Fast algorithm for the visualization of surfaces

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Scientific context and motivation

In several engineering applications such as mechanism design or control theory, it is important to visualize the solution surface of an implicit equation of the form $f(x, y, z) = 0$. The first efficient algorithm to visualize such surface was the marching cube [5], and was enhanced more recently with variants such as surface nets [1, 3] and dual contouring [4]. These algorithms are based on the evaluation of the function f on a regular grid of points in $\mathbb{R}^3$.

However when f is a polynomial of high degree, the time to evaluate f on the grid dominates the computation time of the marching cube algorithm. A goal of this thesis is to use techniques from computer algebra to speedup the evaluation of a polynomial on a grid. In particular, multipoint evaluation techniques [2] allow to evaluate a univariate polynomial of degree n on n numbers in $O(n \log^2 n \log\log n)$ instead of $O(n^2)$ with a direct approach.

Another issue is the reliability of the surface approximation. In [7] and [6], the authors showed that in some cases, interval arithmetic can be used to guarantee that the approximated surface has the same topology as the implicit surface that we want to visualize. In particular, interval arithmetic can guarantee that no component is missed. We would like to adapt the existing fast multipoint evaluation techniques to the reliable algorithms based on interval arithmetic.

Mission

The first challenge for the candidate will be to design, study the complexity and implement an algorithm that speeds up the marching cube algorithm in the case of high degree polynomials. Then the candidate will focus on speeding up reliable algorithm based on interval arithmetic.

During the PhD, the candidate will develop creativity, reading, writing and programming skills. In particular, most algorithms developed during the thesis will be implemented in python or C/C++.

Profile of the candidate

The candidate should have a taste for both mathematics (geometry or numerical analysis) and computer science. Programming skills would be appreciated.

Bibliography

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