convolution product (A.7.12) we have

$$
\begin{aligned}
\widehat{f * h}= & \widehat{f_{a} * h_{a}}+\sum_{n=1}^{\infty} \widehat{f_{n} h_{a}}\left(\cdot-t_{n}\right)+\sum_{n=1}^{\infty} \widehat{h_{n} f_{a}}\left(\cdot-\tau_{n}\right)+ \\
& \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} h_{m} f_{n} \widehat{\delta}\left(\cdot-\left(t_{n}+\tau_{m}\right)\right)
\end{aligned}
$$

and so

$$
\begin{aligned}
(\widehat{f * h})(s)= & \widehat{f_{a}}(s) \widehat{h_{a}}(s)+\sum_{n=1}^{\infty} f_{n} e^{-s t_{n}} \widehat{h_{a}}(s)+ \\
& \sum_{n=1}^{\infty} h_{n} e^{-s \tau_{n}} \widehat{f_{a}}(s)+\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} h_{m} f_{n} e^{-\left(t_{n}+\tau_{m}\right) s}
\end{aligned}
$$

by Lemma A.6.5.c and the definition of the Laplace transform

$$
=\hat{f}(s) \cdot \hat{h}(s) \quad \text { for } s \in \overline{\mathbb{C}_{\beta}^{+}} .
$$

In our applications we shall consider the class of transfer functions given by

$$
\begin{equation*}
\hat{\mathcal{A}}(\beta):=\{\hat{f} \mid f \in \mathcal{A}(\beta)\} . \tag{A.7.19}
\end{equation*}
$$

Corollary A.7.48 $\hat{\mathcal{A}}(\beta)$ is a commutative Banach algebra with identity under pointwise addition and multiplication.

Proof This follows from the properties of $\mathcal{A}(\beta)$ and the Laplace transform as listed in Lemmas A.7.46 and A.7.47, respectively.

We quote two important properties of $\hat{\mathcal{A}}(\beta)$.
Theorem A.7.49 $\hat{f} \in \hat{\mathcal{A}}(\beta)$ is invertible over $\hat{\mathcal{A}}(\beta)$ if and only if

$$
\begin{equation*}
\inf _{s \in \in \mathbb{C}_{\beta}^{+}}|\hat{f}(s)|>0 . \tag{A.7.20}
\end{equation*}
$$

Proof Hille and Phillips [129, theorem 4.18.6].
Since $\mathcal{A}(\beta)$ is an integral domain with identity, we can define coprimeness as in Definition A.7.41. We note that there exist elements in its quotient algebra that do not admit coprime factorizations (Logemann [161] and Vidyasagar, Schneider, and Francis [251]).

Theorem A.7.50 $(\hat{f}, \hat{h})$ are coprime over $\hat{\mathcal{A}}(\beta)$ if and only if

$$
\begin{equation*}
\inf _{s \in \overline{\mathbb{C}_{\beta}^{+}}}(|\hat{f}(s)|+|\hat{h}(s)|)>0 . \tag{A.7.21}
\end{equation*}
$$

Proof Callier and Desoer [36], theorem 2.1.
We need the following facts about almost periodic functions from Corduneau [44] and Bohr [28].

Definition A.7.51 $f$ is almost periodic on the vertical strip $[\beta, \gamma]=\{s \in \mathbb{C} \mid \beta \leq$ $\operatorname{Re}(s) \leq \gamma\}$ if it is continuous there and for any $\varepsilon>0$ there corresponds a $\delta(\varepsilon)>0$ such that any interval of length $\delta(\varepsilon)$ on the imaginary axis contains at least one point, $\jmath \eta$, for which $|f(s+J \eta)-f(s)|<\varepsilon$ for any $s$ in this strip.

It is easy to see that $e^{-s t_{n}}$ is an almost periodic function on any vertical strip. In the next lemma, we shall show that this also holds for infinite sums of these terms.

