Course Notes: A Crash Course on Causality – Week 1: Intro to Causal Effects

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Notations

- We are interested in the causal effect of some treatment A on some outcome Y
- Treatment: A, binary
  - $A = 1$ if receive treatment; and $A = 0$ if receive control
  - Example: $A = 1$ if receive active drug; and $A = 0$ if receive placebo
- Outcome: Y, can be binary or continuous
  - Example: $Y = 1$ if dead; $Y = 0$ otherwise
  - Example: Y can be time until death
- Pre-treatment covariates: X

Potential outcomes

- Potential outcome Y^a is the outcome we would see if treatment was set to $A = a$
- Each person has potential outcome Y^0, Y^1

Counterfactuals

- Conterfactual outcomes: the outcomes would have been observed, had the treatment been different
  - If my treatment is $A = 1$, then my counterfactual outcomes is Y^0
  - If my treatment is $A = 0$, then my counterfactual outcomes is Y^1
- Connection between potential and conterfactuals outcomes
  - **Before** the treatment decision is made, any outcome is a potential outcome, Y^0 and Y^1
  - **After** the study, there is an observed outcome Y^A, and counterfactual outcome Y^{1-A}

Immutable variables

- Variables that we cannot control (or change), such as race, gender, age, are immutable variables
- For immutable variables, causal effects are not well defined
- In this course, we focus on treatments that could be thought of as **interventions**

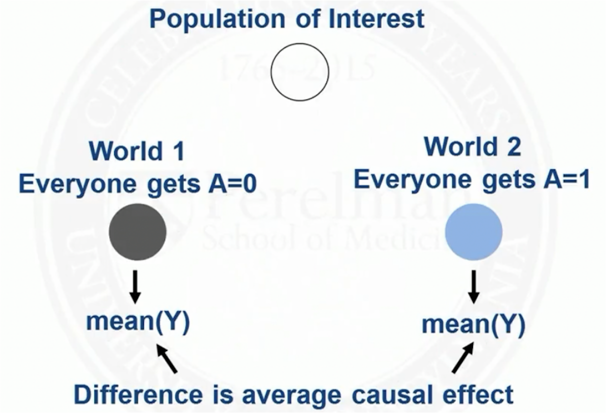
Causal effects

- Definition: treatment A has a causal effect on the outcome Y, if Y^1 differs from Y^0
- Example
  - Y: headache gone one hour from now (yes$=1$, no$=0$)
  - A: take ibuprofen ($A=1$) or not ($A=0$)

Fundamental problem of causal inference

- Fundamental problem of causal inference: we can only observe one potential outcome for each person
- However, with certain assumptions, we can estimate **population level** (average) causal effects $E(Y^1 - Y^0)$
  - Average value of Y if everyone was treated with $A = 1$ minus average value of Y if everyone was treated with $A = 0$
- Headache example:
  - Hopeless: What would have happened to me had I not taken ibuprofen? (Unit level causal effect)
  - Hopeful: What would the rate of headache remission be if everyone took ibuprofen when they had a headache versus if no one did? (Population level causal effect)

Visualization of population average causal effect

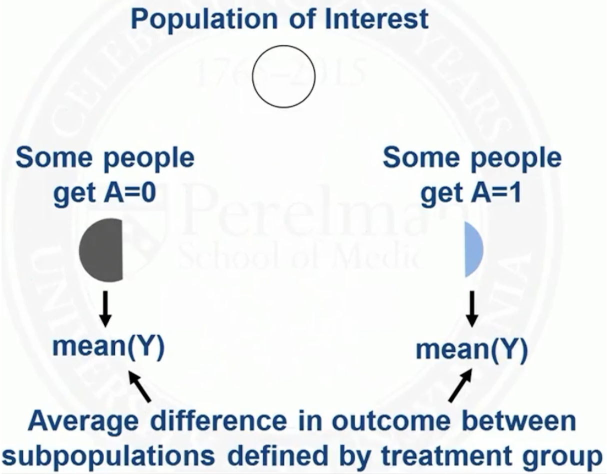


Population average causal effect versus conditioning on treatment/control

$$E(Y^1 - Y^0) \neq E(Y \mid A = 1) - E(Y \mid A = 0)$$

- In the left hand side, $E(Y^1)$ is the mean of Y if the whole population was treated with $A = 1$
- In the right hand side, $E(Y \mid A = 1)$ is restricting to the **subpopulation** of people who actually had $A = 1$
  - This subpopulation may differ from the whole population in important ways
  - For example, people at higher risk for flu are more likely to choose to get a flu shot
- $E(Y \mid A = 1) - E(Y \mid A = 0)$ is not a causal effect, because it is comparing two different populations of people

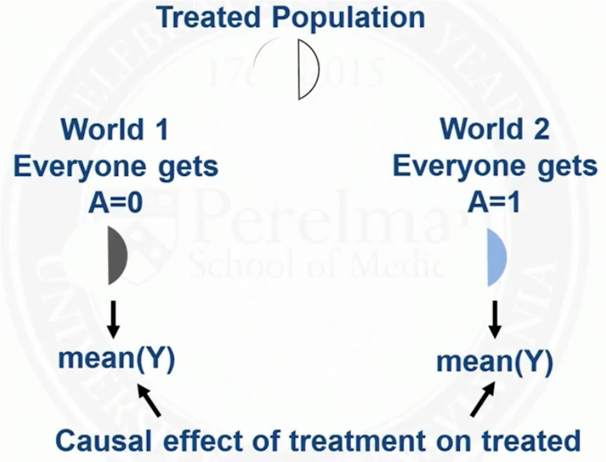
Visualization of real world



Other causal effects

- $E(Y^1/Y^0)$: causal relative risk
- $E(Y^1 - Y^0 \mid A = 1)$: causal effect of treatment on the treated
- $E(Y^1 - Y^0 \mid V = v)$: average causal effect in the subpopulation with covariate $V = v$

