convolution product (A.7.12) we have

$$\begin{aligned}\widehat{f * h} &= \widehat{f_a * h_a} + \sum_{n=1}^{\infty} \widehat{f_n h_a}(\cdot - t_n) + \sum_{n=1}^{\infty} \widehat{h_n f_a}(\cdot - \tau_n) + \\ &\quad \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} h_m f_n \widehat{\delta}(\cdot - (t_n + \tau_m))\end{aligned}$$

and so

$$\begin{aligned}(\widehat{f * h})(s) &= \widehat{f_a}(s)\widehat{h_a}(s) + \sum_{n=1}^{\infty} f_n e^{-st_n}\widehat{h_a}(s) + \\ &\quad \sum_{n=1}^{\infty} h_n e^{-s\tau_n}\widehat{f_a}(s) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} h_m f_n e^{-(t_n+\tau_m)s}\end{aligned}$$

by Lemma A.6.5.c and the definition of the Laplace transform

$$= \hat{f}(s) \cdot \hat{h}(s) \qquad \text{for } s \in \overline{\mathbb{C}_\beta^+}.$$

■

In our applications we shall consider the class of transfer functions given by

$$\hat{\mathcal{A}}(\beta) := \{\hat{f} \mid f \in \mathcal{A}(\beta)\}. \tag{A.7.19}$$

Corollary A.7.48 *$\hat{\mathcal{A}}(\beta)$ is a commutative Banach algebra with identity under pointwise addition and multiplication.*

Proof This follows from the properties of $\mathcal{A}(\beta)$ and the Laplace transform as listed in Lemmas A.7.46 and A.7.47, respectively. ■

We quote two important properties of $\hat{\mathcal{A}}(\beta)$.

Theorem A.7.49 *$\hat{f} \in \hat{\mathcal{A}}(\beta)$ is invertible over $\hat{\mathcal{A}}(\beta)$ if and only if*

$$\inf_{s \in \overline{\mathbb{C}_\beta^+}} |\hat{f}(s)| > 0. \tag{A.7.20}$$

Proof Hille and Phillips [129, theorem 4.18.6]. ■

Since $\mathcal{A}(\beta)$ is an integral domain with identity, we can define coprimeness as in Definition A.7.41. We note that there exist elements in its quotient algebra that do not admit coprime factorizations (Logemann [161] and Vidyasagar, Schneider, and Francis [251]).

Theorem A.7.50 *$(\hat{f}, \hat{h})$ are coprime over $\hat{\mathcal{A}}(\beta)$ if and only if*

$$\inf_{s \in \overline{\mathbb{C}_\beta^+}} (|\hat{f}(s)| + |\hat{h}(s)|) > 0. \tag{A.7.21}$$

Proof Callier and Desoer [36], theorem 2.1. ■

We need the following facts about almost periodic functions from Corduneau [44] and Bohr [28].

Definition A.7.51 f is *almost periodic* on the *vertical strip* $[\beta, \gamma] = \{s \in \mathbb{C} \mid \beta \leq \mathrm{Re}(s) \leq \gamma\}$ if it is continuous there and for any $\varepsilon > 0$ there corresponds a $\delta(\varepsilon) > 0$ such that any interval of length $\delta(\varepsilon)$ on the imaginary axis contains at least one point, $j\eta$, for which $|f(s + j\eta) - f(s)| < \varepsilon$ for any s in this strip. ■

It is easy to see that e^{-st_n} is an almost periodic function on any vertical strip. In the next lemma, we shall show that this also holds for infinite sums of these terms.

Lemma A.7.52 *Suppose that* $\hat{f}(s) = \sum_{n=1}^{\infty} f_n e^{-st_n}$, *where* $f_n \in \mathbb{C}$, $t_n \in \mathbb{R}$ *and* $t_1 = 0$, $t_n > 0$ *for* $n \geq 2$ *and* $\sum_{n=1}^{\infty} |f_n| e^{-\beta t_n} < \infty$ *for a given real* β. *Then* $\hat{f}(s)$ *is holomorphic on* $\mathbb{C}_\beta^+$ *and bounded on* $\overline{\mathbb{C}_\beta^+}$. *Furthermore,* $\hat{f}(s)$ *is an almost periodic function on any vertical strip* $[\beta, \beta + \mu]$, $\mu > 0$.

