An accurate numerical method for computing surface tension forces in CFD codes

Numerical experiments with surface tension

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Accurate numerical method for computing surface tension

1. Context and motivations
   - Context: ocean waves attenuation by falling rain drops
   - The failling drop: a (not so) simple problem
   - Simulate surface tension
2. Numerical solutions
   - Be careful...
   - What we propose
   - Numerical results
   - Applications
3. Conclusion

Context: ocean waves attenuation by falling rain drops

FIGURE : 10^1s, 10^1m

© J. Dekort

FIGURE : 10^0s, 10^0m

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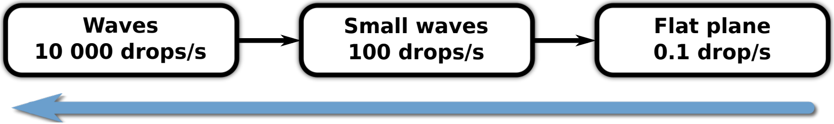
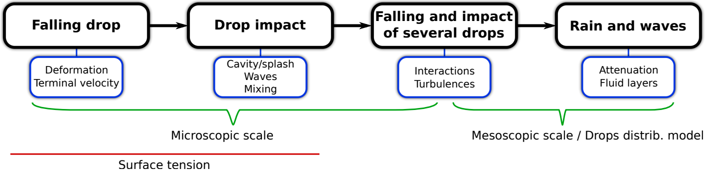
FIGURE : 10^{-2}s, 10^{-2}m



Context: ocean waves attenuation by **falling rain drops**



Difficulties

- Large time and spatial scales
- Sensitive, turbulent
- Measures

Needs (for simulations)

- Meso and micro **numerical models**
- **Appropriate** numerical methods
  - **Accurate** and **efficient**

Project leaders

M. Coquerelle (I2M), S. Glockner (I2M), P. Lubin (I2M), L. Mieussens (IMB), F. Véron (U. Delaware)

A (not so) simple problem

The falling of a rain drop: **surface tension** *dominated*

1. What is its **terminal velocity**?
2. What is the **dynamic of the impact**?

(a) ©Jackson Carson

(b) ©M.-C. Guérout

Classical numerical methods

- Fail to solve (1)
- Introduce errors in (2)

Aparté: what is the numerical convergence?

What we expect

Refine the discretization/*mesh* $\Rightarrow$ Get better results

Precision $\Rightarrow$ **Accuracy**

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What we expect

Refine the discretization/*mesh* ⇒ Get better results

Precision ⇒ **Accuracy**

Example of convergence: approximation of π

h	$h/10^1$	$h/10^2$	$h/10^3$
4	3.3	3.19	3.142

Example of non convergence: approximation of π

h	$h/10^1$	$h/10^2$	$h/10^3$
3	3.2	3.48	4.217

Aparté: what is the numerical convergence?

What we expect

Refine the discretization/*mesh* ⇒ Get better results

Precision ⇒ **Accuracy**

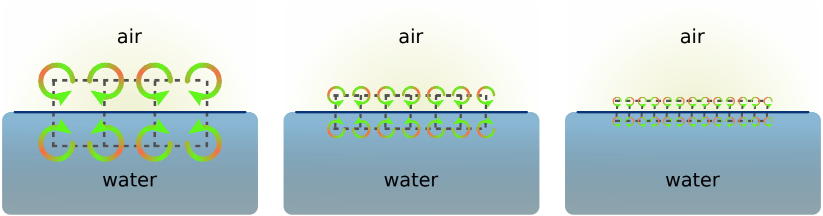


Figure: The equilibrium of a flat surface, **parasitic currents**
Order 1: $h/2 \rightarrow \mathrm{error}/2$

Modeling surface tension

A boundary condition between 2 fluids

Young-Laplace law:

$$\Delta p = \sigma \kappa$$

$\kappa = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) / 2$, the mean curvature, is **purely geometric**