Lemma A.7.52 Suppose that $\hat{f}(s)=\sum_{n=1}^{\infty} f_{n} e^{-s t_{n}}$, where $f_{n} \in \mathbb{C}, t_{n} \in \mathbb{R}$ and $t_{1}=0, t_{n}>0$ for $n \geq 2$ and $\sum_{n=1}^{\infty}\left|f_{n}\right| e^{-\beta t_{n}}<\infty$ for a given real $\beta$. Then $\hat{f}(s)$ is holomorphic on $\mathbb{C}_{\beta}^{+}$and bounded on $\overline{\mathbb{C}_{\beta}^{+}}$. Furthermore, $\hat{f}(s)$ is an almost periodic function on any vertical strip $[\beta, \beta+\mu], \mu>0$.
Proof In Lemma A. 7.47 we proved that $\hat{f}(s)$ is bounded on $\overline{\mathbb{C}_{\beta}^{+}}$. We also proved that it is holomorphic on $\mathbb{C}_{\beta}^{+}$and continuous on the line $s=\beta+\jmath \omega, \omega \in \mathbb{R}$; thus it is continuous on the vertical strip $[\beta, \beta+\mu]$ for $\mu>0$.

The rest of the proof can be found in Corduneau [44] following theorems 3.10 and 3.13. An alternative proof can be found in Bohr [28, appendix II].

That these functions are uniformly continuous on any closed vertical strip $[\beta+\varepsilon, \gamma]$ follows from the following general lemma.

Lemma A.7.53 Consider a function $g(s)$ that is holomorphic on the vertical open strip $(a, b)$ and bounded on any closed vertical strip $\left[a_{1}, b_{1}\right]$ contained in $(a, b)$. Then $g(s)$ is uniformly continuous on the closed vertical strip $\left[a_{1}, b_{1}\right]$.
Proof Corduneau [44, theorem 3.7].
Next we examine the asymptotic behavior of the almost periodic function $\sum_{n=1}^{\infty} f_{n} e^{-s t_{n}}$. Notice that while $e^{-s}$ tends to zero as $\operatorname{Re}(s) \rightarrow \infty$, it does not tend to zero as $|s| \rightarrow \infty$.

Lemma A.7.54 Suppose that $\hat{f}(s)=\sum_{n=1}^{\infty} f_{n} e^{-s t_{n}}$, where $f_{n} \in \mathbb{C}, t_{n} \in \mathbb{R}$ and $t_{1}=0, t_{n}>0$ for $n \geq 2$ and $\sum_{n=1}^{\infty}\left|f_{n}\right| e^{-\beta t_{n}}<\infty$ for a given real $\beta$. $\hat{f}$ satisfies
a. $\left|\hat{f}(s)-f_{1}\right| \rightarrow 0$ as $\operatorname{Re}(s) \rightarrow \infty$ uniformly with respect to $\operatorname{Im}(s)$;
b. $\sup _{s \in \overline{\mathbb{C}_{\beta}^{+}},|s| \geq \rho}|\hat{f}(s)| \rightarrow 0$ as $\rho \rightarrow \infty$ if and only if $\hat{f}(s)=0$ on $\overline{\mathbb{C}_{\beta}^{+}}$.

Proof $a$. The following estimate holds

$$
\left|\hat{f}(s)-f_{1}\right| \leq \sum_{n=2}^{\infty}\left|f_{n}\right| e^{-\operatorname{Re}(s) t_{n}} \leq\left[\sum_{n=2}^{\infty}\left|f_{n}\right| e^{-\beta t_{n}}\right] e^{-(\operatorname{Re}(s)-\beta) t_{\text {min }}}
$$

for $\operatorname{Re}(s)>\beta$, where $t_{\text {min }}$ is the infinum of $t_{n}, n \geq 2$. This establishes a for the case that $t_{\text {min }}$ is positive. For the more general case see Corduneau [44, theorem 3.20] or Bohr [28, p. 106].
b. Let $s_{0}$ be a element in $\overline{\mathbb{C}_{\beta}^{+}}$. We know that given $\varepsilon>0$ there exists $\rho_{1}>0$ such
that $|\hat{f}(s)|<\varepsilon$ for all $s \in\left\{s \in \overline{\mathbb{C}_{\beta}^{+}}\left||s| \geq \rho_{1}\right\}\right.$. Without loss of generality, we may assume that $\left|s_{0}\right|<\rho_{1}$. By Lemma A.7.52, $\hat{f}$ is almost periodic on the vertical strip [ $\beta, \rho_{1}$ ] and so by Definition A. 7.51 for $\varepsilon>0$, there exists a $\delta(\varepsilon)>0$ and a point $\eta \in\left[3 \rho_{1}, 3 \rho_{1}+\delta(\varepsilon)\right]$ such that $\left|\hat{f}\left(s_{1}+J \eta\right)-\hat{f}\left(s_{1}\right)\right|<\varepsilon$ for all $s_{1}$ in the vertical strip $\left[\beta, \rho_{1}\right]$. Since $s_{1}+j \eta \in\left\{s \in \overline{\mathbb{C}_{\beta}^{+}}| | s \mid \geq \rho_{1}\right\}$, we know that $\left|\hat{f}\left(s_{1}+j \eta\right)\right|<\varepsilon$ and consequently $\left|\hat{f}\left(s_{1}\right)\right|<2 \varepsilon$ holds for all $s_{1}$ in this vertical strip $\left[\beta, \rho_{1}\right]$. In particular, we conclude that $\left|\hat{f}\left(s_{0}\right)\right|<2 \varepsilon$. Since $s_{0} \in \overline{\mathbb{C}_{\beta}^{+}}$and $\varepsilon>0$ are arbitrary, it follows that $\hat{f}(s)=0$ on $\overline{\mathbb{C}_{\beta}^{+}}$.