Proof In Lemma A.7.47 we proved that $\hat{f}(s)$ is bounded on $\overline{\mathbb{C}_\beta^+}$. We also proved that it is holomorphic on $\mathbb{C}_\beta^+$ and continuous on the line $s = \beta + j\omega$, $\omega \in \mathbb{R}$; thus it is continuous on the vertical strip $[\beta, \beta + \mu]$ for $\mu > 0$.

The rest of the proof can be found in Corduneau [44] following theorems 3.10 and 3.13. An alternative proof can be found in Bohr [28, appendix II]. ■

That these functions are uniformly continuous on any closed vertical strip $[\beta + \varepsilon, \gamma]$ follows from the following general lemma.

Lemma A.7.53 *Consider a function* $g(s)$ *that is holomorphic on the vertical open strip* (a, b) *and bounded on any closed vertical strip* $[a_1, b_1]$ *contained in* (a, b). *Then* $g(s)$ *is uniformly continuous on the closed vertical strip* $[a_1, b_1]$.

Proof Corduneau [44, theorem 3.7]. ■

Next we examine the asymptotic behavior of the almost periodic function $\sum_{n=1}^{\infty} f_n e^{-st_n}$. Notice that while e^{-s} tends to zero as $\mathrm{Re}(s) \to \infty$, it does not tend to zero as $|s| \to \infty$.

Lemma A.7.54 *Suppose that* $\hat{f}(s) = \sum_{n=1}^{\infty} f_n e^{-st_n}$, *where* $f_n \in \mathbb{C}$, $t_n \in \mathbb{R}$ *and* $t_1 = 0$, $t_n > 0$ *for* $n \geq 2$ *and* $\sum_{n=1}^{\infty} |f_n| e^{-\beta t_n} < \infty$ *for a given real* β. $\hat{f}$ *satisfies*

a. $|\hat{f}(s) - f_1| \to 0$ *as* $\mathrm{Re}(s) \to \infty$ *uniformly with respect to* $\mathrm{Im}(s)$;

b. $\sup_{s \in \overline{\mathbb{C}_\beta^+}, |s| \geq \rho} |\hat{f}(s)| \to 0$ *as* $\rho \to \infty$ *if and only if* $\hat{f}(s) = 0$ *on* $\overline{\mathbb{C}_\beta^+}$.

Proof *a.* The following estimate holds

$$|\hat{f}(s) - f_1| \leq \sum_{n=2}^{\infty} |f_n| e^{-\mathrm{Re}(s) t_n} \leq \left[\sum_{n=2}^{\infty} |f_n| e^{-\beta t_n} \right] e^{-(\mathrm{Re}(s) - \beta) t_{\min}}$$

for $\mathrm{Re}(s) > \beta$, where $t_{\min}$ is the infinum of t_n, $n \geq 2$. This establishes a for the case that $t_{\min}$ is positive. For the more general case see Corduneau [44, theorem 3.20] or Bohr [28, p. 106].

b. Let s_0 be a element in $\overline{\mathbb{C}_\beta^+}$. We know that given $\varepsilon > 0$ there exists $\rho_1 > 0$ such that $|\hat{f}(s)| < \varepsilon$ for all $s \in \{s \in \overline{\mathbb{C}_\beta^+} \mid |s| \geq \rho_1\}$. Without loss of generality, we may assume that $|s_0| < \rho_1$. By Lemma A.7.52, $\hat{f}$ is almost periodic on the vertical strip $[\beta, \rho_1]$ and so by Definition A.7.51 for $\varepsilon > 0$, there exists a $\delta(\varepsilon) > 0$ and a point $\eta \in [3\rho_1, 3\rho_1 + \delta(\varepsilon)]$ such that $|\hat{f}(s_1 + \jmath\eta) - \hat{f}(s_1)| < \varepsilon$ for all s_1 in the vertical strip $[\beta, \rho_1]$. Since $s_1 + \jmath\eta \in \{s \in \overline{\mathbb{C}_\beta^+} \mid |s| \geq \rho_1\}$, we know that $|\hat{f}(s_1 + \jmath\eta)| < \varepsilon$ and consequently $|\hat{f}(s_1)| < 2\varepsilon$ holds for all s_1 in this vertical strip $[\beta, \rho_1]$. In particular, we conclude that $|\hat{f}(s_0)| < 2\varepsilon$. Since $s_0 \in \overline{\mathbb{C}_\beta^+}$ and $\varepsilon > 0$ are arbitrary, it follows that $\hat{f}(s) = 0$ on $\overline{\mathbb{C}_\beta^+}$. ■

Finally, we state an important result on the asymptotic behavior of elements in $\hat{\mathcal{A}}(\beta)$.

