Business Cycles and Financial Crises: A Model of Entrepreneurs and Financiers

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Business Cycles and Financial Crises: A Model of Entrepreneurs and Financiers*

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Abstract

A dynamic general equilibrium model with infinitely lived entrepreneurs and financiers is developed to investigate a possible mechanism that explains business cycles and a financial crisis. The highest growth rate is achievable only if financiers coexist with entrepreneurs, given a certain extent of financial market imperfections. However, if financiers coexist with entrepreneurs, the economy is highly likely to go into a financial crisis for some parameter values. These two-sided implications of the coexistence of entrepreneurs and financiers explain why both instability and high growth are frequently observed in modern economies.

Keywords: Endogenous business cycles; Financial crisis; Economic boom; Financial market imperfections.

JEL Classification Numbers: E32; O16; O40.

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1 Introduction

Over the past twenty years, many researchers, such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Aghion, et al. (1999), and Aghion, et al. (2004), have emphasized the importance of financial market imperfections as a fundamental cause of business cycles. Among those researchers, Woodford (1986) focuses on the interactive relationship between two distinct classes of agents, capitalists and workers, in a financially constrained economy and clarifies a mechanism that creates business cycles.[1] He assumes that capitalists and workers differ in their income resources and their accessibility to credit and shows that endogenous business cycles driven by self-filling rational expectations can emerge.[2] Along the same lines as Woodford (1986), the present paper investigates the macroeconomic implications for business cycles of the existence of two distinct classes of agents by developing a dynamic general equilibrium model with capital accumulation. In contrast to Woodford (1986), who assumes an economy where the class of capitalists is separated from the class of workers in the financial market, we investigate how endogenous business cycles arise in an economy where the class of *entrepreneurs* interacts with the class of *financiers* through the financial market.

Our model is closely related to a pioneering work by Takalo and Toivanen (2012), who develop a formal model that distinguishes the role of *financiers* from that of *entrepreneurs.* They focus on entrepreneurial finance and develop a model in which borrowing entrepreneurs and lending entrepreneurs endogenously appear in equilibrium. However, because their model is static in nature, the model cannot be applied to business cycle problems. In contrast, the model developed in this paper is a dynamic general equilibrium model; thus, we can study the possibilities of endogenous business cycles.

In our model, agents who have inherent entrepreneurial talents are called *entrepreneurs.* Entrepreneurs are able to access a production technology. However, because they receive uninsured idiosyncratic productivity shocks, only highly productive entrepreneurs engage in production in each time period, borrowing financial resources in the financial market at an interest rate lower than their productivity. Less productive entrepreneurs lend their net

[1] The research on the macroeconomic implications for business cycles of two distinct classes in an economy dates back to Kalecki (1937).

[2] See also Woodford (1988a,b).

worth in the financial market at an interest rate greater than their productivity. In other words, borrowing entrepreneurs and lending entrepreneurs endogenously arise in each time period in a similar way to that of Takalo and Toivanen (2012), depending on their received idiosyncratic productivity shocks. Moreover, *financiers* in our model are assumed to have no inherent entrepreneurial talents. Instead, they lend their net worth in the financial market. There are three types of agents in the economy in each time. The first type is entrepreneurs who engage in production, and the second type is entrepreneurs who lend their net worth in the financial market. The third type is financiers who have no entrepreneurial talents but lend their net worth in the financial market. Unlike Takalo and Toivanen (2012), less productive entrepreneurs lending their net worth in the financial market are not called financiers in our model. Instead, agents who have no inherent entrepreneurial talents are called financiers.

Although entrepreneurs who engage in production borrow in the financial market in equilibrium, they face credit constraints. In such a situation, the entrepreneurs who engage in production are not always the "most" productive but the "highly" productive entrepreneurs, implying that production resources are not used in the most efficient way in each time period. Under these circumstances, the existence of financiers has a two-fold importance, given a certain extent of financial market imperfections. The highest growth rate is achievable in a financially constrained economy only if financiers coexist with entrepreneurs. However, if financiers coexist with entrepreneurs, the economy is highly likely to exhibit endogenous business cycles and go into a financial crisis followed by a severe economic depression. That is, provided that the financial market is imperfect in an economy, the existence of financiers contributes to a boost in the growth rate, and, at the same time, the existence of financiers involves a potential peril such that an economy is led to a collapse.

A remarkable characteristic of modern capitalism is the coexistence of entrepreneurs and financiers. As historically observed, financial markets evolve in the process of economic development.[3] The evolution of financial markets yields the class of financiers. As a result, an economy is able to use tremendous financial resources supplied by financiers to complete large investment projects that are otherwise impossible. Financiers lend their net worth to entrepreneurs to propagate their wealth in a financial market and do not engage in produc-

[3] Levine (2005) provides a comprehensive review of research on finance and growth.

tion. Alternately, entrepreneurs, who trust their own entrepreneurial talents, raise funds in the financial markets and invest in projects that produce added values if the investment projects succeed, taking risks involving the investment. Even in an economy with an established financial market, generic agency problems remain and, accordingly, the financial resources are not used in the most efficient way. We model these situations.

In our model, allocative inefficiency has a significant implication for business cycle phenomena. The inefficient use of financial resources and propagated net worth of financiers induce a decrease in the return on financial resources during an economic boom. For some parameter values, a return on financial resources steeply falls at some point in time and a severe depression follows.

To investigate endogenous business cycles, we employ a model of infinitely lived agents. Endogenous business cycles have long been studied by many researchers, using models of infinitely lived agents.[4] Benhabib and Nishimura (1985), who are pioneers of the literature on endogenous business cycles, develop a model of an infinitely lived representative agent with two production sectors and derive sufficient conditions for the economy to exhibit deterministic endogenous business cycles. Boldrin and Denecker (1990) also develop a two-sector dynamic general equilibrium model with specific production technology and agent preferences. They demonstrate that the economy exhibits deterministic endogenous business cycles and even chaotic equilibria for some parameter values. Nishimura and Yano (1995) provide a simple example of a model with capital accumulation in which an economy exhibits ergodically chaotic dynamics. Although these studies demonstrate that deterministic cycles appear in equilibrium, there is no friction in the markets in their models.[5] In contrast, there is financial friction in our model. Although we employ a Ramsey-type of growth model with one production sector, due to financial market imperfections, deterministic endogenous business cycles arise in equilibrium in our model.[6]

[4]Endogenous business cycles also have been studied with overlapping generations models. For instance, Farmer (1986), Reichlin (1986), Benhabib and Laroque (1988), Kitagawa and Shibata (2001), and Rochon and Polemarchakis (2006) derive competitive equilibrium cycles in overlapping generations economies with production sectors.

[5]Endogenous business cycles are also induced by self-fulfilling rational expectations as sunspot equilibria. For sunspot equilibria in growth models of infinitely lived agents, see Woodford (1986), Benhabib and Farmer (1994, 1996), Boldrin and Rustichini (1994), and Gali (1994), among others.

[6]Futagami and Mino (1993) also develop a Ramsey-type of a one-sector growth model with threshold production externalities, and they derive deterministic cycles.

The remainder of this paper is organized as follows. In the next section, we present a growth model in which there are two classes of agents: entrepreneurs and financiers. In section 3, we investigate the equilibrium growth rates and the local and global stabilities of the economy. In section 4, we discuss a financial crisis and in section 5, we present concluding remarks.

2 Model

The economy consists of two classes: one unit measure of infinitely lived entrepreneurs and one unit measure of infinitely lived financiers. Time is discrete and goes from 0 to ∞. Entrepreneurs are ex-ante homogeneous and ex-post heterogeneous because they receive idiosyncratic productivity shocks in each time period. No financiers engage in production because they have no inherent entrepreneurial talents. Instead, financiers lend their net worth in the financial market to obtain their income in each time period.

2.1 Entrepreneurs

An entrepreneur has two types of saving methods. One is lending her net worth in the financial market. If she lends one unit of general goods in the financial market at time $t-1$, she will acquire a claim to r_t units of general goods at time t where r_t is the (gross) real interest rate. The other saving method is starting an investment project. If an entrepreneur invests one unit of general goods in an investment project at time $t-1$, she will create $A\Phi_{t-1}$ units of general goods at time t. An idiosyncratic shock Φ_{t-1} with respect to productivity at time t is realized at time $t-1$, which implies that production takes one gestation period. Accordingly, an entrepreneur at time $t-1$ already knows her productivity at time t. Low productivity cannot be insured because no insurance market exists for the idiosyncratic productivity shocks. If an entrepreneur wants to borrow financial resources in the financial market, she faces a credit constraint due to an agency problem in the financial market. In each period, entrepreneurs consume, lend their net worth, and/or invest in projects borrowing financial resources in the financial market.

