# Hybrid Nonlinear Disturbance Observer Design for Underactuated Bipedal Robots

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***Abstract*— Existence of disturbances in unknown environments is a pervasive challenge in robotic locomotion control. Disturbance observers are a class of unknown input observers that have been extensively used for disturbance rejection in numerous robotics applications. In this paper, we extend a class of widely-used nonlinear disturbance observers to underactuated bipedal robots, which are controlled using hybrid zero dynamics-based control schemes. The proposed hybrid nonlinear disturbance observer provides the autonomous biped robot control system with disturbance rejection capabilities, while the underlying hybrid zero-dynamics based control law remains intact.**

## I. INTRODUCTION

Achieving agile and efficient bipedal locomotion for autonomous biped robots and powered prostheses is of paramount importance in both humanoid and rehabilitation robotics. Hybrid zero dynamics-based (HZD) control is a framework for stable control of underactuated biped robots and powered prosthetic legs with *hybrid dynamics*, where the system trajectories can flow in continuous time and also jump at discrete times due to rigid impacts of the biped leg with the ground [1]–[8]. In this paradigm, stable walking gaits are encoded as re-programmable relations between the robot generalized coordinates, which are enforced via feedback.

Existence of disturbances in unknown environments is a pervasive challenge in HZD-based control of autonomous bipedal robots [9]. Furthermore, in the context of rehabilitation robotics, persistent human inputs might act as disturbances on the wearable robot HZD-based control scheme [10]. These disturbances can deteriorate the performance of the robot motion control systems and even adversely affect their stability. An intuitive idea to counteract the deteriorating effect of disturbances on motion control systems is to estimate unknown disturbances by using the measured outputs and the known control inputs. The estimated disturbance can then be employed for canceling the actual disturbances in a *feedforward* manner. A class of unknown input observers, known as disturbance observers (DOBs), uses this intuitive idea for improving the control system performance in the presence of disturbances [11]–[13].

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The feedforward nature of disturbance compensation in DOB-based control has made it a notable candidate for being employed with previously widely used control schemes, which have been designed for disturbance-free operating conditions. Indeed, if DOBs are properly designed for the application at hand, they will provide the previously designed controller with disturbance rejection capabilities, without requiring to change the nominal control structure [12].

A special class of DOBs and its nonlinear extension, namely, nonlinear disturbance observers (NDOBs), due to Ohnishi, Chen, and collaborators [11]–[15], have recently attracted much attention in numerous robotics applications such as control of upper-limb robotic rehabilitation [16], robotic exoskeletons [17], and robotic teleoperation [18], [19], to name a few. In the context of bipedal locomotion, Sato and collaborators have employed DOBs to reject disturbances in zero moment point (ZMP)-based biped robot control systems [20], [21]. However, this prior body of work has mainly focused on *fully actuated* biped robots. In a recent article by Abe and collaborators [22], DOB-based control has been compared to HZD-based control of an underactuated five-link biped robot for balance recovery, where the two control schemes are contrasted to each other.

Using DOBs along with HZD-based controllers for legged locomotion is still lacking in the literature. Indeed, there exist two inherent challenges in underactuated locomotion settings. First, having less control inputs than degrees-of-freedom (DOFs) in underactuated robots makes it impossible to cancel all disturbances that are acting on the robot DOFs. Second, the underlying hybrid dynamics in biped locomotion necessitates the design of jump maps for NDOB states after each robot leg impact with the ground.

In this article, we extend the class of NDOBs in [14], [15] (also, see [23] for a survey tutorial on NDOBs) to underactuated robotic locomotion and demonstrate that they can provide HZD-based controllers with disturbance rejection capabilities. Indeed, the underlying HZD-based controllers, which are designed under disturbance-free operating conditions, are not required to be changed.

Our proposed hybrid NDOB for underactuated bipedal robots extends the previous class of NDOBs in two important ways. First, through a geometric construction, we design a projection operator for the NDOB output, i.e., the lumped disturbance estimate, so that the projected outputs can be employed to compensate for the adverse effects of disturbances, which are preventing the HZD-based controller from zeroing the outputs. Second, we provide jump maps that update the NDOB states after each leg impact with the ground; thus, making the NDOB a hybrid observer. In the presence of disturbances, we prove the stability properties of the NDOB.

The rest of this paper is organized as follows. In Section II, we provide preliminaries from bipedal robotics and HZD-based control. Next, in Section III, we present the standard NDOB and highlight the challenges of using it in underactuated bipedal robots with hybrid dynamics. In Section IV, we present our hybrid NDOB for underactuated bipedal robots. Thereafter, in Section V, we prove the convergence properties of the proposed hybrid NDOB. In Section VI, we present simulation results for push recovery in a five-link biped robot with point feet. Finally, we conclude the article with remarks and potential research directions in Section VII.

*Remark 1:* There exists another class of hybrid observers that has been employed for estimating the states of bipedal robots controlled using HZD-based controllers (see, e.g., [24], [25]). In this article, however, we are proposing an observer for unknown input, in contrast to state, estimation.

**Notation.** We denote $\mathbb{R}_+ = [0,\infty)$. Given two vectors (matrices) $a$, $b$ of suitable dimensions, we denote by $[a\,;\,b]$ the vector (matrix) $[a^\top, b^\top]^\top$, where $\top$ denotes the transpose operator. Given an integer $N$, we denote by $I_N$ the identity matrix in $\mathbb{R}^{N\times N}$. Given a vector $v$ in $\mathbb{R}^N$, we denote by $\|v\|$ its Euclidean norm. Given a set $\mathcal{A} \subset \mathbb{R}^N$ and a point $x \in \mathbb{R}^N$, we denote by $\|x\|_{\mathcal{A}} = \inf_{y\in\mathcal{A}} \|x-y\|$ the distance of $x$ to $\mathcal{A}$. Given an open and connected set $\mathcal{Q} \subset \mathbb{R}^N$ and a function $h : \mathcal{Q} \to \mathbb{R}^{N-1}$, we denote by $h^{-1}(0)$ its zero level set, i.e., $h^{-1}(0) := \{q \in \mathcal{Q} : h(q) = 0\}$. Given two square matrices $A$, $B \in \mathbb{R}^{N\times N}$, we denote by $B \preceq A$ the positive semi-definiteness of the matrix $A - B$. Given a matrix $A \in \mathbb{R}^{M\times N}$, we denote by $\mathrm{Ker}(A)$ and $\mathrm{Im}(A)$ its kernel and image, respectively. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class-$\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing. A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class-$\mathcal{KL}$ if: (i) for each $t \geq 0$, $\beta(\cdot, t)$ is nondecreasing and $\lim_{s\to 0^+} \beta(s,t) = 0$, and (ii) for each $s \geq 0$, $\beta(s,\cdot)$ is nonincreasing and $\lim_{t\to\infty} \beta(s,t) = 0$.