Finally, we state an important result on the asymptotic behavior of elements in $\hat{\mathcal{A}}(\beta)$.
Corollary A.7.55 The function $\hat{f} \in \hat{\mathcal{A}}(\beta)$ has the limit zero as $s$ goes to infinity in $\overline{\mathbb{C}_{\beta}^{+}}$, i.e., $\sup _{s \in \mathbb{C}_{\beta}^{+}, s \mid \geq \rho}|\hat{f}(s)| \rightarrow 0$ as $\rho \rightarrow \infty$ if and only if $\hat{f}(\cdot)=\hat{f}_{a}(\cdot)$.

Proof This follows from Lemma A.7.54 and Property A.6.2.g.

The subclass of $\hat{\mathcal{A}}(0)$ consisting of Laplace transforms of functions in $L_{1}(0, \infty)$ has another special property.

Theorem A.7.56 The subset of strictly proper, stable, rational transfer functions is dense in the class of Laplace transforms of functions in $\boldsymbol{L}_{1}(0, \infty)$ in the $\boldsymbol{H}_{\infty}$-norm.

Proof For $h \in \boldsymbol{L}_{1}(0, \infty)$, by Property A.6.2 its Laplace transform $\hat{h}$ in $\hat{\mathcal{A}}(0)$ is holomorphic on $\mathbb{C}_{0}^{+}$and continuous on $\overline{\mathbb{C}_{0}^{+}}$. Furthermore, we have that $\lim _{|s| \rightarrow \infty}|\hat{h}(s)|=0$ for $s \in \overline{\mathbb{C}_{0}^{+}}$. We reduce this to an equivalent problem on the unit disc, $\mathbb{D}:=\{z \in \mathbb{C}| | z \mid<1\}$ by introducing the bilinear transformation $\theta: \overline{\mathbb{D}} \rightarrow \overline{\mathbb{C}_{0}^{+}}$defined by

$$
\begin{equation*}
\theta(z):=\frac{1+z}{1-z} \quad \text { for } z \in \overline{\mathbb{D}} \backslash\{1\} . \tag{A.7.22}
\end{equation*}
$$

It is easy to see that $\theta(\mathbb{D})=\mathbb{C}_{0}^{+}$, and it maps the unit circle excluding the point 1 on the imaginary axis. Thus $f_{d}(z):=\hat{h}(\theta(z))$ is holomorphic on $\mathbb{D}$ and continuous on $\overline{\mathbb{D}} \backslash\{1\}$. Furthermore, it is easy to see that

$$
\lim _{z \in \mathbb{D}, z \rightarrow 1} f_{d}(z)=\lim _{s \in \mathbb{C}_{0}^{+},|s| \rightarrow \infty} \hat{h}(s)=0 .
$$

Hence $f_{d}$ is continuous on the unit circle.
It is known from Theorem A.1.12 that the subset of polynomials with complex coefficients is dense in the $\boldsymbol{H}_{\infty}$-norm in the class of complex functions that are holomorphic on $\mathbb{D}$ and continuous on $\overline{\mathbb{D}}$. Hence for every $\varepsilon>0$ there exists a polynomial $Q_{\varepsilon}$ such that

$$
\sup _{z \in \mathbb{D}}\left|f_{d}(z)-Q_{\varepsilon}(z)\right|<\varepsilon .
$$

