Level-Set method for fluid simulation
New numerical method for the reinitialization of the Level-Set

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GDT MFN - ENSCBP 2020

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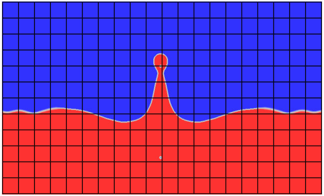
How to represent a multiphase flow ?

Source : Davide Restivo - Wikipedia



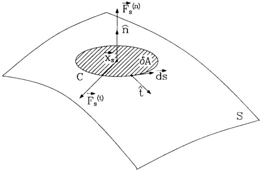
Schematic representation of 2D drop impact on cartesian grid

Navier-Stokes equations :

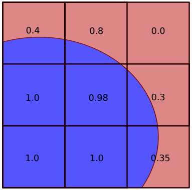
$$\begin{cases} \rho(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}) = \nabla p + \nabla \cdot \tau + \mathbf{f} \\ \mathbf{f} = \rho \mathbf{g} + \sigma \kappa_\Gamma \delta_\Gamma \mathbf{n} \\ \nabla \cdot u = 0 \end{cases}$$

Approche Eulérienne :

How to calculate the physical quantities associated to the interface ?

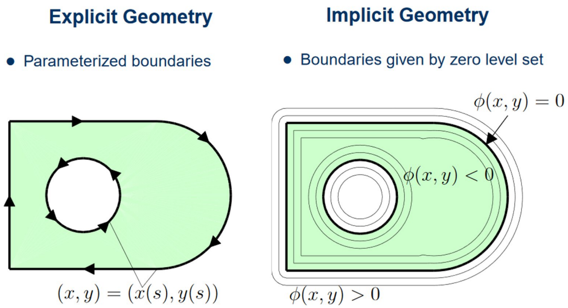


Surface tension force [Brackbill1990]



Volume fraction of each phases

How to represent a multiphase flow ?

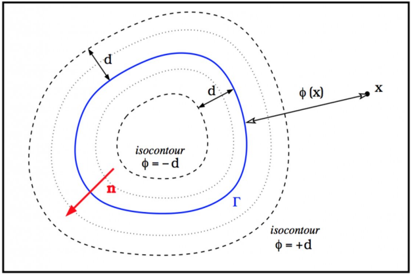


Source : The Level Set Method - Per-Olof Persson - math.mit.edu

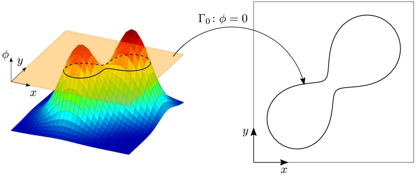
Numerical representation :

- Front tracking
- Volume-of-Fluid
- Moment-of-fluid
- Level-Set

Level-Set defintion



Source : enseeiht.fr



Source : enseeiht.fr

Level-Set definition as signed distance function :

$$\forall x \in \Omega \quad \phi(x) = \begin{cases} -d \text{ if } x \in \Omega^- \\ +d \text{ if } x \in \Omega^+ \\ 0 \text{ if } x \in \Gamma \end{cases}$$

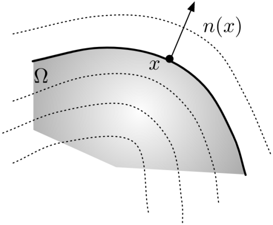
One of the propertie of this definition is :

$$\forall x \in \Omega \quad |\nabla \phi| = 1$$

Level-Set Method : Definition and Properties

Geometric properties

Using the implicite geometry, the eometric properties can be easily calculated.



Source : C.Dapogny, E.Maitre, An introduction to the LevelSet Method

The normal vector **n** :

$$\mathbf{n}(\mathbf{x}) = \frac{\nabla\phi}{|\nabla\phi|} \quad \forall x \in \Gamma$$

The mean curvature κ :

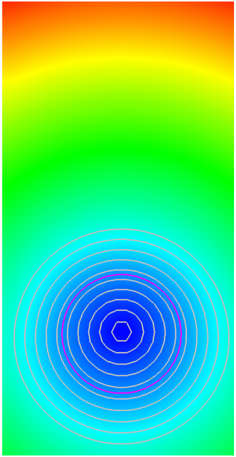
$$\kappa_{Ls}(x) = \nabla \cdot \mathbf{n} \quad \forall x \in \Gamma$$

Level-Set advection

The function ϕ is transported bu the fluid throught the advection equation :

$$\frac{\partial\phi}{\partial t} + \mathbf{u} \cdot \nabla\phi = 0 \quad \forall x \in \Omega$$

which implicity follows the position of the surface over time.