## II. Preliminaries

In this section we briefly review the hybrid dynamical model of underactuated planar biped robots with point feet, the notion of solutions for the biped robot hybrid dynamics, and some standard material from the HZD-based control framework (see, e.g., [1]–[3] for further details).

### A. Hybrid Dynamical Model of Biped Robots

Given an underactuated planar biped robot with point feet that is subject to time-varying disturbances (see Figure 1), its equations of motion during **swing phase** are

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu + d(t), \quad (q,\dot{q}) \notin \mathcal{S}, \quad (1)$$

where the vectors $q = [q_1, \cdots, q_N]^\top \in \mathcal{Q}$ and $\dot{q} = [\dot{q}_1, \cdots, \dot{q}_N]^\top \in \mathbb{R}^N$ denote the joint angles and the joint velocities, respectively. The set $\mathcal{Q}$, called the biped **configuration space**, is assumed to be an open and connected subset of $\mathbb{R}^N$. The state $(q, \dot{q})$ of dynamical system (1) belongs to the state space $\mathcal{X} := \mathcal{Q} \times \mathbb{R}^N$. Moreover, $M(q)$, $C(q,\dot{q})$, and $G(q)$, denote the inertia matrix, the matrix of Coriolis/centrifugal forces, and the vector of gravitational forces, respectively. Having articulated joints,

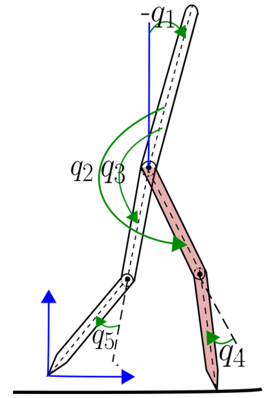


Fig. 1: An example five-link underactuated planar biped robot.

$$\nu_1 I_N \preceq M(q) \preceq \nu_2 I_N, \quad (2)$$

is satisfied for all $q \in \mathcal{Q}$, and some positive real constants $\nu_1$ and $\nu_2$ [26]. The vector of control inputs $u$ belongs to $\mathcal{U}$, an open and connected subset of $\mathbb{R}^{N\text{-}1}$, and $B \in \mathbb{R}^{N\times(N\text{-}1)}$ is assumed to be constant and of full rank $N$-1. Under this assumption, there exists a non-zero row vector $B^\perp \in \mathbb{R}^{1\times N}$ such that $B^\perp B = 0$. We say that system (1) has **one degree of underactuation**. In (1), the vector of time-varying disturbances, which lumps the effect of disturbances that are acting on the robot, is denoted by $d(t)$. We make the following assumption regarding the lumped disturbance signal time derivative, which generalizes the bounded rate of change assumption for disturbances encountered in the NDOB literature (see, e.g., [23]).

**DH1)** We assume that the time derivative of the lumped disturbance signal $d(\cdot)$ is Lebesgue measurable and

$$\|\dot{d}(t)\| \leq \omega_{\mathrm{d}}, \quad (3)$$

for almost all $t \geq 0$ (in Lebesgue sense) and some positive constant $\omega_{\mathrm{d}}$. Moreover, we assume that jumps in the lumped disturbance happen only at biped leg impacts with the ground. We denote

$$\Delta d_j := d(t_j^+) - d(t_j^-), \quad (4)$$

if there is a jump in the disturbance at the $j$-th impact.

The vertical height from the ground and the horizontal position of the swing leg end, with respect to an inertial coordinate frame, are denoted by $p_2^{\mathrm{v}}(q)$ and $p_2^{\mathrm{h}}(q)$, respectively. The set $\mathcal{S}$, called the **switching surface**, is defined as

$$\mathcal{S} := \{(q,\dot{q}) \in \mathcal{X} : p_2^{\mathrm{v}}(q) = 0,\, p_2^{\mathrm{h}}(q) > 0\}. \quad (5)$$

The switching surface in (5) is assumed to be a smooth codimension-one embedded submanifold of the state space $\mathcal{X}$ (see, e.g., [2], [3]). The **double support phase** is assumed to be *instantaneous* and modeled by the rigid impact model

$$[q^+; \dot{q}^+] = [\Delta_q q^-; \Delta_{\dot{q}}(q^-)\dot{q}^-],\ [q^-; \dot{q}^-] \in \mathcal{S}, \quad (6)$$

where $[q^-; \dot{q}^-]$ and $[q^+; \dot{q}^+]$ denote the states of the robot just before and after impact, respectively. Furthermore, the mappings $\Delta_q(\cdot)$ and $\Delta_{\dot{q}}(\cdot)$ are assumed to be smooth (see, e.g., [3]). The biped dynamics are described by the **hybrid dynamical system** in (1)–(6), which can be written as

$$\begin{cases} \dot{x} = f(x) + g(x)u + g_{\rm d}(x)d(t), \text{ for } x \notin \mathcal{S} \\ x^+ = \Delta(x^-), \text{ for } x^- \in \mathcal{S} \end{cases} \tag{7}$$

where $x := [q; \dot{q}] \in \mathcal{X}$, $\Delta(x) := [\Delta_q q; \Delta_{\dot{q}} \dot{q}]$,

$$f(x) := \begin{bmatrix} I_N \\ M^{-1}(q)\{ -C(q,\dot{q})\dot{q} - G(q)\} \end{bmatrix},$$

and

$$g(x) := \left[0; M^{-1}(q)B\right],\ g_{\rm d}(x) := \left[0; M^{-1}(q)\right].$$

Suppose that a state feedback control law of the form $u = u^{\rm fb}(x)$ is given for the dynamical system in (7) and the disturbance signal satisfies DH1. A function $\varphi : [t_0, t_f) \to \mathcal{X}$, $t_f \in \mathbb{R} \cup \{\infty\}$, $t_f > t_0$, is a **solution** of (7) if: 1) $\varphi(t)$ is right continuous on $[t_0, t_f)$, 2) left limits exist at each point of $(t_0, t_f)$, and 3) there exists a closed discrete subset $\mathcal{T} \subset [t_0, t_f)$ such that: a) for every $t \notin \mathcal{T}$, $\varphi(t)$ is differentiable and $(d\varphi(t)/dt) = f(\varphi(t)) + g(\varphi(t))u^{\rm fb}(\varphi(t)) + g_{\rm d}(\varphi(t))d(t)$, and b) for $t \in \mathcal{T}$, $\varphi^-(t) \in \mathcal{S}$ and $\varphi^+(t) = \Delta(\varphi^-(t))$. A solution $\varphi(t)$ of (7) is **periodic** if there exists a finite $t^* > 0$ such that $\varphi(t + t^*) = \varphi(t)$ for all $t \in [t_0, \infty)$. A set $\mathcal{O} \subset \mathcal{X}$ is a **hybrid periodic orbit** of (7) if $\mathcal{O} = \{\varphi(t) \big| t \geq t_0\}$ for some periodic solution $\varphi(t)$. We say that a solution to (7) is **maximal** if it cannot be extended. Given a disturbance signal $d(t)$, we denote the set of all maximal solutions to (7) with initial condition $x_0$ by $\Gamma_d(x_0)$.

