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★**Calculus of variations and optimal control theory.**

A concise introduction.

Princeton University Press, Princeton, NJ, 2012. *xviii*+235 *pp*. $75.00. *ISBN* 978-0-691-15187-8

This is an extremely well-crafted textbook. If you plan to teach a first course to advanced students on the calculus of variations and optimal control and you like the selection of topics that the author has chosen to present (and I do), it is the text you need. What impresses me most is the careful balance between the formal derivations and the explanations that precede or accompany the statements and proofs. These explanations are primarily geometrical, accompanied by suitable graphical illustrations. The role of the key mathematical assumptions is spelled out, which, in my opinion, is essential for understanding and applying the results. Some historical notes and elaboration on interconnections among different results and approaches make the text very attractive and usable also as a research reference.

The text covers classical developments in the calculus of variations and optimal control theory, primarily within the smooth framework. The level of smoothness needed at each point is spelled out, but the text does not present what can be achieved without smoothness, say with nonsmooth analysis. The text focuses on the variational approach, that is, what can be achieved when the candidate for a solution is perturbed. The different types of variations and the respective consequences are clearly displayed, starting with finite-dimensional optimization, through weak and strong variations within the classical theory, and the needle variations of the optimal control setting. The text elaborates on necessary conditions, from the Euler-Lagrange equations to the Pontryagin Maximum Principle. The Hamilton-Jacobi-Bellman equation is presented and its relation to the necessary conditions is discussed. Existence of optimal solutions is elaborated via Filippov's Theorem and its applications to linear systems. More advanced existence results, like the relaxed controls approach, are not presented. Likewise, system theoretic considerations and their relevance to optimization, e.g., stability, sensitivity, approximation, etc., are not presented, with one exception, namely, the linear-quadratic theory and the regulator. Accompanying the text are the classical examples of the calculus of variations and optimal control. Some advanced topics, like variations on manifolds, characteristics of the HJB equation, H_∞ theory and a maximum principle within a hybrid framework, are mentioned, but, intentionally, with considerably fewer details.

The book is aimed at the advanced undergraduate and graduate student level, and can be used as a textbook, including for mathematically inclined students in engineering schools. Exercises are offered, but they are more like mini-research tasks for experienced students. Good students who are at ease with mathematical derivations should be able to solve them, but for less experienced students, the instructor may need to accompany the text with a set of simpler exercises of a drilling type. All in all, it is a first-rate, enjoyable text.

Reviewed by *Zvi Artstein*