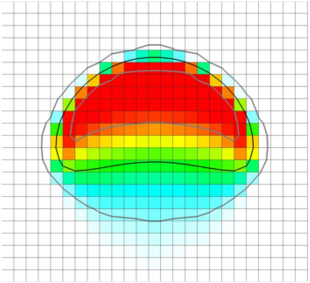
Comes from the advection of the levelset field

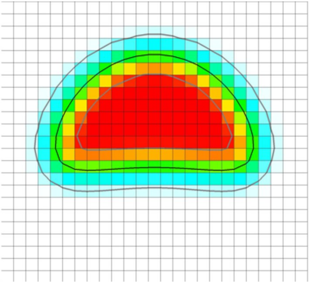
Each Level-Set line does not progress at the same speed

Consequences :

–> Poor resolution of the advection equation

–> lack of precision for the calculation of physical quantities





Need to reinitialize the Level-Set function by finding ϕ_{new} with same zero level set but $|\nabla\phi_{new}| = 1$

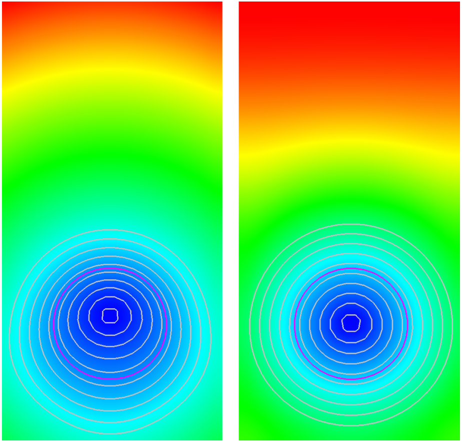
Level-Set Method : Reinitialization - Eikonal equation

Eikonal equation

$$
\begin{cases}
\phi_0 = \phi(x, \tau = 0) \\
\dfrac{\partial \phi}{\partial \tau} + \operatorname{sgn}(\phi_0)(|\nabla \phi| - 1) = 0 \\
\operatorname{sgn}(\phi_0) = \dfrac{\phi}{\sqrt{\phi^2 + \epsilon^2}}
\end{cases}
$$

How to fix the parameters ?

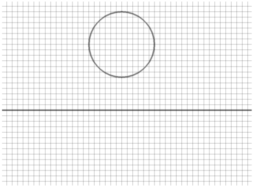
- Integration dummy time ?
- Number of sub-iteration
- **Reinitialization frequency ?**



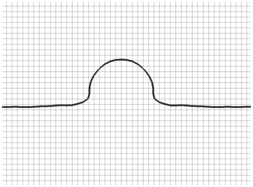
Level-Set Method : Reinitialization - Eikonal equation

Test Case : Drop Impact

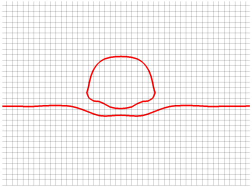
Mesh size : 12 cells by diameter



Initialisation



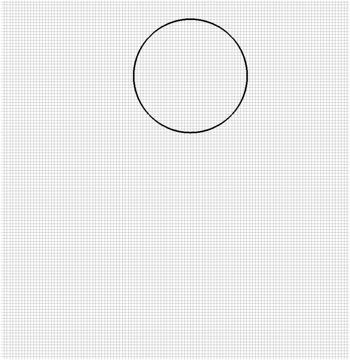
Reinitialization every 10 iterations



Reinitialization every 1 iteration

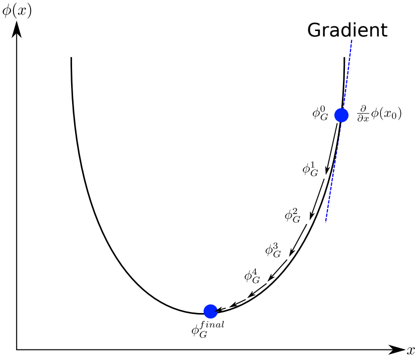
Test Case : Single Vortex

Mesh size : 128×128

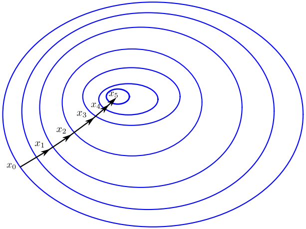


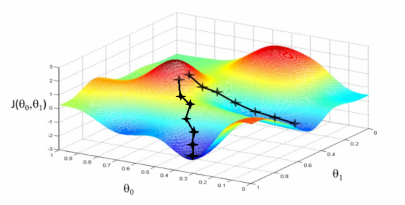
Black :Reinitialization every 10 iterations
Red :Reinitialization every 1 iteration

