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→175
331
519
501
520

34. Originators of the Term Radian.—As long ago as 1910 Thomas Muir pointed out (*Nature*, v. 83, p. 156) that while the earliest recorded use of the term *Radian* in the *New English Dictionary* was in 1879, in the first part of the new edition of the first volume of William Thomson and P. G. Tait's *Treatise on Natural Philosophy*, "my own first use of it was in class-teaching in the College Hall at St. Andrews in 1869, and I possess a notebook, belonging to one of my students of that year, in which the word is used." He hesitated, however, in a definite choice between the terms radial, radian, rad. But he states that as a result of reading a publication of A. J. Ellis (1814–1890) and exchanging letters with him in 1874 "the form radian was definitely adopted by me." Ellis remarks that he had used the term "Radial angle" from his Cambridge undergraduate days, but Muir stated that Ellis approved of radian as a contraction of "*radi*-al *an*-gle." From later correspondence in *Nature*, v. 83, p. 217, 459–460, it appears that, wholly independent of Muir, James Thomson (1822–1892), brother of the above-mentioned William, proposed the name Radian in July 1871 and that he used it in an examination paper at Queen's College, Belfast, on June 5, 1873, published in the college calendar for 1873–74.

Bibliographic reports on the use before 1869 of the term *Radial Angle*, as equivalent to *Radian*, are desired. This term is not listed in *N.E.D.*

R. C. A.

35. *Phil. Mag.* Tables, Suppl. 3 (for Suppl. 1–2, see *MTAC*, p. 201–202).—W. G. Bickley, "Deflexions and vibrations of a circular elastic plate under tension," s. 7, v. 15, 1933, p. 795. The table gives, to 5S, the first two roots of

$$\frac{xJ_{n+1}(x)}{J_n(x)} = -\frac{\sqrt{(x^2+c^2)}\,I_{n+1}\sqrt{(x^2+c^2)}}{I_n\sqrt{(x^2+c^2)}}$$

for $n = 1$, $c = 0, 1, 2, 5, 10, 20$; the values of x and $x^2(x^2+c^2)$ are given, and for $n = 2$ the same quantities are given for the first root. This item was overlooked in the *Guide*, *MTAC* 7.

H. B.

36. Zeros of the Bessel Function $J_\nu(x)$.—If we denote, as usual, the k-th positive zero of $J_\nu(x)$ by $j_{\nu,k}$ then the symmetric function

$$\sigma_{2n}(\nu) = \sum_{k=1}^{\infty} (j_{\nu,k})^{-2n}$$

is, for each positive integer n, a rational function of ν. It was first used by Rayleigh[1] for the calculation of $j_{0,1}$ and $j_{1,1}$ and later by Airey[2] and others for many values of $j_{\nu,1}$. These functions $\sigma_{2n}(\nu)$ are also important as coefficients of the meromorphic functions

$$\tfrac{1}{2}J_{\nu+1}(X)/J_\nu(X) = \sum_{n=1}^{\infty} \sigma_{2n}(\nu)X^{2n-1}$$

$$\tfrac{1}{2}J_\nu(X)/J_{\nu+1}(X) = (\nu+1)X^{-1} - \sum_{n=1}^{\infty} \sigma_{2n}(1+\nu)X^{2n-1}.$$

This last expansion, in effect, was given as far as $n = 4$ by JACOBI[3] in 1849. This is doubtless the first appearance of these functions σ. Later writers have given lists of these functions as follows:

author		range of n
RAYLEIGH[1]	(1874)	1(1)5, 8
GRAF & GUBLER[4]	(1898)	1(1)5
NIELSEN[5]	(1904)	1(1)5
KAPTEYN[6]	(1906)	1(1)6
FORSYTH[7]	(1920)	1(1)3
WATSON[8]	(1922)	1(1)5, 8

As a by-product of a recent investigation the first dozen of the functions were computed and are given below. They have interesting properties which may be discussed elsewhere. We need only the following explanations here. If we use $[x]$ to denote, as usual, the greatest integer $\leqslant x$, and if we define the polynomial $\pi_n(\nu)$ by

$$\pi_n(\nu) = \prod_{k=1}^{n} (k - \nu)^{[n/k]}$$

then the function $\phi_n(\nu)$ defined by

$$\sigma_{2n}(\nu) = 2^{-2n}\phi_n(\nu)/\pi_n(\nu)$$

is a polynomial of degree

$$d = 1 - n + \sum_{k=2}^{n} \left[\frac{n}{k}\right].$$

If we write

$$\phi_n(\nu) = a_0^{(n)} + a_1^{(n)}\nu + \cdots + a_d^{(n)}\nu^d,$$

then the coefficients $a_h^{(n)}$ for $n \leqslant 12$ are given in the following table.

