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4 September 2014

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Abstract

We present an analysis about adoption of new technology by firms in a duopoly with differentiated goods under absolute and relative profit maximization. Technology itself is free, but each firm must expend a fixed set-up cost, for example, for education of its staff. Under absolute profit maximization there are three types of sub-game perfect equilibria depending on the value of set-up cost. Both firms, or one firm, or no firm adopt new technology. On the other hand, under relative profit maximization there are two sub-game perfect equilibria. Both firms, or no firm adopt new technology. And we show that if demand is sufficiently high, it is more probable that both firms adopt new technology under relative profit maximization than that both firms, or one firm adopt new technology under absolute profit maximization.

Keywords: duopoly, relative profit maximization, adoption of new technology
JEL Classification code: D43, L13.
We thank the reviewers for the relevant comments that helped to improve the manuscript.

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1 Introduction

We present an analysis about adoption of new technology by firms in a duopoly with differentiated goods under absolute and relative profit maximization. Technology itself is free, but each firm must expend a fixed set-up cost, for example, for education of its staff.

For analyses of relative profit maximization please see Gibbons and Murphy (1990), Lu (2011), Matsumura, Matsushima and Cato (2013), Satoh and Tanaka (2013), Satoh and Tanaka (2014), Schaffer (1989), Tanaka (2013a), Tanaka (2013b) and Vega-Redondo (1997)[1].

We think that seeking for relative profit or utility is based on the human nature. Even if a person earns big money, he is not happy enough and may be disappointed, if his brother/sister or close friend earns bigger money. On the other hand, even if he is very poor and his neighbor is poorer, he may be consoled by that fact. Also firms in an industry do not only seek to improve their own performance but also want to outperform their rival firms. TV audience-rating race and market share competition by breweries, automobile manufacturers, convenience store chains and mobile-phone carriers, especially in Japan, are examples of such behavior of firms.

We consider the following two stage-game.

1. The first stage: Each firm decides whether it adopts new technology or not.

2. The second stage: Each firm determines the level of its output.

Under absolute profit maximization there are three types of sub-game perfect equilibria depending on the value of set-up cost. Both firms, or one firm, or no firm adopt new technology. On the other hand, under relative profit maximization there are two sub-game perfect equilibria. Both firms, or no firm adopt new technology. And we show that if demand is sufficiently high, it is more probable that both firms adopt new technology under relative profit maximization than that both firms, or one firm adopt new technology under absolute profit maximization. In the last paragraph we consider a case where the set-up costs for the firms are different. Then, there are three types of sub-game perfect equilibria under relative profit maximization. Two, or one or no firm adopt new technology[2].

[1] In Vega-Redondo (1997) it was shown that the equilibrium in a Cournot oligopoly with a homogeneous good under relative profit maximization is equivalent to the competitive equilibrium. But the equilibrium in a Cournot oligopoly with differentiated goods under relative profit maximization is not equivalent to the competitive equilibrium.

In Satoh and Tanaka (2014) and Tanaka (2013a) it was shown that in a duopoly under relative profit maximization Cournot equilibrium and Bertrand equilibrium are equivalent both in symmetric and asymmetric cases.

[2] The theme of Matsumura, Matsushima and Cato (2013) is close to that of our research. It explains the relationship between the competitiveness, which is expressed by the weight of relative profit in the objective function of a firm, and R&D expenditure in oligopoly market. It considers continuous choice of investment levels, but our research deals with selection between adoption and non-adoption of new technology and compare the relative and absolute profit maximizing, and we have an interest in technology transfer rather than R&D expenditure.

2 The model

Two firms, Firm A and B, produce differentiated goods, and consider adoption of new technology from a foreign country. Technology itself is free, but each firm must expend a fixed set-up cost, for example, for education of its staff. Denote the outputs of Firm A and B by x_A and x_B, the prices of their goods by p_A and p_B. The inverse demand functions of the goods are

$$p_A = a - x_A - bx_B,\ p_B = a - x_B - bx_A,$$

where $a > 0$ and $0 < b < 1$. The marginal cost before adoption of new technology is c, and the marginal cost after adoption of new technology is zero. A fixed set-up cost is e. We assume $a > \frac{c}{1-b}$ so that the absolute profit of each firm should be positive.

We compare the incentive of the firms to adopt new technology when the firms maximize their absolute profits and the incentive when they maximize their relative profits.

We assume that if adoption of new technology and non-adoption are indifferent, then the firms adopt new technology.

3 Absolute profit maximization

The profits of Firm A and B before adoption of new technology are

$$\pi_A = (a - x_A - bx_B)x_A - cx_A,\ \pi_B = (a - x_B - bx_A)x_B - cx_B.$$

After adoption of new technology they are

$$\pi_A = (a - x_A - bx_B)x_A - e,\ \pi_B = (a - x_B - bx_A)x_B - e.$$

We assume Cournot type behavior of firms.