Corollary A.7.55 *The function $\hat{f} \in \hat{\mathcal{A}}(\beta)$ has the limit zero as s goes to infinity in $\overline{\mathbb{C}_\beta^+}$, i.e., $\sup_{s \in \overline{\mathbb{C}_\beta^+}, |s| \geq \rho} |\hat{f}(s)| \to 0$ as $\rho \to \infty$ if and only if $\hat{f}(\cdot) = \hat{f}_a(\cdot)$.*

Proof This follows from Lemma A.7.54 and Property A.6.2.g. ■

The subclass of $\hat{\mathcal{A}}(0)$ consisting of Laplace transforms of functions in $\boldsymbol{L}_1(0, \infty)$ has another special property.

Theorem A.7.56 *The subset of strictly proper, stable, rational transfer functions is dense in the class of Laplace transforms of functions in $\boldsymbol{L}_1(0, \infty)$ in the $\boldsymbol{H}_\infty$-norm.*

Proof For $h \in \boldsymbol{L}_1(0, \infty)$, by Property A.6.2 its Laplace transform $\hat{h}$ in $\hat{\mathcal{A}}(0)$ is holomorphic on $\mathbb{C}_0^+$ and continuous on $\overline{\mathbb{C}_0^+}$. Furthermore, we have that $\lim_{|s| \to \infty} |\hat{h}(s)| = 0$ for $s \in \overline{\mathbb{C}_0^+}$. We reduce this to an equivalent problem on the unit disc, $\mathbb{D} := \{z \in \mathbb{C} \mid |z| < 1\}$ by introducing the bilinear transformation $\theta \colon \overline{\mathbb{D}} \to \overline{\mathbb{C}_0^+}$ defined by

$$\theta(z) := \frac{1+z}{1-z} \qquad \text{for } z \in \overline{\mathbb{D}} \backslash \{1\}. \tag{A.7.22}$$

It is easy to see that $\theta(\mathbb{D}) = \mathbb{C}_0^+$, and it maps the unit circle excluding the point 1 on the imaginary axis. Thus $f_d(z) := \hat{h}(\theta(z))$ is holomorphic on $\mathbb{D}$ and continuous on $\overline{\mathbb{D}} \backslash \{1\}$. Furthermore, it is easy to see that

$$\lim_{z \in \overline{\mathbb{D}}, z \to 1} f_d(z) = \lim_{s \in \overline{\mathbb{C}_0^+}, |s| \to \infty} \hat{h}(s) = 0.$$

Hence f_d is continuous on the unit circle.

It is known from Theorem A.1.12 that the subset of polynomials with complex coefficients is dense in the $\boldsymbol{H}_\infty$-norm in the class of complex functions that are holomorphic on $\mathbb{D}$ and continuous on $\overline{\mathbb{D}}$. Hence for every $\varepsilon > 0$ there exists a polynomial Q_ε such that

$$\sup_{z \in \mathbb{D}} |f_d(z) - Q_\varepsilon(z)| < \varepsilon.$$

Since $f_d(1) = 0$, there holds $|Q_\varepsilon(1)| < \varepsilon$. Defining $P_\varepsilon := Q_\varepsilon - Q_\varepsilon(1)$, gives $P_\varepsilon(1) = 0$ and

$$\sup_{z \in \mathbb{D}} |f_d(z) - P_\varepsilon(z)| < 2\varepsilon.$$

Now the bilinear transformation (A.7.22) shows that $\boldsymbol{H}_\infty$ is isometrically isomorphic to $\boldsymbol{H}_\infty(\mathbb{D})$, the space of holomorphic complex functions on $\mathbb{D}$ bounded on $\mathbb{D}$. Thus we see that

$$\sup_{s\in\mathbb{C}_0^+} |\hat{h}(s) - P_\varepsilon(\theta^{-1}(s))| = \sup_{z\in\mathbb{D}} |f_d(z) - P_\varepsilon(z)| < 2\varepsilon.$$