The productivity Φ_{t-1} is a random variable, implying that it is a function of a stochastic event ω_{t-1}, where $\{\omega_{t-1} \in \Omega \mid \Phi_{t-1}(\omega_{t-1}) \leq \Phi\}$ is an element of a σ-algebra $\mathcal{F}$ of a probability space $(\Omega, \mathcal{F}, P)$. As in Angeletos (2007), the stochastic events $\omega_0, \omega_1, ...$ (and the idiosyncratic productivity shocks $\Phi_0(\omega_0), \Phi_1(\omega_1), ...$) are assumed to be independent and identically distributed across both time and agents (the i.i.d. assumption). Φ has support over $[0, h]$, where $h > 0$ is finite. Φ's cumulative distribution function is given by $G(\Phi)$, where $G(\Phi)$ is continuous, differentiable and strictly increasing on the support.

We define the histories of stochastic events and the idiosyncratic productivity shocks until time $t-1$ such that $\omega^{t-1} = \{\omega_0, \omega_1, ...\omega_{t-1}\}$ and $\Phi^{t-1} = \{\Phi_0, \Phi_1, ...\Phi_{t-1}\}$. Then, there exists a probability space $(\Omega^t, \mathcal{F}^t, P^t)$, which is a Cartesian product of t copies of $(\Omega, \mathcal{F}, P)$, where $\Phi^{t-1}(\omega^{t-1})$ is a vector function of the history $\omega^{t-1} \in \Omega^t$.

An entrepreneur at time t maximizes her expected lifetime utility given by:

$$U_t^e = E\left[\sum_{\tau=t}^{\infty} \beta_e^{\tau-t} \ln c_\tau(\omega^\tau) \middle| \Phi^t(\omega^t)\right],$$

subject to:

$$k_\tau(\omega^\tau) + b_\tau(\omega^\tau) = A\Phi_{\tau-1}(\omega_{\tau-1})k_{\tau-1}(\omega^{\tau-1}) + r_\tau b_{\tau-1}(\omega^{\tau-1}) - c_\tau(\omega^\tau) \tag{1}$$

$$b_\tau(\omega^\tau) \geq -\theta a_\tau(\omega^\tau) \tag{2}$$

$$k_\tau(\omega^\tau) \geq 0, \tag{3}$$

for $\tau \geq t \geq 0$, where $\beta_e \in (0, 1)$ is the subjective discount factor, which is common to all entrepreneurs, and $E[.|\Phi^t]$ is an expectation operator given an information set associated with Φ^t at time t. Eq.(1) is the flow budget constraints, where $c_\tau(\omega^\tau)$ is consumption, $k_\tau(\omega^\tau)$ denotes investment in a project, and $b_\tau(\omega^\tau)$ is a debt if negative and credit if positive at time τ. $A\Phi_{\tau-1}(\omega_{\tau-1})k_{\tau-1}(\omega^{\tau-1})$ is the general goods produced by the entrepreneur at time τ. We assume that the general goods are perishable in one period, and thus, $k_\tau(\omega^\tau)$ depreciates entirely in one period. $a_\tau(\omega^\tau) := k_\tau(\omega^\tau) + b_\tau(\omega^\tau)$ is the entrepreneur's net worth and r_τ is the gross interest rate at time τ. Note that $a_\tau(\omega^\tau)$ is equal to her saving because $a_\tau(\omega^\tau) = A\Phi_{\tau-1}k_{\tau-1}(\omega^{\tau-1}) + r_\tau b_{\tau-1}(\omega^{\tau-1}) - c_\tau(\omega^\tau)$. We assume that at $t = 0$, the flow budget constraint is given by $k_0 + b_0 = w_0 - c_0$, where w_0 is the initial endowment that each entrepreneur holds at birth, which is common to all entrepreneurs.

If an entrepreneur borrows financial resources in the financial market, she faces a credit constraint due to an agency problem in the financial market. The credit constraint facing each entrepreneur is given by Eq.(2). As in Aghion et al. (1999), Aghion and Banerjee (2005), Aghion et al. (2005), or Antrás and Caballero (2009), an entrepreneur is able to borrow financial resources in the financial market only up to θ times her net worth.[7] $\theta \in (0, \infty)$ represents the extent of credit constraints where, as θ goes to infinity, the financial market approaches perfection. Finally, Eq.(3) is the non-negativity constraint of investment.

Define $\phi_t := r_{t+1}/A$. From the maximization problem of an entrepreneur, it is optimal for entrepreneurs with $\Phi_t > \phi_t$ to invest in a project, borrow financial resources up to the limit of the credit constraint, and engage in general goods production. Meanwhile, it is optimal for entrepreneurs with $\Phi_t < \phi_t$ to lend their net worth in the financial market and obtain the (gross) interest rate r_{t+1}. Note that ϕ_t is a cutoff of idiosyncratic productivity shocks that divides entrepreneurs into lenders and borrowers at time t. As a result, we obtain a lending-investment-borrowing plan for an entrepreneur who has net worth $a_t(\omega^t)$ at time t as follows:

$$k_t(\omega^t) = \begin{cases} 0 & \text{if } \Phi_t(\omega_t) < \phi_t \\ \frac{a_t(\omega^t)}{1-\mu} & \text{if } \Phi_t(\omega_t) > \phi_t, \end{cases} \tag{4}$$

and

$$b_t(\omega^t) = \begin{cases} a_t(\omega^t) & \text{if } \Phi_t(\omega_t) < \phi_t \\ -\frac{\mu}{1-\mu} a_t(\omega^t) & \text{if } \Phi_t(\omega_t) > \phi_t, \end{cases} \tag{5}$$

where $\mu := \theta/(1+\theta) \in (0,1)$ also measures the extent of credit constraints. Under this lending-investment-borrowing plan, the flow budget constraint at time τ can be rewritten as an intensive form such that:

$$a_\tau(\omega^\tau) = \tilde{R}_\tau a_{\tau-1}(\omega^{\tau-1}) - c_\tau(\omega^\tau), \tag{6}$$

where $\tilde{R}_\tau := \max\{r_\tau, \frac{A\Phi_{\tau-1} - r_\tau \mu}{1-\mu}\}$. We provide a derivation of the budget constraint Eq.(6) in the Appendix. Given the lending-investment-borrowing plan given by Eqs.(4) and (5), an entrepreneur at time t maximizes her lifetime utility U_t^e subject to Eq.(6). The Euler

[7]We present two types of microfoundations for the credit constraint in the Appendix.

equation is given by:

$$\frac{1}{c_t(\omega^t)} = \beta_e E\left[\tilde{R}_{t+1}\frac{1}{c_{t+1}(\omega^{t+1})}\left|\Phi^t(\omega^t)\right.\right]. \tag{7}$$

The lifetime utility function is log-linear, so from Eqs.(6), (7) and the transversality condition, we obtain lemma 1:

Lemma 1 *The law of motion of an entrepreneur's net worth $a_t(\omega^t)$ is given by:*

$$a_{t+1}(\omega^{t+1}) = \beta_e\tilde{R}_{t+1}a_t(\omega^t). \tag{8}$$

Proof. See the Appendix.

2.2 Financiers

Financiers never engage in general goods production because they inherently have no entrepreneurial talents; Financiers acquire income by lending their net worth in the financial market. Each financier is endowed with an initial net worth $W_0 > 0$ at birth.