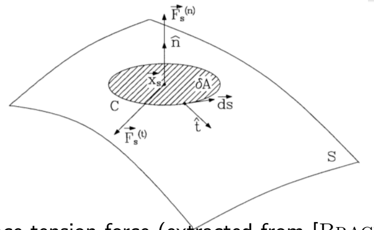


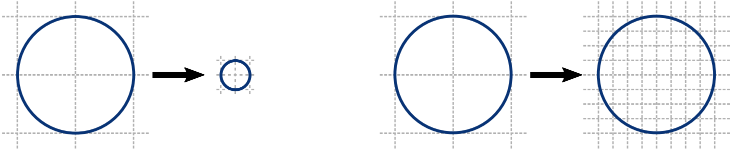
Figure: Surface tension force (extracted from [Brackbill1990])

The *(1-fluid)* incompressible Navier-Stokes equations

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla \mathrm{p} + \nabla \cdot (2\mu \mathbf{D}(\mathbf{u})) + \mathbf{f} + \underline{\sigma \kappa \mathbf{n} \delta_S}$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{and} \quad \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = 0$$

Computing the surface tension forces



Diving into details

$$\text{As } R \to 0, \kappa \to \infty$$

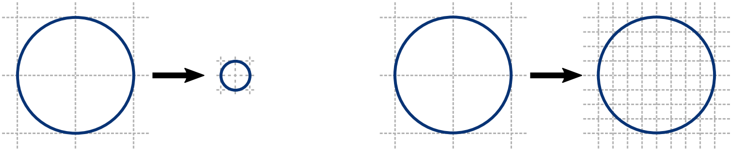
$$\text{Also as h} \to 0, \kappa \to \infty$$

$$\kappa \to \infty \Rightarrow \Delta p \to \infty$$

Barriers

- High gradients/discontinuities
  - **Tough** for numerical methods
  - Errors in computing $\kappa \Rightarrow$ errors in the simulation

Computing the surface tension forces



Diving into details

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In fact, when surface tension is important...

- *Big* errors in κ $\Rightarrow$ **severe** errors in the simulation
  - (numerical) **parasitic/spurious currents** are $O(\kappa^2)$ [Denner et al. 2014]
- Polute simulation results
- Lead to wrong solutions/analysis

Two things to remember

First thing to remember

The **absolute** need to **compute accurately** the curvature

Two things to remember

Geometry memo

1. **Surface** S spatially derivates to...
2. **Normal** vector $\mathbf{n}$ (eq. the tangent plane) spatially derivates to...
3. **Curvature** κ

Moving/Tracking/Transporting the interface

Surface S transported with (spatial) precision $O(h^M)$

$$\Downarrow$$

Curvature κ computed with (spatial) precision $O(h^{M-2})$

Two things to remember

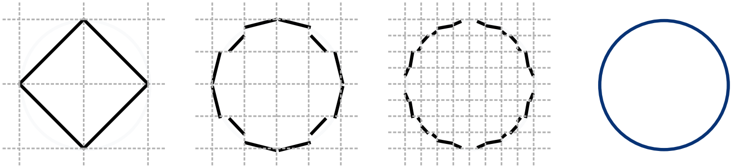
First thing to remember

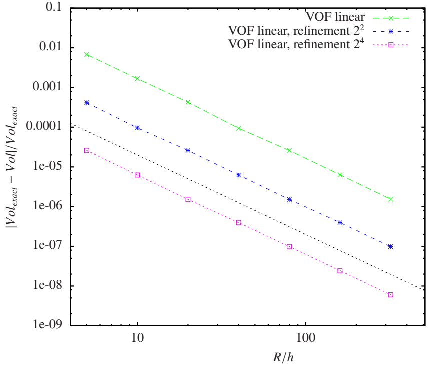
The **absolute** need to **compute accurately** the curvature

