Adding Measurement to van Tonder's Calculus

Computer Science Undegraduate Thesis

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Outline

A brief memory refresher

- van Tonder's calculus provides quantum concepts to the λ-calculus

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- The syntax is the following:

Syntax of λ_q

$t ::=$		term
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	$(t\ t)$	aplication
	c	constant
	$!t$	nonlinear term
	$(\lambda !x.t)$	nonlinear abstraction
$c ::=$		
	$0 \mid 1 \mid H \mid cnot \mid X \mid \ldots$	constants

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- Nonlinear terms are used to distinguish definite terms from not-definite terms

A brief memory refresher (II)

Definition

Let us call a subexpresion definite with respect to a computational basis if it is textually the same in all branches of the superposition.

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Example

$$\frac{1}{\sqrt{2}}(|(\lambda x.0)\ 0\rangle + |(\lambda x.0)\ 1\rangle)$$

- the subexpresion $(\lambda x.0)$ is definite
- the argument $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is non-definite

A brief memory refresher (III)

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All this rules are given formally in a set of well-formedness rules

A brief memory refresher (IV)

- To prevent terms of the form $!t$ from being evaluated, he extends the definition of values as follows:

Values in λ_q

v	$::=$		values:
		x	variable
		c	constant
		$(\lambda x.t)$	linear abstraction
		$(\lambda !x.t)$	nonlinear abstraction
		$!t$	!-suspension

A brief memory refresher (V)

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Example of transition rule with history track

$$\frac{}{\mathcal{H}; ((\lambda x.t)\ v) \rightarrow \mathcal{H}; ((\lambda x.\overline{t}_x)\ \_); t[v/x]} \quad (\beta)$$

where $\overline{t}_x$ is obtained from t by recursively replacing all subterms that do not contain x with the placeholder simbol and keeping x

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- For gates there are some specific rules

Example of Hadamard rule

$H\ 0$ reduces to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

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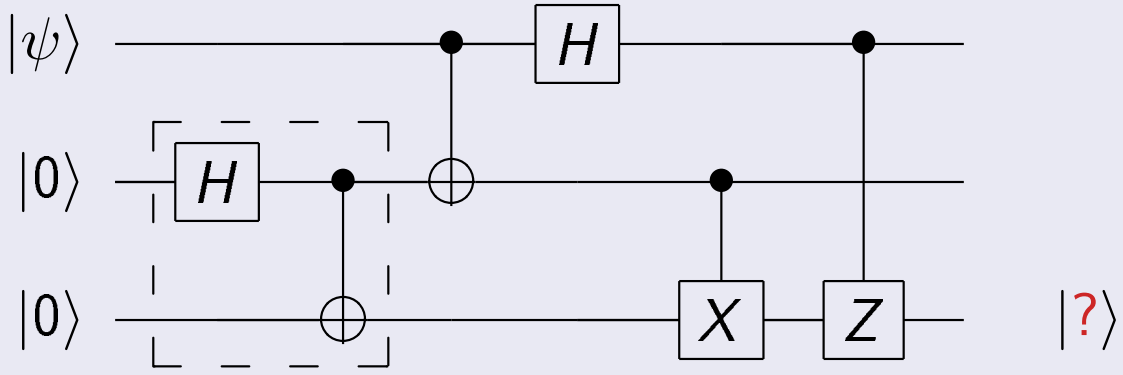
Example: Teleportation with deferred measurement



Here it is not clear that Alice and Bob can be away from each other

Motivation (II)

Same example in λ_q

$$\begin{aligned}\text{teleport } x \longrightarrow\ & \textbf{let } (e_1, e_2) = \text{epr } \textbf{in} \\ & \textbf{let } (x', y') = \text{alice } (x, e_1) \textbf{ in} \\ & \quad \text{bob } (x', y', e_2)\end{aligned}$$

where

$$\text{alice } (x, e_1) \longrightarrow \textbf{let } (x', y') = \mathit{cnot}\ (x, e_1) \textbf{ in } ((H\ x'), y')$$

$$\begin{aligned}\text{bob } (x', y', e_2) \longrightarrow\ & \textbf{let } (y'', e_2') = cX\ (y', e_2) \textbf{ in} \\ & \textbf{let } (x'', e_2'') = cZ\ (x', e_2') \textbf{ in} \\ & \quad (x'', y'', e_2'')\end{aligned}$$