The function $P_\varepsilon(\theta^{-1}(\cdot))$ is a stable rational function in $\overline{\mathbb{C}_0^+}$. Furthermore, we have that

$$\lim_{s\in\overline{\mathbb{C}_0^+}, |s|\to\infty} P_\varepsilon(\theta^{-1}(s)) = \lim_{z\in\overline{\mathbb{D}}, z\to 1} P_\varepsilon(z) = 0,$$

and so $P_\varepsilon(\theta^{-1}(\cdot))$ is strictly proper. ■

In fact, the functions in $\boldsymbol{H}_\infty$ that are approximable by rationals in the $\boldsymbol{H}_\infty$-norm are exactly those that are continuous on the extended imaginary axis. The proof is similar to the analogous result in Lemma A.6.11 on approximation in the $\boldsymbol{L}_\infty$-norm, except that one appeals to Theorem A.1.12 instead of the Weierstrass Theorem. For example, e^{-s} is not approximable by rationals, but $\frac{e^{-s}}{s+1}$ is.

The proof of Theorem A.7.56 is based on Nett [189]. More powerful approximation results can be found in Glover, Curtain, and Partington [112], Glover, Lam, and Partington [113], [114], [115], Ghu, Khargonekar, and Lee [106], Partington et al. [200], Zwart et al. [276] and Makila [174].

Further properties of these convolution algebras can be found in Hille and Phillips [129, sections 4.16–4.18], Callier and Desoer [36]–[38], and Logemann [161] and [162].

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Notation

Symbol	Meaning	Page
$*$	$h * g$, convolution product of h and g	637
$\check{}$	$\check{h}$, Fourier transform of h	637
$\dagger$	$F^\dagger(s) := F(-\overline{s})^*$	415
$\underset{\tau}{\diamondsuit}$	$u \underset{\tau}{\diamondsuit} v$, concatenation of u and v at τ	175
$\oplus$	$Z_1 \oplus Z_2$, direct sum of Z_1 and Z_2	578
$>$	$Q_1 > Q_2$, operator Q_1 larger than Q_2	606
$\geq$	$Q_1 \geq Q_2$, operator Q_1 larger than or equal to Q_2	606
$\hat{}$	$\hat{h}$, Laplace transform of h	635
$\langle \cdot, \cdot \rangle$	$\langle u, v \rangle$, inner product of u and v	576
$\\| \cdot \\|$	$\\|z\\|$, norm of z	572
$\overline{V}$	closure of the set V	574
$\perp$	$V^\perp$, orthogonal complement of V	
	$x \perp y, \Leftrightarrow \langle x, y \rangle = 0$	578
$'$	X', dual space or dual operator of X	589, 594
$''$	X'', second dual of X	592
$*$	Q^*, adjoint operator of Q	601
$\hookrightarrow$	$V \hookrightarrow X$, continuous and dense injection	585
$\mathcal{A}(\beta)$	convolution algebra	661
$\hat{\mathcal{A}}(\beta)$	set of Laplace transforms of $\mathcal{A}(\beta)$	665
$\hat{\mathcal{A}}_-(\beta)$	union of $\hat{\mathcal{A}}(\beta_1)$ over $\beta_1 < \beta$	338
$\hat{\mathcal{A}}_\infty(\beta)$	set of functions in $\hat{\mathcal{A}}_-(\beta)$ that are	
	bounded away from zero at infinity in $\overline{\mathbb{C}_\beta^+}$	338
$\mathcal{B}^\tau$	controllability map on $[0, \tau]$	143

