A Survey of Phase Variable Candidates of Human Locomotion

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Abstract— **Studies show that the human nervous system is able to parameterize gait cycle phase using sensory feedback. In the field of bipedal robots, the concept of a phase variable has been successfully used to mimic this behavior by parameterizing the gait cycle in a time-independent manner. This approach has been applied to control a powered transfemoral prosthetic leg, but the proposed phase variable was limited to the stance period of the prosthesis only. In order to achieve a more robust controller, we attempt to find a new phase variable that fully parameterizes the gait cycle of a prosthetic leg. The angle with respect to a global reference frame at the hip is able to monotonically parameterize both the stance and swing periods of the gait cycle. This survey looks at multiple phase variable candidates involving the hip angle with respect to a global reference frame across multiple tasks including level-ground walking, running, and stair negotiation. In particular, we propose a novel phase variable candidate that monotonically parameterizes the whole gait cycle across all tasks, and does so particularly well across level-ground walking. In addition to furthering the design of robust robotic prosthetic leg controllers, this survey could help neuroscientists and physicians study human locomotion across tasks from a time-independent perspective.**

I. INTRODUCTION

Human locomotion is currently parameterized as a time-dependent process called a gait cycle. The gait cycle is often divided into subsections denoted by percentages representing specific periods of human locomotion. It is often also divided into two gross sections for each leg. The first one, stance, from 0% to 62% is characterized by the sole of the foot in contact with the ground. The second one, swing, from 62% to 100% is characterized by the sole of the foot's lack of contact with the ground [1].

Recent studies show that spinal centers in humans are largely responsible for the control of locomotion [2], [3]. This means that the initiation of locomotion is voluntarily commanded by the brain, but once the motion is started, spinal centers control the muscle activation that makes human locomotion possible. This infers that there exists a low-level programming hierarchy that allows walking without involvement of the brain. Thus, the nervous system can determine at which part of the gait cycle the body is and

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where it should be next. From a mathematical perspective, it is possible to characterize phase using the entire system state [4], [5], but in the case of complex systems, like humans, this can involve measurements of hundreds of variables. Bipedal robots mimic this behavior by parameterizing the gait cycle in a time-independent manner using a single state variable called a phase variable [6].

A phase variable is a mechanical signal that changes monotonically, i.e., it strictly increases or decreases, over time and therefore is able to parameterize a rhythmic process. Given the phase variable at a specific time, the specific state of the process can be determined as well as its next movement. A reliable phase variable candidate must possess specific characteristics, including being monotonic and un-actuated. Monotonicity over time helps distinguish precisely where the process is and underactuation makes the phase variable independent from the controlled process itself. In a gait cycle, a phase variable can be used to control the progression of leg joints. Several phase variables involving joint angles and velocities in the gait cycle have been applied to biped robots [6]–[9]. These phase variables have given biped robots a physiological gait and have yielded outstanding capabilities, including walking, running, and climbing stairs [6]. This is because parameterizing the gait cycle in a time-independent manner gives more robust control over the robot. This simple but rigorous approach has motivated recent studies trying to understand if human locomotion also relies on a phase variable [10].

During recent years there have been successful attempts at translating concepts from the field of bipedal robots into physical rehabilitation. Holgate et al. [11] were the first to use a time-independent phase variable on robotic prostheses to parameterize the gait cycle across different stride lengths. A phase variable was later used to parameterize virtual constraints for nonlinear control of robotic prosthetic legs [12]–[14]. This work used the center of pressure (COP) as a phase variable, which is the location on the sole of the foot where the pressure is concentrated during the stance period of the gait cycle. By using this phase variable, it was possible to control a powered transfemoral prosthesis in a time-independent manner, enabling amputee subjects to walk at different gait speeds using the same controller [15]. Even though the COP works well as a phase variable for controlling the powered transfemoral prosthesis during stance, the information of the COP is lost during the swing period of the gait cycle. This forces the prosthesis to use a different type of controller during this period. Therefore, another phase variable must be found that can control a transfemoral robotic prosthetic leg with information available in both the stance

and swing periods of the gait cycle. In particular, for fully powered transfemoral prostheses, a phase variable cannot be any joint directly actuated by the prosthesis itself. The hip angle with respect to a global reference frame is an angle that parameterizes monotonically both stance and swing periods of the gait cycle. In addition, this angle is not directly actuated by the transfemoral robotic prosthesis. Thus, this angle is a good phase variable candidate to study.