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Goal

The intention of this work is to add the measurement operation to the untyped λ_q, keeping it untyped

1^{st} step: Probabilistic Transition Rules

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$$\frac{\textit{Premises over } P}{\begin{array}{l} P \rightarrow_p Q_1 \\ P \rightarrow_q Q_2 \end{array}}$$

P goes to Q_1 with probability p and to Q_2 with probability q

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- To know the probability of making a transition to each state, we need to know the "shape" of the qubit
- The λ_q syntax has no restriction on the shape of the qubits, they are taken as constants

2^{nd} step: Providing Syntax for Qubits

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Syntax for λ_q^M

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	c_U	*constant for gates*
	q	*constant for qubits*
	M_n	*constant for measurement*

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Syntax for qubits

$q ::=$		*qubits:*
	$\lvert 0\rangle \mid \lvert 1\rangle$	*definite qubits*
	$(q \otimes q)$	*tensorial product*
	$(q + q)$	*superposition*
	$\alpha(q)$	*scalar product*

2^{nd} step: Providing Syntax for Qubits (III)

Problem

- We need to restrict when a string is a qubit and when it is not. We need to add rules for well-formed terms

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$$\frac{\sum_{i=1}^{2^n} |\alpha_i|^2 = 1 \qquad \alpha_i \in \mathbb{C}, i = 1, \ldots, 2^n}{\vdash \alpha_0(!\,|0\rangle \otimes \cdots \otimes !\,|0\rangle) + \cdots + \alpha_{2^n}(!\,|1\rangle \otimes \cdots \otimes !\,|1\rangle)} \; \textit{Superpos}$$

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- The syntax previously defined will be the syntax of "pre-terms"
- A term is a pre-term $\ddot{t}$ if $\vdash \ddot{t}$ can be infered from the rules for well-formedness

Operational Semantic

Now, we can to add a transition rule for measurements

Transition rule for measurements

$$\text{M:}\ \frac{q = \sum_{i=1}^{2^n} \alpha_i q_i}{\mathcal{H}; (M_n\ q) \rightarrow_{|\alpha_i|^2}\ q_i}$$

Where

$$q_i = !q_{1i} \otimes !q_{2i} \otimes \cdots \otimes !q_{ni}$$

with

$$!q_{ki} = !\,|0\rangle \ \text{ or } !\,|1\rangle \quad \forall k = 1 \ldots n$$

This rule discards the history track because measurement operations are irreversible

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  In this case, it is allowed to copy the measurement twice, but this is the only reduction strategy because of the ! mark

Example: Teleportation algorithm

Teleportation in λ_q^M

$$\begin{array}{rl} \text{teleport } q \longrightarrow_1 & \textbf{let } x \otimes y = \text{epr } \textbf{in} \\ & \textbf{let } b_1 \otimes b_2 = M_2 \text{ alice } (q, x) \textbf{ in} \\ & \quad \text{bob } (b_1, b_2, y) \end{array}$$

where

$\text{epr} \equiv \mathit{cnot}\ ((H\ !\,|0\rangle \otimes !\,|0\rangle)$

$\text{alice } (q, x) \longrightarrow_1 \textbf{let } r \otimes w = \mathit{cnot}\ \ q \otimes x \textbf{ in}$

$((H\ r) \otimes w)$

$\text{bob } (b_1, b_2, y) \longrightarrow_1 (\text{zed } b_1)\ (\text{ex } b_2)\ y$

. . . . . .

$\text{ex } b_2 \longrightarrow_1 !\,|0\rangle\ {b_2}^T + !\,|1\rangle\ (X\ b_2)^T$

$\text{zed } b_1 \longrightarrow_1 Z(!\,|0\rangle\ (!\,|0\rangle)^T + b_1 (!\,|1\rangle)^T) - !\,|0\rangle\ (!\,|1\rangle)^T + (X\ b_1)(!\,|1\rangle)^T$

$(\text{ex } b_2)$ is just X^{b_2} and $(\text{zed } b_1)$ is Z^{b_1}

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- To avoid having entanglement between the history track and the computational registry, he used the concepts of linear algebra to distinguish when a term is definite and when it is not
- To prevent cloning, he made rules for well-formed terms
- To add measurement, we used probabilistic transition rules and provided a more detailed syntax with rules for well-formed terms