Unified Phase Variables of Relative Degree Two for Human Locomotion

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***Abstract*— A starting point to achieve stable locomotion is synchronizing the leg joint kinematics during the gait cycle. Some biped robots parameterize a nonlinear controller (e.g., input-output feedback linearization) whose main objective is to track specific kinematic trajectories as a function of a single mechanical variable (i.e., a phase variable) in order to allow the robot to walk. A phase variable capable of parameterizing the entire gait cycle, the hip phase angle, has been used to control wearable robots and was recently shown to provide a robust representation of the phase of human gait. However, this unified phase variable relies on hip velocity, which is difficult to measure in real-time and prevents the use of derivative corrections in phase-based controllers for wearable robots. One derivative of this phase variable yields accelerations (i.e., the equations of motion), so the system is said to be relative degree-one. This means that there are states of the system that cannot be controlled. The goal of this paper is to offer relative degree-two alternatives to the hip phase angle and examine their robustness for parameterizing human gait.**

I. Introduction

Synchronizing the joint kinematics across the gait cycle is a key challenge towards achieving stable locomotion in biped robots, powered prosthetic legs, and exoskeletons. Recently, improvements in hardware (e.g., smaller motors, etc.) have led to wearable robotic applications aimed at helping people recover locomotion after a stroke or an amputation. However, it is still unknown how to best synchronize the lower-limb kinematics of a wearable robot with the human body throughout the gait cycle.

There have been two general approaches towards controlling the lower-limb kinematics of biped robots, powered prosthetic legs, and exoskeletons. A widely used technique is a state machine [1]–[6]. In this technique the kinematic configuration of the leg is changed from one predefined state to another according to multiple switching conditions [3]. One downside of this approach is the number of parameters that have to be tuned [5] as well as its unexpected behavior during non-steady gait (i.e., whenever the switching conditions are faulted). On the other hand, current biped robots are able to achieve stable locomotion by controlling their joint patterns as a function of a single mechanical variable (i.e., phase variable) through the gait cycle [7]–[12]. This concept has been translated to the rehabilitation field and is now being used to control robotic prosthetic legs [13]–[15]. This methodology not only avoids switching conditions but it also makes the controller time-independent. In addition, if the phase variable is chosen correctly, then the desired joint kinematics would match the subject's intention during walking, even during non-steady gait [16].

There is not a clear consensus on which phase variable is capable of robustly representing the lower-limb kinematics throughout the gait cycle. In general, a phase variable needs to have a monotonic trajectory during steady gait and needs to be computed from an unactuated mechanical state of the system [16], [17]. For example, the global stance leg angle (i.e., angle between the hip-to-ankle vector and the vertical axis) has been used as a phase variable in order to synchronize the kinematic patterns on multiple biped robots [7]–[12], [18], [19]. On prosthetic leg applications, however, the choice of phase variable is less obvious since there is a limited amount of feedback available to the leg. In [15] the center of pressure (COP) was used to parameterize the joint trajectories of a transfemoral prosthetic leg. A downside of this choice is that the COP signal is limited to the stance portion of the gait cycle, and thus the swing portion of the gait cycle was not parameterized. The complete gait cycle (i.e., stance and swing portions) needs to be parameterized in order to achieve continuous synchronization between the human and the device.

It has been shown that the hip joint is a major contributor to the synchronization of the gait cycle in mammals [20], [21]. A phase variable computed from the hip joint's phase portrait (angle vs. velocity) was recently shown to robustly represent the phase of human gait during non-steady walking conditions [16]. This phase variable has been used to control the timing of a hip exoskeleton in [22], [23]. However, computing a phase variable as a function of a position and a velocity becomes a limitation for nonlinear controllers [24] whose main objective is to follow a reference trajectory (e.g., enforce specific kinematic patterns). In particular, a phase variable that is function of velocity terms affects the relative degree of a control system [12], [24]. The relative degree is equal to the number of derivatives of the output that must be taken to expose the control input through the dynamics. Lowering the relative degree of a system means that there will be states of the system that cannot be controlled (i.e., hidden dynamics) [24]. The fact that the hip phase angle in [16], [22], [23] is a function of the hip angular velocity greatly limits the options for feedback controllers. Therefore,

This work was supported by the Eunice Kennedy Shriver National Institute of Child Health & Human Development of the National Institutes of Health under Award Number DP2HD080349. The content is solely the responsibility of the authors and does not necessarily represent the official views of the NIH. Robert D. Gregg, IV, Ph.D., holds a Career Award at the Scientific Interface from the Burroughs Wellcome Fund. Dario J. Villarreal holds a Graduate Fellowship from the National Council of Science and Technology (CONACYT) from Mexico.