A representative financier at time t maximizes her lifetime utility as:

$$U_t^c = \sum_{\tau=t}^{\infty}\beta_c^{\tau-t}\ln c_\tau,$$

subject to:

$$W_\tau = r_\tau W_{\tau-1} - c_\tau \tag{9}$$

for $\tau \geq t \geq 0$, where W_τ is her net worth carried over from time τ to time $\tau+1$ and $\beta_c \in (0,1)$ is the subjective discount factor. We assume that $W_0 > 0$. Obtaining the Euler equation is straightforward:

$$\frac{1}{c_t} = \beta_c r_{t+1}\frac{1}{c_{t+1}} \tag{10}$$

Similar to the case of entrepreneurs, from Eqs.(9) and (10) and the transversality condition, we obtain the law of motion of a representative financier's net worth in lemma 2:

Lemma 2 *The law of motion of a representative financier's net worth W_t is given by:*

$$W_{t+1} = \beta_c r_{t+1}W_t. \tag{11}$$

Proof. The proof is omitted because it is essentially the same as in lemma 1. □

2.3 Aggregation

We assume that the law of large numbers can be applied to entrepreneurs. Because $a_t(\omega^t) = \beta_e \tilde{R}_t a_{t-1}(\omega^{t-1})$ from Eq.(8), the net worth $a_t(\omega^t)$ of an entrepreneur who receives a stochastic event ω_t at time t and has a history ω^{t-1} is presented by:

$$a_t(\omega^t) = \beta_e(A\Phi_{t-1}(\omega_{t-1})k_{t-1}(\omega^{t-1}) + r_t b_{t-1}(\omega^{t-1})), \tag{12}$$

where we should note from Eqs.(4) and (5) that for an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) < \phi_{t-1}$, it follows that $k_{t-1}(\omega^{t-1}) = 0$ and $b_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})$. For an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) > \phi_{t-1}$, it follows that $k_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})/(1-\mu)$ and $b_{t-1}(\omega^{t-1}) = -\mu a_{t-1}(\omega^{t-1})/(1-\mu)$. The stochastic event ω_t and the history ω^{t-1} are independent from each other. Therefore, applying the law of large numbers to entrepreneurs, we aggregate the net worth of the entrepreneurs with the stochastic realization ω_t as follows:

$$\tilde{a}_t(\omega_t) := \int_{\Omega^t} a_t(\omega^t) dP^t(\omega^{t-1}) = \beta_e \int_{\Omega^t} (A\Phi_{t-1}(\omega_{t-1})k_{t-1}(\omega^{t-1}) + r_t b_{t-1}(\omega^{t-1}))dP^t(\omega^{t-1}), \tag{13}$$

where we should note that ω^{t-1} is an element of Ω^t. From the financial market clearing condition at time $t-1$, we have:

$$W_{t-1} + \int_{\Omega^t} b_{t-1}(\omega^{t-1}) dP^t(\omega^{t-1}) = 0.$$

The aggregate output at time t is given by:

$$Y_t := \int_{\Omega^t} A\Phi_{t-1}(\omega_{t-1})k_{t-1}(\omega^{t-1}) dP^t(\omega^{t-1}).$$

Therefore, Eq.(13) is rewritten as:

$$\tilde{a}_t(\omega_t) = \beta_e(Y_t - r_t W_{t-1}). \tag{14}$$

Eq.(14) is the aggregate net worth over entrepreneurs who receive a stochastic event ω_t at time t. In some sense, Eq.(14) expresses a distribution of net worth in an economy with respect to ω_t, although the distribution is uniform over ω_t. Note that Eq.(14) is effective for $t \geq 1$. For $t = 0$, it follows that $\tilde{a}_0(\omega_0) = \beta_e w_0$.

As clarified in Eqs.(4) and (5), entrepreneurs with a stochastic event ω_t such that $\Phi_t(\omega_t) > \phi_t$ become producers at time t, whereas entrepreneurs with a stochastic event ω_t such that $\Phi_t(\omega_t) < \phi_t$ become lenders. Therefore, from Eqs.(4) and (5), the aggregate debt or credit $\tilde{b}_t(\omega_t)$ across the entrepreneurs with stochastic realization ω_t is presented by:

$$\tilde{b}_t(\omega_t) = \begin{cases} \tilde{a}_t(\omega_t) = \beta_e(Y_t - r_t W_{t-1}) & \text{if } \Phi_t(\omega_t) < \phi_t \\ -\frac{\mu}{1-\mu}\tilde{a}_t(\omega_t) = -\frac{\mu\beta_e}{1-\mu}(Y_t - r_t W_{t-1}) & \text{if } \Phi_t(\omega_t) > \phi_t. \end{cases} \tag{15}$$

Similarly, the aggregate investment $\tilde{k}_t(\omega_t)$ across the entrepreneurs with stochastic realization ω_t is given by:

$$\tilde{k}_t(\omega_t) = \begin{cases} 0 & \text{if } \Phi_t(\omega_t) < \phi_t \\ \frac{1}{1-\mu}\tilde{a}_t(\omega_t) = \frac{\beta_e}{1-\mu}(Y_t - r_t W_{t-1}) & \text{if } \Phi_t(\omega_t) > \phi_t. \end{cases} \tag{16}$$

From the financial-market clearing condition, we have:

$$\begin{aligned} W_t &= -\int_E \tilde{b}_t(\omega_t) dP(\omega_t) - \int_{\Omega/E} \tilde{b}_t(\omega_t) dP(\omega_t) \\ &= -\beta_e(Y_t - r_t W_{t-1}) \frac{G(\phi_t) - \mu}{1-\mu}, \end{aligned} \tag{17}$$

where $E = \{\omega_t \in \Omega \mid \Phi_t(\omega_t) \leq \phi_t\}$. Multiplying $A\Phi_t(\omega_t)$ on both sides of the second equation of (16) and aggregating the resulting equation across all entrepreneurs who engage in production, we obtain the total output Y_{t+1} as follows:

$$\begin{aligned} & \int_{\Omega/E} A\Phi_t(\omega_t)\tilde{k}_t(\omega_t) dP(\omega_t) = \int_{\Omega/E} \frac{\beta_e A\Phi_t(\omega_t)}{1-\mu}(Y_t - r_t W_{t-1}) dP(\omega_t) \\ \Longleftrightarrow \quad & Y_{t+1} = \frac{\beta_e A F(\phi_t)}{1-\mu}(Y_t - r_t W_{t-1}) \end{aligned} \tag{18}$$

where $F(\phi_t) := \int_{\phi_t}^{\infty} \Phi_t(\omega_t) dG(\Phi_t)$.

3 Equilibrium Dynamics

3.1 Equilibrium

Defining $B := \beta_e/\beta_c$, we derive, from Eqs.(11), (17), and (18), the dynamic equations for $t \geq 1$ with respect to the cutoff ϕ_t and the growth rate of the aggregate output $\Gamma_{t+1}(\phi_t) := Y_{t+1}/Y_t$, respectively, as follows:

$$\frac{B(G(\phi_t) - \mu)}{1 - \mu - B(G(\phi_t) - \mu)} = \frac{\phi_{t-1}(G(\phi_{t-1}) - \mu)}{F(\phi_{t-1})} \tag{19}$$

and

$$\Gamma_{t+1}(\phi_t) = \frac{\beta_c ABF(\phi_t)}{1 - \mu - B(G(\phi_t) - \mu)}. \tag{20}$$

The net worth of the representative financier becomes:

$$W_t = A\beta_c\phi_{t-1}W_{t-1} \tag{21}$$

because $r_t = A\phi_{t-1}$.

$\phi_0 = r_1/A$ is a predetermined variable. To see this predetermination, we consider Eq.(17), which is effective for $t \geq 1$. Because we have $\tilde{a}_0(\omega_0) = \beta_e w_0$ for $t = 0$, Eq.(17) is modified for $t = 0$ such that $W_0 = -\beta_e w_0(G(\phi_0) - \mu)/(1 - \mu)$. Because W_0 and w_0 are predetermined, $\phi_0 = r_1/A$ is also predetermined.[8] In other words, ϕ_0 or r_1 is determined such that the financial market clears at time $t = 0$.

In a competitive equilibrium, the economy is recursively expressed by sequences $\{W_t, \phi_t, Y_{t+1}\}$, such that for all $t \geq 1$, these three sequences satisfy the difference equations (19), (20) and (21), given W_0, ϕ_0, and Y_1, where $Y_1 = \beta_e AF(\phi_0)w_0/(1 - \mu)$.

The dynamic behavior of ϕ_t associated with Eq.(19) provides information about both the dynamic behavior of W_t and the equilibrium growth rates of Y_t from Eqs.(20) and (21). Therefore, we intensively analyze Eq.(19) in what follows.

[8]For $G(\phi_0)$ to be well-defined, it must follow that $\mu - (1 - \mu)W_0/(\beta_e w_0) > 0$. We assume this parameter condition.

3.2 Steady states

We find from lemma 2 that $W_t \geq 0$ for all $t \geq 0$, which implies that $G(\phi_t) \leq \mu$ for all $t \geq 0$ from Eq.(17) because $Y_t - r_t W_{t-1} > 0$ in equilibrium.[9] Therefore, we restrict the domain of the dynamical system (19) to $[0, G^{-1}(\mu)]$.

