convolution product (A.7.12) we have

$$\widehat{f * h} = \widehat{f_a * h_a} + \sum_{n=1}^{\infty} \widehat{f_n h_a} (\cdot - t_n) + \sum_{n=1}^{\infty} \widehat{h_n f_a} (\cdot - \tau_n) + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} h_m f_n \widehat{\delta} (\cdot - (t_n + \tau_m))$$

and so

$$(\widehat{f * h})(s) = \widehat{f_a}(s)\widehat{h_a}(s) + \sum_{n=1}^{\infty} f_n e^{-st_n} \widehat{h_a}(s) + \sum_{n=1}^{\infty} h_n e^{-s\tau_n} \widehat{f_a}(s) + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} h_m f_n e^{-(t_n + \tau_m)s}$$
  
by Lemma A.6.5.c and the definition of the 2

Laplace transform

$$= \hat{f}(s) \cdot \hat{h}(s) \qquad \text{for } s \in \mathbb{C}^+_{\beta}.$$

In our applications we shall consider the class of transfer functions given by

$$\hat{\mathcal{A}}(\beta) := \{ \hat{f} \mid f \in \mathcal{A}(\beta) \}. \tag{A.7.19}$$

**Corollary A.7.48**  $\hat{\mathcal{A}}(\beta)$  is a commutative Banach algebra with identity under pointwise addition and multiplication.

**Proof** This follows from the properties of  $\mathcal{A}(\beta)$  and the Laplace transform as listed in Lemmas A.7.46 and A.7.47, respectively.

We quote two important properties of  $\hat{\mathcal{A}}(\beta)$ .

.

**Theorem A.7.49**  $\hat{f} \in \hat{\mathcal{A}}(\beta)$  is invertible over  $\hat{\mathcal{A}}(\beta)$  if and only if

$$\inf_{s \in \mathbb{C}^+_{\theta}} |\hat{f}(s)| > 0.$$
(A.7.20)

**Proof** Hille and Phillips [129, theorem 4.18.6].

Since  $\mathcal{A}(\beta)$  is an integral domain with identity, we can define coprimeness as in Definition A.7.41. We note that there exist elements in its quotient algebra that do not admit coprime factorizations (Logemann [161] and Vidyasagar, Schneider, and Francis [251]).

**Theorem A.7.50**  $(\hat{f}, \hat{h})$  are coprime over  $\hat{\mathcal{A}}(\beta)$  if and only if

$$\inf_{s \in \mathbb{C}^+_{\hat{h}}} (|\hat{f}(s)| + |\hat{h}(s)|) > 0.$$
(A.7.21)

Proof Callier and Desoer [36], theorem 2.1.

We need the following facts about almost periodic functions from Corduneau [44] and Bohr [28].

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**Definition A.7.51** f is almost periodic on the vertical strip  $[\beta, \gamma] = \{s \in \mathbb{C} \mid \beta \le \operatorname{Re}(s) \le \gamma\}$  if it is continuous there and for any  $\varepsilon > 0$  there corresponds a  $\delta(\varepsilon) > 0$  such that any interval of length  $\delta(\varepsilon)$  on the imaginary axis contains at least one point,  $j\eta$ , for which  $|f(s + j\eta) - f(s)| < \varepsilon$  for any s in this strip.

It is easy to see that  $e^{-st_n}$  is an almost periodic function on any vertical strip. In the next lemma, we shall show that this also holds for infinite sums of these terms.

**Lemma A.7.52** Suppose that  $\hat{f}(s) = \sum_{n=1}^{\infty} f_n e^{-st_n}$ , where  $f_n \in \mathbb{C}$ ,  $t_n \in \mathbb{R}$  and  $t_1 = 0$ ,  $t_n > 0$ for  $n \ge 2$  and  $\sum_{n=1}^{\infty} |f_n| e^{-\beta t_n} < \infty$  for a given real  $\beta$ . Then  $\hat{f}(s)$  is holomorphic on  $\mathbb{C}_{\beta}^+$  and bounded on  $\overline{\mathbb{C}_{\beta}^+}$ . Furthermore,  $\hat{f}(s)$  is an almost periodic function on any vertical strip  $[\beta, \beta + \mu], \mu > 0$ .

