1979 in India. He did his undergraduate studies in the Department of Electrical Engineering at Indian Institute of Technology, Kharagpur, from 1998 to 2002. Currently, he is a doctoral candidate in Electrical and Computer Engineering, University of Illinois at Urbana-Champaign. His research interests include switched and hybrid dynamical systems theory and stochastic control.

Daniel Liberzon was born in the former Soviet Union in 1973. He was a student in the Department of Mechanics and Mathematics at Moscow State University from 1989 to 1993 and received the Ph.D. degree in mathematics from Brandeis University, Waltham, MA, in 1998 (under the supervision of Prof. Roger W. Brockett of Harvard University). Following a postdoctoral position in the Department of Electrical Engineering at Yale University from 1998 to 2000, he joined the University of Illinois at Urbana-Champaign, where he is now an associate professor in the Electrical and Computer Engineering Department and an associate research professor in the Coordinated Science Laboratory. Dr. Liberzon's research interests include nonlinear control theory, analysis and synthesis of switched systems, control with limited information, and uncertain and stochastic systems. He is the author of the book *Switching in Systems and Control* (Birkhauser, 2003). Dr. Liberzon served as an Associate Editor on the IEEE Control Systems Society Conference Editorial Board in 1999–2000. He received the IFAC Young Author Prize and the NSF CAREER Award, both in 2002.