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Shrinkage Estimation for Mean and Covariance Matrices

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Preface

The rapid development of computer technology has started to yield many types of high-dimensional data and to enable us to deal with them well. Indeed, high-dimensional data appear in numerous fields such as web data science, genomics, telecommunication, atmospheric science, financial engineering, and others. With such a background, theory of statistical inference with high dimension has received much attention in recent years.

High-dimensional data in general are hard to handle, and ordinary or traditional methods in statistics are frequently inapplicable for them. This has inspired statisticians to develop new methodology in high dimension from both theoretical and practical aspects. Most statisticians' interests seem to be in development of efficient algorithms for statistical inference and in investigation of their asymptotic properties with the dimension going to infinity. On the other hand, there does not exist much literature in high-dimensional problems from a decision-theoretic point of view.

Statistical decision theory is the study of how to make decisions in the presence of statistical knowledge under uncertainty. It has been studied from around the 1940s and the researchers have already been produced many important and interesting results. Probably the most surprising result in decision-theoretic estimation is the inadmissibility of the sample mean vector to estimate a multivariate normal population mean. In the multivariate normal mean estimation, the sample mean vector is the maximum likelihood estimator and the uniformly minimum variance unbiased estimator, and thus it has been recognized to be optimal for a long time. However, in 1956, Charles Stein showed that the sample mean vector is admissible for the one- and two-dimensional cases but inadmissible for three or more dimensional cases. A little after that, a specific estimator, called a shrinkage estimator, was provided for exactly dominating the sample mean vector. To this day, various extensions of shrinkage estimation have been achieving in other statistical models.

The purpose of this book is to give a brief overview of shrinkage estimation in matrix-variate normal distribution model. More specifically, it includes recent techniques and results in estimation of mean and covariance matrices with a high-dimensional setting that implies singularity of the sample covariance matrix. Such a high-dimensional model can really be analyzed by using the same arguments as for a low-dimensional model. Thus this book takes a unified approach to both high- and low-dimensional shrinkage estimation.

Theory of shrinkage estimation for matrix parameters needs many mathematical tools. In Chap. 1, we begin by briefly introducing basic terminology of decision-theoretic estimation and a mathematical technique in shrinkage estimation. Chapter 2 defines the notation with respect to matrix algebra and collects useful results in terms of the Moore-Penrose inverse, the Kronecker product and matrix decompositions. Chapter 3 provides the definition and some properties of matrix-variate normal distribution and related distributions, including the Wishart distribution and joint distributions corresponding to the Cholesky and the eigenvalue decompositions of the Wishart matrix. With a unified treatment for high- and low-dimensional cases, some related distributions are discussed. Chapter 4 introduces a multivariate linear model and derives its canonical form. To find decision-theoretically optimal estimators, we usually direct our attention to several classes of invariant estimators. Therefore Chap. 4 briefly explains group invariance in the canonical form as well. A key tool in shrinkage estimation is an integration by parts formula, called the Stein identity. Chapter 5 gives a generalized Stein identity on matrix-variate normal distribution. Moreover we list some results on matrix differential operators and in particular show useful differentiation formulae concerning the Moore-Penrose inverse. Chapter 6 addresses the problem of estimating the mean matrix in matrix-variate normal distribution model. A unified result on matricial shrinkage estimation is presented, and extensions and applications are given for more general models. Chapter 7 deals with the problem of estimating the covariance matrix relative to an extended Stein loss and provides various unified estimation procedures for high- and low-dimensional cases. Some related topics to covariance estimation are also touched.

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