The conditions for profit maximization in the second stage when both firms adopt new technology are

$$a - 2x_A - bx_B = 0,\ a - 2x_B - bx_A = 0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a}{2+b}.$$

The profits of the firms are

$$\pi_A = \pi_B = \frac{a^2}{(2+b)^2} - e.$$

The conditions for profit maximization when only Firm B adopts new technology are

$$a - 2x_A - bx_B - c = 0,\ a - 2x_B - bx_A = 0.$$

The equilibrium outputs are

$$x_A = \frac{(2-b)a - 2c}{4-b^2},\ x_B = \frac{(2-b)a + bc}{4-b^2}.$$

The profits of the firms are as follows.

$$\pi_A = \frac{[(2-b)a-2c]^2}{(4-b^2)^2},\ \pi_B = \frac{[(2-b)a+bc]^2}{(4-b^2)^2} - e.$$

Similarly, the profits of the firms when only Firm A adopts new technology are

$$\pi_A = \frac{[(2-b)a+bc]^2}{(4-b^2)^2} - e,\ \pi_B = \frac{[(2-b)a-2c]^2}{(4-b^2)^2}.$$

The conditions for profit maximization when no firm adopts new technology are

$$a - 2x_A - bx_B - c = 0,\ a - 2x_B - bx_A - c = 0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a-c}{2+b}.$$

The profits of the firms are

$$\pi_A = \pi_B = \frac{(a-c)^2}{(2+b)^2}.$$

If

$$\frac{a^2}{(2+b)^2} - e \geq \frac{[(2-b)a-2c]^2}{(4-b^2)^2},$$

the optimal response of each firm when the rival firm adopts new technology is adoption of new technology. Then, we have

$$e \leq \frac{4c[(2-b)a-c]}{(4-b^2)^2}.$$

If

$$\frac{[(2-b)a+bc]^2}{(4-b^2)^2} - e \geq \frac{(a-c)^2}{(2+b)^2},$$

the optimal response of each firm when the rival firm does not adopt new technology is adoption of new technology. Then, we have

$$e \leq \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}.$$

Since $\frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} > \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, we get the following proposition.

Proposition 1. *Under absolute profit maximization the sub-game perfect equilibria of the two-stage game are as follows.*

1. *If $e \leq \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, the sub-game perfect equilibrium is a state such that both firms adopt new technology.*
2. *If $\frac{4c[(2-b)a-c]}{(4-b^2)^2} < e \leq \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, the sub-game perfect equilibrium is a state such that one firm, Firm A or B, adopts new technology.*
3. *If $e > \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, the sub-game perfect equilibrium is a state such that no firm adopts new technology.*

4 Relative profit maximization

We denote the relative profits of Firm A and B by Π_A and Π_B.

When both firms adopt new technology, we have

$$\Pi_A = (a - x_A - bx_B)x_A - e - (a - x_B - bx_A)x_B + e,$$

and

$$\Pi_B = -\Pi_A = (a - x_B - bx_A)x_B - e - (a - x_A - bx_B)x_A + e.$$

The conditions for relative profit maximization are

$$a - 2x_A = 0,\ a - 2x_B = 0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a}{2}.$$

The prices of the goods are

$$p_A = p_B = \frac{(1-b)a}{2}.$$

The absolute profits of the firms are as follows.

$$\pi_A = \pi_B = \frac{(1-b)a^2}{4} - e.$$

The relative profits of the firms are

$$\Pi_A = \Pi_B = 0.$$

When no firm adopts new technology, we have

$$\Pi_A = (a - x_A - bx_B)x_A - cx_A - (a - x_B - bx_A)x_B + cx_B,$$

and

$$\Pi_B = -\Pi_A = (a - x_B - bx_A)x_B - cx_B - (a - x_A - bx_B)x_A + cx_A.$$

The conditions for relative profit maximization are

$$a - 2x_A - c = 0,\ a - 2x_B - c = 0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a-c}{2}.$$

The prices of the goods are

$$p_A = p_B = \frac{(1-b)a + (1+b)c}{2}.$$

The absolute profits of the firms are as follows.

$$\pi_A = \pi_B = \frac{(1-b)(a-c)^2}{4}.$$

The relative profits of the firms are

$$\Pi_A = \Pi_B = 0.$$

When only Firm A adopts new technology, we have

$$\Pi_A = (a - x_A - bx_B)x_A - e - (a - x_B - bx_A)x_B + cx_B,$$

and

$$\Pi_B = -\Pi_A = (a - x_B - bx_A)x_B - cx_B - (a - x_A - bx_B)x_A + e.$$

The conditions for relative profit maximization are

$$a - 2x_A = 0,\ a - 2x_B - c = 0.$$

The equilibrium outputs are

$$x_A = \frac{a}{2},\ x_B = \frac{a-c}{2}.$$

The prices of the goods are

$$p_A = \frac{(1-b)a + bc}{2},\ p_B = \frac{(1-b)a + c}{2}.$$

The absolute profits of the firms are as follows.

