

# ALGORITHMS FOR MINIMIZATION WITHOUT DERIVATIVES

RICHARD P. BRENT

## ABSTRACT

This monograph describes and analyzes some practical methods for finding approximate zeros and minima of functions. Contents include:

1. The use of successive interpolation for finding simple zeros of a function and its derivatives.
2. An algorithm with guaranteed convergence for finding a zero of a function.
3. An algorithm with guaranteed convergence for finding a minimum of a function of one variable.
4. Global minimization given an upper bound on the second derivative.
5. A new algorithm for minimizing a function of several variables without calculating derivatives.
6. Computer programs which implement these algorithms.

## COMMENTS

Only the Abstract is given here. The full work appeared as [4]. A preliminary version appeared as [2]. For related work, see [1, 3].

## REFERENCES

- [1] R. P. Brent, “An algorithm with guaranteed convergence for finding a zero of a function”, *Computer J.* 14 (1971), 422–425. MR 49#4234, Zbl 231.65046. rpb005.
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- [3] R. P. Brent, “A new algorithm for minimizing a function of several variables without calculating derivatives”, in *Optimization* (edited by R. S. Anderssen, L. S. Jennings and D. M. Ryan), University of Queensland Press, Brisbane, 1972, 14–25. MR 52#2574, 52#9601, Zbl 311.90065. rpb007.
- [4] R. P. Brent, *Algorithms for Minimization without Derivatives*, Prentice-Hall, Englewood Cliffs, New Jersey, 1973, 195 pp. (Prentice-Hall Series in Automatic Computation, edited by George Forsythe.) ISBN 0-13-022335-2. Reviewed in: *American Scientist* 61 (May–June 1973), 374; *Mathematical Programming* 4 (1973), 349; *Computer J.* 16 (1973), 314; *Math. Comp.* 28 (1974), 865–866; CR 15#26544; MR 49#4251; Zbl 245.65032. Errata: *Math. Comp.* TE 520, 29 (1975), 1166. MR 51#7283. rpb011.

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