There are at most two steady states in the dynamical system (19). To examine the existence of the steady states in the dynamical system (19), we define ϕ^* and ϕ^{**} such that:

$$G(\phi^*) = \mu$$
$$\tfrac{1-\mu}{B} = \tfrac{F(\phi^{**})}{\phi^{**}} + (G(\phi^{**}) - \mu).$$

A unique value of ϕ^* must exist because $G(.)$ is a strictly increasing function over the support of Φ. To investigate the uniqueness of ϕ^{**}, we define a function such that $H(x) := F(x)/x + (G(x) - \mu)$. $H(x)$ is strictly decreasing in $(0, h)$ because $H'(x) = -F(x)/x^2 < 0$ in $(0, h)$. In addition, $\lim_{x\to 0} H(x) = \infty$ and $\lim_{x\to h} H(x) = 1 - \mu$. Because ϕ^{**} is a solution of $H(x) = (1-\mu)/B$, ϕ^{**} is uniquely determined in $(0, h]$ if and only if $B \leq 1$. ϕ^* and ϕ^{**} can be solved in terms of the parameters of μ and B and the parameters of the distribution of Φ such that $\phi^*(\mu; \Theta)$ and $\phi^{**}(\mu, B; \Theta)$, where Θ is the parameter set of the distribution of Φ; however, we write ϕ^* and ϕ^{**} to save notations.

Because the domain of the dynamical system (19) is $[0, \phi^*]$, the system has a steady-state equilibrium ϕ^{**} in addition to ϕ^* if and only if $\phi^{**} < \phi^*$. Because $H(x) = F(x)/x + (G(x) - \mu)$ is a strictly decreasing function, the condition for $\phi^{**} < \phi^*$ is equivalent to $(1-\mu)/B > H(\phi^*)$. In what follows, to focus our study on interesting cases, we assume this inequality. The inequality $(1-\mu)/B > H(\phi^*)$ is rewritten as in Eq.(22) in Assumption 1.

Assumption 1

$$B < \frac{\phi^*(1 - G(\phi^*))}{F(\phi^*)} =: B^*. \tag{22}$$

Assumption 1 guarantees the existence of two steady-state equilibria in the dynamical system (19). Note that because $F(\phi^*) = \int_{\phi^*}^h \Phi dG(\Phi) > \int_{\phi^*}^h \phi^* dG(\Phi) = \phi^*(1 - G(\phi^*))$, Assumption 1 leads to $B = \beta_e/\beta_c < 1$. This finding implies that the subjective discount

[9] Otherwise, Y_{t+1} becomes negative in Eq.(18).

factor of entrepreneurs is strictly less than that of financiers. In other words, to obtain two steady-state equilibria, financiers need to be more patient than entrepreneurs.

We conclude this subsection with two remarks on the steady states in the economy. First, if μ is arbitrarily close to zero, ϕ^{**} does not exist because the domain of the dynamical system $[0, G^{-1}(\mu)]$ shrinks to the origin. In this case, there is no financial market and financiers are unable to exist because it is impossible for them to lend their net worth in the financial market. Second, if μ is arbitrarily close to one, ϕ^{**} and ϕ^{*} coincide with each other and are equal to h. This can be verified from the definition of ϕ^{*} and ϕ^{**}. If μ is arbitrarily close to one, $G(\phi^{*}) = 1$ holds and thus $\phi^{*} = h$. Likewise, if μ is arbitrarily close to one, $F(\phi^{**}) = \phi^{*}(1 - G(\phi^{**}))$ holds. This equation holds if and only if $\phi^{**} = h$. In this case, all production recourses, including financiers' resources, are intensively and efficiently used by the most talented entrepreneurs.

3.3 Growth rates

The growth rate Γ_{t+1} for $t \geq 1$ in Eq.(20) is a function of ϕ_t. In this section, we demonstrate that the steady state ϕ^{**} provides the highest growth rate in the economy.

Proposition 1 *Suppose that Assumption 1 holds. Then, given the parameter values of β_e, β_c, and μ and the parameter values of the distribution $G(\Phi)$, the growth rate in the steady state ϕ^{**} is the highest for any $\phi_t \in [0, G^{-1}(\mu)]$.*

Proposition 1 is shown below. Differentiating Γ_{t+1} with respect to ϕ_t, we have:

$$\Gamma'_{t+1}(\phi_t) = J(\phi_t)\frac{\beta_c ABG'(\phi_t)}{[1 - \mu - B(G(\phi_t) - \mu)]^2},$$

where $J(\phi_t) := BF(\phi_t) - \phi_t[1 - \mu - B(G(\phi_t) - \mu)]$. It can be easily verified that $J(\phi_t)$ is strictly decreasing and that $J(\phi^{**}) = 0$. Therefore, Γ'_{t+1} is strictly greater than zero if $0 < \phi_t < \phi^{**}$ and it is strictly less than zero if $\phi_t > \phi^{**}$. Therefore, the maximum of Γ_{t+1} is achieved at $\phi_t = \phi^{**}$.

We find from Proposition 1 that the existence of financiers has an important meaning for the economy. Suppose that there are no financiers in this economy. In this case, $W_t = 0$ for all $t \geq 0$ and we only have the steady state $\phi^{*} = G^{-1}(\mu)$. This implies that without financiers, the economy can never attain the highest growth rate, given a certain extent of financial market imperfections. In turn, with the infinitesimal initial net worth $W_0 > 0$ of financiers, the highest growth rate is achievable for some parameter values. The existence of financiers is necessary for the highest growth rate to be achieved provided that the financial market is imperfect.

3.4 Local stability

We investigate the local stability around the steady states of the dynamical system (19). Let us define two functions as $\Psi(\phi_t) := \frac{B(G(\phi_t)-\mu)}{1-\mu-B(G(\phi_t)-\mu)}$, which is the left-hand side of Eq.(19), and $\Lambda(\phi_{t-1}) := \frac{\phi_{t-1}(G(\phi_{t-1})-\mu)}{F(\phi_{t-1})}$, which is the right-hand side. $\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ are, respectively, approximated around the steady state of ϕ^* as follows:

$$\Psi(\phi_t) \approx \frac{BG'(\phi^*)}{1-G(\phi^*)}(\phi_t - \phi^*)$$

and

$$\Lambda(\phi_{t-1}) \approx \frac{\phi^* G'(\phi^*)}{F(\phi^*)}(\phi_{t-1} - \phi^*).$$

From these approximations, we obtain the local dynamical system around the steady state of ϕ^* as follows:

$$\phi_t - \phi^* = \frac{\phi^*(1-G(\phi^*))}{BF(\phi^*)}(\phi_{t-1} - \phi^*). \tag{23}$$

Proposition 2 *Suppose that Assumption 1 holds. Then, the steady state of ϕ^* is locally unstable.*

Proof. It follows from Assumption 1 that $[\phi^*(1-G(\phi^*))]/BF(\phi^*) > 1$. □

On the other hand, $\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ are, respectively, approximated around the steady state of ϕ^{**} as follows:

$$\Psi(\phi_t) \approx \frac{\phi^{**}G'(\phi^{**})F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**})-\mu)}{F(\phi^{**})^2}(\phi_t - \phi^{**})$$

and

$$\Lambda(\phi_{t-1}) \approx \frac{[(G(\phi^{**}) - \mu) + \phi^{**}G'(\phi^{**})]F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**}) - \mu)}{F(\phi^{**})^2}(\phi_{t-1} - \phi^{**}).$$

Therefore, the local dynamical system around the steady state of ϕ^{**} is given by

$$\phi_t - \phi^{**} = \Big[\frac{(G(\phi^{**}) - \mu)F(\phi^{**})}{\phi^{**}G'(\phi^{**})F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**}) - \mu)} + 1\Big](\phi_{t-1} - \phi^{**}).$$

Because ϕ^{**} satisfies $(1-\mu)/B = F(\phi^{**})/\phi^{**} + (G(\phi^{**}) - \mu)$, this equation is rewritten as:

$$\phi_t - \phi^{**} = \Big[\frac{(G(\phi^{**}) - \mu)(1 - \mu - B(G(\phi^{**}) - \mu))}{\phi^{**}G'(\phi^{**})(1-\mu)} + 1\Big](\phi_{t-1} - \phi^{**}). \tag{24}$$

Proposition 3 *Suppose that Assumption 1 holds. Then, the stability around the steady state of ϕ^{**} is ambiguous.*

Proof. Because $G(\phi^{**}) - \mu$ is negative and $1 - \mu - B(G(\phi^{**}) - \mu)$ is positive, the coefficient of $(\phi_{t-1} - \phi^{**})$ in Eq.(24) is less than 1. Note that $G'(\phi^{**})$ is in $(0, \infty)$. Therefore, if $G'(.)$ is sufficiently large around the neighborhood of the steady state of ϕ^{**}, the coefficient of $(\phi_{t-1} - \phi^{**})$ is greater than -1, whereas if $G'(.)$ is sufficiently close to 0 around the neighborhood of the steady state of ϕ^{**}, the coefficient of $(\phi_{t-1} - \phi^{**})$ is smaller than -1. $\square$

We note from Proposition 3 and its proof that the stability of the steady state of ϕ^{**} depends upon the distribution of Φ. If shocks that affect the parameter values of the distribution of Φ occur frequently and the configuration of the distribution is changed, the economy may often lose or restore the stability around the steady state of ϕ^{**}.