Since $f_{d}(1)=0$, there holds $\left|Q_{\varepsilon}(1)\right|<\varepsilon$. Defining $P_{\varepsilon}:=Q_{\varepsilon}-Q_{\varepsilon}(1)$, gives $P_{\varepsilon}(1)=0$ and

$$
\sup _{z \in \mathbb{D}}\left|f_{d}(z)-P_{\varepsilon}(z)\right|<2 \varepsilon .
$$

Now the bilinear transformation (A.7.22) shows that $\boldsymbol{H}_{\infty}$ is isometrically isomorphic to $\boldsymbol{H}_{\infty}(\mathbb{D})$, the space of holomorphic complex functions on $\mathbb{D}$ bounded on $\mathbb{D}$. Thus we see that

$$
\sup _{s \in \mathbb{C}_{0}^{+}}\left|\hat{h}(s)-P_{\varepsilon}\left(\theta^{-1}(s)\right)\right|=\sup _{z \in \mathbb{D}}\left|f_{d}(z)-P_{\varepsilon}(z)\right|<2 \varepsilon
$$

The function $P_{\varepsilon}\left(\theta^{-1}(\cdot)\right)$ is a stable rational function in $\overline{\mathbb{C}_{0}^{+}}$. Furthermore, we have that

$$
\lim _{s \in \mathbb{C}_{0}^{+} \cdot|s| \rightarrow \infty} P_{\varepsilon}\left(\theta^{-1}(s)\right)=\lim _{z \in \overline{\mathbb{D}} \cdot z \rightarrow 1} P_{\varepsilon}(z)=0,
$$

and so $P_{\varepsilon}\left(\theta^{-1}(\cdot)\right)$ is strictly proper.
In fact, the functions in $\boldsymbol{H}_{\infty}$ that are approximable by rationals in the $\boldsymbol{H}_{\infty}$-norm are exactly those that are continuous on the extended imaginary axis. The proof is similar to the analogous result in Lemma A.6.11 on approximation in the $\boldsymbol{L}_{\infty}$-norm, except that one appeals to Theorem A.1.12 instead of the Weierstrass Theorem. For example, $e^{-s}$ is not approximable by rationals, but $\frac{e^{-s}}{s+1}$ is.

The proof of Theorem A. 7.56 is based on Nett [189]. More powerful approximation results can be found in Glover, Curtain, and Partington [112], Glover, Lam, and Partington [113], [114], [115], Ghu, Khargonekar, and Lee [106], Partington et al. [200], Zwart et al. [276] and Makila [174].

Further properties of these convolution algebras can be found in Hille and Phillips [129, sections 4.16-4.18], Callier and Desoer [36]-[38], and Logemann [161] and [162].

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## Notation

Symbol Meaning ..... Page

* $\quad h * g$, convolution product of $h$ and $g$ ..... 637
$\breve{h}$, Fourier transform of $h$ ..... 637
$\dagger \quad F^{\dagger}(s):=F(-\bar{s})^{*}$ ..... 415
$\diamond \quad u \diamond v$, concatenation of $u$ and $v$ at $\tau$ ..... 175
$\oplus \quad Z_{1} \oplus Z_{2}$, direct sum of $Z_{1}$ and $Z_{2}$ ..... 578
$>\quad Q_{1}>Q_{2}$, operator $Q_{1}$ larger than $Q_{2}$ ..... 606
$\geq \quad Q_{1} \geq Q_{2}$, operator $Q_{1}$ larger than or equal to $Q_{2}$ ..... 606
$\hat{h}$, Laplace transform of $h$ ..... 635
$\langle\cdot, \cdot\rangle \quad\langle u, v\rangle$, inner product of $u$ and $v$ ..... 576
$\|\cdot\| \quad\|z\|$, norm of $z$ ..... 572
closure of the set $V$ ..... 574
$\perp \quad V^{\perp}$, orthogonal complement of $V$ $x \perp y, \Leftrightarrow\langle x, y\rangle=0$ ..... 578
, $\quad X^{\prime}$, dual space or dual operator of $X$ ..... 589, 594
" $\quad X^{\prime \prime}$, second dual of $X$ ..... 592
* $Q^{*}$, adjoint operator of $Q$ ..... 601
$\hookrightarrow \quad V \subset X$, continuous and dense injection ..... 585
$\mathcal{A}(\beta) \quad$ convolution algebra ..... 661
$\hat{\mathcal{A}}(\beta) \quad$ set of Laplace transforms of $\mathcal{A}(\beta)$ ..... 665
$\hat{\mathcal{A}}(\beta) \quad$ union of $\hat{\mathcal{A}}\left(\beta_{1}\right)$ over $\beta_{1}<\beta$ ..... 338
$\hat{\mathcal{A}}_{\infty}(\beta) \quad$ set of functions in $\hat{\mathcal{A}}_{-}(\beta)$ that are
bounded away from zero at infinity in $\overline{\mathbb{C}_{\beta}^{+}}$ ..... 338
$\mathcal{B}^{\tau} \quad$ controllability map on $[0, \tau]$ ..... 143

