

# Optimally Biomimetic Passivity-Based Control of a Lower-Limb Exoskeleton over the Primary Activities of Daily Life: supplemental material

## 1. DESIGNED BASIS FUNCTIONS

PHI has  $\phi$  incorporated with 67 basis functions, where the list of basis functions for the ankle and the knee joints  $\xi_{\text{ankle}}, \xi_{\text{knee}} \in \mathbb{R}^{67 \times 1}$  are shown as follows

$$\begin{aligned} \xi_{\text{ankle}} = & [1, 0, \sin(\theta_a), \cos(\theta_a), \dots \\ & 0, 0, \sin(\theta_a + \theta_k), \cos(\theta_a + \theta_k), \dots \\ & \sin(2\theta_a), \cos(2\theta_a), 0, 0, \dots \\ & \sin(2\theta_a + 2\theta_k), \cos(2\theta_a + 2\theta_k), 2\sin(2\theta_a + \theta_k), 2\cos(2\theta_a + \theta_k), \dots \\ & \sin(\theta_a + 2\theta_k), \cos(\theta_a + 2\theta_k), \sin(\phi + \theta_a), \cos(\phi + \theta_a), \dots \\ & 0, 0, \sin(\phi + \theta_a + \theta_k), \cos(\phi + \theta_a + \theta_k), \dots \\ & \sin(\phi + 2\theta_a), \cos(\phi + 2\theta_a), 0, 0, \dots \\ & \sin(\phi + 2\theta_a + 2\theta_k), \cos(\phi + 2\theta_a + 2\theta_k), 2\sin(\phi + 2\theta_a + \theta_k), 2\cos(\phi + 2\theta_a + \theta_k), \dots \\ & \sin(\phi + \theta_a + 2\theta_k), \cos(\phi + \theta_a + 2\theta_k), \dot{\theta}_k, \dots \\ & \sin(\theta_a)\dot{\theta}_k, \cos(\theta_a)\dot{\theta}_k, \sin(\theta_k)\dot{\theta}_k, \cos(\theta_k)\dot{\theta}_k, \dots \\ & \sin(\theta_a + \theta_k)\dot{\theta}_k, \cos(\theta_a + \theta_k)\dot{\theta}_k, \sin(2\theta_a)\dot{\theta}_k, \cos(2\theta_a)\dot{\theta}_k, \dots \\ & \sin(2\theta_k)\dot{\theta}_k, \cos(2\theta_k)\dot{\theta}_k, \sin(2\theta_a + 2\theta_k)\dot{\theta}_k, \cos(2\theta_a + 2\theta_k)\dot{\theta}_k, \dots \\ & \sin(2\theta_a + \theta_k)\dot{\theta}_k, \cos(2\theta_a + \theta_k)\dot{\theta}_k, \sin(\theta_a + 2\theta_k)\dot{\theta}_k, \cos(\theta_a + 2\theta_k)\dot{\theta}_k, \dots \\ & \sin(\phi + \theta_a)\dot{\theta}_k, \cos(\phi + \theta_a)\dot{\theta}_k, \sin(\phi + \theta_k)\dot{\theta}_k, \cos(\phi + \theta_k)\dot{\theta}_k, \dots \\ & \sin(\phi + \theta_a + \theta_k)\dot{\theta}_k, \cos(\phi + \theta_a + \theta_k)\dot{\theta}_k, \sin(\phi + 2\theta_a)\dot{\theta}_k, \cos(\phi + 2\theta_a)\dot{\theta}_k, \dots \\ & \sin(\phi + 2\theta_k)\dot{\theta}_k, \cos(\phi + 2\theta_k)\dot{\theta}_k, \sin(\phi + 2\theta_a + 2\theta_k)\dot{\theta}_k, \cos(\phi + 2\theta_a + 2\theta_k)\dot{\theta}_k, \dots \\ & \sin(\phi + 2\theta_a + \theta_k)\dot{\theta}_k, \cos(\phi + 2\theta_a + \theta_k)\dot{\theta}_k, \dots \\ & \sin(\phi + \theta_a + 2\theta_k)\dot{\theta}_k, \cos(\phi + \theta_a + 2\theta_k)\dot{\theta}_k]^T, \end{aligned}$$

