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Lee Middleton and Jayanthi Sivaswamy

Hexagonal Image Processing

A Practical Approach

With 116 Figures

Springer

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To my parents,
Lee

To Munni and to the loving memory of Appa,
Jayanthi

Foreword

The sampling lattice used to digitize continuous image data is a significant determinant of the quality of the resulting digital image, and therefore, of the efficacy of its processing. The nature of sampling lattices is intimately tied to the tessellations of the underlying continuous image plane. To allow uniform sampling of arbitrary size images, the lattice needs to correspond to a regular - spatially repeatable - tessellation. Although drawings and paintings from many ancient civilisations made ample use of regular triangular, square and hexagonal tessellations, and Euler later proved that these three are indeed the only three regular planar tessellations possible, sampling along only the square lattice has found use in forming digital images. The reasons for these are varied, including extensibility to higher dimensions, but the literature on the ramifications of this commitment to the square lattice for the dominant case of planar data is relatively limited. There seems to be neither a book nor a survey paper on the subject of alternatives. This book on hexagonal image processing is therefore quite appropriate.

Lee Middleton and Jayanthi Sivaswamy well motivate the need for a concerted study of hexagonal lattice and image processing in terms of their known uses in biological systems, as well as computational and other theoretical and practical advantages that accrue from this approach. They present the state of the art of hexagonal image processing and a comparative study of processing images sampled using hexagonal and square grids. They address the hexagonal counterparts of a wide range of issues normally encountered in square lattice-based digital image processing - data structures for image representation, efficient pixel access, geometric and topological computations, frequency domain processing, morphological operations, multiscale processing, feature detection, and shape representation. The discussions of transformations between square and hexagonal lattice-based images and of hybrid systems involving both types of sampling are useful for taking advantage of both in real-life applications. The book presents a framework that makes it easy to implement hexagonal processing systems using the square grid as the base,

e.g., to accommodate existing hardware for image acquisition and display, and gives sample computer code for some commonly encountered computations.

This book will serve as a good reference for hexagonal imaging and hexagonal image processing and will help in their further development. I congratulate the authors on this timely contribution.

Professor Narendra Ahuja
August, 2004

Preface

The field of image processing has seen many developments in many fronts since its inception. However, there is a dearth of knowledge when it comes to one area namely the area of using alternate sampling grids. Almost every textbook on Digital Image Processing mentions the possibility of using hexagonal sampling grids as an alternative to the conventional square grid. The mention, however, is usually cursory, leading one to wonder if considering an alternative sampling grid is just a worthless exercise. Nevertheless, the cursory mention also often includes a positive point about a hexagonal grid being advantageous for certain types of functions. While it was curiosity that got us interested in using hexagonal grids, it was the positive point that spurred us to study the possibility of using such a grid further and deeper. In this process we discovered that while many researchers have considered the use of hexagonal grids for image processing, most material on this topic is available only in the form of research papers in journals or conference proceedings. In fact it is not possible to find even a comprehensive survey on this topic in any journal. Hence the motivation for this monograph.

In writing this book, we were mindful of the above point as well as the fact that there are no hardware resources that currently produce or display hexagonal images. Hence, we have tried to cover not only theoretical aspects of using this alternative grid but also the practical aspects of how one could actually perform hexagonal image processing. For the latter, we have drawn from our own experience as well that of other researchers who have tried to solve the problem of inadequate hardware resources.

A large part of the work that is reported in the book was carried out when the authors were at the Department of Electrical and Electronic Engineering, The University of Auckland, New Zealand. The book took its current shape and form when the authors had moved on to the University of Southampton (LM) and IIIT-Hyderabad (JS). Special thanks to Prof. Narendra Ahuja for readily agreeing to write the foreword. Thanks are due to the anonymous reviewers whose feedback helped towards making some key improvements to the book.

Lee: Thanks are first due to Prof. Mark Nixon and Dr John Carter who were understanding and provided me time to work on the book. Secondly thanks go to my, then, supervisor Jayanthi for believing in the idea I came to her office with. Thirdly, I would like to thank the *crew* at Auckland University for making my time there interesting: adrian, anthony, bev, bill, brian, brad, bruce, colin, david, dominic, evans, evan, geoff (×2), jamie, joseph, nigel, russell m, and woei. Finally, thanks go to Sylvia for being herself the whole time I was writing the manuscript.

Jayanthi: Thanks to Richard Staunton for many helpful comments and discussions, to Prof. Mark Nixon for the hospitality. I am also grateful to Bikash for clarifications on some of the finer points and to Professors K Naidu, V U Reddy, R Sangal and other colleagues for the enthusiastic encouragement and support. The leave from IIIT Hyderabad which allowed me to spend concentrated time on writing the book is very much appreciated. The financial support provided by the DST, Government of India, the Royal Society and the British Council partly for the purpose of completing the writing is also gratefully acknowledged. Finally, I am indebted to Prajit for always being there and cheering me on.

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