If the steady state of ϕ^{**} is unstable, the economy exhibits endogenous business cycles. If μ is arbitrarily close to zero, $\lim_{\mu\to 0}\phi^* = 0$ because $\lim_{\mu\to 0} G(\phi^*) = 0$. In this case, from Assumption 1, we do not have the steady state of ϕ^{**}, and thus the economy never exhibits endogenous business cycles. Alternately, if μ is arbitrarily close to one, it follows from the definition of ϕ^{**} that $\lim_{\mu\to 1}\phi^{**} = h$ because $\lim_{\mu\to 1} G(\phi^{**}) = 1$. In this case, assuming that $\lim_{\mu\to 1} G'(\phi^{**})\partial\phi^{**}/\partial\mu$ is bounded above, we are able to demonstrate, using L'Hospital's rule, that the coefficient of $(\phi_{t-1} - \phi^{**})$ in Eq.(24) is arbitrarily close to one as μ is arbitrarily close to one. Again, in this case, the economy never exhibits endogenous business cycles.

Therefore, it is when the extent of financial market imperfections is at the intermediate level that the economy experiences endogenous business cycles. This consequence is consistent with the existing literature (e.g., Aghion et al., 2004; Kunieda and Shibata, 2011).

3.5 Global dynamics: Phase diagram analysis

From Propositions 2 and 3, we find that there are various patterns of the dynamic behavior of the economy, depending upon the configurations of the functions of $\Lambda(\phi)$ and $\Psi(\phi)$. However, it is impossible to analyze those patterns comprehensively. In this section, we investigate two typical cases of the dynamic behavior of the economy, using phase diagrams.

We first consider the features of the functions of $\Lambda(\phi)$ and $\Psi(\phi)$. We easily obtain $\Lambda(0) = 0$ and $\Lambda(\phi^*) = 0$. Because $\Lambda'(\phi) = [(G(\phi)-\mu)(F(\phi)+\phi^2 G'(\phi))+\phi G'(\phi)F(\phi)]/F(\phi)^2$, we obtain:

$$\lim_{\phi \to 0} \Lambda'(\phi) < 0$$

and

$$\lim_{\phi \to \phi^*} \Lambda'(\phi) > 0.$$

Therefore, from the continuity of $\Lambda(\phi)$, there is a minimum value of $\Lambda(\phi)$ in $(0, \phi^*)$. Let the value of ϕ that gives the minimum value be $\bar{\phi}$. Then, $\bar{\phi}$ satisfies $\Lambda'(\bar{\phi}) = 0$ or equivalently:

$$G(\bar{\phi}) - \mu = \frac{-\bar{\phi}G'(\bar{\phi})F(\bar{\phi})}{F(\bar{\phi}) + \bar{\phi}^2 G'(\bar{\phi})}.$$

From this, we obtain the minimum value of $\Lambda(\phi)$:

$$\Lambda(\bar{\phi}) = \frac{-\bar{\phi}^2 G'(\bar{\phi})}{F(\bar{\phi}) + \bar{\phi}^2 G'(\bar{\phi})} =: M,$$

where $M \in (-1, 0)$.

On the other hand, it is easily shown that $\Psi(\phi)$ is an increasing function. We also know that $\Psi(\phi^*) = 0$ and

$$\Psi(0) = \frac{-B\mu}{1 - (1 - B)\mu},$$

where we note that $\Psi(0)$ is decreasing with respect to B and that $\lim_{B \to 0} \Psi(0) = 0$ and

$\lim_{B \to B^*} \Psi(0) = -B^*\mu/[1-(1-B^*)\mu] < 0$. Here, we impose an assumption regarding the relationship between M and $\Psi(0)$ so that we always obtain equilibrium.

Assumption 2

$$M \geq \Psi(0).$$

Assumption 2 guarantees that any sequence of $\{\phi_t\}$ that is generated from the dynamical system (19) with an initial value of $\phi_0 \in [0, \phi^*]$ is an equilibrium path. Both M and $\Psi(0)$ are negative. Therefore, if β_e is very small relative to β_c, Assumption 2 does not hold because $|\Psi(0)|$ is very small in such a case.[10]

[Figure 1 around here]

Figure 1 provides two phase diagrams for two typical cases of the dynamic behavior of the economy. In both cases, we assume that the initial value of ϕ_0 is close to ϕ^*. Panel A provides the case in which the steady state of ϕ^{**} is stable. Because ϕ^{**} gives the highest growth rate, as ϕ_t decreases from ϕ_0, the growth rate increases and the economy experiences an economic boom. Eventually, the economy converges to the steady state that gives the highest growth rate because the steady state of ϕ^{**} is stable. Panel B provides the case in which the steady state of ϕ^{**} is unstable. As in the case of Panel A, the growth rate increases and the economy experiences an economic boom as ϕ_t decreases from ϕ_0. However, because the steady state of ϕ^{**} is unstable, ϕ_t does not converge to the steady state of ϕ^{**}. Accordingly, the economy fluctuates forever, and it may even exhibit a complex dynamic behavior, depending upon the configurations of $\Lambda(\phi)$ and $\Psi(\phi)$.

4 Discussion about a financial crisis

In this section, we discuss a financial crisis accompanied by a credit contraction and followed by a severe depression, using the current model. Let us suppose that M and $\Psi(0)$ are very close under Assumption 2. Figure 2 illustrates an equilibrium path of ϕ_t that leads to a financial crisis. As observed in Figure 2, the initial value of ϕ_0 is close to ϕ^*. As in the

[10]Meanwhile, there are configurations of $\Lambda(\phi)$ and $\Psi(\phi)$ that satisfy Assumption 2 because $\lim_{B \to B^*} \Lambda'(\phi^*) = \lim_{B \to B^*} \Psi'(\phi^*)$.

examples in Figure 1, the growth rate gradually increases and the economy experiences an economic boom. However, if ϕ_t happens to be close to $\bar{\phi}$ in some period, the cutoff ϕ_t (equivalently the interest rate) steeply falls down to a very small value in the next period, which is close to zero. In the subsequent period, ϕ_t suddenly goes up. Because ϕ^* deviates far from ϕ^{**} that gives the highest growth rate, if ϕ_t happens to be very close to ϕ^* when it suddenly increases, the growth rate suddenly decreases and an economic collapse follows. In such a case, the economy goes into a severe depression because it takes so much time for ϕ_t to start to increase steadily. Figure 3 illustrates the growth rates that corresponds to each value of ϕ_t in Figure 2.

[Figure 2 around here]

[Figure 3 around here]

While an economic boom is ongoing, ϕ_t (or the interest rate) keeps decreasing, as observed in Figure 3. This decrease occurs because the supply of financial resources by financiers keeps increasing in the financial market in the process of the economic boom. When ϕ_t is relatively large, the equilibrium interest rate is also large, and the financiers' net worth thus propagates. As the financiers' net worth propagates, the supply of financial resources increases. The increase in the supply of financial resources exerts downward pressure on the equilibrium interest rate. While the interest rates continue to decrease during the boom, unproductive projects are executed by the unproductive entrepreneurs. Because the most productive entrepreneurs face financial constraints, even increases in investment in the unproductive projects boost the growth rates during the boom. At the end of the boom, the increase in the supply of financial resources by the financiers causes a steep fall in the interest rate in the financial market.[11] The net interest rate could even become negative at the end of

[11] One might argue that there may be another pressure that reduces the equilibrium interest rate. The burden of repayment facing producing entrepreneurs becomes heavier and heavier as the financiers' net worth evolves during the boom. We find from Eqs.(19)-(21) and the function $J(\phi_t)$ that when $\phi_t > \phi^{**}$, the growth rate of W_t is greater than the growth rate of Y_t. This finding implies that the total net worth held by entrepreneurs is likely to shrink, provided that ϕ_t is even larger than ϕ^{**}. As a result, the demand for borrowing would decrease because of the financial constraints associated with the entrepreneurs' net worth. Although this decreased demand for borrowing may also exert downward pressure on the equilibrium interest rate, the effect of the decreased demand for borrowing on a steep fall in the interest rate is limited because

the boom. If a negative net interest rate is achieved, the total net worth held by financiers shrinks. Then, the supply of financial resources is significantly reduced in the next period and the equilibrium interest rate significantly rises. As a result, credit contraction occurs in the financial market and the unproductive investment projects, which induce the high growth rate during the boom, are not conducted any more. Accordingly, the economy goes into a severe depression.