**Proof** In Lemma A.7.47 we proved that  $\hat{f}(s)$  is bounded on  $\overline{\mathbb{C}_{\beta}^{+}}$ . We also proved that it is holomorphic on  $\mathbb{C}_{\beta}^{+}$  and continuous on the line  $s = \beta + j\omega, \omega \in \mathbb{R}$ ; thus it is continuous on the vertical strip  $[\beta, \beta + \mu]$  for  $\mu > 0$ .

The rest of the proof can be found in Corduneau [44] following theorems 3.10 and 3.13. An alternative proof can be found in Bohr [28, appendix II].

That these functions are uniformly continuous on any closed vertical strip  $[\beta + \varepsilon, \gamma]$  follows from the following general lemma.

**Lemma A.7.53** Consider a function g(s) that is holomorphic on the vertical open strip (a, b) and bounded on any closed vertical strip  $[a_1, b_1]$  contained in (a, b). Then g(s) is uniformly continuous on the closed vertical strip  $[a_1, b_1]$ .

Proof Corduneau [44, theorem 3.7].

Next we examine the asymptotic behavior of the almost periodic function  $\sum_{n=1}^{\infty} f_n e^{-st_n}$ . Notice that while  $e^{-s}$  tends to zero as  $\text{Re}(s) \to \infty$ , it does not tend to zero as  $|s| \to \infty$ .

**Lemma A.7.54** Suppose that  $\hat{f}(s) = \sum_{n=1}^{\infty} f_n e^{-st_n}$ , where  $f_n \in \mathbb{C}$ ,  $t_n \in \mathbb{R}$  and  $t_1 = 0$ ,  $t_n > 0$ for  $n \ge 2$  and  $\sum_{n=1}^{\infty} |f_n| e^{-\beta t_n} < \infty$  for a given real  $\beta$ .  $\hat{f}$  satisfies a.  $|\hat{f}(s) - f_1| \to 0$  as  $Re(s) \to \infty$  uniformly with respect to Im(s); b.  $\sup_{s \in \overline{\mathbb{C}_{\beta}^+}, |s| \ge \rho} |\hat{f}(s)| \to 0$  as  $\rho \to \infty$  if and only if  $\hat{f}(s) = 0$  on  $\overline{\mathbb{C}_{\beta}^+}$ .

**Proof** *a*. The following estimate holds

$$|\hat{f}(s) - f_1| \le \sum_{n=2}^{\infty} |f_n| e^{-\operatorname{Re}(s)t_n} \le \left[\sum_{n=2}^{\infty} |f_n| e^{-\beta t_n}\right] e^{-(\operatorname{Re}(s) - \beta)t_{\min}}$$

for  $\operatorname{Re}(s) > \beta$ , where  $t_{\min}$  is the infinum of  $t_n$ ,  $n \ge 2$ . This establishes a for the case that  $t_{\min}$  is positive. For the more general case see Corduneau [44, theorem 3.20] or Bohr [28, p. 106].

b. Let  $s_0$  be a element in  $\overline{\mathbb{C}_{\beta}^+}$ . We know that given  $\varepsilon > 0$  there exists  $\rho_1 > 0$  such

that  $|\hat{f}(s)| < \varepsilon$  for all  $s \in \{s \in \overline{\mathbb{C}_{\beta}^+} \mid |s| \ge \rho_1\}$ . Without loss of generality, we may assume that  $|s_0| < \rho_1$ . By Lemma A.7.52,  $\hat{f}$  is almost periodic on the vertical strip  $[\beta, \rho_1]$  and so by Definition A.7.51 for  $\varepsilon > 0$ , there exists a  $\delta(\varepsilon) > 0$  and a point  $\eta \in [3\rho_1, 3\rho_1 + \delta(\varepsilon)]$  such that  $|\hat{f}(s_1 + j\eta) - \hat{f}(s_1)| < \varepsilon$  for all  $s_1$  in the vertical strip  $[\beta, \rho_1]$ . Since  $s_1 + j\eta \in \{s \in \overline{\mathbb{C}_{\beta}^+} \mid |s| \ge \rho_1\}$ , we know that  $|\hat{f}(s_1 + j\eta)| < \varepsilon$  and consequently  $|\hat{f}(s_1)| < 2\varepsilon$  holds for all  $s_1$  in this vertical strip  $[\beta, \rho_1]$ . In particular, we conclude that  $|\hat{f}(s_0)| < 2\varepsilon$ . Since  $s_0 \in \overline{\mathbb{C}_{\beta}^+}$  and  $\varepsilon > 0$  are arbitrary, it follows that  $\hat{f}(s) = 0$ on  $\overline{\mathbb{C}_{\beta}^+}$ .