$$\begin{aligned} \xi_{\text{knee}} = & [0, 1, 0, 0, \dots \\ & \sin(\theta_k), \cos(\theta_k), \sin(\theta_a + \theta_k), \cos(\theta_a + \theta_k), \dots \\ & 0, 0, \sin(2\theta_k), \cos(2\theta_k), \dots \\ & \sin(2\theta_a + 2\theta_k), \cos(2\theta_a + 2\theta_k), \sin(2\theta_a + \theta_k), \cos(2\theta_a + \theta_k), \dots \\ & 2\sin(\theta_a + 2\theta_k), 2\cos(\theta_a + 2\theta_k), 0, 0, \dots \\ & \sin(\phi + \theta_k), \cos(\phi + \theta_k), \sin(\phi + \theta_a + \theta_k), \cos(\phi + \theta_a + \theta_k), \dots \\ & 0, 0, \sin(\phi + 2\theta_k), \cos(\phi + 2\theta_k), \dots \\ & \sin(\phi + 2\theta_a + 2\theta_k), \cos(\phi + 2\theta_a + 2\theta_k), \sin(\phi + 2\theta_a + \theta_k), \cos(\phi + 2\theta_a + \theta_k), \dots \\ & 2\sin(\phi + \theta_a + 2\theta_k), 2\cos(\phi + \theta_a + 2\theta_k), -\dot{\theta}_a, \dots \end{aligned}$$

$$\begin{aligned}
& -\sin(\theta_a)\dot{\theta}_a, -\cos(\theta_a)\dot{\theta}_a, -\sin(\theta_k)\dot{\theta}_a, -\cos(\theta_k)\dot{\theta}_a, \dots \\
& -\sin(\theta_a + \theta_k)\dot{\theta}_a, -\cos(\theta_a + \theta_k)\dot{\theta}_a, -\sin(2\theta_a)\dot{\theta}_a, -\cos(2\theta_a)\dot{\theta}_a, \dots \\
& -\sin(2\theta_k)\dot{\theta}_a, -\cos(2\theta_k)\dot{\theta}_a, -\sin(2\theta_a + 2\theta_k)\dot{\theta}_a, -\cos(2\theta_a + 2\theta_k)\dot{\theta}_a, \dots \\
& -\sin(2\theta_a + \theta_k)\dot{\theta}_a, -\cos(2\theta_a + \theta_k)\dot{\theta}_a, -\sin(\theta_a + 2\theta_k)\dot{\theta}_a, -\cos(\theta_a + 2\theta_k)\dot{\theta}_a, \dots \\
& -\sin(\phi + \theta_a)\dot{\theta}_a, -\cos(\phi + \theta_a)\dot{\theta}_a, -\sin(\phi + \theta_k)\dot{\theta}_a, -\cos(\phi + \theta_k)\dot{\theta}_a, \dots \\
& -\sin(\phi + \theta_a + \theta_k)\dot{\theta}_a, -\cos(\phi + \theta_a + \theta_k)\dot{\theta}_a, -\sin(\phi + 2\theta_a)\dot{\theta}_a, -\cos(\phi + 2\theta_a)\dot{\theta}_a, \dots \\
& -\sin(\phi + 2\theta_k)\dot{\theta}_a, -\cos(\phi + 2\theta_k)\dot{\theta}_a, -\sin(\phi + 2\theta_a + 2\theta_k)\dot{\theta}_a, -\cos(\phi + 2\theta_a + 2\theta_k)\dot{\theta}_a, \dots \\
& -\sin(\phi + 2\theta_a + \theta_k)\dot{\theta}_a, -\cos(\phi + 2\theta_a + \theta_k)\dot{\theta}_a, \dots \\
& -\sin(\phi + \theta_a + 2\theta_k)\dot{\theta}_a, -\cos(\phi + \theta_a + 2\theta_k)\dot{\theta}_a]^T.
\end{aligned}$$

## 2. WEIGHTING FACTORS FOR OPTIMIZATION PROBLEM

The optimization problem is re-stated below

$$\begin{aligned}
& \arg \min_{\alpha} \sum_j \{ [\text{vGRF} \cdot U(q_j, p_j, \alpha) - Y_j]^T \cdot W_j(U, Y_j) \cdot [\text{vGRF} \cdot U(q_j, p_j, \alpha) - Y_j] \\
& \quad + [U^B(q_j, p_j, \alpha) - Y_j^B]^T W_k [U^B(q_j, p_j, \alpha) - Y_j^B] \} \\
& \quad + U(q_0, p_0, \alpha)^T W_r U(q_0, p_0, \alpha) + \Lambda \|W_s \alpha\|_1.
\end{aligned}$$