Historically, financial markets evolve in the process of economic development. The development of financial markets produces the financier class. We have investigated the macroeconomic implications of the coexistence of entrepreneurs and financiers. In section 3.3, we have shown that the coexistence of entrepreneurs and financiers is likely to lead the economy to the highest growth rate, given a certain extent of financial market imperfections. This section, however, has clarified that the coexistence of financiers and entrepreneurs is highly likely to cause a severe depression for some parameter values. These two-side implications of the coexistence of entrepreneurs and financiers explain why both instability and high growth are frequently observed in modern economies.

To conclude this section, we present remarks on output distribution in each time period between the class of entrepreneurs and the class of financiers. From Eqs. (19)-(21) and Proposition 1, we find that if $\phi_t > \phi^{**}$, the growth rate of W_t is higher than the growth rate of Y_t, whereas if $\phi_t < \phi^{**}$, the growth rate of W_t is smaller than the growth rate of Y_t. Assuming that W_0 is smaller than w_0,[12] during the boom before a financial crisis, the net worth inequality between the entrepreneurial class and the financier class shrinks. The total net worth held by the financier class may even overtake the total net worth held by the entrepreneurial class before a financial crisis. However, if ϕ_t becomes close to zero when a financial crisis occurs, W_t also becomes close to zero, while Y_t remains a certain positive value that is significantly greater than W_t, which implies that when a financial crisis occurs, the net worth inequality widens.

ϕ_t is close to ϕ^{**} at the end of the boom.

[12]Because we have assumed that ϕ_0 is close to ϕ^*, the assumption that W_0 is smaller than w_0 is plausible.

5 Concluding Remarks

Over the past twenty years, many countries have suffered from financial crises followed by severe economic depressions. However, the reasons why such severe crises occurred repeatedly in modern economies remain unclear Is capitalism inherently unstable? Our dynamic general equilibrium model provides a possible answer to this question, generating endogenous business cycles and a financial crisis.

Our model has demonstrated that in a financially constrained economy, the coexistence of entrepreneurs and financiers has a two-fold importance. On the one hand, economic growth is accelerated and the highest growth rate is achievable only when financiers coexist with entrepreneurs. On the other hand, because of the coexistence of financiers and entrepreneurs, a financial crisis followed by a severe depression is highly likely to occur for some parameter values. If a financial market becomes perfect, no financial crises occur in our model. Therefore, it is important to consider a policy to establish a financial market that is close to perfection. However, it seems very difficult to enact a complete policy to obtain a perfect financial market because of the potential agency problems remaining in a financial market. As such, it is also important to consider a policy to avoid financial crises given a certain extent of financial market imperfections. This topic is left for future research.

Appendix

Derivation of Eq.(6)

We should note that when making a lending-investment-borrowing decision at time $t-1$, an entrepreneur has information about her productivity at time t, which is given by $\Phi_{t-1}(\omega_{t-1})$. From Eqs.(4) and (5), the lending-investment-borrowing plan at time $t-1$ of an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) > \phi_{t-1} := r_t/A$ is given such that $b_{t-1}(\omega^{t-1}) = -\mu k_{t-1}(\omega^{t-1})$ and $k_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})/(1-\mu)$. Therefore, her budget constraint at time t is given by

$$k_t(\omega^t) + b_t(\omega^t) = (A\Phi_{t-1}(\omega_{t-1}) - r_t\mu)k_{t-1}(\omega^{t-1}) - c_t(\omega^t),$$

or equivalently,

$$a_t(\omega^t) = \frac{A\Phi_{t-1}(\omega_{t-1}) - r_t\mu}{1-\mu} a_{t-1}(\omega^{t-1}) - c_t(\omega^t). \tag{A.1}$$

Similarly, from Eqs.(4) and (5), the lending-investment-borrowing plan at time $t-1$ of an entrepreneur with $\Phi_{t-1}(\omega_{t-1}) < \phi_{t-1} := r_t/A$ is given such that $b_{t-1}(\omega^{t-1}) = a_{t-1}(\omega^{t-1})$ and $k_{t-1}(\omega^{t-1}) = 0$. Therefore, her budget constraint at time t is given by:

$$k_t(\omega^t) + b_t(\omega^t) = r_t b_{t-1}(\omega^{t-1}) - c_t(\omega^t),$$

or equivalently,

$$a_t(\omega^t) = r_t a_{t-1}(\omega^{t-1}) - c_t(\omega^t). \tag{A.2}$$

From Eqs.(A.1) and (A.2), the flow budget constraints for $\tau \geq t$ are given by Eq.(6).

Proof of lemma 1

From the flow budget constraint (6), we have:

$$E\left[\frac{a_{t+1}(\omega^{t+1})}{c_{t+1}(\omega^{t+1})}\bigg|\Phi^t(\omega^t)\right] = a_t(\omega^t)E\left[\frac{\tilde{R}_{t+1}}{c_{t+1}(\omega^{t+1})}\bigg|\Phi^t(\omega^t)\right] - 1. \tag{B.1}$$

Substituting Eq.(7) into Eq.(B.1), we have:

$$\frac{a_t(\omega^t)}{c_t(\omega^t)} = \beta_e E\left[\frac{a_{t+1}(\omega^{t+1})}{c_{t+1}(\omega^{t+1})}\bigg|\Phi^t(\omega^t)\right] + \beta_e.$$

From this equation and the law of iterated expectations, we obtain:

$$\frac{a_t(\omega^t)}{c_t(\omega^t)} = \beta_e^\tau E\left[\frac{a_{t+\tau}(\omega^{t+\tau})}{c_{t+\tau}(\omega^{t+\tau})}\bigg|\Phi^t(\omega^t)\right] + \beta_e + \beta_e^2 + ... + \beta_e^\tau.$$

From the transversality condition, we have $\lim_{\tau\to\infty}\beta_e^\tau E[a_{t+\tau}(\omega^{t+\tau})/c_{t+\tau}(\omega^{t+\tau})|\Phi^t(\omega^t)] = 0$. Therefore, $a_t(\omega^t)/c_t(\omega^t) = \beta_e/(1-\beta_e)$ for all $t \geq 0$ and thus $a_{t+1}(\omega^{t+1}) = \beta_e\tilde{R}_{t+1}a_t(\omega^t)$ from Eq.(6). $\square$

Microfoundations for the credit constraint (2)

Microfoundation I

Following Aghion et al. (1999), Aghion and Banerjee (2005), and Aghion et al. (2005), we assume that financial market imperfections arise simply from the possibility that borrowers may not repay their obligations.

Let us consider an entrepreneur who borrows financial resources in the financial market. The net worth that the entrepreneur prepares for her own investment project is a_t. If she borrows $-b_t$ in the financial market, her total resources to invest are $k_t = a_t - b_t$ at time t. The return on one unit of investment at time t is $A\Phi_t$. If the entrepreneur earnestly repays her obligations, then she will acquire a net income, $A\Phi_t k_t + r_{t+1} b_t$ at time $t+1$. Meanwhile, if the entrepreneur does not repay her obligations, she will incur a cost δk_t to hide her revenue. In this case, the lender monitors the entrepreneur and is able to capture the entrepreneur with a probability of p_{t+1}. Thus, her expected income is given by $A\Phi_t k_t - \delta k_t + p_{t+1} r_{t+1} b_t$.

Under this lending contract, the incentive compatibility constraint for the entrepreneur not to default on her loan is given by:

$$A\Phi_t k_t + r_{t+1} b_t \geq [A\Phi_t - \delta] k_t + p_{t+1} r_{t+1} b_t, \tag{C1}$$

or equivalently,

$$b_t \geq -\frac{\delta}{r_{t+1}(1-p_{t+1})} k_t, \tag{C2}$$

The left-hand side of Eq. (C1) represents the revenue that the entrepreneur obtains when she invests in a project and consistently repays her obligations. The right-hand side is the gain when she defaults.