This survey looks at multiple phase variable candidates involving the hip angle with respect to a global reference frame across tasks normally performed on a daily basis, e.g., level-ground walking at different cadences, running, sprinting, and stair negotiation. Finding a good phase variable candidate has important implications in the field of physical rehabilitation, since it would allow more robust control of powered transfemoral prostheses. In addition, it would increase bipedal robot applications by providing parameterizations of different tasks that can later be used for time-independent nonlinear control. It could also allow neuroscientists and physicians to study human locomotion across tasks from a time-independent perspective.

II. METHODS

The pelvis, hip, knee, and ankle angles across seven different locomotion tasks were taken from data available in the literature, see Section II-A. They were analyzed to find phase variable candidates directly related to the hip angle with respect to a global reference frame. A total of five phase variable candidates were studied and evaluated in the sagittal plane, where most gait motion takes place. The main criteria to evaluate the phase variable candidates was monotonicity, which was determined by inspection in this preliminary work.

A. Tasks

Tasks often performed on a daily basis were considered for the analysis of each phase variable candidate. In total, seven tasks were included in the study: normal cadence [16], slow cadence [16], fast cadence [16], running [17], [18], sprinting [17], [18], stair ascent [19], and stair descent [20].

B. Phase Variable Candidates

Five phase variable candidates directly related to the hip angle with respect to a global reference frame were analyzed.

1) Angle Relative to the Pelvis (θ_1): Angle θ_1 in Fig. 1 (left) is the hip angle with respect to the pelvis frame and the thigh. This is the angle often measured in gait studies.

2) Global Angle with Respect to the Thigh (θ_2): In order to get a global reference frame, the pelvic tilt α was subtracted from θ_1. Therefore, angle θ_2 in Fig. 1 (right) is the global angle with respect to the thigh.

3) Global Angle with Respect to the Ankle (θ_3): Using triangular geometry, vector $\overrightarrow{d_a}$ from the hip to the ankle was found from Fig. 2 and its magnitude is given by the equation

$$d_a = \sqrt{T^2 + S^2 - 2TS\cos(\gamma)}, \qquad (1)$$

where T and S are the average length of the thigh and shank, respectively, for a person whose height is 1.80 m [16], and
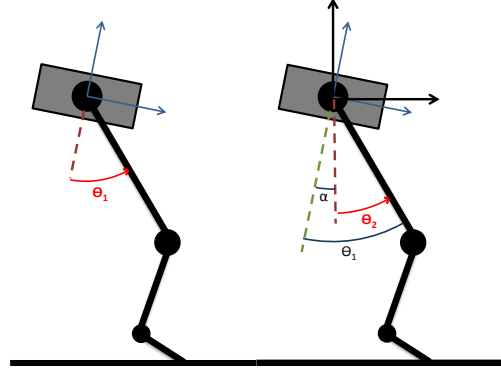

Fig. 1.   Angle θ_1 relative to the pelvis (left) and global angle θ_2 with respect to the thigh (right).

γ is the complementary angle of the knee directly measured in gait studies. Similarly, angle θ_3 with respect to the global reference frame shown on the left of Fig. 2 was calculated by the equation

$$\theta_3 = \theta_2 - s', \qquad (2)$$

where s' is given by the equation

$$s' = \arccos\left(\frac{d_a^2 + T^2 - S^2}{2d_a T}\right). \qquad (3)$$

This phase variable candidate θ_3 has been used in the past to control biped robots with point feet for level-ground walking [6], [9], although it was calculated in a different manner.
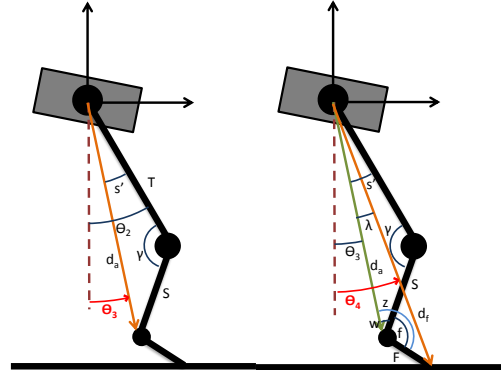

Fig. 2.   Global angle θ_3 with respect to the ankle (left) and global angle θ_4 with respect to the tip of the foot (right).