We set

$$w_{\text{ankle}} = \begin{bmatrix} I(N_{\text{stance,early}}) & 0 & 0 & 0 \\ 0 & I(N_{\text{stance,middle}}) & 0 & 0 \\ 0 & 0 & 2I(N_{\text{stance,late}}) & 0 \\ 0 & 0 & 0 & 4I(N_{\text{swing}}) \end{bmatrix} \quad (\text{S1})$$

$$w_{\text{knee}} = \begin{bmatrix} I(N_{\text{stance,early}}) & 0 & 0 & 0 \\ 0 & I(N_{\text{stance,middle}}) & 0 & 0 \\ 0 & 0 & 5I(N_{\text{stance,late}}) & 0 \\ 0 & 0 & 0 & 2I(N_{\text{swing}}) \end{bmatrix} \quad (\text{S2})$$

$$W_j = \gamma_j \cdot \zeta \cdot \begin{bmatrix} w_{\text{ankle}} & 0 \\ 0 & w_{\text{knee}} \end{bmatrix}, \quad (\text{S3})$$

where  $I(\cdot)$  represents the identity matrix,  $N_{\text{stance,early}}$  and  $N_{\text{stance,late}}$  represent the initial 15% and late 15% of stance phase boundaries. The coefficient  $\gamma_j$  changes according to the tasks.  $\zeta$  depends on the exoskeleton and human inputs  $(U, Y_j)$  and equals to 10 if  $\text{sign}(U(i) \cdot Y_j(i)) < 0$  for all sample index  $i$ , otherwise,  $\zeta = 1$ .

We also have

$$w^k = \begin{bmatrix} I(N_{\text{stance,early}}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I(N_{\text{stance,late}}) & 0 \\ 0 & 0 & 0 & I(N_{\text{swing}}) \end{bmatrix} \quad (\text{S4})$$

$$W_k = \gamma_j \cdot \begin{bmatrix} w^k & 0 \\ 0 & w^k \end{bmatrix} \quad (\text{S5})$$

