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Differential Equations and Data Analysis

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To the giants on whose shoulders we stand.

Preface

Having taught Ordinary Differential Equations (ODE) for nearly 20 years to engineering and science majors, I have come to the conclusion that the first thing they need to learn is modeling real-world data with linear ODE with constant coefficients. That is what this book is about. Nonlinear ODE are, of course, very important and interesting, arguably more so than their linear counterparts. However, nonlinear theory is the *second* thing that should be learned, and only after linear theory has been mastered.

The book begins with simple population modeling examples in Chap. 1 and ends with frequency response analysis in Chap. 10. Hopefully, the eight chapters in between make the elevation gain somewhat gradual, but that depends on the reader's preparation. As a minimum, I expect multivariate calculus, some familiarity with matrices, programming literacy that is sufficient for parsing the included MATLAB code, and a bit of probability theory. Linear algebra will be very beneficial as well.

With the exception of Chaps. 5 and 10, each chapter contains at least one section where physical data is modeled and analyzed. Some chapters contain three or four such sections. In this respect, the book is similar to Martin Braun's *Differential Equations and Their Applications*, published by Springer-Verlag in the early 1980s, but in the following respects it is different.

Firstly, there is a narrow focus on linear theory. This is justified in Chaps. 3 and 4 where it is shown that many ODE that are relevant in science and engineering are either linearized or are linear to begin with. Besides making the book coherent and concise, the exclusion of nonlinear theory created space for topics in linear theory and data analysis that are rarely found in ODE texts, such as the connection between nonlinear least squares and maximum likelihood estimates in Chap. 2, a thorough discussion of vector spaces in Chap. 6, and multidimensional convolution in Chap. 7.

Secondly, data analysis is treated on equal footing with linear ODE theory. It has two dedicated chapters—Chap. 2 on estimation of model parameters using nonlinear least squares and Chap. 8 on discrete Fourier transform (DFT)—and is consistently practiced in other chapters. I tried to include either full or downsampled data sets as tables whenever possible; when I could not find good quality data, I simulated it in MATLAB and included code. Some authors urge their readers to have pen and paper at the ready. I urge mine to have MATLAB (or its equivalent) running on their computer: the mathematics discussed in this book is best learned as it is translated into code.

Thirdly, I abandoned traditional "definition-lemma-theorem" style of mathematical exposition in favor of a more conversational style. My main goal is not to show "how" but to explain "why": Why does one separate variables? Why do complex exponentials make sense? Why are Fourier coefficients what they are? Why does the model deviate from experiment? Also, there are no citations in the main text: all attributions are in the "Comments and bibliography" sections that end each chapter.

Answering the "why" questions is more difficult than answering the "how" questions, often because of misconceptions that stand in the way. Chapter 5 addresses one popular misconception about the exponential function—that it is a strange number e multiplied by itself x times; in my experience, most sophomores have trouble accepting that complex and matrix exponentials make sense because of that. The "linear algebra" Chap. 6 has abstract algebra content aimed at clearing two other misconceptions: that imaginary numbers are "figments of our imagination" and that vectors are "things with magnitude and direction." I took the unusual step of defining number fields to dispel all doubts surrounding complex numbers. That is followed by an abstract definition of a vector space whose purpose is to show that vectors are defined by vector operations rather than their geometric attributes.

While most of the material in the book is standard, the presentation often is not. For instance, the derivation of the convolution formula in Chap. 7 uses only simple Calculus and does not rely on Dirac's delta function—I have not seen this in other books. The exposition of Fourier analysis begins with DFT, presented as a way of approximating functions with trigonometric polynomials, and proceeds to Fourier series, derived as limits of DFT-based approximations, and then to Fourier transform, derived as a limiting case of Fourier series: this may not be a new idea, but it is unorthodox.

In summary, this book presents linear ODE theory and immerses the reader into modeling of real data, with all its warts. By itself, it may not be suitable for a conventional ODE course, however, I hope that it will serve as a useful supplement.

Stockton, California Aleksei Beltukov
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