### B. Hybrid Zero Dynamics Control Framework

HZD framework relies on the concept of **virtual constraints**. Virtual constraints are relations of the form $h(q) = 0$ among the joint variables of a legged robot that encode stable walking gaits [1]–[4]. Using a given virtual constraint, an output of the form

$$y = h(q) = H_0 q - h_{\rm d} \circ \theta(q), \tag{8}$$

is considered for the biped hybrid dynamics (1)–(6), where $H_0 \in \mathbb{R}^{(N-1)\times N}$ is a matrix of full rank $N-1$ and $H_0 q$ represents a set of body coordinates for the biped. Moreover, $h_{\rm d}(\cdot) = \left[h_{\rm d}^1(\cdot); \cdots; h_{\rm d}^{N-1}(\cdot)\right]$ is a vector of $N-1$ Bézier polynomial functions $h_{\rm d}^i : \mathcal{Q} \to \mathbb{R}$, $1 \leq i \leq N-1$. Furthermore, the function $\theta : \mathcal{Q} \to \mathbb{R}$, $\theta(q) = c_0 q$, is called the **phase function** (see Figure 1). In the above, the row vector $c_0 \in \mathbb{R}^{1\times N}$ is chosen such that $[H_0; c_0] \in \mathbb{R}^{N\times N}$ is invertible. Moreover, the zero level set $h^{-1}(0)$ is a one-dimensional curve in $\mathcal{Q}$ with no self-intersections. We make the following standard assumptions, following the HZD framework, regarding the output function $y = h(q)$ in (8).

**OH1)** The output function $y = h(q)$ in (8) is designed to have **well-defined vector relative degree** $\{2, \cdots, 2\}$ for all $q \in h^{-1}(0)$. The vector well-defined relative degree condition holds if and only if

$$A(q) := \frac{\partial h}{\partial q} M^{-1}(q) B, \tag{9}$$

is an invertible matrix for all $q \in h^{-1}(0)$. The matrix $A(q) \in \mathbb{R}^{(N-1)\times(N-1)}$ is called the **decoupling matrix** associated with the virtual constraint in (8). △

**OH2)** The output function $y = h(q)$ in (8) is designed to be invariant with respect to impacts with the ground. In particular, let the post-impact and pre-impact biped joint configurations and velocities be related to each other through (6). We say that the output function $y = h(q)$ is **hybrid invariant**, if whenever $h(q_0^-) = 0$ and $(\partial h/\partial q)(q^-)\dot{q}^- = 0$, then $h(q^+) = 0$ and $(\partial h/\partial q)(q^+)\dot{q}^+ = 0$. △

Once the hybrid invariant outputs in (8) are zeroed using proper control inputs, the biped configuration variables and joint velocities evolve in the set

$$\mathcal{Z} := \left\{(q, \dot{q}) \in T\mathcal{Q} : h(q) = 0,\ \tfrac{\partial h}{\partial q}\dot{q} = 0\right\}, \tag{10}$$

which is called the **hybrid zero dynamics manifold** associated with the outputs in (8). It can be shown that

$$\left[\xi_1; \xi_2\right] = \Xi(\left[q; \dot{q}\right]) := \left[\theta(q); \gamma_0(q)\dot{q}\right], \tag{11}$$

is a valid change of coordinates on $\mathcal{Z}$, where $\gamma_0(q) := B^\perp M(q)$. Given the coordinate $\xi_1$, the configuration

$$q(\xi_1) = \left[H_0; c_0\right]^{-1} \cdot \left[h_{\rm d}(\xi_1); \xi_1\right], \tag{12}$$

zeros the output $y = h(q)$ in (8).

Once the states are constrained to $\mathcal{Z}$ via feedback, the resulting closed-loop motion is governed by lower-dimensional dynamics, called the **hybrid zero dynamics (HZD)**. The HZD in $[\xi_1; \xi_2]$ coordinates are given by

$$\begin{cases} [\dot{\xi}_1; \dot{\xi}_2] = [\kappa_1(\xi_1)\xi_2; \kappa_2(\xi_1)], \text{ for } (\xi_1, \xi_2) \in [\theta^+, \theta^-) \times \mathbb{R} \\ [\xi_1^+; \xi_2^+] = [\theta^+; \delta_{\rm zero}\xi_2^-], \text{ for } (\xi_1^-, \xi_2^-) \in \{\theta^-\} \times \mathbb{R} \end{cases} \tag{13}$$

for a proper real constant $\delta_{\rm zero}$ (see Equation (5.67) in [3]). Moreover, $\theta^- := \theta(q_0^-)$ and $\theta^+ := \theta(q_0^+)$ are the values of the phase function just before and just after the ground impacts. Also,

$$\kappa_1(\xi_1) := B^\perp M(q(\xi_1)) \frac{\partial h}{\partial q}\bigg|_{q(\xi_1)},\ \kappa_2(\xi_1) := -B^\perp \frac{\partial V}{\partial q}\bigg|_{q(\xi_1)},$$

where $q(\xi_1)$ is given by (12) and $V(q)$ is the potential energy.

The following hypothesis provides the relationship between hybrid periodic orbits that represent stable walking gaits with hybrid invariant outputs for the biped robot dynamics.