| Symbol | Meaning | Page |
| :---: | :---: | :---: |
| $\mathcal{B}^{\infty}$ | controllability map on $[0, \infty)$ | 159 |
| $\hat{\mathcal{B}}(\beta)$ | $\hat{\mathcal{A}}_{-}(\beta)\left[\hat{\mathcal{A}}_{\infty}(\beta)\right]^{-1}$ | 340 |
| C | set of complex numbers |  |
| $\mathbb{C}(s)$ | class of rational functions | 653 |
| $\mathbb{C}_{p}(s)$ | class of proper rational functions | 653 |
| $\underline{C}^{+}$ | all complex numbers with real part larger than $\beta$ | 636 |
| $\overline{\mathbb{C}_{\beta}^{+}}$ | all complex numbers with real part larger than or equal to $\beta$ | 635 |
| $\mathbb{C}_{\beta}^{-}$ | all complex numbers with real part less than $\beta$ | 229 |
| $C[0,1]$ | class of continuous functions from $[0,1] \text { to } \mathbb{C}$ |  |
| $\boldsymbol{C}([a, b] ; X)$ | class of continuous functions from |  |
|  | [ $a, b$ ] to $X$ | 586 |
| $C^{1}([0, \tau] ; Z)$ | class of continuously differentiable |  |
|  | functions from $[0, \tau]$ to $Z$ | 101 |
| $\mathcal{C}^{\text {r }}$ | observability map on [0, $\tau]$ | 154 |
| $\mathcal{C}^{\infty}$ | observability map on $[0, \infty)$ | 159 |
| D( $T$ ) | domain of $T$ | 582 |
| D | unit disc | 450 |
| $\mathcal{F}_{L}(P, Q)$ | lower linear fractional transformation | 430 |
| $\mathcal{F}_{U}(P, Q)$ | upper linear fractional transformation | 430 |
| $H_{G}$ | Hankel operator associated with symbol $G$ | 387 |
| $\boldsymbol{H}_{\infty}$ | Hardy space of bounded holomorphic functions on $\mathbb{C}_{0}^{+}$with values in $\mathbb{C}$ | 643 |
| $\boldsymbol{H}_{\infty}(\mathbb{D})$ | Hardy space of bounded holomorphic function on $\mathbb{D}$ with values in $\mathbb{C}$ | 450 |
| $\boldsymbol{H}_{\infty}\left(\mathbb{D} ; \mathbb{C}^{k \times m}\right)$ | Hardy space of bounded holomorphic function on $\mathbb{D}$ with values in $\mathbb{C}^{k \times m}$ | 450 |
| $\boldsymbol{H}_{\infty}(X)$ | Hardy space of bounded holomorphic functions on $\mathbb{C}_{0}^{+}$with values in $X$ | 643 |
| $\boldsymbol{H}_{\infty}^{-}(\beta)$ | subset of $\boldsymbol{H}_{\infty}$ | 377 |
| $\boldsymbol{H}_{\infty}\left[\boldsymbol{H}_{\infty}\right]^{-1}$ | quotient field of $\boldsymbol{H}_{\infty}$ | 654 |
| $\mathrm{H}_{2}$ | Hardy space of square integrable functions on $\mathbb{C}_{0}^{+}$with values in $\mathbb{C}$ | 643 |
| $\mathrm{H}_{2}(\mathbb{D})$ | Hardy space of square intergrable |  |
|  | functions on $\mathbb{D}$ with values in $\mathbb{C}$ | 450 |
| $\boldsymbol{H}_{2}\left(\mathbb{D} ; \mathbb{C}^{m}\right)$ | Hardy space of square intergrable functions on $\mathbb{D}$ with values in $\mathbb{C}^{m}$ | 450 |
| $\mathrm{H}_{2}(\mathrm{Z})$ | Hardy space of square integrable |  |
|  | functions on $\mathbb{C}_{0}^{+}$with values in $Z$ | 643 |
| $I_{\delta}$ | approximate identity | 534 |
| $J\left(z_{0} ; t_{0}, t_{e}, u\right)$ | cost functional on the interval [ $\left.t_{0}, t_{e}\right]$ | 269 |
| ker $T$ | kernel of $T$ | 583 |
| $L_{B}^{\tau}$ | controllability gramian of $\Sigma(A, B,-)$ on $[0, \tau]$ | 144 |
| $L_{C}^{\tau}$ | observability gramian of $\Sigma(A,-, C)$ on $[0, \tau]$ | 154 |