Second thing to remember

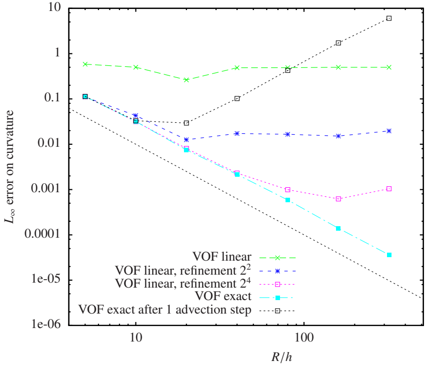
The surface (transport methods) have to be **at least** 3^{rd} **order accurate** for κ to converge

Linear Volume Of Fluid (VOF-PLIC)





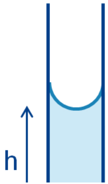
(a) Distance to the surface



(b) Curvature error

Figure: (non) convergence of geometric computations

Capillary rise with VOF-PLIC + CA



Remarks

- Static contact angle model: questionable
- Errors in $\kappa \Rightarrow$ error in equilibrium pressure $\Rightarrow$ error in height

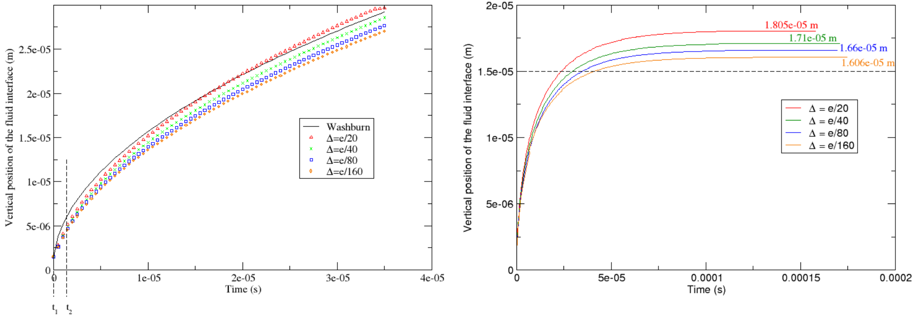


Figure: Numerical results

What we propose

Model choice

- 1-fluid incompressible Navier-Stokes equations
- With **Continuum Surface-Force (CSF)** [Brackbill1990]

$$\sigma \kappa \mathbf{n} \delta_S \Rightarrow \sigma \kappa \nabla c$$

Interface/Surface

Level Set representation

- transport: 5^{th} order accurate
- curvature: 4^{th} order accurate based on the *Closest Point* method

Achievement

(at least) 3^{rd} **order accurate surface tension force computation**

More details

M. Coquerelle, S. Glockner: A fourth-order accurate curvature computation in a level set framework for two-phase flows subjected to surface tension forces. JCP 2016

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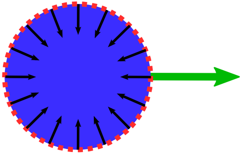
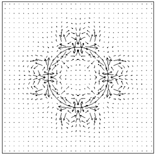
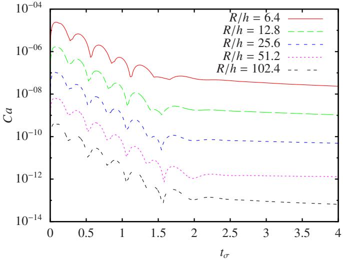


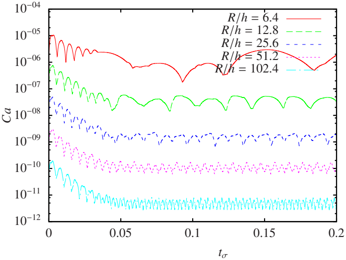


Study case: static and translating column at equilibrium

1. No gravity ⇒ equilibrium state ⇒ **null velocity field** in its ref. frame
2. Numerical errors on κ ⇒ **parasitic currents**