**CH1)** Given the underactuated biped robot dynamics in (1)–(6), we assume that there exists an output of the form $y = h(q)$ satisfying OH1 and OH2, and an input-output feedback linearizing controller $u_{\rm n}(q, \dot{q})$ associated with the given output, such that when $d(t) \equiv 0$, $u_n(\cdot)$ drives the output $y = h(q)$ to zero and by doing so makes the hybrid periodic orbit

$$\mathcal{O} := \left\{\left(q^*(t), \dot{q}^*(t)\right) | 0 \leq t < t^*\right\}, \tag{14}$$

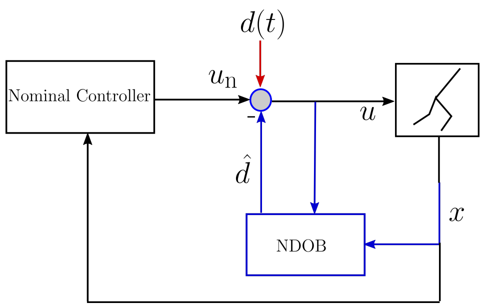


Fig. 2: Block diagram of a DOB-based control system.

with period $t^* > 0$, exponentially stable for the resulting closed-loop hybrid dynamics. The hybrid periodic orbit $\mathcal{O}$ in (14) represents a stable robot walking gait. Designing such feedback controllers and outputs has been extensively investigated in, e.g, [2]–[5]. △

## III. Standard NDOB Structure

In this section we briefly present the structure of the standard NDOB and its limitations for being employed in underactuated biped robot control systems with hybrid dynamics.

Given a nonlinear control system and a nominal control law $u_{\rm n}(x)$, the underlying mechanism of NDOB operation can be explained as follows (see also Figure 2). Using the state and input information, the NDOB estimates the disturbance $d(t)$, which is deteriorating the performance of the nominal controller. The NDOB output $\hat{d}$ is then added to the nominal control input in the following feedforward manner:

$$u = u_{\rm n}(x) - \hat{d}. \tag{15}$$

In the ideal case, when $\hat{d} = d(t)$, the disturbances acting on the system would be canceled out and the nominal control input $u_{\rm n}(x)$ would achieve the desired objectives, without the need to modify the nominal controller. The NDOB dynamics for the robot swing phase dynamical equations in (1) are given by (see [14], [15], [23] for the details of its derivation)

$$\begin{aligned} \dot{z} &= -L^{\varepsilon}_{\rm d}(q)z + L^{\varepsilon}_{\rm d}(q)\big\{C(q,\dot{q})\dot{q} + G(q) - Bu - p^{\varepsilon}(\dot{q})\big\}, \\ \hat{d} &= z + p^{\varepsilon}(\dot{q}), \end{aligned} \tag{16}$$

where $z \in \mathbb{R}^N$ is the *state* of the NDOB. The *auxiliary vector* and the *gain matrix* of the NDOB are given by

$$p^{\varepsilon}(\dot{q}) = X_{\varepsilon}^{-1}\dot{q},\ L^{\varepsilon}_{\rm d}(q) = X_{\varepsilon}^{-1}M^{-1}(q). \tag{17}$$

respectively, where $X_{\varepsilon} \in \mathbb{R}^{N\times N}$ is a constant symmetric and positive definite matrix depending on a positive constant $\varepsilon$. For simplicity of exposition, we let $X_{\varepsilon} = (\varepsilon/\nu_2)I_N$, where $\nu_2$ is the positive constant in (2). Finally, NDOB disturbance tracking error is defined to be

$$e_{\rm d} := \hat{d} - d(t). \tag{18}$$

When $\hat{d} = d(t)$, namely, in the ideal case, $e_{\rm d} = 0$. We have the following proposition regarding the NDOB error dynamics.

*Proposition 1 ([14], [23]):* Consider the biped robot swing phase dynamics in (1). Consider the NDOB in (16) with auxiliary vector and gain matrix given by (17). The NDOB error dynamics, during the swing phase, are governed by

$$\dot{e}_{\rm d} = -L^{\varepsilon}_{\rm d}(q)e_{\rm d} + \dot{d}. \tag{19}$$

Moreover,

$$-e_{\rm d}^{\top}L^{\varepsilon}_{\rm d}(q)e_{\rm d} \le -\frac{1}{\varepsilon}\|e_{\rm d}\|^2,\ \text{for all } q \in \mathcal{Q},\ e_{\rm d} \in \mathcal{E}, \tag{20}$$

where $\mathcal{E} \subset \mathbb{R}^N$ is open, connected, and contains the origin.

**Limitations of the standard NDOB.** There are two main challenges in employing the conventional NDOB in (16) for an underactuated biped robot with hybrid dynamics given by (1)–(6). First, the underactuated dynamics of the biped make it impossible to use the feedforward disturbance compensation according to (15) because the NDOB output $\hat{d}$ belongs to $\mathbb{R}^N$, while there are only $N-1$ motor torque inputs. Therefore, there is a need for projecting the NDOB output onto the space of control inputs in a proper manner so that the disturbance components that are preventing the output $y = h(q)$ to be zeroed are compensated for. Second, because of the hybrid nature of the biped robot dynamics, it is non-trivial how the NDOB state and output should be updated after each impact of the biped robot leg with the ground.

## IV. Hybrid NDOB Design

In this section we extend the NDOBs, due to Chen *et al.* [12], [14], [15], that were previously used for fully actuated robots. The resulting extended hybrid NDOBs can be employed along with previously designed HZD-based controllers for underactuated biped robots subject to disturbances.

### A. NDOB Output for Underactuated Robots

The following technical lemma will be useful for designing a projection operator for the hybrid NDOB.

*Lemma 1:* Consider the underactuated biped dynamics given by (1)–(6). Suppose that the output $y = h(q)$ for the biped dynamics satisfies OH1. Given any $v \in \mathbb{R}^N$, there exists a neighborhood of $h^{-1}(0)$ such that for all $q$ in it, there exist vectors $v^{\pitchfork} \in \mathbb{R}^{N-1}$ and $v^{\parallel} \in \mathrm{Ker}(\partial h/\partial q)\big|_q$ such that

$$v = Bv^{\pitchfork} + M(q)v^{\parallel}. \tag{21}$$

*Proof.* See the Appendix.

*Remark 2:* The geometric interpretation of Lemma 1, when $q$ belongs to $h^{-1}(0)$, is shown in Figure 3.