1D Gradient descent



2D Gradient descent



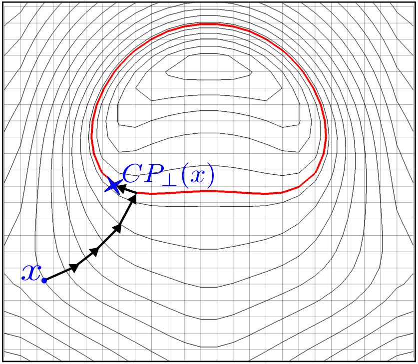


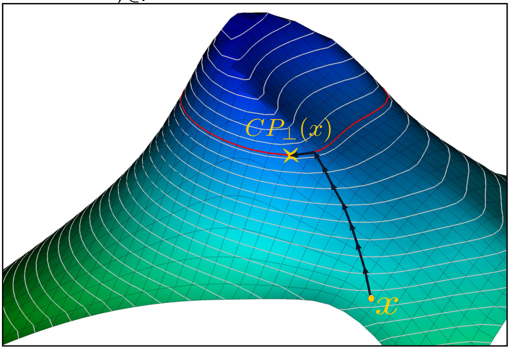
3D representation of 2D gradient descent
source : A.Avati Machine Learning Stanford

Closest-Points Method : Reinitalization of the Level-Set

Closest-Points applied to the set level field

$$\forall x \in \Omega, y = CP(x), \|\overrightarrow{xy}\| = \min_{y \in \Gamma}(\|x - y\|)$$

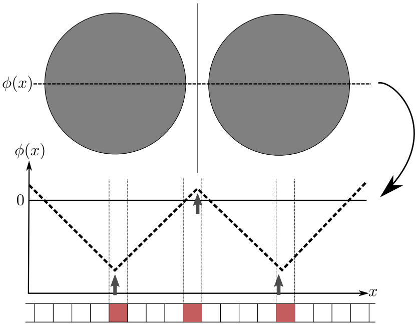




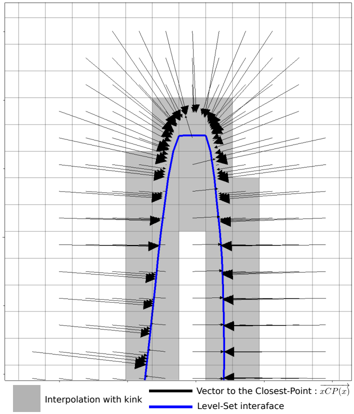
$$\phi(x) = \begin{cases} -\|\overrightarrow{xCP(x)}\| \text{ if } x \in \Omega^- \\ +\|\overrightarrow{xCP(x)}\| \text{ if } x \in \Omega^+ \end{cases} \Rightarrow \phi(x) = \begin{cases} -d \text{ if } x \in \Omega^- \\ +d \text{ if } x \in \Omega^+ \\ 0 \text{ if } x \in \Gamma \end{cases}$$

We get the initial Level-Set definition !

Kink : A point at equidistance from at least two interfaces



Schematic representation of a fold and impact on interpolations



Error on the interpolations which has a kink in the interpolation stencil :

$\rightarrow$ Error on the gradient descent and the location of the Closest-point

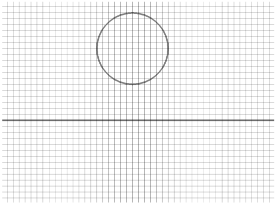
$\rightarrow$ Huge impact on cells at the interface, negligible elsewhere

$\rightarrow$ **Decide to not reinitialize this cells**

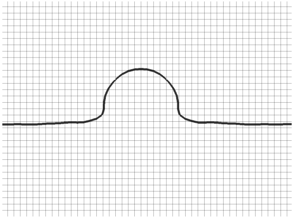
Reinitialization using Closest-Points Method : Test Cases

Test Case : Drop Impact

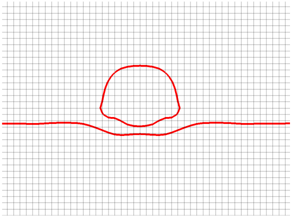
Mesh size : 12 cells by diameter



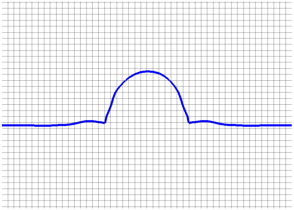
Initialisation



Eikonal Equation
every 10 iterations



Eikonal Equation
every 1 iteration



Closest-Points Method
every 1 iteration

Reinitialization using Closest-Points Method : Test Cases

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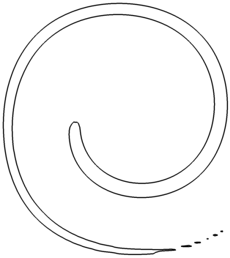
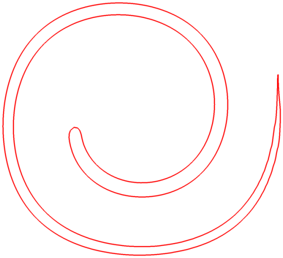
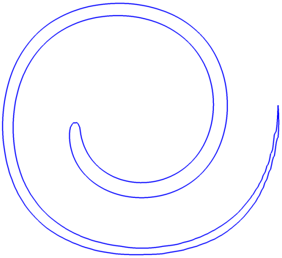