4) Global angle with respect to the tip of the foot (θ_4): Using triangular geometry, vector $\overrightarrow{d_f}$ from the hip to the tip of the foot was found from Fig. 2 and its magnitude is given by the equation

$$d_f = \sqrt{d_a^2 + F^2 - 2d_a F\cos(z)}, \qquad (4)$$

where F is the average length of the foot for a person whose height is 1.80 m [16], and z is the angle between the foot and the vector $\overrightarrow{d_a}$ previously calculated in (1). Angle z was calculated by adding the angles f, which is the ankle angle directly measured in gait studies, and angle w, which is given by the equation

$$w = 180 - (s' + \gamma),$$

where γ is the complementary angle of the knee directly measured in gait studies and s' was given by (3). Similarly, angle θ_4 with respect to the global reference frame shown in Fig. 2 (right) was calculated by the equation

$$\theta_4 = \theta_3 + \lambda, \quad (5)$$

where λ is given by the equation

$$\lambda = \arccos\left(\frac{d_f^2 + d_a^2 - F^2}{2 d_f d_a}\right). \quad (6)$$

5) Global angle with respect to the vector addition of $\overrightarrow{d_a}$ and the thigh vector (θ_5): The vector $\overrightarrow{d_{sum}}$ was calculated from Fig. 3 by adding the vector $\overrightarrow{T}$ of constant length T, starting at the hip and ending at the knee, and the vector $\overrightarrow{d_a}$ from (1). Similarly, angle θ_5 with respect to the global reference frame, shown on Fig. 3, was calculated by the equation

$$\theta_5 = \arcsin(\frac{d_{sumx}}{\|d_{sum}\|}), \quad (7)$$

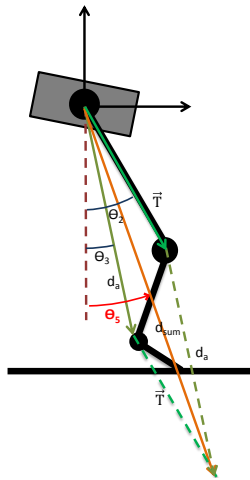where d_{sumx} is the projection of the vector $\overrightarrow{d_{sum}}$ on the horizontal component.

Fig. 3. Global angle θ_5 with respect to $\overrightarrow{d_{sum}}$.

III. RESULTS

Figs. 4, 5, and 6 show the different phase variable candidates defined in Section II-B across the tasks previously discussed in Section II-A. It can be seen in Fig. 4 (top) that the angle θ_1 is piecewise monotonic during the whole gait cycle across the tasks of stair ascent and level-ground walking for normal, slow, and fast cadence. Across other tasks, in particular during stair descent, this angle is not monotonic. In Fig. 4 (bottom), the global angle θ_2 with respect to the thigh is shown across different tasks. This angle follows the same trajectory as the angle θ_1 relative to the pelvis. The difference between angles θ_1 and θ_2 is that angle θ_2 has a small offset that represents the pelvic tilt during locomotion. Hence, in terms of monotonicity, angle θ_2 has the same properties as angle θ_1.
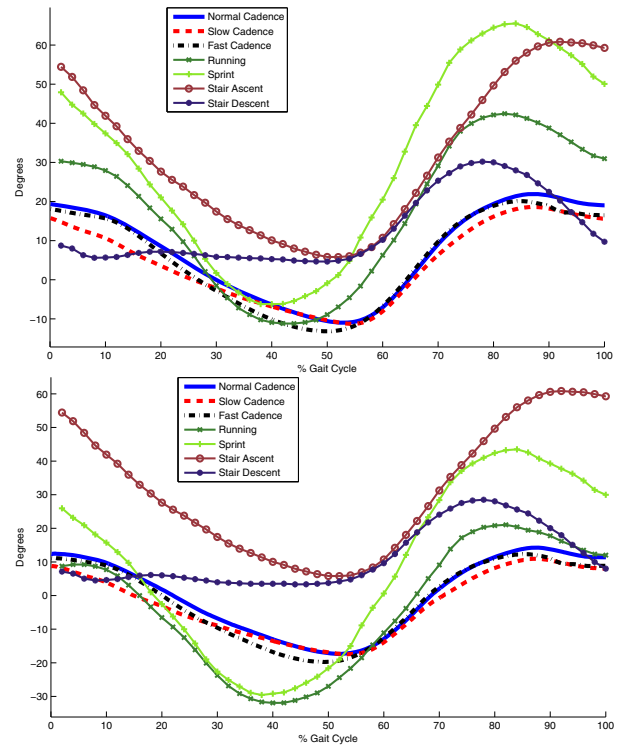

Fig. 4. Angle θ_1 relative to the pelvis (top) and global angle θ_2 with respect to the thigh (bottom) across tasks.