*Proposition 2:* Consider the underactuated biped dynamics given by (1)–(6). Suppose that the output $y = h(q)$ satisfies OH1. Consider the projection operator $\Pi_{\rm d} : \mathcal{Q} \to \mathbb{R}^{N-1}$,

$$\Pi_{\rm d}(q) := A^{-1}(q)\Big(\frac{\partial h}{\partial q}M^{-1}(q)\Big), \tag{22}$$

where $A(q)$ is given by (9). Consider an arbitrary vector $v \in \mathbb{R}^N$ with its decomposition in (21). There exists a

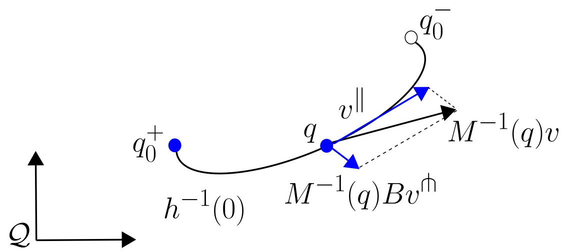


Fig. 3: Geometric interpretation of Lemma 1 when $q \in h^{-1}(0)$. The vector $v^{\parallel}$ is tangent to $h^{-1}(0)$ at $q$.

neighborhood of $h^{-1}(0)$ such that for all $q$ in it, we have

$$v^{\pitchfork} = \Pi_{\mathrm{d}}(q)v. \tag{23}$$

*Proof:* Given an arbitrary vector $v$, there exists a neighborhood of $h^{-1}(0)$ such that (21) holds, due to Lemma 1. Multiplying both sides of (21) by $\Pi_{\mathrm{d}}(q)$, we get

$$\Pi_{\mathrm{d}}(q)v = A^{-1}(q)\big(\frac{\partial h}{\partial q}M^{-1}(q)\big)\big(Bv^{\pitchfork} + M(q)v^{\parallel}\big)$$

$$= A^{-1}(q)\frac{\partial h}{\partial q}M^{-1}(q)Bv^{\pitchfork} + A^{-1}(q)\frac{\partial h}{\partial q}v^{\parallel}.$$

Since $v^{\parallel} \in \mathrm{Ker}(\partial h/\partial q)\big|_q$ and $\frac{\partial h}{\partial q}M^{-1}(q)B = A(q)$, it can be easily seen that (23) holds. ■

Using the projection operator in (22), we project the standard NDOB output in the following way

$$\hat{d}^{\pitchfork} = \Pi_{\mathrm{d}}(q)\hat{d}, \tag{24}$$

and apply the DOB-based control input

$$u(q, \dot{q}, \hat{d}^{\pitchfork}) = u_{\mathrm{n}}(q, \dot{q}) - \hat{d}^{\pitchfork}, \tag{25}$$

to the underactuated biped robot in (1)–(6). The following proposition gives the biped robot output error dynamics when the NDOB-based control law in (25) is used.

*Proposition 3:* Consider the biped robot dynamical system in (1)–(6) and the NDOB in (16), with the auxiliary vector and the gain matrix in (17), and the projected output given by (24). Given an input-output feedback linearizing control law $u_{\mathrm{n}}(q, \dot{q})$ for the output $y = h(q)$, which satisfies OH1, and applying the NDOB-based control input in (25), the output error dynamics during the swing phase are given by

$$\ddot{y} = -K_p y - K_d \dot{y} - \frac{\partial h}{\partial q}M^{-1}(q)e_{\mathrm{d}}, \tag{26}$$

where $K_p$ and $K_d$ are the PD gains of the feedback linearizing controller, and $e_{\mathrm{d}}$ is the disturbance tracking error in (18).

*Proof:* We only provide a sketch of the proof. Taking two derivatives of the output $y = h(q)$, we have

$$\ddot{y} = \frac{\partial h}{\partial q}M^{-1}(q)B(u_{\mathrm{n}} - \hat{d}^{\pitchfork}) + \frac{\partial h}{\partial q}M^{-1}(q)d(t) + \delta(q, \dot{q}),$$

where $\delta(q, \dot{q})$ is a smooth function of $q$ and $\dot{q}$ independent of $u_{\mathrm{n}}$, $\hat{d}^{\pitchfork}$, and $d$. Note that $A(q)u_{\mathrm{n}}(q, \dot{q}) + \delta(q, \dot{q})$ is equal to $-K_p y - K_d \dot{y}$ for an input-output feedback linearizing controller $u_{\mathrm{n}}$. Using the identity (23) and the definition of $\Pi_{\mathrm{d}}(q)$ in (22), we get

$$\ddot{y} = A(q)u_{\mathrm{n}} + \delta(q, \dot{q}) - \frac{\partial h}{\partial q}M^{-1}(q)B\Pi_{\mathrm{d}}(q)\hat{d} + \frac{\partial h}{\partial q}M^{-1}(q)d$$

$$= -K_p y - K_d \dot{y} - \frac{\partial h}{\partial q}M^{-1}(q)\big(d(t) - \hat{d}\big).$$

Using (18) in the above identity concludes the proof. ■

Having obtained the output error dynamics, the following proposition shows the disturbance effect on the biped robot swing phase zero dynamics.

*Proposition 4:* Consider the biped robot hybrid dynamics in (1)–(6), and the NDOB given by (16), (17), and (24), and the DOB-based control law in (25), respectively. Given a disturbance signal $d(t)$, consider its decomposition according to (21). The tangential component of the disturbance $d(\cdot)$ perturbs the swing phase zero dynamics in (13) according to

$$\begin{cases} \dot{\xi}_1 = \kappa_1(\xi_1)\xi_2 \\ \dot{\xi}_2 = \kappa_2(\xi_1) + \zeta(\xi_1)d^{\parallel}(t) \end{cases} \tag{27}$$

where

$$\zeta(\xi_1) := B^{\perp}M(q(\xi_1)) \tag{28}$$

and $d^{\parallel}(t) \in \mathrm{Ker}(\partial h/\partial q)\big|_{q(\xi_1)}$, where $q(\xi_1)$ is given by (12).

*Proof.* See the appendix.

*Remark 3:* Proposition 4 should be of no surprise, as we have less control inputs than the biped robot DOFs and cannot cancel the disturbances in all directions. Indeed, we have already used all the $N-1$ actuated directions, according to (25), to compensate for disturbance components that are preventing the output $y = h(q)$ from being zeroed.

### *B. Design of the NDOB Jump Map*

In this section we address the issue of updating the NDOB states after each impact of the robot swing leg with the ground. Since $C(q, \dot{q})\dot{q} + G(q) - Bu = -M(q)\ddot{q} + Bd(t)$ during the swing phase, it can be seen from the NDOB dynamical equations in (16) that the NDOB states depend on the joint accelerations. Also, the biped robot joint accelerations are affected by impulsive forces after each rigid impact of the swing leg with the ground. Indeed, at any instance of time $t_{\mathrm{I}}$ at which a rigid impact with the ground happens, we have $\int_{t_{\mathrm{I}}^-}^{t_{\mathrm{I}}^+} \ddot{q}(\tau)\, d\tau = \dot{q}^+ - \dot{q}^-$, where $t_I^-$ and $t_I^+$ represent the time instants just before and just after the impact, respectively. The joint velocities $\dot{q}^-$ and $\dot{q}^+$ at these two time instants are related to each other through (6). Before finding the relationship between the pre-impact and post-impact NDOB states, we assume the following regarding the NDOB states, which *will be formally proved in the next section* (see Theorem 1).