Eikonal Equation
every 10 iterations



Eikonal Equation
every 1 iteration

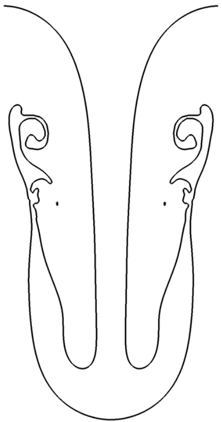


Closest-Points Method
every 1 iteration

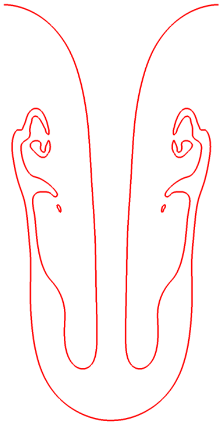
Reinitialization using Closest-Points Method : Test Cases

Test Case : Rayleigh-Taylor Instability

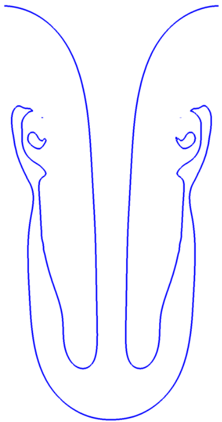
Mesh size : 96×384



Eikonal Equation
every 10 iterations



Eikonal Equation
every 1 iteration



Closest-Points Method
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Reinitialization using Closest-Points Method : Test Cases

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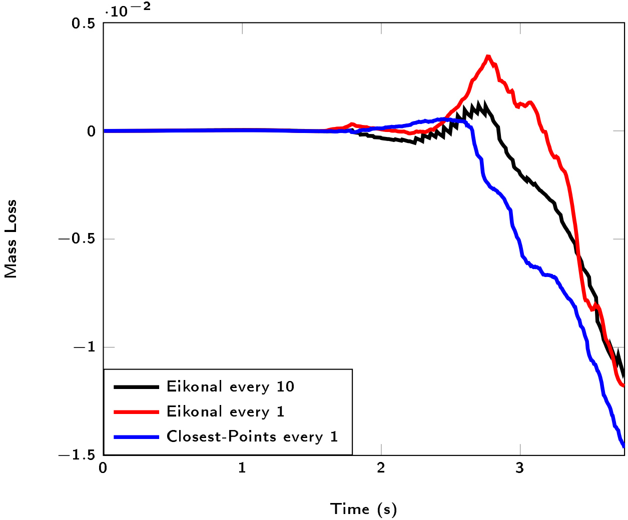
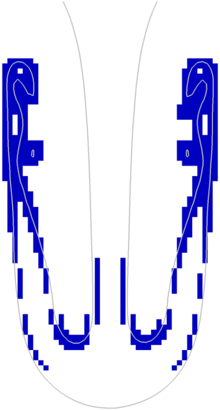
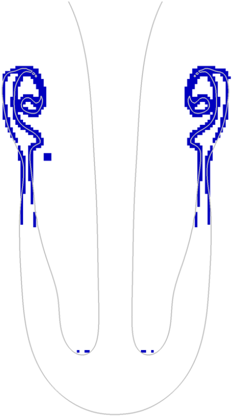


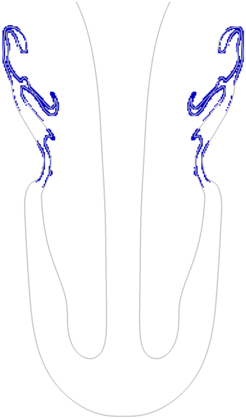
Test Case : Rayleigh-Taylor Instability



Mesh size : 48×192



Mesh size : 96 ×384



Mesh size : 192×768

Conclusion and Perspectives

Conclusions

Main points of this new method of reinitializations :

Polyvalent	⇒ Reintialization can be done every iteration whithout impacting topological changes
Robust	⇒ Works without restriction, for all type of test cases
Simple	⇒ Relie only on a gradient descent, can be easily extend to any type of mesh

Perspectives

⇒ Algorithm optimization to improve the computation time
⇒ Improving gradient descent close to kink to improve the precision