Finally, we state an important result on the asymptotic behavior of elements in  $\hat{\mathcal{A}}(\beta)$ .

**Corollary A.7.55** The function  $\hat{f} \in \hat{\mathcal{A}}(\beta)$  has the limit zero as s goes to infinity in  $\overline{\mathbb{C}}_{\beta}^+$ , *i.e.*,  $\sup_{s \in \overline{\mathbb{C}}_{\beta}^+, |s| \ge \rho} |\hat{f}(s)| \to 0$  as  $\rho \to \infty$  if and only if  $\hat{f}(\cdot) = \hat{f}_a(\cdot)$ .

Proof This follows from Lemma A.7.54 and Property A.6.2.g.

The subclass of  $\hat{\mathcal{A}}(0)$  consisting of Laplace transforms of functions in  $L_1(0, \infty)$  has another special property.

**Theorem A.7.56** The subset of strictly proper, stable, rational transfer functions is dense in the class of Laplace transforms of functions in  $L_1(0, \infty)$  in the  $H_{\infty}$ -norm.

**Proof** For  $h \in L_1(0, \infty)$ , by Property A.6.2 its Laplace transform  $\hat{h}$  in  $\hat{\mathcal{A}}(0)$  is holomorphic on  $\mathbb{C}_0^+$  and continuous on  $\overline{\mathbb{C}_0^+}$ . Furthermore, we have that  $\lim_{|s|\to\infty} |\hat{h}(s)| = 0$  for  $s \in \overline{\mathbb{C}_0^+}$ . We reduce this to an equivalent problem on the unit disc,  $\mathbb{D} := \{z \in \mathbb{C} \mid |z| < 1\}$  by introducing the bilinear transformation  $\theta: \overline{\mathbb{D}} \to \overline{\mathbb{C}_0^+}$  defined by

$$\theta(z) := \frac{1+z}{1-z} \qquad \text{for } z \in \overline{\mathbb{D}} \setminus \{1\}.$$
(A.7.22)

It is easy to see that  $\theta(\mathbb{D}) = \mathbb{C}_0^+$ , and it maps the unit circle excluding the point 1 on the imaginary axis. Thus  $f_d(z) := \hat{h}(\theta(z))$  is holomorphic on  $\mathbb{D}$  and continuous on  $\overline{\mathbb{D}} \setminus \{1\}$ . Furthermore, it is easy to see that

$$\lim_{z\in\overline{\mathbb{D}},z\to 1}f_d(z)=\lim_{s\in\overline{\mathbb{C}}^+_0,|s|\to\infty}\hat{h}(s)=0.$$

Hence  $f_d$  is continuous on the unit circle.

It is known from Theorem A.1.12 that the subset of polynomials with complex coefficients is dense in the  $H_{\infty}$ -norm in the class of complex functions that are holomorphic on  $\mathbb{D}$  and continuous on  $\overline{\mathbb{D}}$ . Hence for every  $\varepsilon > 0$  there exists a polynomial  $Q_{\varepsilon}$  such that

$$\sup_{z\in\mathbb{D}}|f_d(z)-Q_{\varepsilon}(z)|<\varepsilon.$$

Since  $f_d(1) = 0$ , there holds  $|Q_{\varepsilon}(1)| < \varepsilon$ . Defining  $P_{\varepsilon} := Q_{\varepsilon} - Q_{\varepsilon}(1)$ , gives  $P_{\varepsilon}(1) = 0$ and

$$\sup_{z\in\mathbb{D}}|f_d(z)-P_{\varepsilon}(z)|<2\varepsilon.$$

Now the bilinear transformation (A.7.22) shows that  $H_{\infty}$  is isometrically isomorphic to  $H_{\infty}(\mathbb{D})$ , the space of holomorphic complex functions on  $\mathbb{D}$  bounded on  $\mathbb{D}$ . Thus we see that

$$\sup_{s\in\mathbb{C}^+_0}|\hat{h}(s)-P_{\varepsilon}(\theta^{-1}(s))|=\sup_{z\in\mathbb{D}}|f_d(z)-P_{\varepsilon}(z)|<2\varepsilon.$$

The function  $P_{\varepsilon}(\theta^{-1}(\cdot))$  is a stable rational function in  $\overline{\mathbb{C}_0^+}$ . Furthermore, we have that

$$\lim_{\varepsilon \overline{\mathbb{C}}_{0}^{+}, |s| \to \infty} P_{\varepsilon}(\theta^{-1}(s)) = \lim_{z \in \overline{\mathbb{D}}, z \to 1} P_{\varepsilon}(z) = 0.$$

and so  $P_{\varepsilon}(\theta^{-1}(\cdot))$  is strictly proper.