In Fig. 5 (top) the global angle θ_3 with respect to the ankle is shown across multiple tasks. This angle is piecewise monotonic in all tasks except for stair ascent. In Fig. 5 (bottom) the global angle θ_4 with respect to the tip of the foot is shown across tasks. The calculation of this phase variable candidate involves indirectly the pelvis, hip, knee, and ankle angles. This angle looks piecewise monotonic across most of the tasks, specifically during level-ground walking at different cadences. Nevertheless, this phase variable candidate also fails in terms of monotonicity during stair ascent. Thus, by inspection angle θ_3 and angle θ_4 share the same properties of monotonicity.

Fig. 6 shows the global angle θ_5 with respect to the vector addition of $\overrightarrow{T}$ and $\overrightarrow{d_a}$, i.e., $\overrightarrow{d_{sum}}$, across different tasks. This phase variable candidate looks more piecewise monotonic across all tasks than the previous candidates. A reason this might be the case is that the overall angle θ_5 with respect to $\overrightarrow{d_{sum}}$ changes depending on the magnitude of $\overrightarrow{d_a}$, which depends directly on the knee angle γ that varies across the gait cycle. Thus, the dependence on the knee angle may correct violations in monotonicity that were present in the other phase variable candidates.

Finally, considering that level-ground walking is the most performed daily locomotion task, Fig. 7 shows a comparison between all phase variable candidates across this task in particular. It can be seen how the global angle θ_5 with respect to $\overrightarrow{d_{sum}}$ is the most linear piecewise monotonic phase variable candidate during the whole gait cycle for this task.
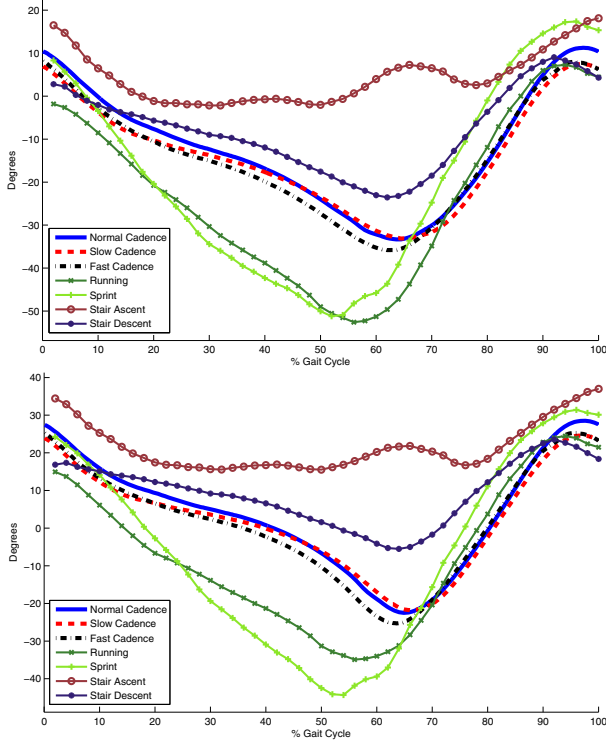
Fig. 5. Global angle θ_3 with respect to the ankle (top) and global angle θ_4 with respect to the tip of the foot (bottom) across tasks.
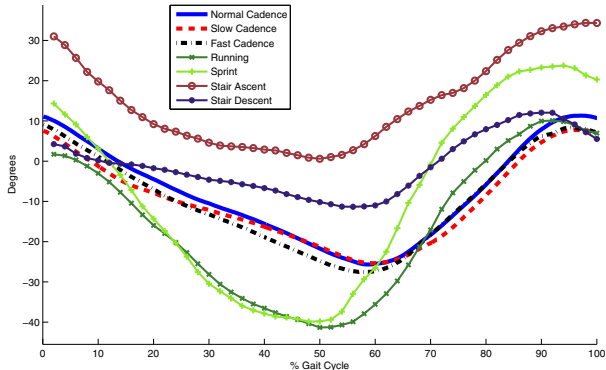

Fig. 6. Global angle θ_5 with respect to $\overrightarrow{d_{sum}}$ across tasks.
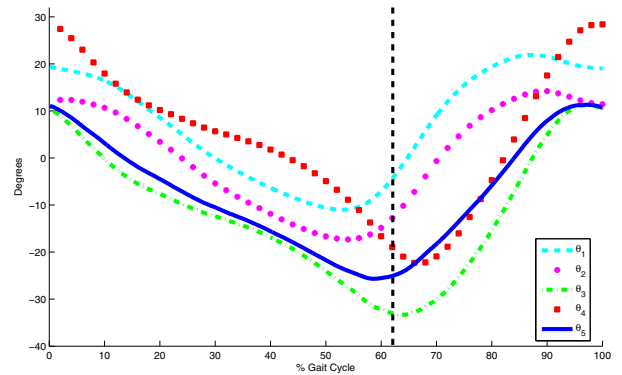

Fig. 7. Phase variable candidates during level ground walking at normal cadence. The vertical dashed line represents the division between stance and swing period in the gait cycle.