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in this paper we propose alternative phase variables that do not depend on velocities and are similarly capable of correctly parameterizing the joint kinematics of the stance and swing portions.

To evaluate the robustness of these phase variables, we examine data from able-bodied human subject experiments (N = 10) where a phase-shifting perturbation is applied to the person walking [25]. A phase-shifting perturbation slows or advances the overall progression of the gait cycle (i.e., decelerating or accelerating through the leg joint patterns). We analyze alternative phase variables and compare their ability to parameterize non-steady lower-limb joint trajectories with that of the previous phase variable analyzed in [16].

Some key concepts used throughout the paper are introduce in Section II-A. The alternative phase variable candidates are proposed in Section II-B. The experimental methods and statistical analyses used to study these phase variables are presented in Section II-C and Section II-D. We finish by discussing the caveats of our results and how these phase variables might perform when implemented in a powered prosthetic leg.

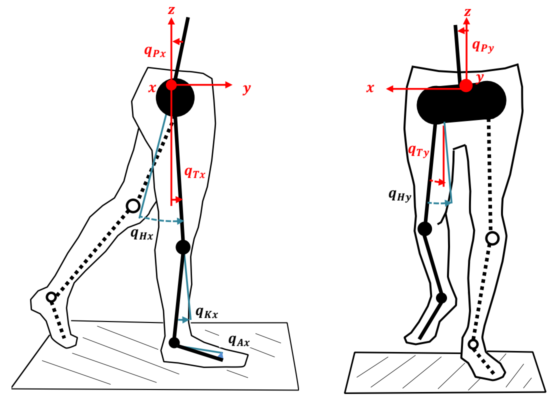

Fig. 1. The body diagrams of the sagittal (left) and frontal (right) planes of a person walking in 3D space. The configuration vector for this model is $q = (q_{Px}, q_{Tx}, q_{Hx}, q_{Kx}, q_{Ax}, q_{Py}, q_{Ty}, q_{Hy})^T$. The first letter in the subscript notation stands for the joint this variable represents (e.g., $_P$ - Pelvis, $_T$ - Thigh, $_H$ - Hip, $_K$ - Knee, and $_A$ - Ankle). The second letter in the subscript notation stands for the plane where the variable is measured. The variables in red represent variables measured from a global frame whereas the variables in blue represent variables measured from a relative frame.

II. Methods

A. Definitions and Preliminaries

In order to simplify the notation throughout the paper we denote the configuration vector of a dynamical system as $q(t) \in \mathrm{R}^d$ (where d is the number of degrees of freedom) and its time integral as $\tilde{q} \triangleq \int_0^t q(\tau) d\tau$. In addition, we define the partial derivative of a function $h(\cdot)$ with respect to a vector $z(t)$ to be $H_z \triangleq \frac{\partial h}{\partial z} = \nabla_z h$.

Without loss of generality and following the Euler-Lagrange equation, the equations of motion of any mechanical system can be represented by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q) = B\mathbf{u}.$$

The matrices M, C, and N represent the mass/inertia forces, Coriolis forces, and gravitational forces of the system, respectively. The vector $\mathbf{u}$ represents the inputs to the system. These inputs are torques or forces acting on the configuration vector through the mapping B. If we were to compute the value of the acceleration terms from this equation in order to represent these dynamics as a system of differential equations, then it would yield the following equation

$$\ddot{q} = M(q)^{-1}B\mathbf{u} - M(q)^{-1}[C(q, \dot{q})\dot{q} + N(q)],$$

where $M(q)^{-1}$ exists for any well-defined mechanical system [26]. For simplicity we express this highly nonlinear equation as $\ddot{q} = F(q, \dot{q}) + G(q)\mathbf{u}$.