In fact, the functions in  $H_{\infty}$  that are approximable by rationals in the  $H_{\infty}$ -norm are exactly those that are continuous on the extended imaginary axis. The proof is similar to the analogous result in Lemma A.6.11 on approximation in the  $L_{\infty}$ -norm, except that one appeals to Theorem A.1.12 instead of the Weierstrass Theorem. For example,  $e^{-s}$  is not approximable by rationals, but  $\frac{e^{-s}}{s+1}$  is.

The proof of Theorem A.7.56 is based on Nett [189]. More powerful approximation results can be found in Glover, Curtain, and Partington [112], Glover, Lam, and Partington [113], [114], [115], Ghu, Khargonekar, and Lee [106], Partington et al. [200], Zwart et al. [276] and Makila [174].

Further properties of these convolution algebras can be found in Hille and Phillips [129, sections 4.16–4.18], Callier and Desoer [36]–[38], and Logemann [161] and [162].

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# Notation

Symbol	Meaning	Page
*	h * g, convolution product of $h$ and $g$	637
~	$\check{h}$ , Fourier transform of $h$	637
†	$F^{\dagger}(s) := F(-\overline{s})^*$	415
$\diamondsuit_{\mathbf{T}}$	$u \diamondsuit v$ , concatenation of $u$ and $v$ at $\tau$	175
Ф	$Z_1 \oplus Z_2$ , direct sum of $Z_1$ and $Z_2$	578
>	$Q_1 > Q_2$ , operator $Q_1$ larger than $Q_2$	606
2	$Q_1 \ge Q_2$ , operator $Q_1$ larger than or equal to $Q_2$	606
^	$\hat{h}$ , Laplace transform of $h$	635
$\langle \cdot, \cdot \rangle$	$\langle u, v \rangle$ , inner product of u and v	576
·	z  , norm of z	572
$\overline{V}$	closure of the set V	574
$\bot$	$V^{\perp}$ , orthogonal complement of V	
	$x \perp y, \Leftrightarrow \langle x, y \rangle = 0$	578
/	X', dual space or dual operator of X	589, 594
//	X'', second dual of X	592
*	$Q^*$ , adjoint operator of $Q$	601
$\hookrightarrow$	$V \subset X$ , continuous and dense injection	585
$\mathcal{A}(eta)$	convolution algebra	661
$\hat{\mathcal{A}}(\beta)$	set of Laplace transforms of $\mathcal{A}(\beta)$	665
$\hat{\mathcal{A}}_{-}(eta)$	union of $\hat{\mathcal{A}}(\beta_1)$ over $\beta_1 < \beta$	338
$\hat{\mathcal{A}}_{\infty}(eta)$	set of functions in $\hat{\mathcal{A}}_{-}(\beta)$ that are	
	bounded away from zero at infinity in $\overline{\mathbb{C}_{\beta}^{+}}$	338
$\mathcal{B}^{\tau}$	controllability map on $[0, \tau]$	143