| Symbol | Meaning | Page |
| :---: | :---: | :---: |
| $L(\Omega ; Z)$ | class of Lebesgue measurable functions |  |
|  | from $\Omega$ to $Z$ | 626 |
| $\boldsymbol{L}_{\infty}(a, b)$ | class of bounded measurable functions |  |
|  | from $[a, b]$ to $\mathbb{C}$ | 573 |
| $L_{\infty}(\Omega ; Z)$ | class of bounded measurable functions |  |
|  | from $\Omega$ to $Z$ | 626 |
| $L_{\infty}\left(\partial \mathbb{D} ; \mathbb{C}^{k \times m}\right)$ | class of bounded measurable functions |  |
|  | from $2 \mathbb{D}$ to $\mathbb{C}^{k \times m}$ | 450 |
| $L_{p}(a, b)$ | class of Lebesgue measurable complex- |  |
|  | valued functions with $\int_{a}^{b}\|f(t)\|^{p} d t<\infty$ | 573 |
| $L_{p}(\Omega ; Z)$ | class of Lebesgue measurable $Z$-valued |  |
|  | functions with $\int_{\Omega}\|f(t)\|^{p} d t<\infty$ | 626 |
| $L_{2}((-\jmath \infty, j \infty) ; Z)$ | $L_{p}(\Omega ; Z)$ with $p=2$ and $\Omega=(-\jmath \infty, j \infty)$ | 639 |
| $L_{2}(2 \mathrm{D})$ | $L_{p}(\Omega ; Z)$ with $p=2, \Omega=\partial \mathbb{D}$ and $Z=\mathbb{C}$ | 450 |
| $L_{2}\left(\partial \mathbb{D} ; \mathbb{C}^{m}\right)$ | $L_{p}(\Omega ; Z)$ with $p=2, \Omega=\partial \mathbb{D}$ and $Z=\mathbb{C}^{m}$ | 450 |
| $\boldsymbol{L}_{2}^{\text {loc }}([0, \infty) ; U)$ | class of functions which are in |  |
|  | $L_{2}((a, b) ; U)$ for all $a, b \in[0, \infty)$ | 175 |
| $\mathcal{L}(X)$ | bounded linear operators from $X$ to $X$ | 584 |
| $\mathcal{L}(X, Y)$ | bounded linear operators from $X$ to $Y$ | 584 |
| $\ell_{p}$ | complex-valued sequences with |  |
|  | $\sum^{\infty}\left\|x_{n}\right\|^{p}<\infty$ | 572 |
|  | $\sum_{n=1}^{\infty}\left\|x_{n}\right\|<\infty$ |  |
| $\ell_{\infty}$ | bounded complex-values sequences | 573 |
| $\mathcal{M A}$ | class of matrices with elements in $\mathcal{A}$ | 656 |
| $\mathcal{M} \hat{\mathcal{A}}(\beta)$ | class of matrices with elements in $\hat{\mathcal{A}}(\beta)$ | 349 |
| $\mathcal{M} \hat{\mathcal{A}}_{-}(\beta)$ | class of matrices with elements in $\hat{\mathcal{A}}_{-}(\beta)$ | 349 |
| $\mathcal{M} \hat{\mathcal{B}}(\beta)$ | class of matrices with elements in $\hat{\mathcal{B}}(\beta)$ | 349 |
| $\boldsymbol{M}_{2}\left(\left[-h_{p}, 0\right] ; \mathbb{C}^{n}\right)$ | $\mathbb{C}^{n} \oplus \boldsymbol{L}_{2}\left(\left(-h_{p}, 0\right) ; \mathbb{C}^{n}\right)$ | 56 |
| $\mathcal{N}$ | nonobservable subspace | 157 |
| $\mathbb{N}$ | set of positive integers |  |
| $\boldsymbol{P}\left(\Omega ; \mathcal{L}\left(Z_{1}, Z_{2}\right)\right)$ | class of weakly measurable |  |
|  | functions from $\Omega$ to $\mathcal{L}\left(Z_{1}, Z_{2}\right)$ | 626 |
| $\boldsymbol{P}_{p}\left(\Omega ; \mathcal{L}\left(Z_{1}, Z_{2}\right)\right)$ | functions in $\boldsymbol{P}\left(\Omega ; \mathcal{L}\left(Z_{1}, Z_{2}\right)\right.$ with |  |
|  | $\int_{\Omega}\\|F(t)\\|^{p} d t<\infty$ | 626 |
| $\boldsymbol{P}_{\infty}\left(\Omega ; \mathcal{L}\left(Z_{1}, Z_{2}\right)\right)$ | class of bounded weakly measurable |  |
|  | functions from $\Omega$ to $\mathcal{L}\left(Z_{1}, Z_{2}\right)$ | 626 |
| $\boldsymbol{P}_{\infty}((-\jmath \infty, \jmath \infty) ; \mathcal{L}(U, Y))$ | class of weakly measurable bounded |  |
|  | functions from $(-\jmath \infty, \jmath \infty)$ to $\mathcal{L}(U, Y)$ | 639 |
| $\mathbb{R}$ | the set of real numbers |  |
| $\mathcal{R}$ | reachable subspace | 157 |
| $\mathcal{R}(\beta)$ | $\beta$-stable, proper, rational functions | 653 |
| $\mathcal{R}^{r}(\beta)$ | $\beta$-stable, real, proper, rational functions | 653 |
| $\mathcal{R}_{\infty}(\beta)$ | $\beta$-stable, biproper, rational functions | 653 |
| $\mathcal{R}_{\infty}^{r}{ }^{(\beta)}$ | $\beta$-stable, real, biproper, rational functions | 653 |