Let $x_1 = q$ and $x_2 = \dot{q}$ define the state of the corresponding nonlinear dynamical system such that

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= F(x_1, x_2) + G(x_1)\mathbf{u} \\ y &= x_1 = h(q) \end{aligned} \tag{1}$$

where y is defined as an output function to be regulated by the input $\mathbf{u}$. Notice that function y is dependent only on the configuration vector (i.e., q) of the system and not on velocity terms. The goal of this controller is to follow a trajectory. In particular, we want to follow a trajectory where the output function is equal to zero (i.e., $y = h(q) = 0$).

By taking twice the time derivative of the output function we get the following equations:

$$\begin{aligned} \dot{y} &= \dot{x_1} = x_2 \\ \ddot{y} &= \dot{x_2} = F(x_1, x_2) + G(x_1)\mathbf{u}. \end{aligned} \tag{2}$$

This system is said to have relative degree 2 (i.e., $r = 2$) because we had to differentiate the output function (y) twice before the input of our system ($\mathbf{u}$) appeared. Since the original system is second order ($n = 2$), there are zero hidden dynamics in the system (i.e., $n - r = 0$). Notice that if the input of the system appeared in the first time derivative of our output function, then the relative degree of the system would have decreased (i.e., $r = 1$). This would imply that there are some dynamics in the system that cannot be controlled and the best option for controlling the system would be a proportional controller.

B. Phase Variable Candidates

A total of two alternative phase variable candidates for control applications are derived in this paper. These phase variables are derived such that they are not functions of velocities. Using the same procedure as in [16], these phase variables were evaluated on their ability to parameterize perturbed joint kinematics through the gait cycle. The performance of these phase variable candidates is compared to the phase variable previously analyzed in [16].

*1) Phase Hip Velocity (**PHV** [γ]):* This phase variable was previously derived in [16]. In the most general case, this phase variable is computed as follows: $\gamma = \arctan2(\frac{1}{\omega}\dot{q}_{Tx}, q_{Tx})$ where q_{Tx} and $\dot{q}_{Tx}$ correspond to the global thigh angle and global thigh angular velocity (Fig. 1), and ω represents the gait cycle cadence. This phase variable exploits the fact that the phase portrait of the global thigh angle (i.e., q_{Tx} vs. $\dot{q}_{T_x}$) is a periodic orbit resembling an ellipse, Fig. 2. This particular shape is achieved thanks to the cosine-like trajectory of the global thigh angle during the gait cycle, and thus its time derivative traces a sine-like trajectory (i.e., these two signals are 90° out of phase from each other). As shown in [16] this phase variable theoretically yields a monotonic, bounded, and linear phase variable. However, due to the fact that an angular velocity is used in the calculation of this phase variable it changes the relative degree of the system to one (i.e., $r = 1$). The time derivatives of an output function of the form $y = g(\gamma(q, \dot{q})) = h(q, \dot{q})$, where $h = g \circ \gamma$, can be computed to show that the input $\mathbf{u}$ appears in the first time-derivative rather than in the second one:

$$\begin{aligned}\dot{y} &= H_q \dot{q} + H_{\dot{q}} \ddot{q} \\ &= H_q \dot{q} + H_{\dot{q}}(F(q, \dot{q}) + G(q)\mathbf{u}).\end{aligned}$$

Therefore, it is not possible to use a proportional-derivative (PD) controller. Only a proportional controller could be used for trajectory tracking. This implies that there will be states of the system that cannot be controlled.

*2) Phase Hip Integral (**PHI** [Φ]):* This phase variable was computed also exploiting the fact that the motion of the global thigh angle (q_{Tx}) is correlated to a cosine-like trajectory during human locomotion. Using this fact and the knowledge that the derivative of a cosine function has a linear relationship to its integral (i.e., $x(t) = A\cos(\omega t) \Rightarrow \dot{x}(t) = -\omega^2 \int_0^t x(\tau) d\tau$, where ω is the frequency of the signal) then we can compute a phase variable that is a function of the integral of the global thigh angle rather than of its angular velocity. Using the same procedure as in [16], we compute the phase variable PHI as $\Phi = \arctan2(\omega \tilde{q}_{Tx}, q_{Tx})$, where we have defined $\tilde{q} \triangleq \int_0^t q(\tau) d\tau$. The variable Φ also yields a monotonic, bounded, and linear phase variable, Fig. 2. The implication of using the integral of a state of our dynamical system in this phase variable calculation is that we extend the order of our system plus one (i.e., $\bar{n} = n+1$). We notice that the input of the dynamical system ($\mathbf{u}$) appears at the second time derivative of the output function $y = g(\Phi(q, \tilde{q})) = h(q, \tilde{q})$, where $h = g \circ \Phi$. In other words,