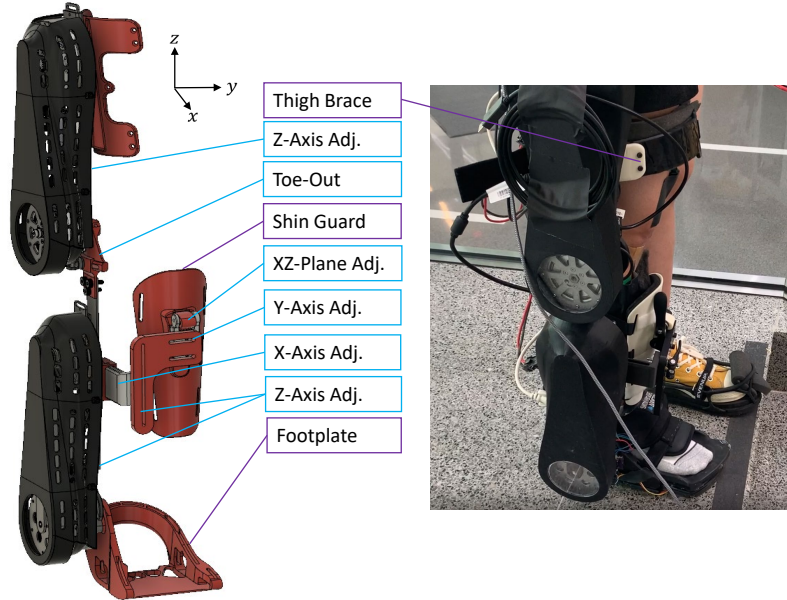
$$W_r = 5I \quad (\text{S6})$$

$$\Lambda = 0.5 \quad (\text{S7})$$

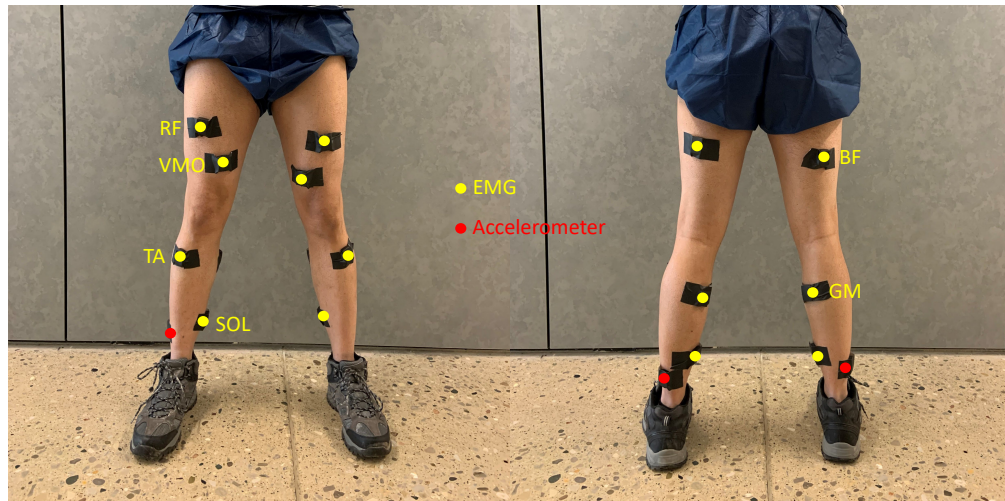
$$W_s = \begin{bmatrix} I(N_{\text{Pot}}) & 0 \\ 0 & 10I(N_{\text{Gyro}}) \end{bmatrix}, \quad (\text{S8})$$

where  $N_{Pot}$  and  $N_{Gyro}$  represents the number of basis functions associated with the shaped potential energy and the shaped gyroscopic terms.