To achieve the probability p_{t+1} to capture a defaulting entrepreneur, the lender incurs an effort cost, $b_t C(p_{t+1})$, which is increasing and convex with respect to p_{t+1}. As in Aghion and Banerjee (2005), we assume $C(p_{t+1}) = \kappa \log(1 - p_{t+1})$, where κ is strictly greater than δ so that our study is meaningful.[13] The lender can choose an optimal probability by solving

[13] If $\delta \geq \kappa$, no entrepreneurs face binding credit constraints.

a maximization problem such that:

$$\max_{p_{t+1}} \; -p_{t+1}r_{t+1}b_t - \kappa \log(1-p_{t+1})b_t.$$

Because $-b_t > 0$, this maximization problem is rewritten as:

$$\max_{p_{t+1}} \; p_{t+1}r_{t+1} + \kappa \log(1-p_{t+1}).$$

From the first-order condition, we have

$$r_{t+1} = \frac{\kappa}{1-p_{t+1}}. \tag{C3}$$

As the interest rate r_{t+1} increases, the lender chooses a higher probability to detect a defaulting entrepreneur. From Eqs. (C2) and (C3), we obtain:

$$b_t \geq -\frac{\delta}{\kappa}k_t,$$

or equivalently,

$$b_t \geq -\frac{\delta}{\kappa-\delta}a_t. \tag{C4}$$

Because the entrepreneur's productivity Φ_t is not observable, the lender does not impose entrepreneur-specific credit constraints. The lender must know the entrepreneurs' net worth, a_t. As long as the lender imposes a credit constraint given by inequality (C4) on entrepreneurs who borrow financial resources, no entrepreneurs will default in equilibrium. Because $\delta < \kappa$, we can let $\theta := \delta/(\kappa-\delta) \in [0,\infty)$, and thus,

$$b_t \geq -\theta a_t,$$

which is a credit constraint in the main text. δ and κ are associated with a default cost and a monitoring cost, respectively. θ represents the extent of the credit constraint.

Microfoundation II

We extend the microfoundation for a credit constraint presented by Antràs and Caballero (2009) in a manner suitable for our model. We consider the participation constraint faced by a lender and the incentive compatibility constraint of entrepreneurs such that they do not back out of their investment projects.

It is assumed that at the end of time t and after investment has occurred, any entrepreneur can back out of her investment project at no cost, taking some fraction of her investments, $(1-\mu)(a_t - b_t)$, where $0 < \mu < 1$, and does not repay her obligations to the lender. In this case, the entrepreneur will engage in general goods production somewhere in the economy.

If an entrepreneur absconds at the end of time t, the lender can reclaim the remainder of investments, $\mu(w_t - b_t)$. It is assumed that the lender can relend the remainder of the investments in the financial market. Thus, when making a financial contract with an entrepreneur, the lender faces a participation constraint such that:

$$r_{t+1}\mu(a_t - b_t) \geq -r_{t+1}b_t,$$

or equivalently

$$b_t \geq -\frac{\mu}{1-\mu}a_t.$$

On the other hand, the incentive compatibility constraint for an entrepreneur not abscond from her project at the end of time t is given by:

$$A\Phi_t(a_t - b_t) + r_{t+1}b_t \geq A\Phi_t(1-\mu)(a_t - b_t). \tag{C5}$$

For entrepreneurs with Φ_t such that $r_{t+1} - \mu A\Phi_t \leq 0$, Eq. (C5) always holds. Therefore, we focus on entrepreneurs with Φ_t such that $r_{t+1} - \mu A\Phi_t > 0$. Then, Eq. (C5) is rewritten as:

$$b_t \geq -\frac{\mu}{(\phi_t/\Phi_t) - \mu}a_t. \tag{C6}$$

Because $\phi_t/\Phi_t \leq 1$ in equilibrium, it follows that $-\mu/((\phi_t/\Phi_t) - \mu) \leq -\mu/(1-\mu)$, implying that Eq.(C6) is redundant. In other words, if the lender imposes a credit constraint $b_t \geq$

$-\mu a_t/(1-\mu)$, which is the participation constraint of the lender, entrepreneurs never default. By letting $\mu/(1-\mu) := \theta$, we obtain the credit constraint $b_t \geq -\theta a_t$, as shown in the main text. As μ, or equivalently θ, increases, it becomes more difficult for the entrepreneurs to withdraw their investment without repaying their obligations.

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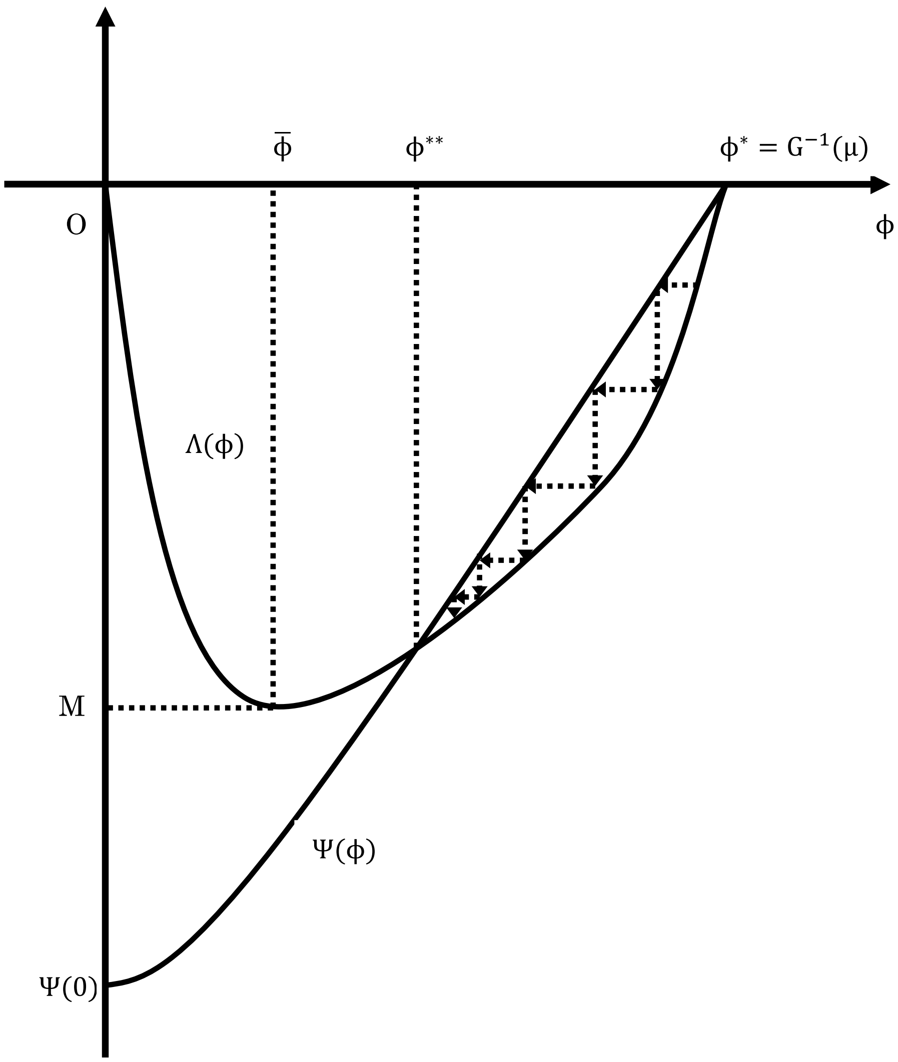
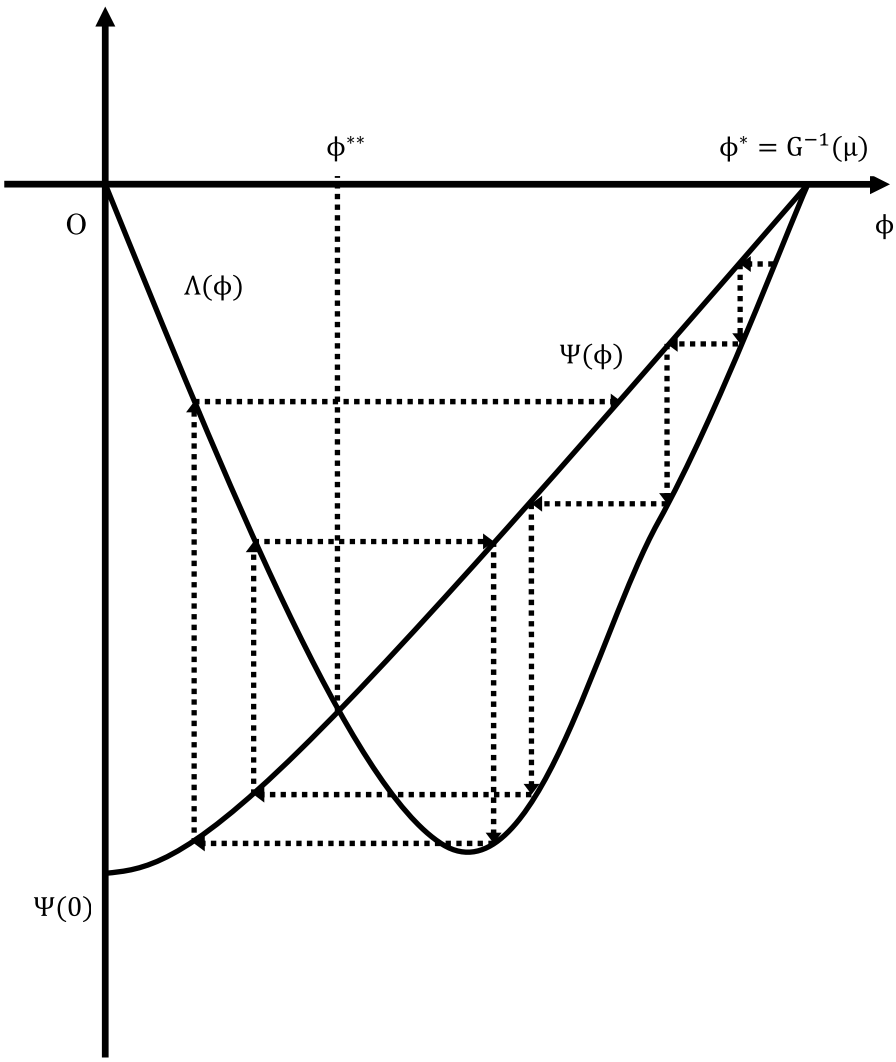