Visualization of causal effect of treatment on the treated



Most common causal assumptions

- Stable Unit Treatment Value Assumption (SUTVA)
- Consistency
- Ignorability
- Positivity

Stable Unit Treatment Value Assumption (SUTVA)

- SUTVA involves two assumptions

1. No interference
   - Units do not interfere with each other
   - Treatment assignement of one unit does not affect that outcome of another unit
   - Spillover or contagion are also terms for interference
2. One version of treatment

- SUTVA allows us to write potential outcome for a person in terms of only that person's treatments

Consistency assumption

- Consistency assumption: the potential outcome under treatment $A = a$, Y^a, is equal to the observed outcome if the actual treatment received is $A = a$

$$Y = Y^a \text{ if } A = a, \text{ for all } a$$

Ignorability assumption

- Ignorability assumption: given pre-treatment covariates X, treatment assignment is independent from the potential outcomes

$$Y^0, Y^1 \perp A \mid X$$

- **Among people with the same values of X, we can think of treatment A as being randomly assigned**
- Example: Y^0 and Y^1 are not independent from A marginally, but within levels of X, treatment might be randomly assigned
  - X: age; can take values 'younger' or 'older'
  - Y: hip fracture
  - Older people are more likely to get treatment $A = 1$
  - Older people are also more likely to have the outcome, regardless of treatment

Positivity assumption

- Positivity assumption: for every set of values of X, treatment assignment was not deterministic

$$P(A = a \mid X = x) > 0, \text{ for all } a \text{ and } x$$

- If for some values of X, treatment was deterministic, then we would have no observed values of Y for one of the treatment groups for those values of X

Observed data and potential outcomes

- Under all above assumptions, the observed data average outcome $E(Y \mid A = a, X = x)$ equals the potential outcomes $E(Y^a \mid X = x)$

$$\begin{aligned} E(Y \mid A = a, X = x) &= E(Y^a \mid A = a, X = x) \text{ by consistency} \\ &= E(Y^a \mid X = x) \text{ by ignorability} \end{aligned}$$

- If we want a marginal causal effect, we can average over X

$$E(Y^a) = \sum_x E(Y \mid A = a, X = x) P(X = x)$$

Standardization

- Standardization involves stratifying and then averaging
  - First obtain the mean treatment effect within each stratum $E(Y \mid A = a, X = x)$
  - Then pool across stratum, weighing by the probability (size) of each stratum $P(X = x)$

Standardization example: two diabetes treatments

- Treatments: saxagliptin (new medicine) vs sitagliptin
- Outcome: major adverse cardiac event (MACE)
- Covariate: past use of oral antidiabetic (OAD) drug
- Challenge
  - Saxa users were more likely to have past use of OAD drug
  - Patients with past use of OAD drugs are at higher risk of MACE
- Stratify parents in two subpopulations by whether having prior OAD use
  - Within levels of the prior OAD use variable, treatment can be thought of as randomized (ignorability)

Example continued: unstratified

Table 1: Unstratified raw data

	MACE=yes	MACE=no	Total
Saxa=yes	350	3650	4000
Saxa=no	500	6500	7000
Total	850	10150	11000

$$P(\text{MACE} \mid \text{Saxa} = \text{yes}) = 350/4000 = 0.088$$
$$P(\text{MACE} \mid \text{Saxa} = \text{no}) = 500/7000 = 0.071$$

Example continued: subpopulation without prior OAD use

Table 2: Prior OAD use = no

	MACE=yes	MACE=no	Total
Saxa=yes	50	950	1000
Saxa=no	200	3800	4000
Total	250	4750	5000

$$P(\text{MACE} \mid \text{Saxa} = \text{yes}) = 50/1000 = 0.05$$
$$P(\text{MACE} \mid \text{Saxa} = \text{no}) = 200/4000 = 0.05$$

Example continued: subpopulation with prior OAD use

Table 3: Prior OAD use = yes

	MACE=yes	MACE=no	Total
Saxa=yes	300	2700	3000
Saxa=no	300	2700	3000
Total	600	5400	6000

$$P(\text{MACE} \mid \text{Saxa} = \text{yes}) = 300/3000 = 0.10$$
$$P(\text{MACE} \mid \text{Saxa} = \text{no}) = 300/3000 = 0.10$$

Example continued: mean potential outcome for Saxa

$$
\begin{aligned}
& E(Y^{\text{saxa}}) \\
= \;& E(Y \mid A = \text{saxa}, X = \text{prior OAD use yes}) P(\text{prior OAD use yes}) \\
& + E(Y \mid A = \text{saxa}, X = \text{prior OAD use no}) P(\text{prior OAD use no}) \\
=& (300/3000)(6000/11000) + (50/1000)(5000/11000) \\
=& 0.077
\end{aligned}
$$

- Similarly, $E(Y^{\text{sita}}) = 0.077$
- Hence, the treatment Saxa or not has no causal effects on the MACE outcome

Problems with standardization

- There will be many X variables needed to achieve ignorability
- Stratification would lead to many empy cells
- Alternative to standardization: matching inverse probability of treatment weighting (IPTW), etc

References

- Coursera class: “A Crash Course on Causality: Inferring Causal Effects from Observational Data”, by Jason A. Roy (University of Pennsylvania)
  - https://www.coursera.org/learn/crash-course-in-causality