$$\begin{aligned}\dot{y} &= H_q \dot{q} + H_{\tilde{q}} q \\ \ddot{y} &= \dot{H}_q \dot{q} + H_{\tilde{q}} \dot{q} + \dot{H}_{\tilde{q}} q + H_q \ddot{q} \\ &= \dot{H}_q \dot{q} + H_{\tilde{q}} \dot{q} + \dot{H}_{\tilde{q}} q + H_q (F(q, \dot{q}) + G(q)\mathbf{u}).\end{aligned}$$

Using this phase variable for control still yields non-controllable dynamics (i.e., $\bar{n} - r = 3 - 2 = 1$) but the relative degree of the system is two. However, replacing the time derivative in the phase variable function with the integral allows us to control the system in a more robust way since a PD controller can now be used for trajectory tracking.

*3) Sagittal and Frontal Hip Angle (**SFH** [Θ]):* This phase variable function was derived to take only angles that could be measured from a human subject walking as inputs. As previously mentioned, one could say that during locomotion a human traces a cosine-like trajectory using his/her thigh on

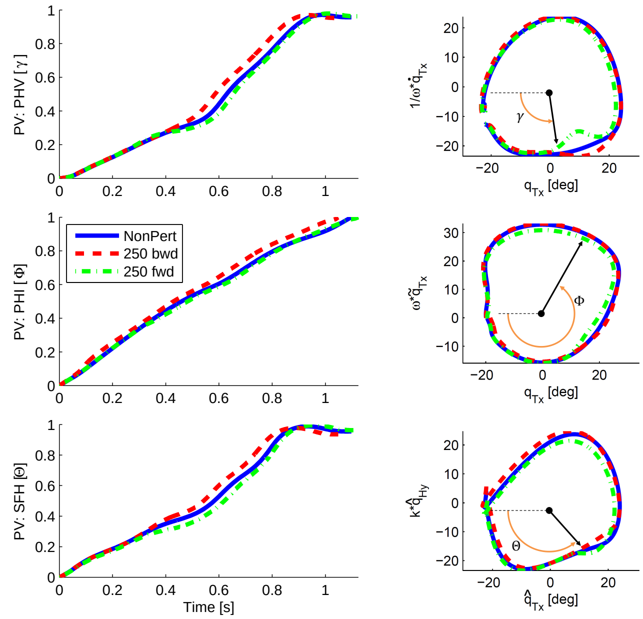

Fig. 2. Three phase variables (PV) are shown as well as the phase planes from which they were computed. On top, the Phase Hip Velocity (PHV) [γ] is shown. On the middle, the Phase Hip Integral (PHI) [Φ] is shown. On the bottom, the Sagittal/Frontal Hip Angle (SFH) [Θ] is shown. Each phase variable and phase plane is shown under two perturbation conditions (a backward and a forward perturbation occurring 250 ms after initial contact with the force plate).

the sagittal plane. Similarly, the relative angle of the hip with respect to the pelvis on a frontal plane (q_{Hy} in Fig. 1) traces a cosine-like trajectory with a constant phase shift with respect to q_{Tx}. When these two variables are plotted one against the other, the result is a tilted oval shape because these signals have a relative phase offset less than 90 deg. Ideally, we would like these two signals to form a circle in order to achieve a linear phase trajectory as in PV γ and Φ. Thus, we use a principal component analysis (PCA) approach in order to find the independent basis vectors that allow us to compute two new variables ($\hat{q}_1$ and $\hat{q}_2$) that are uncorrelated from each other. In other words, a linear transformation $\mathbf{T} \in \mathrm{SO}(2)$ can be found in order to compute $\hat{q} = \mathbf{T}[q_{Hy}, q_{Tx}]^T$, where $\hat{q} = [\hat{q}_1, \hat{q}_2]^T$ is a pair of uncorrelated measurements. On a polar coordinate system, this transformation results in two signals that are 90° out of phase from each other. After computing these new variables (i.e., $\hat{q}_1$ and $\hat{q}_2$) the phase variable SFH can be computed similarly to the PHV phase variable, i.e., $\Theta = \arctan2(k\hat{q}_2, \hat{q}_1)$, where k is a scaling factor that gives the same amplitude to both signals in order to achieve a linear phase variable [16]). The consequence of using this phase variable, which is only a function of the configuration variables of our system, is that the relative degree is equal to the order of the system (i.e., $n = r$). The input of the system will appear in the second time-derivative of the output function $y = g(\Theta(q)) = h(q)$, where $h = g \circ \Theta$:

$$\begin{aligned}\dot{y} &= H_q \dot{q} \\ \ddot{y} &= \dot{H}_q \dot{q} + H_q \ddot{q} \\ &= \dot{H}_q \dot{q} + H_q (F(q, \dot{q}) + G(q)\mathbf{u}).\end{aligned}$$

However, in order to calculate this phase variable a higher-dimensional (3D) model needs to be considered and thus the number of states that need to be measured increases.

C. Experimental Protocol

The experimental protocol was approved by the Institutional Review Board at the University of Texas at Dallas. A total of ten able-bodied subjects (4 women, height: 175.44 cm $\pm$ 6.10 cm, weight: 67.25 kg $\pm$ 7.40 kg) gave written informed consent of the experimental protocol prior to experimentation. A 10 camera motion capture system (Vicon T20s, Oxford, UK) was used to record kinematic data. Anthropomorphic measurements (e.g., leg length, hip width, knee width, etc.) were taken from each subject before the experiment and later entered into the motion capture software Nexus to create a 3D kinematic model with the help of the Plug-in-Gait module.

The experimental procedure was the same as in [25]. In summary, the experiment contained four sets of 72 trials, where each trial consisted of the subject walking from a fixed starting point, stepping with their right foot on the force plate in the middle of the walkway, and continuing to walk until the end of the walkway. The perturbations were randomized (50% probability of occurrence) as well as the onset times of the perturbation (i.e., 100 ms or 250 ms after initial contact with the force plate). Whenever a perturbation happened, the force plate traveled a distance of 5 cm over 100 ms in either direction (i.e., in the walking direction or against the walking direction of the human subject).

D. Statistical Analysis

The correlation coefficients between average perturbed and average non-perturbed joint angle trajectories were computed using MATLAB (MathWorks, Massachusetts, USA) for each perturbation condition per subject. The correlation coefficient averaged across all types of perturbations was calculated in order to have a unique metric per subject capable of quantifying the performance of each parameterization (i.e., phase variable candidate). We consider that despite the type of perturbation the correlation coefficient can measure how well each of the leg joint angles matched the nominal kinematics, Fig. 3. An upper-tail t-test was used to statistically compare the correlation coefficients of the time-based and each of the phase variable parameterizations of the joint angle trajectories between perturbed and non-perturbed conditions. A p-value less than 0.05 in this test would correspond to a statistically greater correlation coefficient for one parameterization than another parameterization.

A lower-tail t-test was used to compare the transient error observed between parameterizations. The observed transient error was quantified by the RMS error between the average

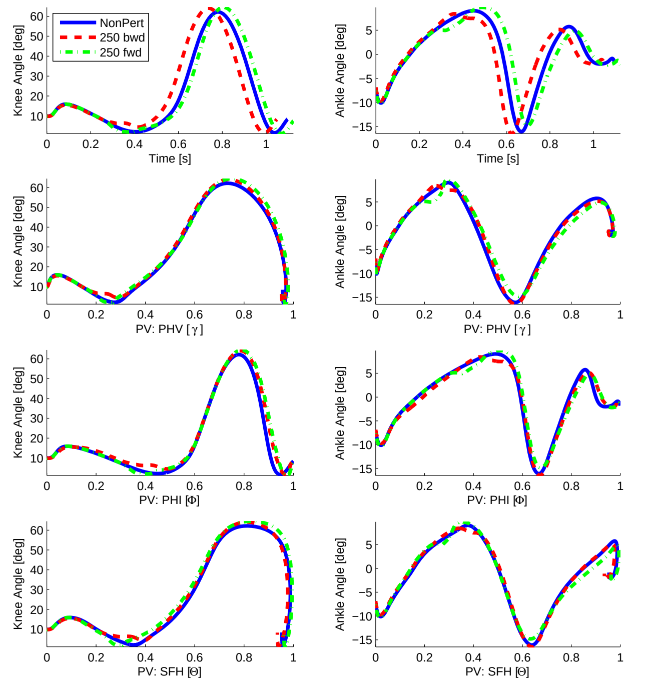

Fig. 3. The non-perturbed and perturbed (backward and forward perturbation occurring 250 ms after initial contact with the force plate) averaged knee angle (left) and averaged ankle angle (right) across subjects are shown under different parameterizations.