Symbol	Meaning	Page
$\mathcal{B}^{\infty}$	controllability map on $[0, \infty)$	159
$\hat{\mathcal{B}}(\beta)$	$\hat{\mathcal{A}}_{-}(\beta)[\hat{\mathcal{A}}_{\infty}(\beta)]^{-1}$	340
$\mathbb{C}$	set of complex numbers	
$\mathbb{C}(s)$	class of rational functions	653
$\mathbb{C}_p(s)$	class of proper rational functions	653
$\mathbb{C}_\beta^+$	all complex numbers with real part larger than β	636
$\overline{\mathbb{C}_\beta^+}$	all complex numbers with real part larger than or equal to β	635
$\mathbb{C}_\beta^-$	all complex numbers with real part less than β	229
$\mathcal{C}[0, 1]$	class of continuous functions from $[0, 1]$ to $\mathbb{C}$	574
$\boldsymbol{C}([a, b]; X)$	class of continuous functions from $[a, b]$ to X	586
$\boldsymbol{C}^1([0, \tau]; Z)$	class of continuously differentiable functions from $[0, \tau]$ to Z	101
$\mathcal{C}^{\tau}$	observability map on $[0, \tau]$	154
$\mathcal{C}^{\infty}$	observability map on $[0, \infty)$	159
$\boldsymbol{D}(T)$	domain of T	582
$\mathbb{D}$	unit disc	450
$\mathcal{F}_L(P, Q)$	lower linear fractional transformation	430
$\mathcal{F}_U(P, Q)$	upper linear fractional transformation	430
H_G	Hankel operator associated with symbol G	387
$\boldsymbol{H}_{\infty}$	Hardy space of bounded holomorphic functions on $\mathbb{C}_0^+$ with values in $\mathbb{C}$	643
$\boldsymbol{H}_{\infty}(\mathbb{D})$	Hardy space of bounded holomorphic function on $\mathbb{D}$ with values in $\mathbb{C}$	450
$\boldsymbol{H}_{\infty}(\mathbb{D}; \mathbb{C}^{k\times m})$	Hardy space of bounded holomorphic function on $\mathbb{D}$ with values in $\mathbb{C}^{k\times m}$	450
$\boldsymbol{H}_{\infty}(X)$	Hardy space of bounded holomorphic functions on $\mathbb{C}_0^+$ with values in X	643
$\boldsymbol{H}_{\infty}^-(\beta)$	subset of $\boldsymbol{H}_{\infty}$	377
$\boldsymbol{H}_{\infty}[\boldsymbol{H}_{\infty}]^{-1}$	quotient field of $\boldsymbol{H}_{\infty}$	654
$\boldsymbol{H}_2$	Hardy space of square integrable functions on $\mathbb{C}_0^+$ with values in $\mathbb{C}$	643
$\boldsymbol{H}_2(\mathbb{D})$	Hardy space of square intergrable functions on $\mathbb{D}$ with values in $\mathbb{C}$	450
$\boldsymbol{H}_2(\mathbb{D}; \mathbb{C}^m)$	Hardy space of square intergrable functions on $\mathbb{D}$ with values in $\mathbb{C}^m$	450
$\boldsymbol{H}_2(Z)$	Hardy space of square integrable functions on $\mathbb{C}_0^+$ with values in Z	643
I_δ	approximate identity	534
$J(z_0; t_0, t_e, u)$	cost functional on the interval $[t_0, t_e]$	269
$\ker T$	kernel of T	583
L_B^{τ}	controllability gramian of $\Sigma(A, B, -)$ on $[0, \tau]$	144
L_C^{τ}	observability gramian of $\Sigma(A, -, C)$ on $[0, \tau]$	154