$$\pi_A = \frac{a[(1-b)a + bc]}{4} - e,$$

and

$$\pi_B = \frac{(a-c)[(1-b)a - c]}{4}.$$

The relative profits of the firms are

$$\Pi_A = \frac{a[(1-b)a + bc]}{4} - \frac{(a-c)[(1-b)a - c]}{4} - e = \frac{c(2a-c)}{4} - e,$$

and

$$\Pi_B = -\frac{c(2a-c)}{4} + e.$$

By the assumption that $a > \frac{c}{1-b}$ the absolute profit of each firm is positive. If $e < \frac{c(2a-c)}{4}$, we have $\Pi_A > 0$ and $\Pi_B < 0$, if $e > \frac{c(2a-c)}{4}$, we have $\Pi_A < 0$ and $\Pi_B > 0$. If $e = \frac{c(2a-c)}{4}$, $\Pi_A = \Pi_B = 0$. When only Firm B adopts new technology, we obtain the converse results.

The game in the first stage is depicted as follows.

		B	
		adoption of new technology	non-adoption
A	adoption of new technology	0, 0	$\frac{c(2a-c)}{4} - e, -\frac{c(2a-c)}{4} + e$
	non-adoption	$-\frac{c(2a-c)}{4} + e, \frac{c(2a-c)}{4} - e$	0, 0

From this table we get the following proposition.

Proposition 2. *Under relative profit maximization the sub-game perfect equilibria of the two-stage game are as follows.*

1. *If $e \leq \frac{c(2a-c)}{4}$, the sub-game perfect equilibrium is a state such that both firms adopt new technology*
2. *If $e > \frac{c(2a-c)}{4}$, the sub-game perfect equilibrium is a state such that no firm adopts new technology.*

Note that in the case of absolute profit maximization if $e < \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, the sub-game perfect equilibrium is a state such that both firms adopt new technology. Comparing $\frac{c(2a-c)}{4}$ with $\frac{4c[(2-b)a-c]}{(4-b^2)^2}$ yields

$$\frac{c(2a-c)}{4} - \frac{4c[(2-b)a-c]}{(4-b^2)^2} = \frac{2abc(8-8b+b^3)+b^2c^2(8-b^2)}{4(4-b^2)^2} > 0.$$

Comparing $\frac{c(2a-c)}{4}$ with $\frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$ yields

$$\frac{c(2a-c)}{4} - \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} = \frac{2abc(8-8b+b^3)+bc^2(8b-b^3-16)}{4(4-b^2)^2}.$$

By the assumption that $a > \frac{c}{1-b}$

$$\frac{c(2a-c)}{4} - \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} > \frac{b^2c^2(8-8b+b^2+b^3)}{4(1-b)(4-b^2)^2} > 0.$$

Thus, we obtain the following results.

Proposition 3.

1. *In a duopoly, it is more probable that both firms adopt new technology under relative profit maximization than that both firms adopt new technology under absolute profit maximization.*
2. *In a duopoly, it is more probable that both firms adopt new technology under relative profit maximization than that one firm adopts new technology under absolute profit maximization.*

Different set-up costs As a reference we consider a case where the set-up costs of the firms are different. Denote the set-up costs of Firm A and B by e_A and e_B, and assume $e_B - e_A > 0$. Then, the game is depicted as follows.

B

		adoption of new technology	non-adoption
A	adoption of new technology	$e_B - e_A, e_A - e_B$	$\frac{c(2a-c)}{4} - e_A, -\frac{c(2a-c)}{4} + e_A$
	non-adoption	$-\frac{c(2a-c)}{4} + e_B, \frac{c(2a-c)}{4} - e_B$	0, 0

If $\frac{c(2a-c)}{4} - e_A \geq 0$, the strategy to adopt new technology is a dominant strategy for Firm A, and if $\frac{c(2a-c)}{4} - e_A < 0$, the strategy not to adopt new technology is a dominant strategy for Firm A. Similarly, if $\frac{c(2a-c)}{4} - e_B \geq 0$, the strategy to adopt new technology is a dominant strategy for Firm B, and if $\frac{c(2a-c)}{4} - e_B < 0$, the strategy not to adopt new technology is a dominant strategy for Firm B. Since $e_B > e_A$, the sub-game perfect equilibria are as follows.

1. If $e_B \leq \frac{c(2a-c)}{4}$, the sub-game perfect equilibrium is a state where both firms adopt new technology.

2. If $e_A < \frac{c(2a-c)}{4} \leq e_B$, the sub-game perfect equilibrium is a state where only Firm A adopts new technology.

3. If $e_A > \frac{c(2a-c)}{4}$, the sub-game perfect equilibrium is a state where no firm adopts new technology.

5 Concluding Remarks

In the future research we would like to analyze economic welfare relating to technology adoption by firms and the optimal policies by the government to subsidize or tax adoption of new technology, in particular, under relative profit maximization.

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