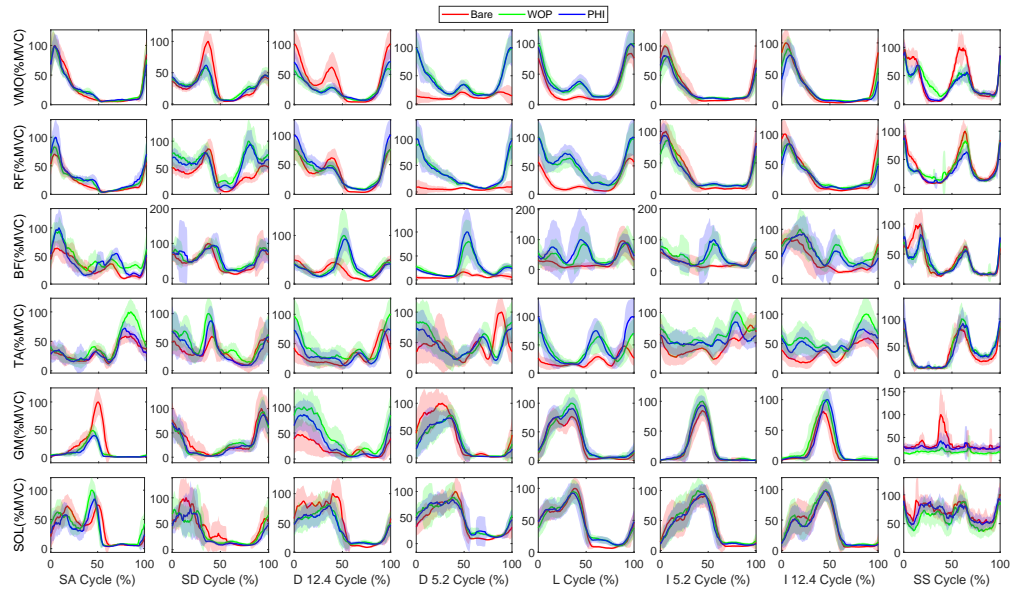
### 3. HUMAN SUBJECT RESULTS



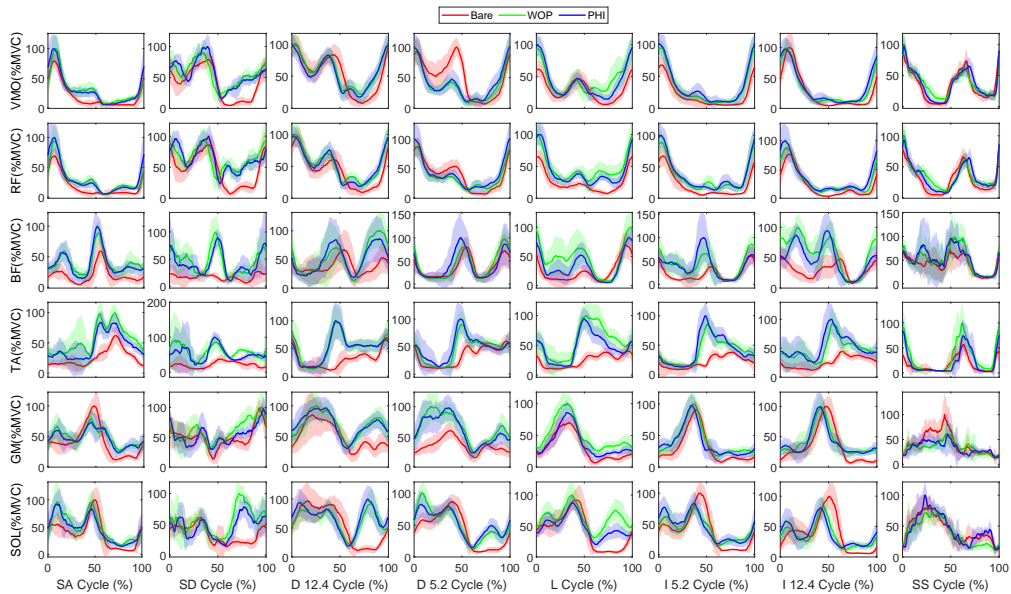
**Fig. S1.** CAD model of *Comex* knee-ankle exoskeleton. Four attachments and eight adjustments are capable of adjustment to secure the exoskeleton to the subject.



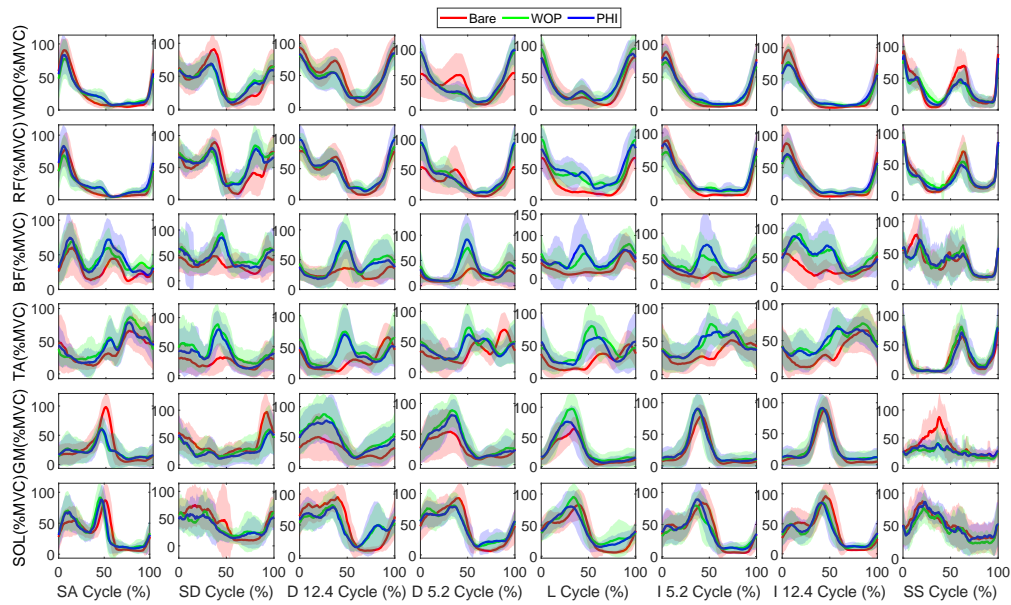
**Fig. S2.** Pictures with EMG measurement setup. The skin was prepared by removing the top layer of dead cells to reduce skin to electrode impedance. EMGs were placed and secured with tapes.