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Symbol	Meaning	Page
$\mathcal{B}^\infty$	controllability map on $[0,\infty)$	159
$\hat{\mathcal{B}}(\beta)$	$\hat{A}_{-}(B)[\hat{A}_{++}(B)]^{-1}$	340
C	set of complex numbers	540
$\mathbb{C}(s)$	class of rational functions	652
$\mathbb{C}_{n}(s)$	class of proper rational functions	652
$\mathbb{C}_{p}^{+}$	all complex numbers with real part larger than $\beta$	636
$\frac{c_{\beta}}{C^+}$	all complex numbers with real part larger than $p$	030
$\mathbb{C}_{\beta}$	an complex numbers with real part larger than or equal to $\theta$	() <b>7</b>
<u> </u>	equal to p	635
$C_{\beta}$	all complex numbers with real part less than $\beta$	229
C[0, 1]	class of continuous functions from	
$C([a, b], \mathbf{V})$		574
C([a, b]; X)	class of continuous functions from	
$\mathbf{C}^{\dagger}(0 = 1, \mathbf{z})$	[a, b] to X	586
$C^{*}([0,\tau];Z)$	class of continuously differentiable	
0T	functions from $[0, \tau]$ to Z	101
$\mathcal{C}^{i}$	observability map on $[0, \tau]$	154
C∞ D(T)	observability map on $[0, \infty)$	159
D(T)	domain of T	582
	unit disc	450
$\mathcal{F}_L(P, Q)$	lower linear fractional transformation	430
$\mathcal{F}_U(P, Q)$	upper linear fractional transformation	430
$H_G$	Hankel operator associated with symbol G	387
$H_{\infty}$	Hardy space of bounded holomorphic	
	functions on $\mathbb{C}_0^+$ with values in $\mathbb{C}$	643
$oldsymbol{H}_{\infty}(\mathbb{D})$	Hardy space of bounded holomorphic	
	function on $\mathbb D$ with values in $\mathbb C$	450
$H_{\infty}(\mathbb{D};\mathbb{C}^{\kappa imes m})$	Hardy space of bounded holomorphic	
	function on $\mathbb{D}$ with values in $\mathbb{C}^{k \times m}$	450
$H_{\infty}(X)$	Hardy space of bounded holomorphic	
	functions on $\mathbb{C}_0^+$ with values in X	643
$oldsymbol{H}^{-}_{\infty}(oldsymbol{eta})$	subset of $oldsymbol{H}_{\infty}$	377
$oldsymbol{H}_{\infty}[oldsymbol{H}_{\infty}]^{-1}$	quotient field of $oldsymbol{H}_\infty$	654
$H_2$	Hardy space of square integrable	
	functions on $\mathbb{C}_0^+$ with values in $\mathbb{C}$	643
$H_2(\mathbb{D})$	Hardy space of square intergrable	
	functions on $\mathbb{D}$ with values in $\mathbb{C}$	450
$H_2(\mathbb{D};\mathbb{C}^m)$	Hardy space of square intergrable	
	functions on $\mathbb{D}$ with values in $\mathbb{C}^m$	450
$H_2(Z)$	Hardy space of square integrable	
	functions on $\mathbb{C}_0^+$ with values in Z	643
$I_{\delta}$	approximate identity	534
$J(z_0;t_0,t_e,u)$	cost functional on the interval $[t_0, t_e]$	269
ker T	kernel of T	583
$L_B^{\tau}$	controllability gramian of $\Sigma(A, B, -)$ on $[0, \tau]$	144
$L_C^{\tau}$	observability gramian of $\Sigma(A, -, C)$ on $[0, \tau]$	154