perturbed and non-perturbed joint trajectories for each parameterization per subject. An overall error was calculated by averaging the RMS errors across all types of perturbations. This averaged RMS error provides a single metric capable of describing the observed transients of one subject across all the perturbations, Fig. 3. A p-value less than 0.05 in this analysis would correspond to a statistically smaller transient response for a parameterization.

III. RESULTS

The correlation coefficients between perturbed and non-perturbed gait cycles were computed for each phase variable as stated in Section II-D. Fig. 3 shows the averaged knee and ankle joint trajectories across subjects parameterized with all phase variable candidates for a forward or backward perturbation that occurred 250 ms after initial contact with the force plate. Table I shows the mean correlation coefficients for each phase variable and time across all subjects. For the phase variable that is a function of velocities (PHV [γ]), the correlation coefficients for each joint are greater than that of the time parameterization. As a consequence, parameterizing the joint kinematics using this phase variable is statistically more robust than parameterizing it by time, Table II.

The alternative phase variable candidates presented in this paper (SFH [Θ] and PHI [Φ], Fig. 2) are statistically better than time at parameterizing the kinematics of most joints. Table II shows that the correlation coefficient calculated between the perturbed and non-perturbed knee joint trajectories using PV: PHI [Φ] was not statistically greater than the time parameterization. In a similar manner, the correlation

TABLE I
AVERAGE CORRELATION COEFFICENTS AND RMS ERRORS

	Hip	Knee	Ankle
Correlation Coefficients			
t	0.983	0.969	0.884
$\gamma(\cdot)$	0.996	0.995^{+}	0.958
$\Phi(\cdot)$	0.997^{+}	0.984	0.958
$\Theta(\cdot)$	0.989	0.994	0.969^{+}
RMS Error			
t	2.920	4.967	3.786
$\gamma(\cdot)$	1.767	2.731^{-}	2.698
$\Phi(\cdot)$	1.439^{-}	4.019	2.377^{-}
$\Theta(\cdot)$	2.181	3.128	2.536

The mean correlation coefficients and RMS errors across subjects for each parameterizations are presented. The symbol "$^{+}$"/"$^{-}$" next to a number, (on the correlation coefficients) represents the greatest/smallest value per column.

TABLE II
P-VALUES

	Hip	Knee	Ankle
Correlation Coefficient Hypothesis			
$\gamma(\cdot) > t$	0.006*	0.002*	0.006*
$\Phi(\cdot) > t$	0.001*	0.098	0.001*
$\Theta(\cdot) > t$	0.054	0.011*	0.002*
RMS Error Hypothesis			
$\gamma(\cdot) < t$	0.017*	0.012*	0.002*
$\Phi(\cdot) < t$	0.011*	0.076	0.002*
$\Theta(\cdot) < t$	0.128	0.019*	0.002*

The p-values computed from an upper tail t-test (correlation coefficeints) and a lower tail t-test (RMS error) are shown. The alternative hypotheses from these statistical tests are shown on the first column. The symbol "*" denotes the numbers that are smaller than 0.05.

coefficient of hip joint parameterized by PV: SFH [Θ] was not statistically greater than the time parameterization.

The RMS error values were similar for each of the phase variables, Table I. The phase variable PHV [γ] was able to statistically reduce the error observed between perturbed and non-perturbed kinematics, Table II. The phase variable PHI [Φ] was only able to statistically reduce the error observed between the perturbed and non-perturbed joint trajectories of the hip and ankle. The phase variable SFH [Θ] was able to statistically reduce the error observed between the perturbed and non-perturbed kinematics of the knee and ankle joints.

IV. DISCUSSION

The alternative phase variables presented in this paper are not functions of velocity states and are able to parameterize the stance and swing portions of the gait cycle. These alternative phase variables improve the objective of trajectory tracking (e.g., enforcing a kinematic pattern) when an input-output feedback linearization control is used on a robotic prosthetic leg or exoskeleton. Using these phase variables open the possibility of parameterizing PD controllers to control robotic platforms used for rehabilitation.