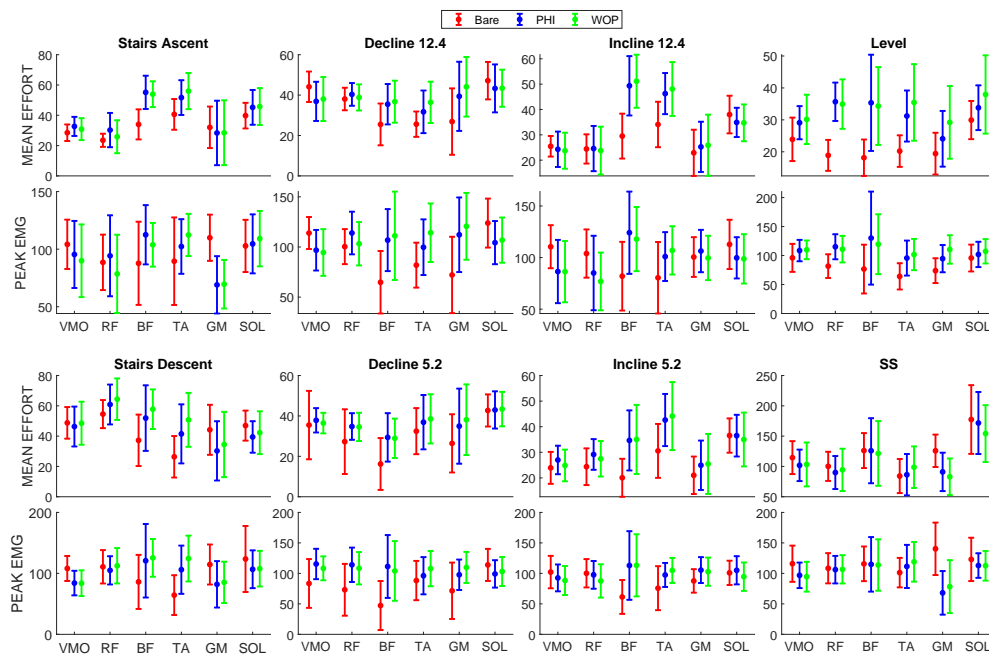
**Fig. S3.** Subject 2 EMG comparisons between bare and active modes (PHI and WOP methods) for each muscle (VMO, RF, BF, TA, GM and SOL) and task {Stairs Ascent/Descent (7in step height), Decline ( $-5.2^\circ$ ,  $-12.4^\circ$ ) at 0.6 m/s, level ground (1 m/s), Incline ( $5.2^\circ$ ,  $12.4^\circ$ ) at 0.6 m/s, and Sit-Stand cycle (45 BPM)}. The red solid (bare), blue solid (PHI method), and green solid (WOP method) lines represent the time-normalized ensemble averages across all repetitions.