Panel A: ϕ^{**} is locally stable.

Figure 1



Panel B: ϕ^{**} is locally unstable.

Figure 1

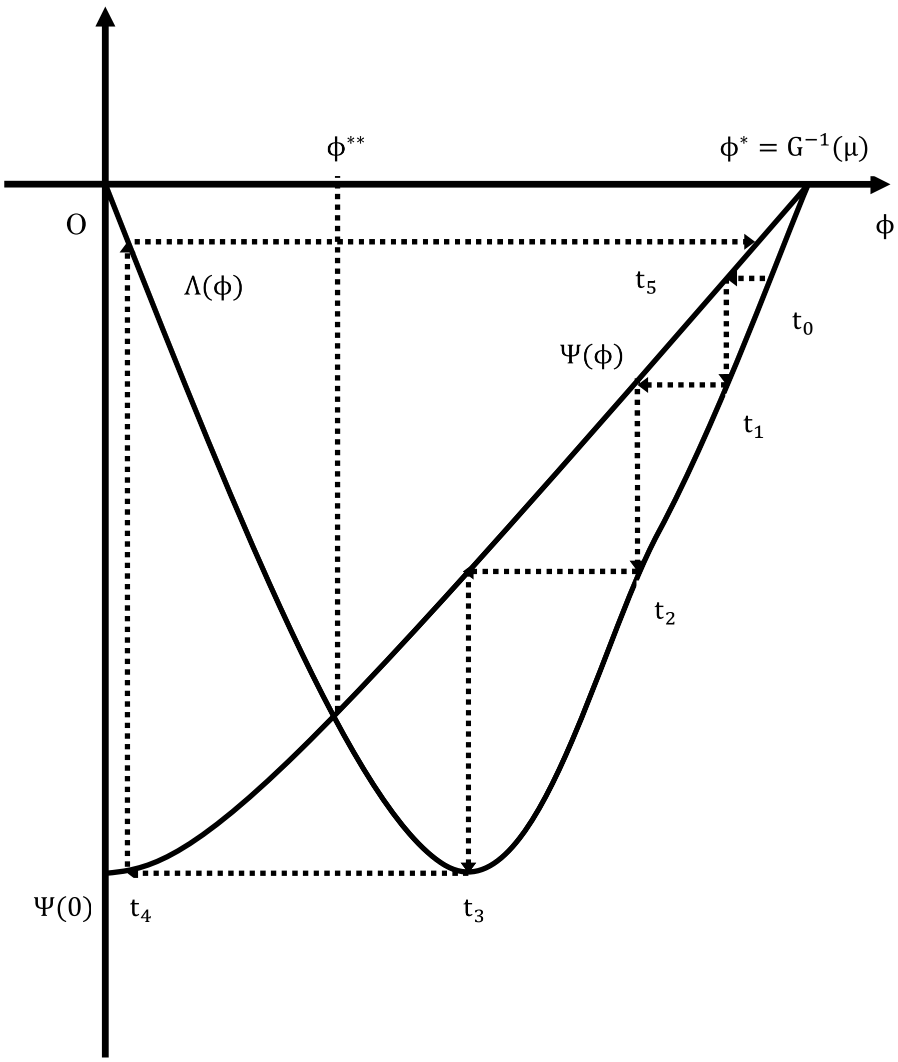


Figure 2: Financial Crisis

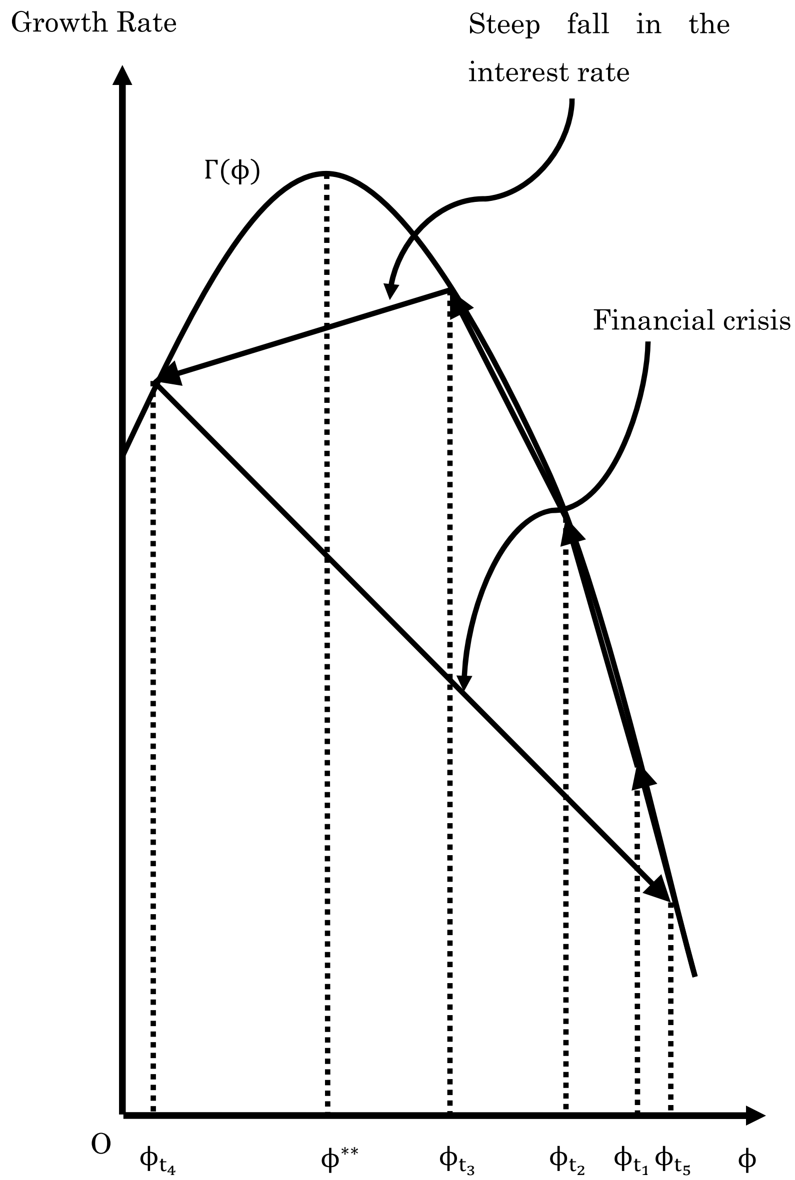


Figure 3: Financial Crisis and Growth rates