Symbol	Meaning	Page
$\boldsymbol{L}(\Omega; Z)$	class of Lebesgue measurable functions from Ω to Z	626
$\boldsymbol{L}_\infty(a, b)$	class of bounded measurable functions from $[a, b]$ to $\mathbb{C}$	573
$\boldsymbol{L}_\infty(\Omega; Z)$	class of bounded measurable functions from Ω to Z	626
$\boldsymbol{L}_\infty(\partial\mathbb{D}; \mathbb{C}^{k\times m})$	class of bounded measurable functions from $\partial\mathbb{D}$ to $\mathbb{C}^{k\times m}$	450
$\boldsymbol{L}_p(a, b)$	class of Lebesgue measurable complex-valued functions with $\int_a^b \lvert f(t)\rvert^p dt < \infty$	573
$\boldsymbol{L}_p(\Omega; Z)$	class of Lebesgue measurable Z-valued functions with $\int_\Omega \lvert f(t)\rvert^p dt < \infty$	626
$\boldsymbol{L}_2((-\jmath\infty, \jmath\infty); Z)$	$\boldsymbol{L}_p(\Omega; Z)$ with $p = 2$ and $\Omega = (-\jmath\infty, \jmath\infty)$	639
$\boldsymbol{L}_2(\partial\mathbb{D})$	$\boldsymbol{L}_p(\Omega; Z)$ with $p = 2$, $\Omega = \partial\mathbb{D}$ and $Z = \mathbb{C}$	450
$\boldsymbol{L}_2(\partial\mathbb{D}; \mathbb{C}^m)$	$\boldsymbol{L}_p(\Omega; Z)$ with $p = 2$, $\Omega = \partial\mathbb{D}$ and $Z = \mathbb{C}^m$	450
$\boldsymbol{L}_2^{loc}([0, \infty); U)$	class of functions which are in $\boldsymbol{L}_2((a, b); U)$ for all $a, b \in [0, \infty)$	175
$\mathcal{L}(X)$	bounded linear operators from X to X	584
$\mathcal{L}(X, Y)$	bounded linear operators from X to Y	584
ℓ_p	complex-valued sequences with $\sum_{n=1}^{\infty} \lvert x_n\rvert^p < \infty$	572
ℓ_∞	bounded complex-values sequences	573
$\mathcal{MA}$	class of matrices with elements in $\mathcal{A}$	656
$\mathcal{M}\hat{\mathcal{A}}(\beta)$	class of matrices with elements in $\hat{\mathcal{A}}(\beta)$	349
$\mathcal{M}\hat{\mathcal{A}}_-(\beta)$	class of matrices with elements in $\hat{\mathcal{A}}_-(\beta)$	349
$\mathcal{M}\hat{\mathcal{B}}(\beta)$	class of matrices with elements in $\hat{\mathcal{B}}(\beta)$	349
$\boldsymbol{M}_2([-h_p, 0]; \mathbb{C}^n)$	$\mathbb{C}^n \oplus \boldsymbol{L}_2((-h_p, 0); \mathbb{C}^n)$	56
$\mathcal{N}$	nonobservable subspace	157
$\mathbb{N}$	set of positive integers	
$\boldsymbol{P}(\Omega; \mathcal{L}(Z_1, Z_2))$	class of weakly measurable functions from Ω to $\mathcal{L}(Z_1, Z_2)$	626
$\boldsymbol{P}_p(\Omega; \mathcal{L}(Z_1, Z_2))$	functions in $\boldsymbol{P}(\Omega; \mathcal{L}(Z_1, Z_2))$ with $\int_\Omega \lVert F(t)\rVert^p dt < \infty$	626
$\boldsymbol{P}_\infty(\Omega; \mathcal{L}(Z_1, Z_2))$	class of bounded weakly measurable functions from Ω to $\mathcal{L}(Z_1, Z_2)$	626
$\boldsymbol{P}_\infty((-\jmath\infty, \jmath\infty); \mathcal{L}(U, Y))$	class of weakly measurable bounded functions from $(-\jmath\infty, \jmath\infty)$ to $\mathcal{L}(U, Y)$	639
$\mathbb{R}$	the set of real numbers	
$\mathcal{R}$	reachable subspace	157
$\mathcal{R}(\beta)$	β-stable, proper, rational functions	653
$\mathcal{R}^r(\beta)$	β-stable, real, proper, rational functions	653
$\mathcal{R}_\infty(\beta)$	β-stable, biproper, rational functions	653
$\mathcal{R}^r_\infty(\beta)$	β-stable, real, biproper, rational functions	653

Symbol	Meaning	Page
$\mathbb{R}(s)$	real, rational functions	653
$\mathbb{R}_p(s)$	real, proper, rational functions	653
ran T	range of the operator T	582
$r_\sigma(T)$	spectral radius of T	614
$u^{min}(\cdot; z_0, t_0, t_e)$	optimal input trajectory	272
$y^{min}(\cdot; z_0, t_0, t_e)$	optimal output trajectory	272
$\mathbb{Z}$	set of integers	
$z^{min}(\cdot; z_0, t_0, t_e)$	optimal state trajectory	272
$\partial\mathbb{D}$	unit circle	450
$\vec{\delta}_T(G, G_\Delta)$	directed gap	558
$\Delta(\lambda)$	characteristic function of delay system	58
Γ_h	Hankel operator associated with impulse response h	396
$\rho(A)$	resolvent set of A	608
$\rho_\infty(A)$	component of $\rho(A)$ that contains an interval $[r, \infty)$, $r \in \mathbb{R}$	70
$\Sigma(A, B, C, D)$	state linear system	141
$\Sigma(A, B, C)$	state linear system with $D = 0$	141
$\Sigma(A, B, -)$	state linear system with C undefined	141
$\Sigma(A, -, C)$	state linear system with B undefined	141
$\Sigma_d(A, B, C, D)$	discrete-time state linear system	211
$\sigma(A)$	spectrum of A	610
$\sigma_c(A)$	continuous spectrum of A	610
$\sigma_p(A)$	point spectrum of A	610
$\sigma_r(A)$	residual spectrum of A	610
$\sigma_\delta^+(A)$	$\sigma(A) \cap \overline{\mathbb{C}_\delta^+}$	229
$\sigma_\delta^-(A)$	$\sigma(A) \cap \mathbb{C}_\delta^-$	229

Index

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