IV. DISCUSSION

The phase variable candidates investigated in this survey across multiple tasks help visualize the gait cycle from a time-independent perspective. They show how the combination of several joint angles with respect to a global reference frame can parameterize the gait cycle as a piecewise monotonic variable. In addition, it can be inferred that an ideal piecewise monotonic variable that parameterizes the gait cycle across several tasks may exist. Finding it may involve an optimization algorithm technique, where constraints are applied to ensure strict monotonicity.

Amongst all tasks, phase variable candidate θ_5 in Fig. 6 showed the best monotonicity, making it close to ideal. This means that this phase variable candidate by itself could potentially be used to parameterize all tasks. A remarkable feature of Fig. 6 is that phase variable candidate θ_5 can become completely piecewise monotonic across all tasks if the gait cycle is shifted 10% to the right. This raises the possibility of redefining the beginning of the gait cycle. Indeed, it may be more convenient from the hip perspective to consider the last part of swing as the beginning of the gait cycle rather than the heel strike.

This survey only looked at joint positions as phase variable candidates rather than velocities and accelerations, which rarely fulfill the monotonicity requirement across the gait cycle. By definition accelerations and velocities vary faster than positions, and their measurements are more susceptible to noise. This means that joint patterns parameterized by a phase variable involving velocities or accelerations may oscillate more than a phase variable involving only positions. Therefore, positions are likely more robust choices for controlling the progression of joint patterns. The use of velocity and acceleration variables also has negative theoretical implications in feedback linearization control strategies since they decrease the relative degree of the closed-loop system [6], [21].

All the phase variable candidates evaluated in this survey were defined in the sagittal plane, where most gait motion takes place. Recent experiments in biped robots show that it is possible to use a single phase variable from the sagittal plane to control 3D walking [22]. However, it has also been demonstrated through optimization algorithms that pairing a sagittal-plane phase variable with a frontal-plane phase variable could help improve stability in 3D walking [23].

Finally, this survey offers a novel approach in gait analysis and neuroscience to study locomotion since it introduces a time-independent parametrization of the gait cycle. Lately, experiments have been conducted to try to understand if there exists a phase variable in human locomotor control [10]. Future work will experimentally test these phase variable candidates and analyze how well they parameterize the gait cycle. If future studies show that a phase variable parameterizes human locomotion, this could inform research on human joint impedances, which are highly time-varying parameters during walking [24]. This research could also

lead to great improvements for robotic prosthetic legs, which often control time-varying joint impedances [25] or phase-dependent outputs [11], [15]. As a consequence, amputees will be able to walk more anthropomorphically at different speeds across different tasks. In addition, clinicians will take less time to configure powered transfemoral prostheses since they will not be constrained to time-dependent parameters at specific gait speeds [15]. From a neuroscience perspective, it would mean that the spinal cord contains all the information necessary for humans to control progression of joint patterns during locomotion. This could imply that a central pattern generator in the nervous system relies only on local sensory feedback for human locomotion.

V. CONCLUSION

This survey aids in translating time-independent concepts from the robotic field into the gait analysis and the neuroscience fields. The behavior of several phase variable candidates in the human leg across several locomotion tasks were analyzed. It was shown that several piecewise monotonic variables related to the hip angle with respect to a global reference frame parameterized the gait cycle. Some of the phase variable candidates examined behave better for specific tasks than others. The most piecewise monotonic phase variable candidate found across all tasks is the global angle θ_5 with respect to $\overrightarrow{d_{sum}}$ shown in Fig. 3.

Finally, the major motivation behind this survey is to help design more robust time-independent controllers for powered transfemoral prostheses that can operate across different locomotion tasks. We hope the new phase variables defined in this paper will enable bipedal robots to achieve additional locomotor tasks, as well as encourage neuroscientists and physicians to observe human locomotion from a time-independent perspective.

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