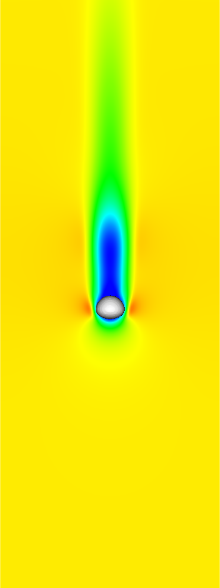


(a) Static column

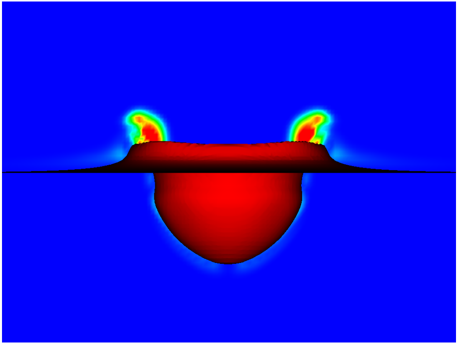


(b) Translating column

Application to our project

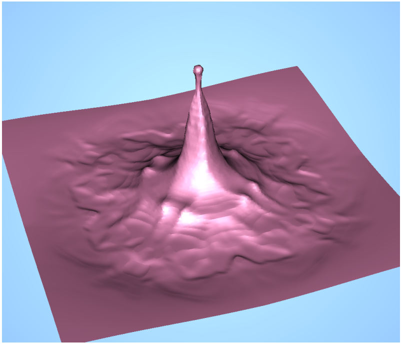


Falling drop, $64 \times 64 \times 640$, 32 comp. nodes

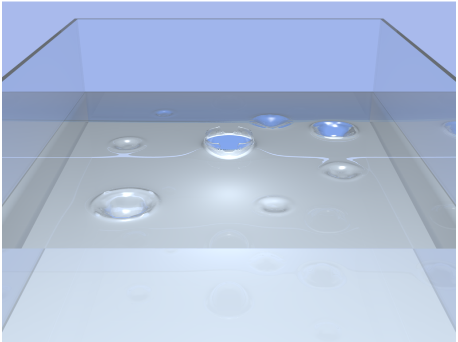


Cavity formation after impact, $256 \times 256 \times 128$, 64 comp. nodes

Application to our project



Falling drop on a surface, $150 \times 150 \times 75$, 32 comp. nodes



Falling drops on a surface, $400 \times 400 \times 200$, 128 comp. nodes

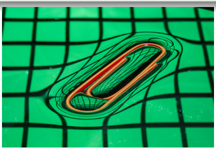
Conclusion

Warning...

- Numerical convergence is **mandatory** for simulation analysis
    - (most) state of the art surface tension methods **do not converge**
        - ... industrial codes as well
        - the **smaller the scale**, the **more severe the problem**
- **Reliability** of studies?
- No *all inclusive* solution, level set methods have drawbacks

... but don't worry!

- Solutions (will) exist...
    - Test your software: easy minimal translating column test
- Still an opened research field
    - Next step: triple line models (Ph.D. starting)



(c) Neil Fried

notus
Computational Fluid Dynamics

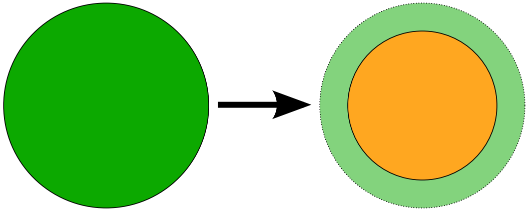
Errors on curvature ⇒ wrong interface dynamic

CSF methods rely on the accurate computation of curvature

3 criteria

1. Accuracy against exact curvature
2. Minimal deviation along the surface
3. Minimal variation along the normal

Effects on surface dynamic :



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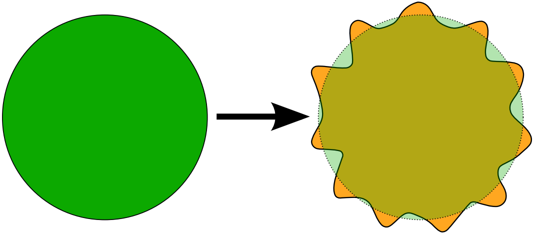

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