The phase variable PHV (γ) is the only phase variable capable of parameterizing all the leg joint trajectories (i.e., hip, knee, and ankle) statistically better than time across perturbations. This might be due to the fact that this phase variable (i.e., PHV) is calculated from a phase portrait. A phase portrait is a commonly used method to represent the state of a second-order dynamical system. Thus, this phase variable has a physical meaning and may contain the most relevant information of the system dynamics. The phase variable PHI acts more as a filter for high frequency dynamics since the integral of a state is not susceptible to small disturbances.

Even though the phase variable PHV was better at parameterizing non-steady gait, it may not be the best phase variable for control applications. One of the biggest inconveniences of using the phase variable PHV in real time applications is the fact that it is a function of the global thigh angular velocity. This fact not only limits its use to proportional controllers for trajectory tracking (see Section II-A) but it also increases the noise in the system. A controller parameterized by this phase variable, in application, is subject to high frequency noise due to numerical differentiation. This noise could indeed be filtered, but doing this adds a delay on the system, thus affecting the wearable robot's synchronization to the user. This phase variable also yields a non-monotonic and non-linear phase variable, which further undermines the performance of the controller.

In application, the phase variable PHI is a better alternative to estimate the phase of the gait cycle. As previously stated, this phase variable acts as a filter, avoiding numerical and sensor measurement noise that could affect the computation of the phase variable. In Fig. 2 it can be noticed that this phase variable yields the most linear phase variable amongst all others. Even if this phase variable is slightly less representative of human locomotion (Section III), its linearity helps in control applications by improving the controller's performance when tracking a reference trajectory. In real-time control applications, a non-linear phase variable (such as PHV and SFH) is more sensitive to measurement noise during steep regions of the phase trajectory. Therefore, the phase variable PHI has an advantage over the other phase variables (i.e., PHV and SFH) in real-time control applications. This phase variable also allows the use of a PD controller for the objective of trajectory tracking (i.e., commanding a desired kinematic pattern on the prosthetic leg or exoskeleton).

An inconvenience of using the phase variable SFH is that this phase variable is not consistent across all subjects. In fact, one subject had to be removed from our analysis since it was not possible to compute a linear and monotonic phase variable from her thigh motion. An interesting observation was that this variable worked better with men than female subjects. This could be due to a physiological difference in the frontal plane kinematics between male and female subjects. A female subject may abduct and adduct her thigh in a different manner than men, which increases her pelvic motion on the frontal plane. These variances on the walking patterns between subjects yield an unreliable phase variable. Nevertheless, Hamed et al. have found that coupling the sagittal and frontal planes in a phase variable optimizes stable 3D walking for biped robots [7], [8]. Thus, even if the phase variable SFH is not consistent across all subjects, it could provide stability benefits that are not captured in our study.

The phase variables PHV [γ] and PHI [Φ] discussed in this paper are viable options for controlling prosthetic leg and exoskeletons. Some tradeoffs have to be made whenever choosing one phase variable over the other. For example, the phase variable PHV theoretically offers a robust representation of the gait cycle's phase but sacrifices controllability of velocity terms in a controller (i.e., can only use a proportional controller) and is more susceptible to noise. By adding one additional state with trivial dynamics, the phase variable PHI is less sensitive to noise and allows the use of derivative corrections in the feedback control scheme.

V. Conclusion

This paper presented two alternative phase variable candidates, that are functions of position alone and are capable of parameterizing the entire gait cycle. These phase variables are especially useful when an input-output feedback linearization control is applied to a robotic prosthetic leg or an exoskeleton where the objective is to enforce specific kinematic trajectories. The phase variable PHV parameterized the lower-limb joint kinematics during non-steady gait statistically better than time. However, for real time applications it limits the controller to a proportional gain and is more susceptible to sensor noise. The phase variable PHI is a good estimator of the phase of the gait cycle and yields a monotonic and linear phase variable for real time applications. Phase variable SFH is only a function of configuration variables of our system but does not generalize to all subjects. Deviations in gait kinematics can vastly alter this phase variable, thus it is likely to have non-monotonic behavior and nonlinearities. Variable PHI is the best option for control applications due to its linearity and reliability, whereas variable PHV is better for offline gait analysis due to its superior ability to correlate perturbed and non-perturbed kinematics.

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