Notation	687	

Symbol	Meaning	Page
$L(\Omega; Z)$	class of Lebesgue measurable functions	
	from $\Omega$ to Z	626
$L_{\infty}(a,b)$	class of bounded measurable functions	
	from $[a, b]$ to $\mathbb{C}$	573
$L_{\infty}(\Omega; Z)$	class of bounded measurable functions	
	from $\Omega$ to Z	626
$oldsymbol{L}_{\infty}(\partial \mathbb{D};\mathbb{C}^{k imes m})$	class of bounded measurable functions	
	from $\partial \mathbb{D}$ to $\mathbb{C}^{k \times m}$	450
$\boldsymbol{L}_p(a,b)$	class of Lebesgue measurable complex-	
	valued functions with $\int_{a}^{b}  f(t) ^{p} dt < \infty$	573
$L_p(\Omega; Z)$	class of Lebesgue measurable Z-valued	
r · · · ·	functions with $\int_{\Omega}  f(t) ^p dt < \infty$	626
$L_2((-1\infty, 1\infty); Z)$	$L_p(\Omega; Z)$ with $p = 2$ and $\Omega = (-1\infty, 1\infty)$	639
$L_2(\partial \mathbb{D})$	$L_p(\Omega; Z)$ with $p = 2, \Omega = \partial \mathbb{D}$ and $Z = \mathbb{C}$	450
$L_2(\partial \mathbb{D}; \mathbb{C}^m)$	$L_p(\Omega; Z)$ with $p = 2$ , $\Omega = \partial \mathbb{D}$ and $Z = \mathbb{C}^m$	450
$L_2^{\overline{loc}}([0,\infty);U)$	class of functions which are in	
2 ((-)	$L_2((a, b); U)$ for all $a, b \in [0, \infty)$	175
$\mathcal{L}(X)$	bounded linear operators from $X$ to $X$	584
$\mathcal{L}(X, Y)$	bounded linear operators from X to Y	584
l.	complex-valued sequences with	
°p		
	$\sum_{n=1}  x_n ^p < \infty$	572
$\ell_{\infty}$	bounded complex-values sequences	573
$\mathcal{MA}$	class of matrices with elements in $\mathcal{A}$	656
$\mathcal{M}\hat{\mathcal{A}}(eta)$	class of matrices with elements in $\hat{\mathcal{A}}(\beta)$	349
$\mathcal{M}\hat{\mathcal{A}}_{-}(eta)$	class of matrices with elements in $\hat{\mathcal{A}}_{-}(\beta)$	349
$\mathcal{M}\hat{\mathcal{B}}(eta)$	class of matrices with elements in $\hat{\mathcal{B}}(\beta)$	349
$M_2([-h_p, 0]; \mathbb{C}^n)$	$\mathbb{C}^n \oplus L_2((-h_p, 0); \mathbb{C}^n)$	56
$\mathcal{N}$	nonobservable subspace	157
N	set of positive integers	
$P(\Omega; \mathcal{L}(Z_1, Z_2))$	class of weakly measurable	
	functions from $\Omega$ to $\mathcal{L}(Z_1, Z_2)$	626
$\boldsymbol{P}_p(\Omega; \mathcal{L}(Z_1, Z_2))$	functions in $P(\Omega; \mathcal{L}(Z_1, Z_2))$ with	
	$\int_{\Omega} \ F(t)\ ^p dt < \infty$	626
$\boldsymbol{P}_{\infty}(\Omega; \mathcal{L}(Z_1, Z_2))$	class of bounded weakly measurable	
	functions from $\Omega$ to $\mathcal{L}(Z_1, Z_2)$	626
$P_{\infty}((-j\infty,j\infty);\mathcal{L}(U,Y))$	class of weakly measurable bounded	
	functions from $(-j\infty, j\infty)$ to $\mathcal{L}(U, Y)$	639
$\mathbb{R}$	the set of real numbers	
$\mathcal{R}$	reachable subspace	157
$\mathcal{R}(eta)$	$\beta$ -stable, proper, rational functions	653
$\mathcal{R}^r(oldsymbol{eta})$	$\beta$ -stable, real, proper, rational functions	653
$\mathcal{R}_{\infty}(oldsymbol{eta})$	$\beta$ -stable, biproper, rational functions	653
$\mathcal{R}^r_\infty(eta)$	$\beta$ -stable, real, biproper, rational functions	653

# 688 Notation

Symbol	Meaning	Page
$\mathbb{R}(s)$	real, rational functions	653
$\mathbb{R}_p(s)$	real, proper, rational functions	653
ran T	range of the operator T	582
$r_{\sigma}(T)$	spectral radius of T	614
$u^{min}(\cdot;z_0,t_0,t_e)$	optimal input trajectory	272
$y^{min}(\cdot;z_0,t_0,t_e)$	optimal output trajectory	272
$\mathbb{Z}_{+}$	set of integers	
$z^{min}(\cdot; z_0, t_0, t_e)$	optimal state trajectory	272
∂D	unit circle	450
$\delta_T(G, G_\Delta)$	directed gap	558
$\Delta(\lambda)$	characteristic function of delay system	58
$\Gamma_h$	Hankel operator associated with	
	impulse response h	396
$\rho(A)$	resolvent set of A	608
$ \rho_{\infty}(A) $	component of $\rho(A)$ that contains an	
	interval $[r, \infty), r \in \mathbb{R}$	70
$\Sigma(A, B, C, D)$	state linear system	141
$\Sigma(A, B, C)$	state linear system with $D = 0$	141
$\Sigma(A, B, -)$	state linear system with C undefined	141
$\Sigma(A, -, C)$	state linear system with B undefined	141
$\Sigma_d(A, B, C, D)$	discrete-time state linear system	211
$\sigma(A)$	spectrum of A	610
$\sigma_c(A)$	continuous spectrum of A	610
$\sigma_p(A)$	point spectrum of A	610
$\sigma_r(A)$	residual spectrum of A	610
$\sigma^+_\delta(A)$	$\sigma(A) \cap \overline{\mathbb{C}_{\delta}^+}$	229
$\sigma_{\delta}^{-}(A)$	$\sigma(A)\cap \check{\mathbb{C}_{\delta}^{-}}$	229

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