| Symbol | Meaning | Page |
| :---: | :---: | :---: |
| $\mathbb{R}(s)$ | real, rational functions | 653 |
| $\mathbb{R}_{p}(s)$ | real, proper, rational functions | 653 |
| $\operatorname{ran} T$ | range of the operator $T$ | 582 |
| $r_{\sigma}(T)$ | spectral radius of $T$ | 614 |
| $u^{\text {min }}\left(\cdot ; z_{0}, t_{0}, t_{e}\right)$ | optimal input trajectory | 272 |
| $y^{\text {min }}\left(\cdot ; z_{0}, t_{0}, t_{e}\right)$ | optimal output trajectory | 272 |
| $\mathbb{Z}$ | set of integers |  |
| $z^{\min }\left(\cdot ; z_{0}, t_{0}, t_{e}\right)$ | optimal state trajectory | 272 |
| $\partial \mathbb{D}$ | unit circle | 450 |
| $\vec{\delta}_{T}\left(G, G_{\Delta}\right)$ | directed gap | 558 |
| $\Delta(\lambda)$ | characteristic function of delay system | 58 |
| $\Gamma_{h}$ | Hankel operator associated with impulse response $h$ | 396 |
| $\rho(A)$ | resolvent set of $A$ | 608 |
| $\rho_{\infty}(A)$ | component of $\rho(A)$ that contains an interval $[r, \infty), r \in \mathbb{R}$ | 70 |
| $\Sigma(A, B, C, D)$ | state linear system | 141 |
| $\Sigma(A, B, C)$ | state linear system with $D=0$ | 141 |
| $\Sigma(A, B,-)$ | state linear system with $C$ undefined | 141 |
| $\Sigma(A,-, C)$ | state linear system with $B$ undefined | 141 |
| $\Sigma_{d}(A, B, C, D)$ | discrete-time state linear system | 211 |
| $\sigma(A)$ | spectrum of $A$ | 610 |
| $\sigma_{c}(A)$ | continuous spectrum of $A$ | 610 |
| $\sigma_{p}(A)$ | point spectrum of $A$ | 610 |
| $\sigma_{r}(A)$ | residual spectrum of $A$ | 610 |
| $\sigma_{\delta}^{+}(A)$ | $\sigma(A) \cap \overline{\mathbb{C}_{\delta}^{+}}$ | 229 |
| $\sigma_{\delta}^{-}(A)$ | $\sigma(A) \cap \mathbb{C}_{\delta}^{-}$ | 229 |

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