**DOBH1)** The NDOB states in (16) and the robot joint velocities remain essentially bounded for all $t \geq 0$. △

We consider the impact time instant $t_{\mathrm{I}}$ and integrate both sides of (16) from $t_{\mathrm{I}}^-$ to $t_{\mathrm{I}}^+$. We have

$$\int_{t_{\mathrm{I}}^-}^{t_{\mathrm{I}}^+} \dot{z}(\tau)\, d\tau = -\int_{t_{\mathrm{I}}^-}^{t_{\mathrm{I}}^+} L_{\mathrm{d}}(q(\tau))z(\tau)\, d\tau +$$

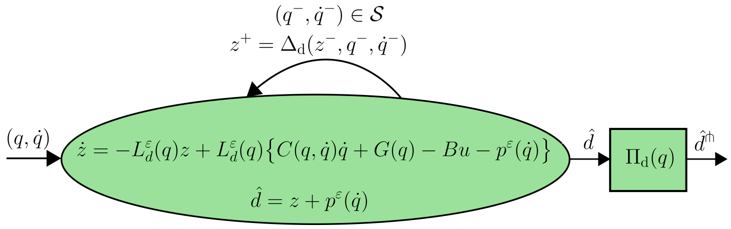


Fig. 4: The proposed hybrid NDOB structure.

$$\int_{t_{\rm I}^-}^{t_{\rm I}^+} L_{\rm d}(q(\tau))\Big\{ - M(q(\tau))\ddot q(\tau) - p\big(\dot q(\tau)\big) + d(\tau)\Big\}\, d\tau.$$

Due to DH1 and DOBH1, we have

$$z^+ - z^- = -\int_{t_{\rm I}^-}^{t_{\rm I}^+} L_{\rm d}(q(\tau)) M(q(\tau))\ddot q(\tau).$$

Since NDOB gain matrix $L_{\rm d}(q)$ is given by (17), we have

$$z^+ - z^- = -X_{\varepsilon}^{-1}(\dot q^+ - \dot q^-).$$

Using the above calculation, we propose to use the jump map

$$z^+ = \Delta_{\rm d}(z^-, q^-, \dot q^-), \tag{29a}$$

$$\Delta_{\rm d}(z^-, q^-, \dot q^-) = z^- - X_{\varepsilon}^{-1}(\Delta_{\dot q}(q^-)\dot q^- - \dot q^-), \tag{29b}$$

for the NDOB states after each impact of the biped swing leg with the ground, where $\Delta_{\dot q}(\cdot)$ is given by (6).

Using the definition of $\hat d$, we have

$$\hat d^+ = z^+ + p^{\varepsilon}(\dot q^+) \underbrace{=}_{(17)} z^- - X_{\varepsilon}^{-1}(\underbrace{\Delta_{\dot q}(q^-)\dot q^-}_{\dot q^+} - \dot q^-) + X_{\varepsilon}^{-1}\dot q^+.$$

Therefore, $\hat d^+ = z^- + X_{\varepsilon}^{-1}\dot q^-$. From (16), it can be easily seen that

$$\hat d^+ = \hat d^-. \tag{30}$$

Moreover, given an impact time instant $t_j$, corresponding to the $j$-th leg impact with the ground and under DH1, we can integrate both sides of (20) from $t_{\rm I}^-$ to $t_{\rm I}^+$ to obtain

$$e_{\rm d}^+ = e_{\rm d}^- + \Delta d_j, \tag{31}$$

where $\Delta d_j$ is defined in (4).

## V. NDOB-Based Control Stability Analysis

For the underactuated biped robot hybrid dynamical system given by (1)–(6), we have proposed the following hybrid NDOB, whose overall structure is depicted in Figure 4,

$$\begin{cases} \dot z = -L_{\rm d}^{\varepsilon}(q) z + L_{\rm d}^{\varepsilon}(q)\{N - Bu - p^{\varepsilon}(\dot q)\}, & \text{for } (q, \dot q) \notin \mathcal{S} \\ z^+ = \Delta_{\rm d}(z^-, q^-, \dot q^-), & \text{for } (q^-, \dot q^-) \in \mathcal{S} \\ \hat d = z + p^{\varepsilon}(\dot q) \end{cases} \tag{32}$$

where $N := C(q, \dot q)\dot q + G(q)$, and $p_{\rm d}^{\varepsilon}(\dot q)$, $L_{\rm d}^{\varepsilon}(q)$ are given by (17). Furthermore, NDOB jump map $\Delta_{\rm d}(\cdot)$ is given by (29). Given the nominal control input $u_{\rm n}(q, \dot q)$ in CH1, we propose to employ the DOB-based control law given by (25) with the projection operator $\Pi_{\rm d}(q)$ given by (22) and the disturbance estimate $\hat d$ given by (32).

For the closed-loop dynamics of the biped robot under the DOB-based control law in (25), we consider the coordinates

$$\eta_{\rm d} := [\bar x; e_{\rm d}], \tag{33}$$

where

$$\bar x := [\eta_1; \eta_2; \xi_1; \xi_2], \tag{34}$$

with $[\eta_1; \eta_2] := [y; \dot y]$, $y = h(q)$ is given by (8), $\dot y = (\partial h/\partial q)\dot q$, and $[\xi_1; \xi_2]$ are the zero dynamics manifold coordinates given by (11). We have the following proposition regarding the closed-loop dynamics of the biped robot.