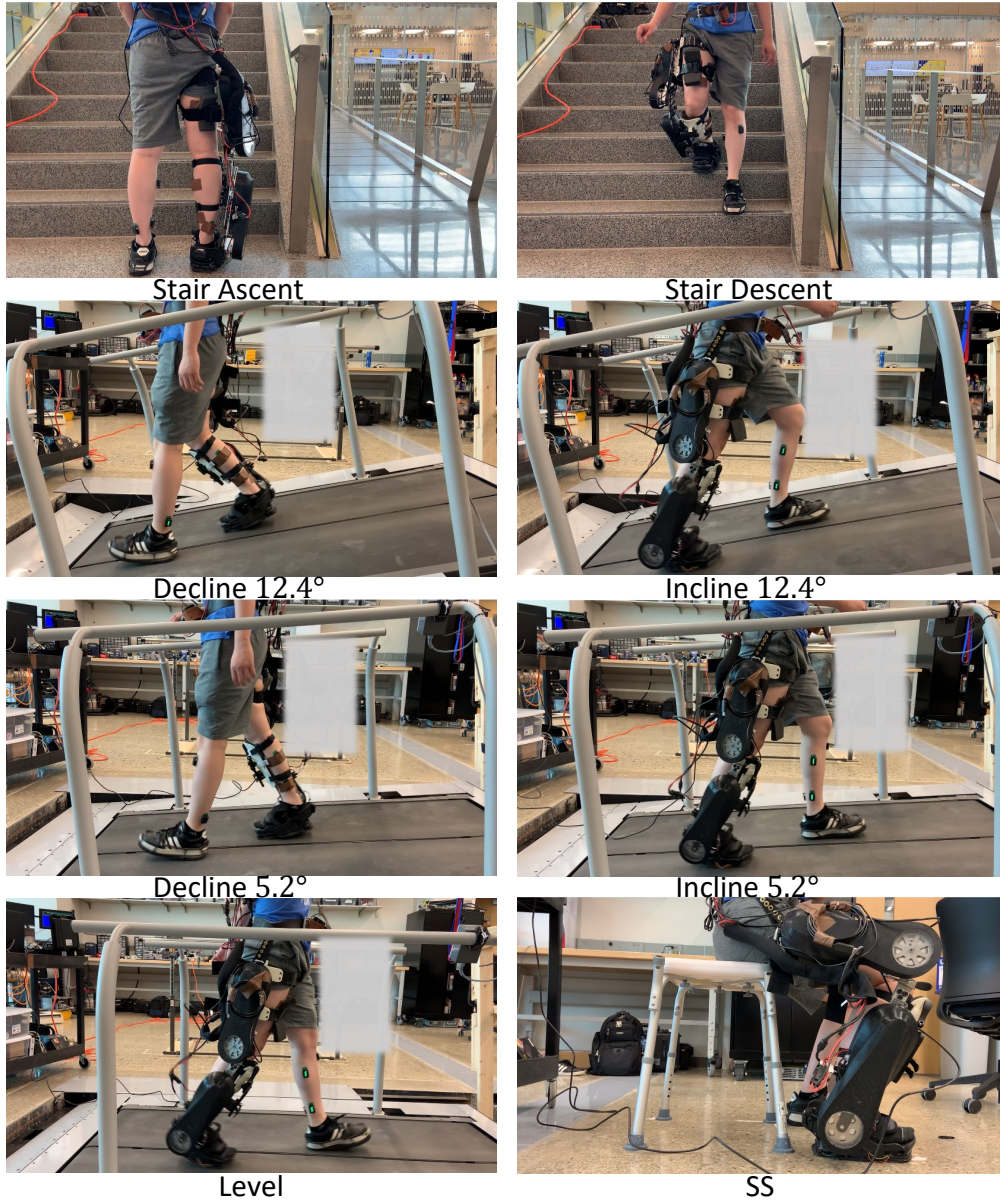
**Fig. S4.** Subject 3 EMG comparisons between bare and active modes (PHI and WOP methods) for each muscle (VMO, RF, BF, TA, GM and SOL) and task {Stairs Ascent/Descent (7in step height), Decline ( $-5.2^\circ$ ,  $-12.4^\circ$ ) at 0.6 m/s, level ground (0.8 m/s), Incline ( $5.2^\circ$ ,  $12.4^\circ$ ) at 0.6 m/s, and Sit-Stand cycle (45 BPM)}. The red solid (bare), blue solid (PHI method), and green solid (WOP method) lines represent the time-normalized ensemble averages across all repetitions.



**Fig. S5.** Across-subjects averaged EMG comparisons between bare and active modes (PHI and WOP methods) for each muscle (VMO, RF, BF, TA, GM and SOL) and task {Stairs Ascent/Descent (7in step height), Decline ( $-5.2^\circ$ ,  $-12.4^\circ$ ) at 0.6 m/s, level ground (combined 0.8 m/s and 1 m/s), Incline ( $5.2^\circ$ ,  $12.4^\circ$ ) at 0.6 m/s, and Sit-Stand cycle (45 BPM)}. The red solid (bare), blue solid (PHI method), and green solid (WOP method) lines represent the time-normalized ensemble averages across all repetitions.



**Fig. S6.** Across-subjects mean effort (%MVC.S) and peak EMG (%MVC) comparisons for VMO, RF, BF, TA, GM and SOL: showing mean ( $\pm$  SD) for different tasks and muscles (columns).



**Fig. S7.** The subject wore *Comex* knee-ankle exoskeleton and conducted tests for different tasks (Stairs Ascent/Descent (7in step height), Decline ( $-5.2^{\circ}$ ,  $-12.4^{\circ}$ ) at 0.6 m/s, Incline ( $5.2^{\circ}$ ,  $12.4^{\circ}$ ) at 0.6 m/s, level ground (1 m/s), and Sit-Stand cycle (45 BPM)).

## **MEDIA**

This supplementary material includes 1 video clip. The accompanying video demonstrates the versatility of the controller in providing biomimetic assistance across multiple activities.