SUJET STAGE M2 - Computational geometry
Meshing Singular Surfaces - Application in Robotics

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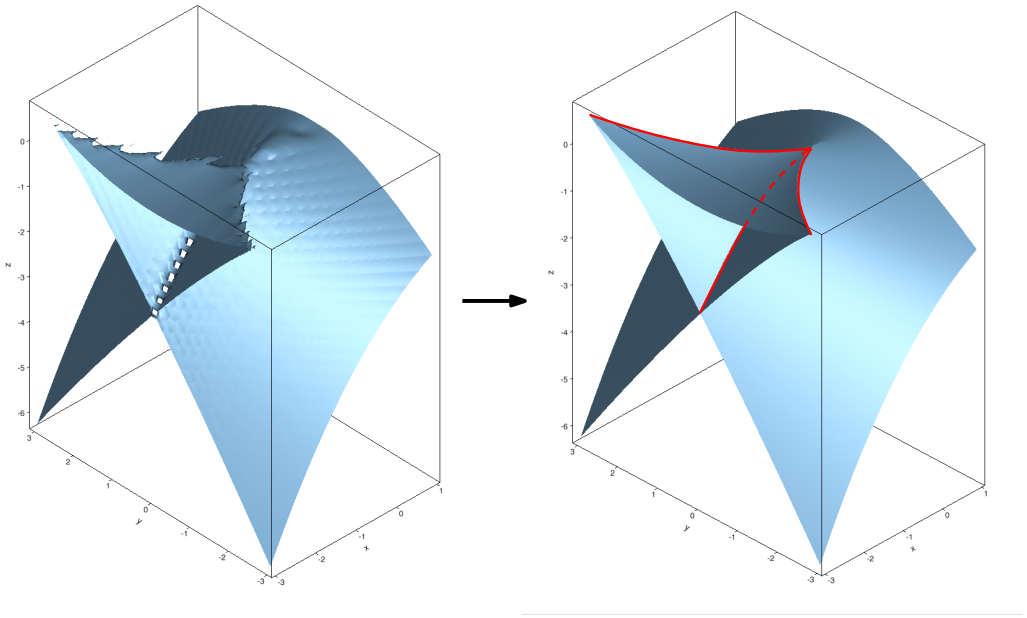

Figure 1: Swallowtail singularity. Left: numerical non-certified triangulation. Right: Topologically correct triangulation.

1 Context

Scientific problems, for instance in mechanical design or control theory, can often be modelled by a system of equations and inequalities. For example, given a robot with three degrees of freedom, the set of points it can reach is bounded by a surface. In this case, being able to provide a plot of this surface, which can evolve in real time when the designer modifies the shape of the mechanism, would be an invaluable help. Moreover, those surfaces can have self-intersections, and other kinds of more degenerate singularities (see Figure 1).

To compute the mesh of non-singular surfaces, several efficient algorithms have been proposed, starting by the famous Marching Cube algorithm [LC87] and later improved notably by dual contouring [JLSW02]. In this case, some approaches also guarantee that the resulting mesh is topologically correct [PV04, BW21].

On the other hand, when the surface is singular those methods fail in the neighborhood of the singular curve (see Figure 1). Such singular surfaces naturally appear when studying the configuration space of mechanism in robotics.

2 Objectives

A classical way to represent a singular surface is to decompose it in smooth cells of dimension 0, 1 and 2. A topologically correct meshing of a singular surface must respect these features. The 2-dimensional cells are subdivided by triangles and the singularities (0 and 1-dimensional cells) are identified as special vertices and edges.

The internship will focus on singular surfaces obtained as projections in $\mathbb{R}^3$ of smooth surfaces in $\mathbb{R}^4$. In this setting, previous work studied their singular curves [DMP19]. The main objective of this internship is to design an algorithm to mesh the smooth part and correctly connect it to the singular curve.

The algorithm will be implemented in python, C or C++ and tested on robotics applications. In particular, the meshing package of the CGAL library (CGAL 3D Mesh Generation) could be a starting point.

Profile of the internship candidate

- Interest in geometry and algorithms design
- Willing to develop in python, C or C++

References

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[JLSW02] Tao Ju, Frank Losasso, Scott Schaefer, and Joe Warren. Dual contouring of hermite data. In *Proceedings of the 29th Annual Conference on Computer Graphics and Interactive Techniques*, SIGGRAPH '02, pages 339–346, New York, NY, USA, 2002. Association for Computing Machinery.

[LC87] W. E. Lorensen and H. E. Cline. Marching cubes: A high resolution 3d surface construction algorithm. *SIGGRAPH Comput. Graph.*, 21:163–169, August 1987.

[PV04] Simon Plantinga and Gert Vegter. Isotopic approximation of implicit curves and surfaces. In *Proceedings of the 2004 Eurographics/ACM SIGGRAPH Symposium on Geometry Processing*, SGP '04, pages 245–254, New York, NY, USA, 2004. ACM.