*Proposition 5:* Consider the biped robot hybrid dynamics in (1)–(6), under the DOB-based control law in (25), with the hybrid NDOB in (32). Then, in a neighborhood of the zero dynamics manifold associated with the output $y = h(q)$, $[x;\, e_d] \mapsto [\bar x;\, e_d]$ is a valid change of coordinates and the closed-loop dynamics of the biped robot and the NDOB can be written as

$$\begin{cases} \dot{\bar x} = F(\bar x, e_{\rm d}) + G_{\rm d}(\bar x) d^{\parallel}(t) & \text{for } (q^-, \dot q^-) \notin \mathcal{S} \\ \dot e_{\rm d} = \hat F^{\varepsilon}(\bar x, e_{\rm d}) + \dot d(t) & \text{for } (q^-, \dot q^-) \notin \mathcal{S} \\ \bar x^+ = \Delta(\bar x^-) & \text{for } (q^-, \dot q^-) \in \mathcal{S} \\ e_{\rm d}^+ = \hat\Delta(x^-, e_{\rm d}^-) + \Delta d_j & \text{for } (q^-, \dot q^-) \in \mathcal{S} \end{cases} \tag{35}$$

where $\Delta d_j$ and $\bar x$ are defined in (4) and (34), respectively. Furthermore, $F(\bar x, e_{\rm d}) := [\eta_2; -K_p\eta_1 - K_d\eta_2 - (\partial h/\partial q)M^{-1}e_{\rm d}; \kappa_1(\xi_1); \kappa_2(\xi_1)\xi_2]$, $G_{\rm d}(\bar x) := [0; 0; 0; \zeta(\xi_1)]$ with $\zeta(\xi_1)$ defined in (28), $\hat F^{\varepsilon}(\bar x, e_{\rm d}) := -L_{\rm d}^{\varepsilon}(\bar x)e_{\rm d}$, and $\hat\Delta(x^-, e_{\rm d}^-) = e_{\rm d}^-$.

*Proof:* The proof follows directly from Propositions 3, 4, and the NDOB tracking error jump map given by (31). ■

The next proposition and the theorem following it state that the biped robot and the hybrid NDOB interconnected dynamics have an exponentially stable periodic orbit, which is induced by the periodic orbit in (14), in the presence of constant disturbances with zero tangential component. Moreover, in the presence of time varying disturbances with bounded tangential component, the induced periodic orbit is locally input-to-state stable for the biped robot and the hybrid NDOB interconnected dynamics.

*Proposition 6:* Consider the biped robot and the hybrid NDOB interconnected dynamics given by (35). Consider the periodic orbit $\mathcal{O}$ in CH1. If $d^{\parallel}(t) = 0$, $\dot d(t) = 0$, and $\Delta d_j = 0$, for all $t \geq 0$ and all impact moments $t_j$, then there exists $\varepsilon^*$ such that for all $\varepsilon \in \big(0, \varepsilon^*\big]$, the orbit

$$\mathcal{O}_{\rm ext} := \mathcal{O} \times \{0\} \subset \mathcal{X} \times \mathcal{E}, \tag{36}$$

is an exponentially stable periodic orbit of (35).

*Proof:* Proof is removed for the sake of brevity. ■

*Theorem 1:* Consider the biped robot and the NDOB closed-loop hybrid dynamics given by (35) with disturbance inputs $d(\cdot)$ satisfying DH1. Consider the exponentially stable hybrid periodic orbit $\mathcal{O}_{\rm ext}$ in (36). Then, $\mathcal{O}_{\rm ext}$ is locally input-to-state stable for the biped robot and the NDOB closed-loop dynamics. Namely, there exist $r > 0$ and class-$\mathcal{KL}$ function

$\iota$ such that for each $\eta_{\rm d,0} \in \{\|\eta_{\rm d}\|_{\mathcal{O}_{\rm ext}} \leq r\}$, the maximal solutions to (35) with initial condition $\eta_{\rm d}(0) = \eta_{\rm d,0}$ satisfy

$$\|\eta_{\rm d}(t)\|_{\mathcal{O}_{\rm ext}} \leq \max\left\{\iota(\|\eta_{\rm d}(0)\|_{\mathcal{O}_{\rm ext}}, t), \omega_{\rm d}\right\}, \ \forall t \geq 0. \tag{37}$$

*Proof:* The closed-loop dynamics of the biped robot and the NDOB are given by (35). From Proposition 6, it follows that the set $\mathcal{O}_{\rm ext}$ is pre-asymptotically stable for the dynamics in (35) with inputs $d^{\parallel}(t)$, $\dot{d}(t)$, and $\Delta d_j$. Using Proposition 2.4 and Definition 2.3 in [27], (37) follows. ■

## VI. Simulation Studies

In this section we present simulation results for push recovery in a five-link biped robot with point feet corresponding to the biped robot RABBIT [28] (see Figure 1). The physical parameters of the robot are taken from [28] and will not be presented here for the sake of brevity. The hybrid invariant outputs for the biped robot dynamcis have the form given by (8) and are taken from [28]. Initiating the biped robot states on the hybrid periodic orbit $\mathcal{O}$ induced by the outputs in (8), we start applying a push disturbance $F_{\rm d}(t) = [f_{\rm x}(t);\ 0]$ at the joint connecting the torso and the two legs, where $f_{\rm x}(t)$ is a trapezoidal signal that starts from zero at $t = 2$ seconds, reaches to $-10$ N at $t = 3$ seconds, lasts for three seconds, and goes from $-10$ N to zero in one second. The time-varying disturbance acting on the biped is then equal to $d(t) = J^{\top}(q(t))F_{\rm d}(t)$, where $J(q)$ is the proper Jacobian matrix [29].

When no NDOB is used along with the nominal control law, the biped robot stops walking at about 5.5 sec. When the NDOB-based control law in (25) with $\varepsilon = 0.1$ in (32) is employed, however, the biped robot manages to recover from the applied push disturbance. Figure 5 depicts the 2-norm of the motor torques, the 2-norm of the output tracking error, and the 2-norm of the disturbance tracking error, when the NDOB-based control law in (32) is used. Figure 6 depicts phase plots of the biped robot in the absence of push disturbance and in the presence of push disturbance with and without NDOB.

## VII. Conclusion and Further Remarks

In this paper, we extended a class of NDOBs to underactuated bipedal robots, which are controlled using HZD-based control schemes. The proposed hybrid NDOB addresses the issues of underactuation and hybrid dynamics in biped robots. The proposed extension to NDOBs further motivates investigation of disturbance observers for robust control of powered prostheses used in rehabilitation robotics. An interesting question, as first posed by [10], is whether to treat persistent human input as a disturbance in HZD-based control of powered prostheses.