h	$n = 1$	2	3	4	5	6	7	8	9
0	1	1	2	11	38	946	4580	202738	3786092
1				5	14	1026	4324	311387	6425694
2						362	1316	185430	4434158
3						42	132	53752	1596148
4								7640	317136
5								429	33134
6									1430

h	$n = 10$	11	12	
0	261868876	1992367192	2381255244240	175
1	579783114	4028104212	7315072725560	331
2	547167306	3458238276	10093635442028	519
3	287834558	1649756012	8275251041478	501
4	92481350	479550668	4491836314618	520
5	18631334	87264812	1701744523728	
6	2305702	9748732	461790600920	
7	160850	614228	90534103564	
8	4862	16796	12743301316	
9			1257916872	
10			82812812	
11			3271518	
12			58786	

Thus for $n = 6$ we have

$$\sigma_{12}(\nu) = \frac{42\nu^3 + 362\nu^2 + 1026\nu + 946}{2^{12}(\nu+1)^6(\nu+2)^3(\nu+3)^2(\nu+4)(\nu+5)(\nu+6)}.$$

D. H. L.

[1] Rayleigh, London Math. So. *Proc.*, s. 1, v. 5, 1874, p. 119–124; *Scientific Papers*, v. 1, 1899, p. 192, 195. The entry for $n=8$ is due to A. Cayley; see also his *Collected Papers*, v. 9, 1896, p. 20.
[2] J. R. Airey, *Phil. Mag.*, s. 6, v. 41, 1921, p. 200–203.
[3] C. G. J. Jacobi, *Astr. Nachrichten*, v. 28, 1849, cols. 93–94; *Gesammelte Werke*, Berlin, v. 7, 1891, p. 173 [*for* $10i + 32$, *read* $10i + 22$].
[4] J. H. Graf & E. Gubler, *Einleitung in die Theorie der Bessel'schen Funktionen*, v. 1, Bern, 1898, p. 130.
[5] N. Nielsen, *Handbuch der Theorie der Cylinderfunktionen*, Leipzig, 1904, p. 360.
[6] W. Kapteyn, *Archives Néerlandaises d. Sci. exactes et nat.*, s. 2, v. 11, 1906, p. 149, 168.
[7] A. R. Forsyth, *Mess. Math.*, s. 2, v. 50, 1920, p. 135.
[8] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, Cambridge, 1922 or 1944, p. 502.

QUERIES

13. Tables of Integrals.—We are now interested in evaluating integrals of the following forms: $\int_x^\infty e^{-t}dt/t^n$, $\int_x^\infty e^{-t^2}dt/t^{2n}$. Are there published tables of these functions?

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Editorial Note: Among many tables of $\int_x^\infty e^{-t}dt/t = -Ei(-x)$ reference may be given to NYMTP, *Tables of Sine, Cosine and Exponential Integrals*, 2v., 1940, for $x = [0(.0001)1.9999; 9D], [0(.001)10; 9S], [10(.1)15; 14D]$. There are useful Bibliographies in the volumes. When n is a positive integer $\int_x^\infty e^{-t}dt/t^n$ may be made to depend upon $Ei(-x)$. For the cases $n = +2(-1)-2$ tables were published by W. L. Miller & T. R. Rosebrugh, R. So. Canada, *Proc. and Trans.*, series 2, section III, v. 9, 1903, p. 80–101, for $x = [.1(.001)1(.01)2; 9D]$. There are also tables (p. 80–81) of $-\int_x^\infty e^{-t}dt/t^2 + 1/x + \ln x$, and $-\int_x^\infty e^{-t}dt/t - \ln x$, for $x = [0(.001).1; 9D]$. In the case of the second integral, when $n = 0$ we have the error function of which the most extensive table is that of A. A. Markov, *Table des Valeurs de l'Intégrale* $\int_x^\infty e^{-t^2}dt$, St. Petersburg, 1888, for $x = [0(.001)3(.01)4.8; 11D]$ with Δ^3; see *MTAC*, p. 136. However a more extensive table of the closely related function $H(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ has been published in NYMTP, *Tables of Probability Functions*, v. 1, 1941, $x = [0(.0001)1(.001)5.6; 15D]$. This table can be used to evaluate the above integral by means of the relation $\int_x^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}[1 - H(x)]$. Are there other tables of the first function than for $-2 > n > 2$, and of the second for $n \neq 0$?

QUERIES—REPLIES

14. Tables of $N^{3/2}$ (Q 5, p. 131; QR 8, p. 204; 11, p. 336; 13, p. 375).—We have ms. tables, to 10S, as follows for:

$$N = 100(1)1000,\ 1000(10)10\,000,\ 1005(10)1565, \text{ and also}$$
$$N = [1.0001(.0001)1.0099; 9D].$$