## Appendix

**Proof of Lemma 1.** Due to OH1 and by continuity, $A(q)$ is also invertible in a neighborhood of $h^{-1}(0)$. Consider an arbitrary point $q$ in this neighborhood. Define $v' := M^{-1}(q)v$. First, we claim that $\mathrm{Ker}(\partial h/\partial q)\big|_q \cap \mathrm{Im}(M^{-1}(q)B) = \{0\}$. Suppose, by way of contradiction, that there exists $v \neq 0$ such that $v \in \mathrm{Ker}(\partial h/\partial q)\big|_q \cap \mathrm{Im}(M^{-1}(q)B)$. Then, $v = M^{-1}(q)Bw$ for some $w \neq 0$. Since $v \in \mathrm{Ker}(\partial h/\partial q)\big|_q$, then $(\partial h/\partial q)v = 0$. Therefore, we have

$$\frac{\partial h}{\partial q}\Big|_q M^{-1}(q)Bw = 0 \Rightarrow A(q)w = 0.$$

This is a contradiction. Therefore, $\mathrm{Ker}(\partial h/\partial q)\big|_q \cap \mathrm{Im}(M^{-1}(q)B) = \{0\}$. Moreover, $\dim(\mathrm{Im}(M^{-1}(q)B)) = N-1$ and $\dim(\mathrm{Ker}(\partial h/\partial q)\big|_q) = 1$. Thus, there exist vectors $v^{\pitchfork} \in \mathbb{R}^{N-1}$ and $v^{\parallel} \in \mathrm{Ker}(\partial h/\partial q)\big|_q$ such that

$$v' = v^{\parallel} + M^{-1}(q)Bv^{\pitchfork}.$$

Multiplying both sides by $M(q)$ concludes the proof. ■

**Proof of Proposition 4.** The proof can be carried out exactly to the proof of Theorem 5.1 in [3, Chapter 5]. We only show that $\dot{\xi}_2 = \kappa_2(\xi_1) + \zeta_1(\xi_1)d^{\parallel}(t)$. Since

$$\dot{\xi}_2 = \begin{bmatrix}\dot{q}^{\top}\frac{\partial(B^{\perp}M)^{\top}}{\partial q} & B^{\perp}M\end{bmatrix}\begin{bmatrix}\dot{q}\\ -M^{-1}(q)\left[C\dot{q} + G - d(t)\right]\end{bmatrix},$$

(see [3, pp. 121–122]), we have

$$\dot{\xi}_2 = \kappa_2(\xi_1) + B^{\perp}M(q(\xi_1))M^{-1}(q(\xi_1))d(t),$$

where $q(\xi_1)$ is given by (12). Since $d(t) = Bd^{\pitchfork}(t) + M(q(\xi_1))d^{\parallel}(t)$ on $\mathcal{Z}$, due to Lemma 1, we have

$$\dot{\xi}_2 = \kappa_2(\xi_1) + B^{\perp}Bd^{\pitchfork}(t) + B^{\perp}M(q(\xi_1))d^{\parallel}(t).$$

Setting $B^{\perp}B = 0$ concludes the proof. ■

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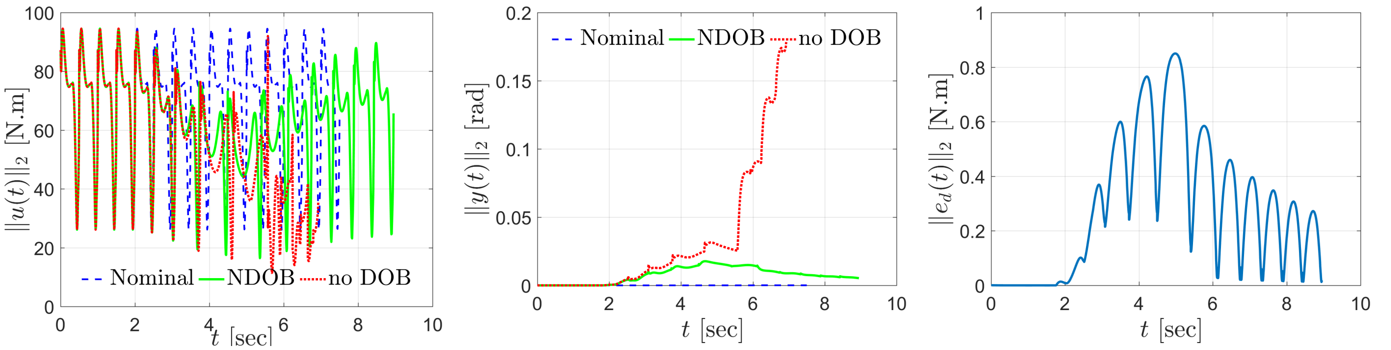
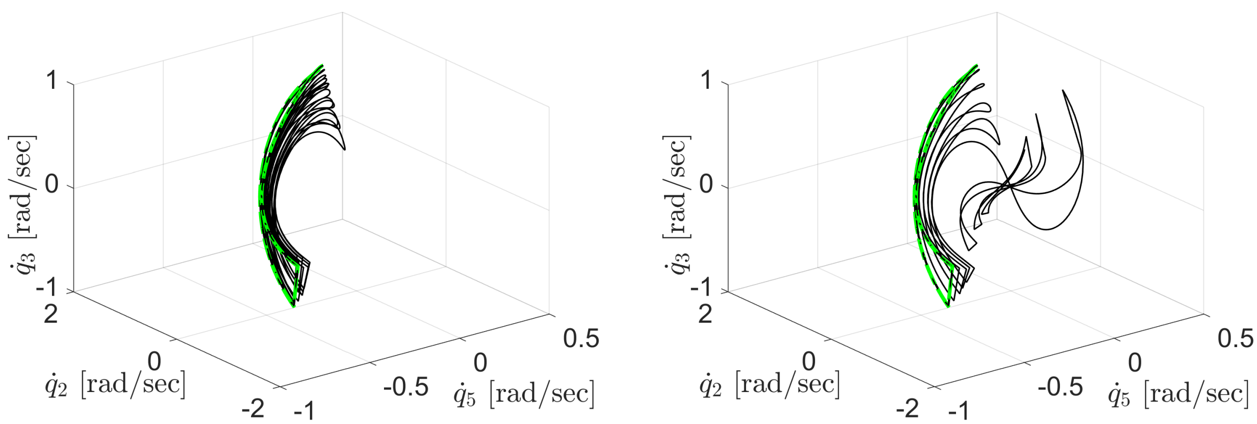



(a) (b) (c)

Fig. 5: The control input and tracking errors of the biped robot: (a) the 2-norm of the motor control torques $u$, (b) the 2-norm of the output tracking error, and (c) the 2-norm of the disturbance tracking error with the NDOB-based control law.




(a) (b)

Fig. 6: The projected phase portrait of the five-link biped robot: (a) (green) nominal controller with no disturbance, (black) NDOB-based control law with push disturbance, and (b) (green) nominal controller with no disturbance, (black) nominal control law with push disturbance.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